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Exploring the Geometric Design of a 3T1R Parallel-based Machine Tool to Achieve a Targeted Workspace

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ABSTRACT

This study presents a novel approach to the design of a 3T1R parallel-based machine tool by incorporating local and global dexterity indices. The methodology focuses on the synthesis of geometric dimensions to optimize the mechanism's performance. Initially, the paper provides a brief overview of the inverse kinematics of the mechanism and develops a 2-degrees-of-freedom (2-DoF) design based on the desired workspace, which is defined as the largest enclosed operational area. The research also evaluates the kinematic precision and distance singularity of the manipulator using global and local dexterity indices. Subsequently, the optimal dimensions for the individual components of the 3T1R parallel-based machine tool are determined by combining two 2-DoF mechanisms that have undergone design improvements. The study then discusses various conditions related to each component and establishes enhanced dimensions for the 3T1R manipulator to achieve the desired workspace. The design process considers the appropriate range of global and local dexterity indices to improve the performance of the 3T1R manipulator components within the targeted workspace. With this enhanced design, the 3T1R manipulator is capable of achieving four degrees of motion at any given position, while avoiding singular points outside the workspace boundaries. This methodology can be applied to a wide range of parallel-based machine tools.

Keywords: parallel-based machine tool; dimensional synthesis; geometric design; Local Dexterity Index (LDI); Global Dexterity Index (GDI).

I. INTRODUCTION

Designing parallel-based machine tools with the desired workspace is crucial due to their more constrained workspace than serial manipulators. To achieve this, dimensional synthesis is used to design parallel mechanisms. Thus, an enhanced parallel mechanism design must identify the optimal geometry that satisfies the design specifications and required workspace [1-3]. This necessitates a comprehensive understanding of the mechanism's workspace. Various methods have been proposed in the literature to analyse the workspace of manipulators, as reported by Merlet [1], including discretization techniques [4-6], numerical approaches [7-9], and geometrical methods [10, 11]. However, exploring new methods that can lead to the best design for the desired workspace is crucial, which would contribute significantly to the literature in this area.

Designing parallel manipulators has gained significant research interest, leading to the proposal of various dimensional synthesis approaches to tackle diverse design challenges [12-16]. These methods aim to design manipulators that meet specific workspace requirements. For instance, Boudreau and Gosselin [17] formulated a cost function as their objective. They utilized genetic algorithms to generate a workspace closely matching the desired one. Similarly, Laribi et al. [18] determined the DPR dimensions required to create the most compact workspace encompassing a predetermined region. Wang et al. [19] explored a 3-DOF parallel manipulator with actuation redundancy. Their study concentrates on assessing the manipulator's workspace and singularity points. Huang et al [20] also studied the design and synthesis of a 3-DOF module in the innovative TriVariant hybrid robot with 5-DOF. Their study concentrates on the module's conceptual design and dimensional synthesis, providing valuable insights into reconfigurable hybrid robot systems. Karimi et al. [21] tackled dimensional synthesis for a 3-RPR planar parallel manipulator, incorporating a workplace, geometric, and singularity constraints using the branch-and-prune algorithm and

McCormick relaxations. Schappler et al. [22] investigated the combination of structural and dimensional synthesis through global optimization techniques. Che et al. [23] applied particle swarm optimization, genetic algorithms, and DE algorithms to perform dimensional analysis on the Rec4 Schönflies motion generator. Zhao et al. [24] employed a successive approximation iteration method to conduct kinematics and dimensional analysis on a hybrid parallel manipulator, combining the Pareto-optimal solution with a genetic algorithm. Song et al. [25] performed dimensional synthesis and optimized the dimensions of a 2R1T 3-UPU parallel mechanism using a performance atlas. Their approach can assist in the design and optimization of parallel mechanisms. Rao et al. [26] used the Pareto-optimal solution for the dual objective optimization of hexaslides, considering the trade-off between workspace and dexterity through SQP. Fomin et al. [27] addressed the dimension optimization issue for a specific hexapod reconfigurable manipulator using a non-intelligent algorithmic approach, which resulted in a set of equations solved numerically. Analytical solutions are often preferred for their reliability and minimal local avoidance. Liu et al. [28] introduced a geometric design method for parallel manipulators using the maximum inscribed workspace, while Le et al. [29] suggested a design method based on a geometrical analysis of the workspace shape. Yang et al. [30] utilized principle component analysis to achieve optimal comprehensive performance in the dimensional synthesis of parallel manipulators. Peng et al. [14] performed dimensional synthesis on an innovative 2R1T parallel manipulator with an asymmetric configuration and investigated motion/force transmissibility evaluation indices, providing calculation methods and definitions for excellent transmission orientation workspace and good transmission orientation capacity (GTOC). Gregorio [31, 32] examined and designed the position analysis of the 3-URU mechanism using the Jacobian properties method, applicable to all parallel mechanisms with lower mobility and block diagonal Jacobians.

Ji et al. [33] assessed the mechanism's performance using local and global transmission indices and optimized the design parameters to enhance transmission performance. Dastjerdi et al. [34] proposed a theoretical model to design a delta robot with a defined workspace, aiming to minimize the mechanism's size while considering prescribed workspace and workspace constraints as subjective factors. Laryushkin et al. [35] investigated the kinematic properties and singularity analysis of a 4-DOF Delta-type parallel robot. They analyzed the robot's motion capabilities and identifies the configurations where it loses dexterity. Toz et al. [36] also investigated dexterity analysis of a 6-DOF Delta Robot with an Orthogonal Base Platform. Puglisi et al. [37] introduced novel concepts of global performance and compound index to identify optimal parameters for parallel manipulators, providing improved workspace capabilities, isotropy, and dexterity. Rezania and Ebrahimi [38] investigated the relationship between global and local condition indices and manipulability ellipsoids for specific moving platform orientations. Kucuk [39] proposed a new design method with two objectives: increasing the workspace area and reducing the likelihood of interference. Wu and Yin [40] discussed novel 4-DOF parallel manipulator known as H4. They mentioned that the GDI index can be used as an important criterion for the selection and optimization of parallel mechanisms in various applications. Additionally, they compared dexterity among seven planar parallel manipulators with two kinematic chains and three degrees of freedom. Researchers have utilized various indices to address issues related to parallel mechanisms' design, workspace, and dimensional factors, as documented in previous studies [41-47].

The paper proposes a novel method for dimensional synthesis of the 3T1R manipulator with a desired workspace as its primary contribution. The proposed geometric solution is both fast and easy to implement. The design approach utilizes global and local dexterity indices for the theoretical dimensional synthesis of the mechanism. The paper's structure is as follows: In section II, the 3T1R manipulator is introduced as a member of the Schönflies motion

generators family with four degrees of freedom [48]. Sections III and IV provide a brief overview of the mathematical modelling for the mechanism's kinematics and singularity analysis. The local and global dexterity indices examine the manipulator's singularity and kinematic accuracy. Section VI of the paper presents the dimensional synthesis and design process for a 2-degrees-of-freedom (2-DoF) mechanism with a desired workspace. Subsequently, in Section VII, an analytical synthesis method is developed for a 3T1R parallel-based machine tool by combining two improved 2-DoF mechanisms. The discussion further encompasses various conditions for different parts of the mechanism, followed by identifying the best possible dimensions for the 3T1R manipulator with the desired workspace. The dimensions are chosen based on the best global dexterity index and the most extensive range of the local dexterity index.

II. MANIPULATOR DESCRIPTION

The 3T1R manipulator is composed of four links connected to an end-effector. These links have a constant length and enable three linear displacements in the X, Y, and Z directions, along with a rotation around the X-axis. The proposed parallel-based machine tool consists of two similar pairs of links, namely 2-PR(Pa)U and 2-PR(Pa)R, where P, R, Pa, and U refer to a prismatic, revolute, parallelogram, and universal joints, respectively (Fig. 1). According to [49], the parallelogram setup provides higher stiffness compared to a standard chain. A mechanism's stably independent motion capability is defined as its degree of freedom or mobility [50-52]. Therefore, considering $DOF = 6 - \text{Rank}(\$^T_r)$ [51], the 3T1R manipulator has four degrees of freedom, with three translational and one rotational motion around the horizontal axis X.

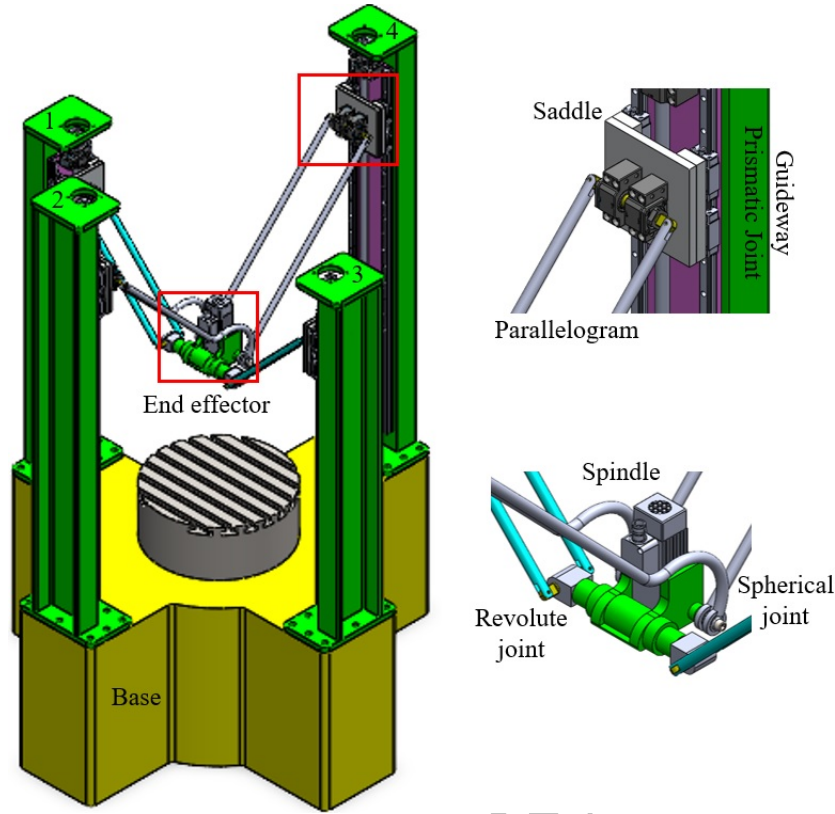


Fig. 1. Schematic view of 3T1R parallel-based machine tool.

III. KINEMATIC ANALYSIS

This section provides a brief overview of the mathematical modelling of the kinematics of the 3T1R manipulator, which has been discussed in [46]. Inverse kinematics involves determining the joint coordinates corresponding to the configuration of the end-effector, and it is a crucial aspect of this parallel-based machine tool. The proposed parallel-based machine tool can be represented in vector form, where the moving frame $\{\mathbf{P}\}$ is attached to the end-effector and the fixed frame $\{\mathbf{O}\}$ is considered at the midpoint of the line connecting links 2 and 4. The origins of these frames coincide with the centre of the end-effector and base, respectively, within the Cartesian coordinate system (Fig. 2).


$$\mathbf{d}_i = \mathbf{q}_i - \mathbf{a}_i - \mathbf{L}_i \quad (1)$$

In velocity analysis, the goal is to obtain the velocity vector of actuators or end-effector when the velocity of the other is known. Substituting $\mathbf{q}_i = \mathbf{P} + \mathbf{R}^p \mathbf{b}_i$ in Eq. (1) and taking the time derivative on both sides, the resulting equation gives:

$$\mathbf{P}\mathbf{C} + \mathbf{b}\mathbf{C} + (\mathbf{b}_i \times \omega_{\mathbf{P}}) = \mathbf{a}\mathbf{C} + \mathbf{d}_{io}d_i\mathbf{C} + d_{io}d_i\mathbf{C} + L_i\mathbf{C}_{io} + L_i(\mathbf{L}_{io} \times \omega_i) \quad (2)$$

where $\mathbf{P}'$ is the linear velocity of the end-effector centre in frame $\{\mathbf{O}\}$. w_i and w_p are also the angular velocities of the links and end-effector centre, respectively. While d'_i and L'_i are

respectively the time derivatives of d_i and L_i . $\mathbf{d}_{io}$ and $\mathbf{L}_{io} = [L_{iox} \ L_{ioy} \ L_{ioz}]$ are also the unit vectors for the sliding saddle displacement and parallelogram length, respectively.

As $\mathbf{a}'_i$, $\mathbf{b}'_i$, $\mathbf{d}'_{io}$ and $\mathbf{L}'_i$ are zero for each guideway. By performing algebraic operations and simplifications, and considering that the rotation is just around X axis, Eq. (2) can be written in a matrix form as the following:

$$\begin{bmatrix} \dot{\mathbf{a}}_{1ox} & L_{1oy} & L_{1oz} & (b_{1y}L_{1oz} - b_{1z}L_{1oy}) \\ \dot{\mathbf{a}}_{2ox} & L_{2oy} & L_{2oz} & (b_{2y}L_{2oz} - b_{2z}L_{2oy}) \\ \dot{\mathbf{a}}_{3ox} & L_{3oy} & L_{3oz} & (b_{3y}L_{3oz} - b_{3z}L_{3oy}) \\ \dot{\mathbf{a}}_{4ox} & L_{4oy} & L_{4oz} & (b_{4y}L_{4oz} - b_{4z}L_{4oy}) \end{bmatrix} \begin{bmatrix} \ddot{\theta}_x \\ \ddot{\theta}_y \\ \ddot{\theta}_z \\ \dot{\omega}_x \end{bmatrix} + \begin{bmatrix} \mathbf{d}_{1o} \times \mathbf{L}_{1o} & 0 & 0 & 0 \\ \mathbf{d}_{2o} \times \mathbf{L}_{2o} & 0 & 0 & 0 \\ \mathbf{d}_{3o} \times \mathbf{L}_{3o} & 0 & 0 & 0 \\ \mathbf{d}_{4o} \times \mathbf{L}_{4o} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{a}}'_1 \\ \dot{\mathbf{a}}'_2 \\ \dot{\mathbf{a}}'_3 \\ \dot{\mathbf{a}}'_4 \end{bmatrix} \quad (3)$$

where V_x , V_y , V_z and ω_x are respectively velocities in X, Y, Z axes and the angular velocity around X axis.

Considering $\mathbf{J}_x = \frac{\partial \mathbf{L}_{io}}{\partial \theta_i}^T (\mathbf{b}_i' \times \mathbf{L}_{io})^T \times \dot{\theta}_i$ and $\mathbf{J}_q = \begin{bmatrix} \mathbf{d}_{1o} \times \mathbf{L}_{1o} & 0 & 0 & 0 \\ \mathbf{d}_{2o} \times \mathbf{L}_{2o} & 0 & 0 & 0 \\ \mathbf{d}_{3o} \times \mathbf{L}_{3o} & 0 & 0 & 0 \\ \mathbf{d}_{4o} \times \mathbf{L}_{4o} & 0 & 0 & 0 \end{bmatrix}$ as the

Jacobian matrices of the end effector and actuators, we have.

$$\mathbf{J}_x \dot{\mathbf{X}} = \mathbf{J}_q \dot{\mathbf{q}} \quad (4)$$

The Jacobian matrix is defined as $\mathbf{J} = \mathbf{J}_x^{-1} \mathbf{J}_q$, and the inverse Jacobian matrix takes the form $\mathbf{J}^{-1} = \mathbf{J}_q^{-1} \mathbf{J}_x$.

Finally, the universal velocity mapping formula from joint space to end-effector space can be obtained as:

$$\dot{\mathbf{q}} = \mathbf{J}^{-1} \dot{\mathbf{X}} \quad \text{or} \quad \dot{\mathbf{X}} = \mathbf{J} \dot{\mathbf{q}} \quad (5)$$

IV. SINGULARITY ANALYSIS

Singularity arises when the determinant of matrices $\mathbf{J}_q$ and $\mathbf{J}_x$ in Eq. (4) approaches zero, resulting in three singularity types: The first is called the inverse singularity and occurs when $|\mathbf{J}_q| = 0$ or when vectors $\mathbf{d}_{io}$ and $\mathbf{L}_{io}$ are perpendicular, which happens when the links are normal to the guideways. As a result, the links may have small movements while the end-

effector is fixed. The second type of singularity is a direct singularity, which happens when $|\mathbf{J}_x| = 0$. In this case, the mechanism is under constraint, and the end-effector will displace even when the links are locked or fixed, resulting in more degrees of freedom than intended. The third singularity type is a combination of both inverse and direct singularities.

The Global Conditioning Index (GCI) is employed to explore the singularity of the mechanism within its workspace [53]. When the Jacobian matrix is evaluated using the GCI, it assesses the dexterity of the mechanism [54], indicating the mechanism's kinematic performance. Additionally, it can be used to assess the manipulator's stiffness. Calculating this index requires obtaining the condition number of the Jacobian matrix. The error magnifying factor, which is also referred to as the condition number, illustrates how the errors in the actuators amplify in different directions and lead to the ultimate pose error of the mechanism [55]. Moreover, the condition number can assess the minimum distance required to avoid the detrimental effects of singular points. It can provide a measure of the precision of the Cartesian velocity of the end-effector. The condition number is computed as the quotient of the highest singular value of the Jacobian matrix and the lowest singular value, which can be expressed as follows [56, 57]:

$$K(\mathbf{J}) = \frac{\sigma_2}{\sigma_1} \quad (6)$$

Here, σ_1 and σ_2 denote the highest and lowest determinants of the Jacobian matrix, respectively. The condition number is always equal to or greater than 1. When K approaches 1, the Jacobian matrix transforms into an orthogonal matrix, signifying an ideal kinematic condition termed kinematic isotropy. This enhances the kinematic dexterity of the mechanism and helps avoid the singularity. GCI assesses the overall kinematic performance and can be computed using the following formula:

$$\text{GCI} = \frac{\partial^K d_w}{\partial t_w} \quad (7)$$

The parameter $\int dw$ represents the total volume of the workspace considered for analysis of singularities, where w denotes the number of selected end-effector positions.

The local dexterity index (LDI) is expressed as the reciprocal of the condition number [55-57] and can be computed as follows:

$$\text{LDI} = \frac{1}{K} \quad (8)$$

LDI ranges from 0 to 1, and a value of 1 indicates that the system can perform at that specific point with high accuracy. Similarly, the global dexterity index (GDI) can be calculated from one to zero as the following:

$$\text{GDI} = \frac{\int_K \frac{1}{\lambda} dw}{\int dw} \quad (9)$$

Overall, to calculate the workspace and singularity algorithm, we follow a procedure similar to the flowchart below:

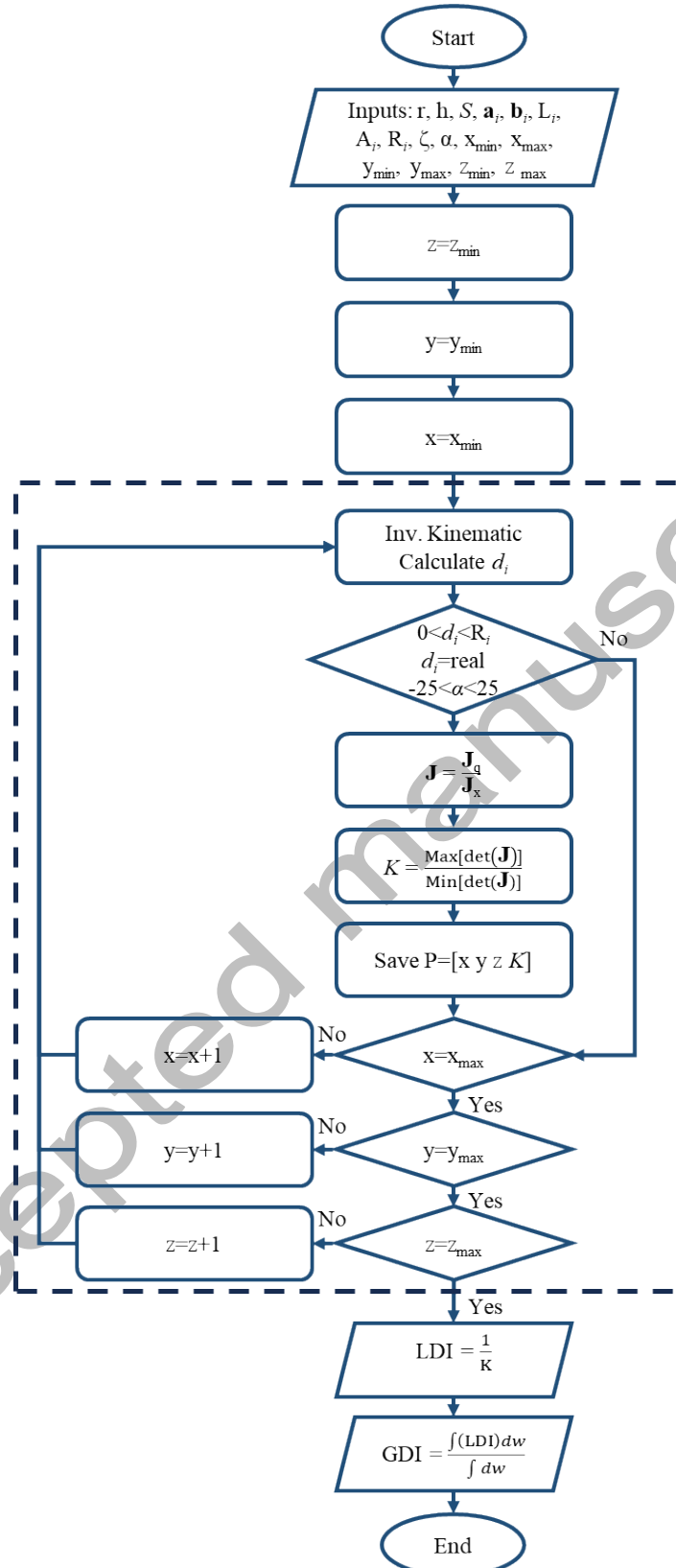


Fig. 3. Flowchart for calculating the workspace, GDI and LDI.

Considering the flowchart in Fig. 3, and the relationship between the Jacobian matrix and dimensional design parameters (Eqs. 1-5) on one side, and GDI and LDI on the other, it can

be considered that the impact of the parameters is also taken into account through the influence of the Jacobian matrix on the LDI and GDI of the mechanism. Therefore, the way LDI is distributed, considering each parameter individually, has been taken as a criterion for stability and optimal mechanism design.

V. DIMENSIONAL SYNTHESIS OF A TYPICAL 2-DOF MECHANISM FOR THE DESIRED WORKSPACE

This section performs the dimensional synthesis and design improvement process for a 2-DoF PRR mechanism [58] with a desired workspace based on the best obtained GDI and LDI. The relations between the dexterity indices and the geometric parameters of the PRR manipulator are studied to obtain the best possible design parameters of the manipulator for the given workspace.

Fig. shows a schematic view of a PRR mechanism. A , R , L , and ζ in Fig. are the distance between guideways, the motion course of the guideways, the length of the links, and the angle between the end-effector joints, respectively. Fig. also shows the obtained overall workspace of the mechanism that covers the desired cylindrical workspace, illustrated in red.

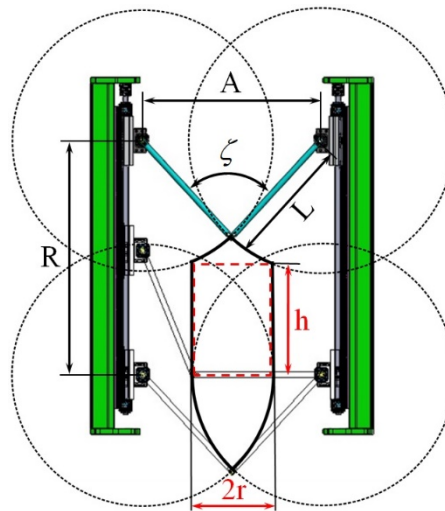


Fig. 4. The dimensions of the 2-DoF PRR mechanism with the desired workspace specifications.

To investigate the impact of change in ζ from 60 to 120 degrees on the GDI of the mechanism, a guideway length of $R = 1000$ (mm) and a distance of $A = 1000$ (mm) between the two guides were arbitrarily set. Fig. 5 displays the angle ζ with the highest GDI value, which was identified. As per Fig. 5, the GDI value of the mechanism rises as the angle between the two links increases from 60 to 88 degrees, with a peak value of 0.6629. The GDI value then decreases after 88 degrees. The indicator on the graph represents the maximum GDI value, whereas the maximum LDI value is 1 at 90 degrees, with a minimum value of 0.0024. Moreover, it should be noted that the minimum LDI values at various angles, highlighted in yellow on Fig. 3, exhibit different degrees of fluctuation. This is primarily due to the presence of a few extremely low LDI values within the workspace.

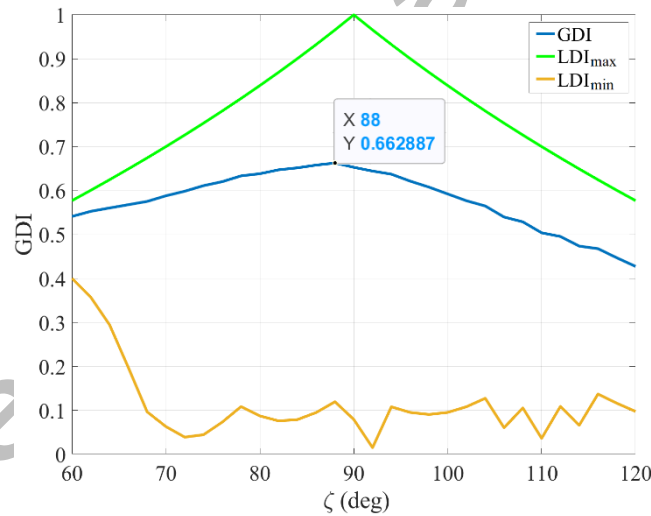


Fig. 5. GDI and LDI analyses of the PRR mechanism with assumed dimensions and change in ζ .

According to Fig. , the maximum GDI is obtained when $\zeta_{opt} = 88^\circ$. Suppose the desired workspace of the under-study mechanism is considered a cylindrical workspace with a height of h and a radius of r . In that case, the optimal design parameters for the PRR mechanism can be obtained according to Fig. as the following equations:

$$A_{opt} = \frac{2r \cos \frac{\alpha - \zeta_{opt}}{2}}{1 - \cos \frac{\alpha - \zeta_{opt}}{2}} \quad (10)$$

$$L_{opt} = \frac{A_{opt}}{2 \sin \frac{\alpha - \zeta_{opt}}{2}} \quad (11)$$

$$R_{opt} = h + \sqrt{L_{opt}^2 - \frac{A_{opt}^2}{4} - r^2} \quad (12)$$

where A_{opt} , L_{opt} , and R_{opt} are the optimum distance between the guideways, the optimum motion course of the guideways, and the optimum length for the links, with the best GDI and LDI, respectively.

Therefore, for the assumed mechanism dimensional parameters, i.e., $A = 1000$ (mm) and $R = 1000$ (mm), obtained $\zeta_{opt} = 88^\circ$. For the required radius and height of the desired workspace that are $r = 150$ (mm) and $h = 400$ (mm), the optimum mechanism dimensions will be $A_{opt} = 682.5$ (mm), $L_{opt} = 491.25$ (mm), and $R_{opt} = 852.5$ (mm) according to Eqs. (10) to (12). Moreover, considering the obtained optimal dimensional parameters, GDI, the maximum LDI, and the minimum LDI will be 0.6537, 0.9657, and 0.1494, according to Eqs. (9) and (8), respectively.

In order to better illustrate the LDI distribution throughout the overall workspace of the mechanism, the following conditions are assumed:

$$\begin{aligned} & \text{Black} \leq \text{LDI}_{min} \\ & \text{LDI}_{min} < \text{Yellow} \leq (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.3 + \text{LDI}_{min} \\ & (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.3 + \text{LDI}_{min} < \text{Magenta} \leq (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.45 + \text{LDI}_{min} \\ & (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.45 + \text{LDI}_{min} < \text{Cyan} \leq (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.6 + \text{LDI}_{min} \\ & (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.6 + \text{LDI}_{min} < \text{Red} \leq (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.75 + \text{LDI}_{min} \\ & (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.75 + \text{LDI}_{min} < \text{Green} \leq (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.9 + \text{LDI}_{min} \\ & \text{Blue} \geq (\text{LDI}_{max} - \text{LDI}_{min}) \cdot 0.9 + \text{LDI}_{min} \end{aligned} \quad (13)$$

The distribution of LDI throughout the manipulator workspace has been illustrated in Fig. , based on Eq. (13). According to Fig. , higher amounts of LDI occur when the end-effector

approaches closer to the centre of the workspace gamut. Almost 80% of the workspace has an LDI of more than 0.4.

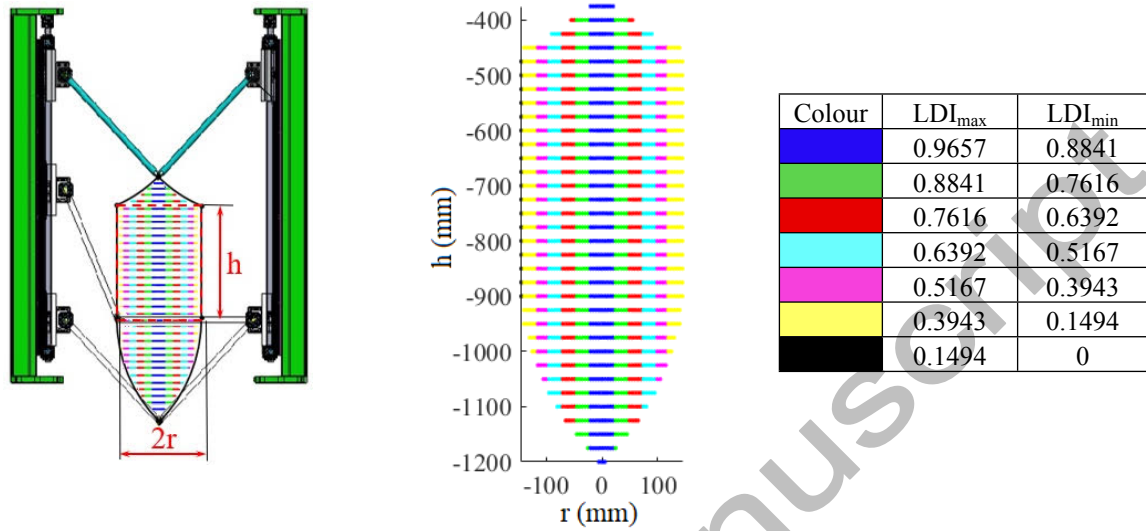


Fig. 6. The distribution of LDI throughout the overall workspace of the mechanism.

As most mechanisms need to have a wider end-effector, a fixed horizontal end-effector of width B has been assumed between the links of the mechanism as the end-effector (Fig.).

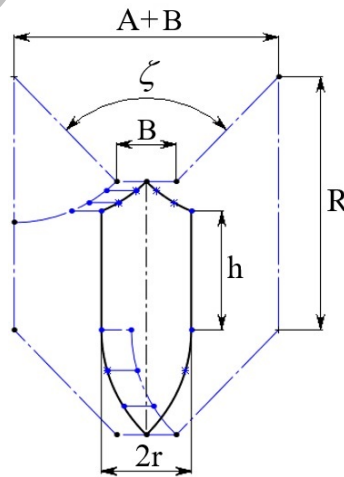


Fig. 7. The details for the overall workspace gamut of the mechanism with the horizontal end-effector of width B .

For the mechanism with the flat end-effector of width $B = 200$ (mm) and optimal dimensions of $A_{opt} = 682.5 + 200$ (mm), $L_{opt} = 491.25$ (mm), and $R_{opt} = 852.5$ (mm), and for the required radius and height of the desired workspace that are $r = 150$ (mm) and $h = 400$ (mm), the same dexterity analysis has been performed and illustrated in Fig. 8.

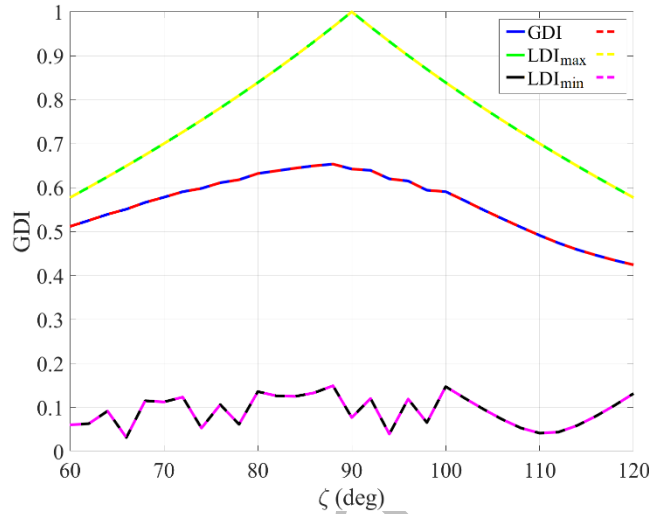


Fig. 8. GDI and LDI analyses of the improved PRR mechanism with and without the end horizontal effector of width B.

In Fig. 8, the lines show the GDI and LDI of the mechanism without a flat end-effector, and the dashed lines are for the mechanism with an end-effector of length B. The obtained GDI results have been compared to those obtained for the optimal PRR mechanism without the flat end-effector, and according to Fig. , both the obtained results are consistent.

The constraints should be checked to obtain the overall workspace borders of the mechanism. α the angle between the desired workspace radius, r , and the projection of the link on the YZ plane can be calculated using the obtained optimum length for the links and the desired workspace dimensions as follows (Fig.):

$$\alpha = \sin^{-1} \left(\frac{r}{L \cos(44^\circ)} \right) \quad (14)$$

Therefore, considering Eq. (14), for $\zeta_{opt} = 88^\circ$, and the optimum link length of $L_{opt} = 491.25$ (mm), for the desired cylindrical workspace of radius $r = 150$ (mm), the minimum angle of the joints located on the guideways should be $\alpha = 25^\circ$.

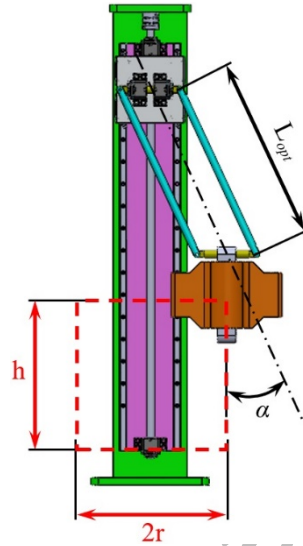


Fig. 9. Maximum angle of rotation of universal joints located on guideways.

In Fig. a, the workspace of the mechanism is shown along with the desired workspace. Based on Fig. b, which represents a cross-sectional view of the workspace, a significant portion of the desired workspace is eliminated due to the motion constraints of the links. To calculate the minimum length of the link, L , or the suitable angle for parallelogram pendulum motion capability, α , an optimization equation for the size of L needs to be determined considering the mechanism's motion in space. In Fig. c, the blue lines of the workspace borders indicate the path of the end link's motion, creating a section composed of four similar surfaces.

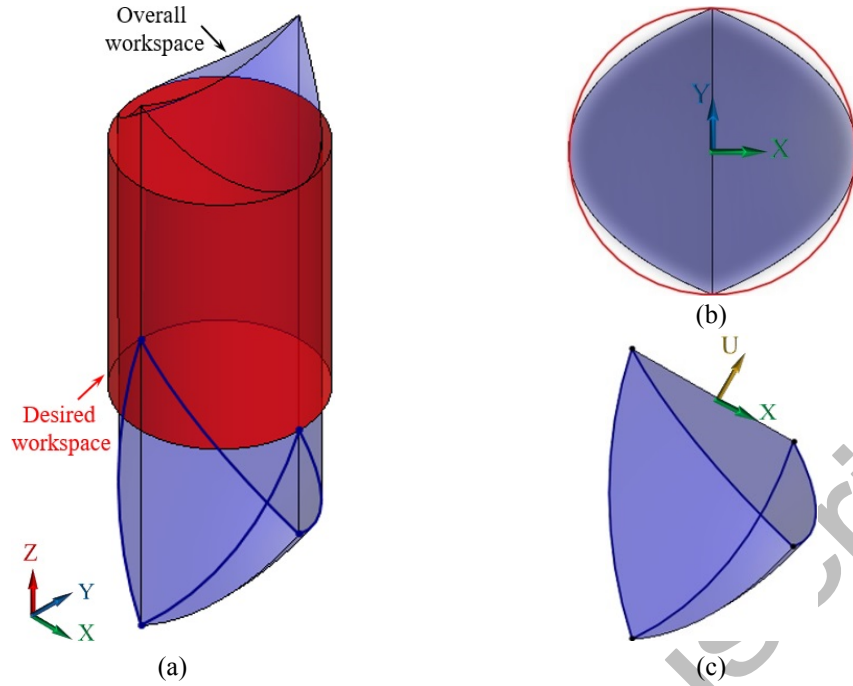


Fig. 10. a. three-dimensional view of the workspace of the mechanism, within blue; b. cross-sectional view of the workspace; c. the sectioned-off workspace resulting from the motion of the links, indicated by blue highlighted lines representing the outermost borders of the workspace.

With a view perpendicular to the section shown in Fig. c, Fig. 11 is created. In Fig. 11, the blue-colour circle represents the movement of links in the free state, and the red-colour area represents the desired workspace. The link with length L has also been shown in blue. The intersection point between the link movement path and the desired workspace, as well as the most negligible value of L that can form the intersection point of the workspace, have also been illustrated in Fig. . Based on Fig. 1, we have:

$$x^2 + u^2 = L^2 \quad (15)$$

$$\frac{\partial}{\partial x} \left(L \sin \frac{\partial \theta_{opt}}{\partial x} \right) + (u \sin a)^2 = r^2 \quad (16)$$

According to Fig. , the following relationships hold:

$$x = L \cos g \quad (17)$$

$$u = L \sin g \quad (18)$$

Where γ is the angle between the link and the image is parallel to the horizon and is reflected on the intersected surface.

By substituting Eqs. (17) and (18) in Eq. (16), L can be obtained as:

$$L = \frac{r}{\sqrt{\frac{a}{e} \cos g + \sin \frac{a}{e} \frac{\ddot{\theta}}{2} + \sin^2 g \sin^2 a}} \quad (19)$$

Taking the derivative of Eq. (20) concerning γ and setting the resulting equation equal to zero yields:

$$\frac{\partial}{\partial \gamma} \left(\frac{r}{\sqrt{\frac{a}{e} \cos g + \sin \frac{a}{e} \frac{\ddot{\theta}}{2} + \sin^2 g \sin^2 a}} \right) = 0 \Rightarrow \cos g = \frac{\sin \frac{a}{e} \frac{\ddot{\theta}}{2}}{\sin^2 a - 1} \quad (20)$$

By substituting $a = 25^\circ$ into Eq. (20), two solutions $\gamma = 0^\circ$ and $\gamma = 32.25^\circ$, are obtained. Therefore, for the mechanism to cover the desired workspace, L can be obtained by Eq. (19) and Eqs. (11) and (12) are utilized to calculate A and R, which yields $L = 552.63$ (mm), $A = 767.77$ (mm), and $R = 900.69$ (mm).

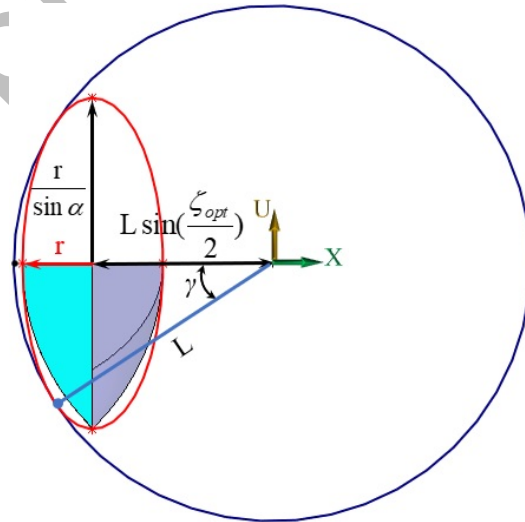


Fig. 11. A view perpendicular to the section shown in Fig. c with the related equations.

Considering the obtained optimum mechanism dimensions and $\alpha = 25^\circ$, both with and without flat end effector, 2-DoF mechanisms have been designed using SolidWorks software. The desired workspace within the overall workspace borders of both mechanisms (without/with platform) has been demonstrated in Fig. . There is no considerable difference between the workspace gamut of the mentioned mechanisms, as shown in Fig. 12.

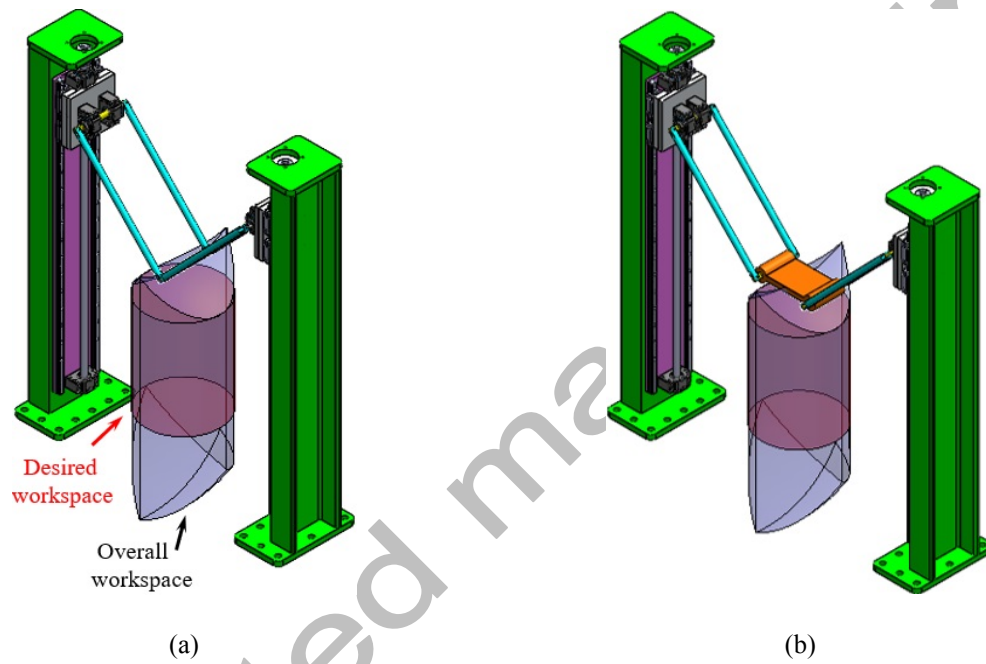


Fig. 12. Desired workspace within the overall workspace borders: a. mechanism without flat end-effector; b. mechanism with the horizontal end effector of width $B = 200$ (mm).

A search algorithm based on the inverse position kinematics is utilized to find the overall workspace borders in which a user-defined domain is swept. At each spatial point, the geometrical and kinematic constraints are considered. The kinematic constraints considered are the collision constraint between links and guideways and the collision constraint for the end-effector joints. If the conditions are met, the point is validated. After the completion of the search algorithm, the available workspace is produced [57]. In other words, when the end-effector is in a specified location in Cartesian space, the required displacement of actuators is

obtained. Then, the specified constraints are checked, and if satisfied, the point is selected as a valid pose of the end-effector and added to the workspace. Sweeping is done by dividing the selected domain into horizontal x-y planes with different levels of z. The code has been developed under MATLAB for the workspace of the mechanism, and the obtained workspace is illustrated in Fig. .

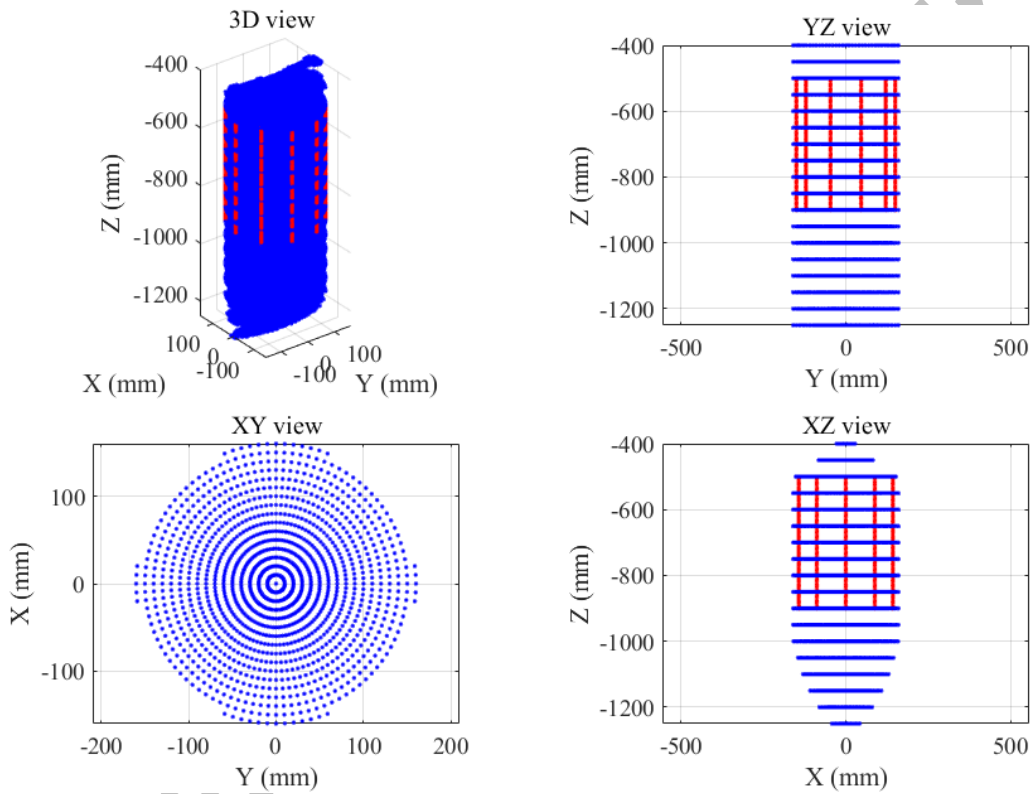


Fig. 13. The workspace borders of the mechanism obtained by the code written in MATLAB.

VI. 3T1R PARALLEL-BASED MACHINE TOOL DESIGN FOR THE DESIRED WORKSPACE

After the design for the desired workspace of a 2-DoF mechanism with the best LDI and GDI, a novel geometric design method for the 3T1R 4-DoF parallel-based machine tool, which is obtained by merging two improved 2-DoF mechanisms, is developed. The different

conditions for combining two mechanisms are discussed, and the best-merged 3T1R manipulator is introduced based on the dexterity indices. Finally, the various conditions for the different mechanism parts are discussed. The best possible dimensions for the design of the 3T1R manipulator with the desired workspace, which has the best global dexterity index and the best range of the local dexterity index, are identified. Fig. 14 shows a schematic view of the two merged 2-DoF mechanisms, which gives a 3T1R parallel-based machine tool.

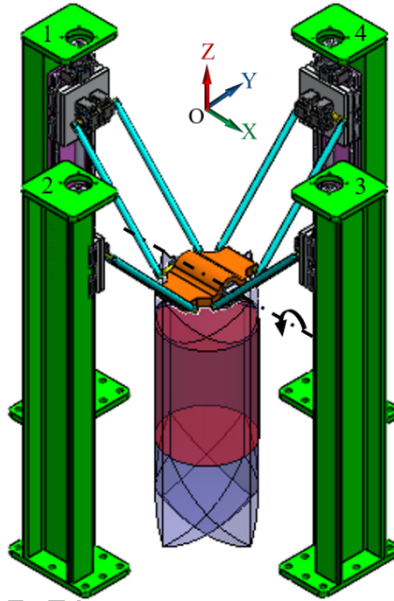


Fig. 1. The workspace of 3T1R manipulator obtained from combining two 2-DoF mechanisms.

1. *The First Design Scenario*

Two mechanisms are aligned orthogonally in the most basic scenario. Additionally, the rotational joint is positioned between the first and third joints in the positive and negative orientations of the X-axis to form the rotational axis, which is indeed the mechanism created by Liu [49]. In the first scenario, depicted in Fig. 2a, the distances of axes 1, 2, 3, and 4 from the end-factor's centre are identical. Therefore, $b_{ix} = 150$ (mm) for $i = 1, 3$ and $b_{iy} = 150$ (mm) for $i = 2, 4$ are considered when calculating the singularity's size and examining how the LDI

distribution is spread. To determine how LDI is distributed in the mechanism's workspace, the assumed $\mathbf{b}_i$ and computed $\mathbf{a}_i$ are as follows:

$$\begin{aligned}\mathbf{b}_1 &= [-150 \ 0 \ 0] & \mathbf{a}_1 &= [-383.88 - 150 \ 0 \ 0] \\ \mathbf{b}_2 &= [0 \ -150 \ 0] & \mathbf{a}_2 &= [0 \ -383.88 - 150 \ 0] \\ \mathbf{b}_3 &= [150 \ 0 \ 0] & \mathbf{a}_3 &= [383.88 + 150 \ 0 \ 0] \\ \mathbf{b}_4 &= [0 \ 150 \ 0] & \mathbf{a}_4 &= [0 \ 383.88 + 150 \ 0]\end{aligned}$$

Fig. 2b illustrates an analysis of the distribution of LDI in the mechanism's workspace. Fig. 2b shows that the mechanism's GDI value is 6.12×10^{-4} and its minimum and maximum LDI values are 4.2×10^{-20} and 7.1×10^{-3} , respectively. Fig. 2b divides the LDI values acquired at various workspace locations according to colour by considering Eq. (13).

Fig. 2b shows that the rotational axis leads to a singularity in the mechanism because the yellow sections, which have very small LDI, occupy a significant portion of the working area along the Y axis and are formed in the shape of planes in the directions perpendicular to the rotational axis. The device is ineffective because there is a singularity in the workspace.

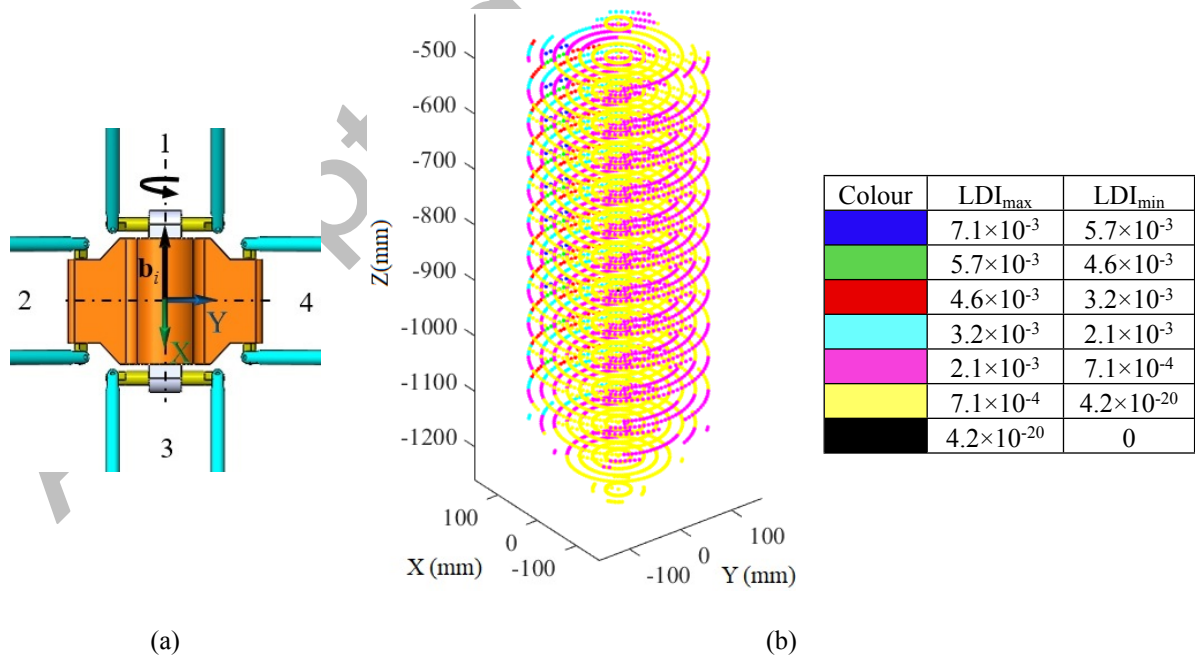


Fig. 2. The first design scenario: a. connection between the links and the end-effector; b. LDI distribution throughout the mechanism workspace.

2. *The Second Design Scenario*

The second design mode maintains the same spacing between the second and fourth axes while moving the rotational axis between joints 1 and 3 in the positive and negative directions of the Y axis and the negative and positive directions of the X axis, respectively. The magnitudes of vectors $\mathbf{b}_1$ and $\mathbf{b}_3$ are equivalent, as are the magnitudes of vectors $\mathbf{b}_2$ and $\mathbf{b}_4$. To determine how LDI is distributed in the mechanism's working area, the assumed $\mathbf{b}_i$ and computed $\mathbf{a}_i$ are as follows:

$$\begin{aligned}\mathbf{b}_1 &= [-150 \ 150 \ 0] & \mathbf{a}_1 &= [-383.88 -150 \ 150 \ 0] \\ \mathbf{b}_2 &= [0 \ -150 \ 0] & \mathbf{a}_2 &= [0 \ -383.88 -150 \ 0] \\ \mathbf{b}_3 &= [150 \ -150 \ 0] & \mathbf{a}_3 &= [383.88 +150 \ -150 \ 0] \\ \mathbf{b}_4 &= [0 \ 150 \ 0] & \mathbf{a}_4 &= [0 \ 383.88 +150 \ 0]\end{aligned}$$

Fig. 36a show the connection of the links to end-effector using the second scenario. The connection of the first and second links in this scenario is diagonal. Fig. 3b divides the LDI values acquired at various workspace locations according to colour by considering Eq. (13). The second design mode's LDI distribution in the workspace is depicted in Fig. 3b, where the only distinction between the second and first design scenarios is the changed singularity axis. The workspace's yellow regions of minimal LDI are distributed diagonal throughout the workspace. The mechanism's lowest and highest LDI values are 2.3×10^{-35} and 2.8×10^{-3} , respectively, and the GDI value for the second mode equals 4.2×10^{-4} . Similar to the first instance, the mechanism is rendered ineffective by a singularity in the workspace's centre.

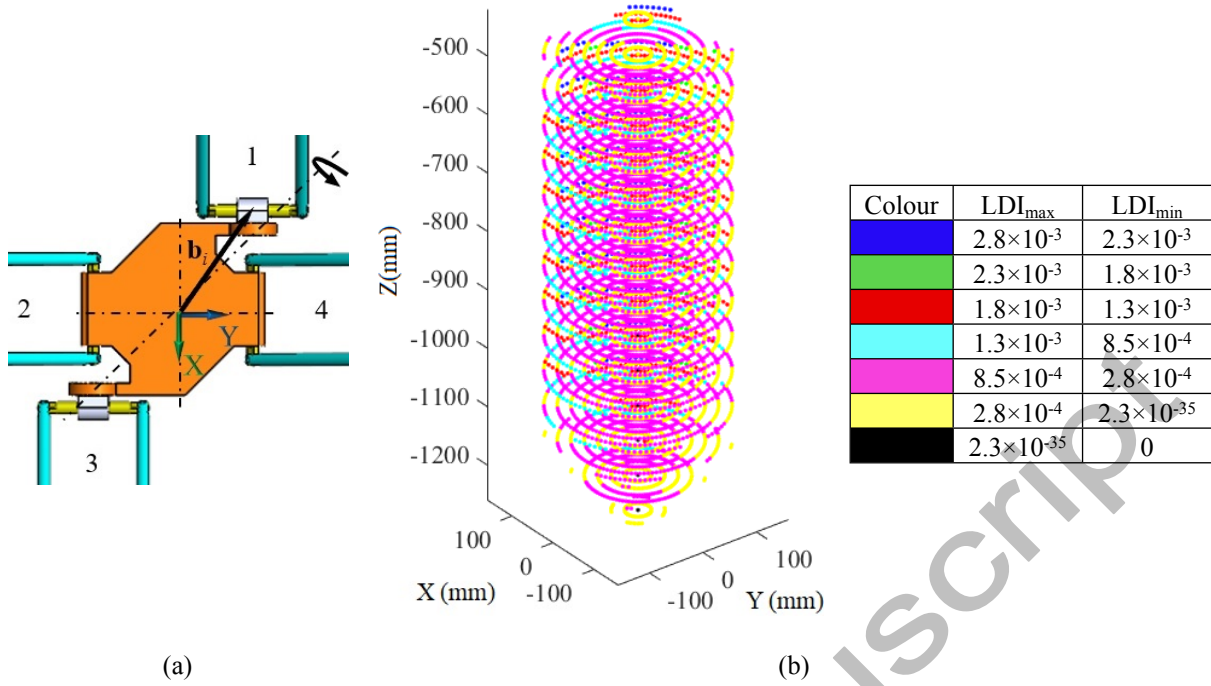


Fig. 3. The second design scenario: a. connection between the links and the end-effector; b. LDI distribution throughout the mechanism workspace.

3. The Third Design Scenario

The third design scenario involves positioning the rotational axis between joints 1 and 3, located on the X-axis, and their Y-value is zero. Links 2 and 4 are also positioned in the positive or negative Y-direction. As depicted in Fig. 4a, the rotational axis is outside the end-effector plane. In contrast, the average distance between axis 2 and 4 remains constant. In this configuration, the magnitudes of the $\mathbf{b}_1$ and $\mathbf{b}_3$ are equivalent, but the magnitudes of the $\mathbf{b}_2$ and $\mathbf{b}_4$ vectors are not. Specifically, the average distance between links 2 and 4 with the rotational axis is denoted as S and assumed to be 150 (mm).

$$S = \frac{b_{2y} + b_{4y}}{2} \quad (21)$$

This design aims to better guide the singularities out of the workspace. Consequently, the chosen sizes for $\mathbf{b}_i$ and the resulting calculated values of $\mathbf{a}_i$ are as follows to analyse the distribution of LDI in the mechanism's workspace:

$$\mathbf{b}_1 = [-150 \ 0 \ 0]$$

$$\mathbf{b}_2 = [0 \ 0 \ 0]$$

$$\mathbf{b}_3 = [150 \ 0 \ 0]$$

$$\mathbf{b}_4 = [0 \ 300 \ 0]$$

$$\mathbf{a}_1 = [-383.88 - 150 \ 0 \ 0]$$

$$\mathbf{a}_2 = [0 \ -383.88 \ 0]$$

$$\mathbf{a}_3 = [383.88 + 150 \ 0 \ 0]$$

$$\mathbf{a}_4 = [0 \ 383.88 + 300 \ 0]$$

In the third design mode, the workspaces of the two mechanisms of the 2-DoF are not fully engaged. The desired workspace is not fully achieved in the combined space of the two mechanisms because of the distance between the connecting axis of links 1 and 3 and the axis of links 2 and 4 (Fig. 4b).

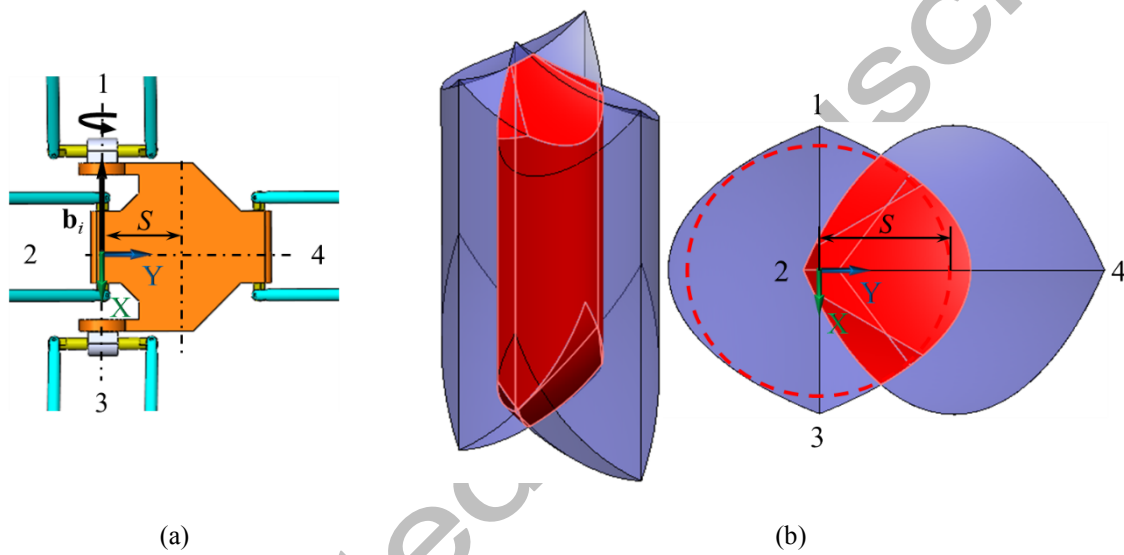


Fig. 4. The third design scenario: a. connection between the links and the end-effector; b. the engaged workspace of the combined 2-DoF mechanisms.

The optimal dimensions of one of the 2-DoF mechanisms (the mechanism containing links 1 and 3) are kept unchanged to address this problem. The new optimal dimensions of that mechanism are determined to cover the common workspace of the two combined mechanisms of the desired workspace by examining the dimensions of another mechanism.

The method introduced in Fig. has also been used to calculate the minimum value of L for links 2 and 4. The new workspace that can cover the desired workspace for the third scenario

is shown in Fig. 5. Given Fig. 5, displacing the ellipse in the y-axis direction by $\frac{S}{\sin a}$ results in:

$$\frac{\partial}{\partial \xi} \left(L \sin \frac{\xi_{opt}}{2} + \frac{S}{\sin a} \right) \frac{\partial}{\partial \theta} = r^2 \quad (22)$$

Therefore, the minimum value of L can be obtained from the following:

$$L^2 \frac{\partial}{\partial \xi} \cos^2 g + 2 \cos g \sin \frac{\xi_{opt}}{2} + \sin^2 \frac{\xi_{opt}}{2} + \sin^2 g \sin^2 a + S^2 - r^2 = 0 \quad (23)$$

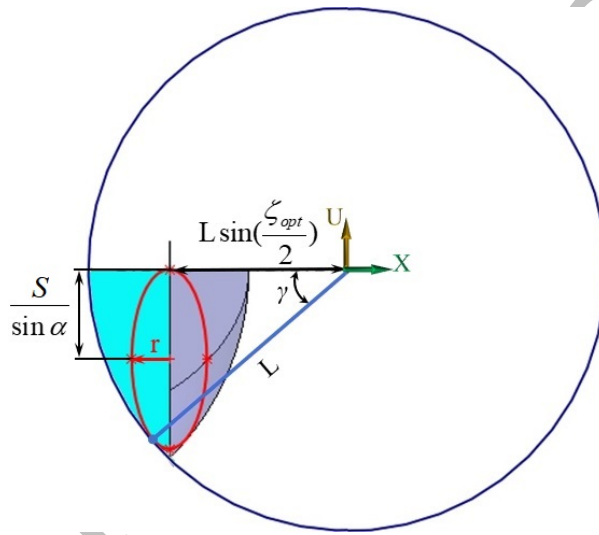


Fig. 5. The engaged workspace of the combined 2-DoF mechanisms in the third design condition.

Taking into account $Q = \cos^2 g + 2 \cos g \sin \frac{\xi_{opt}}{2} + \sin^2 \frac{\xi_{opt}}{2} + \sin^2 g \sin^2 a$, $W = 2S \sin g \sin a$, and

$T = S^2 - r^2$, Eq. (23) is rewritten as follows:

$$QL^2 + WL + T = 0 \quad (24)$$

The maximum value of L can be calculated by taking the derivative of Eq. (24) concerning γ and setting the resulting equation equal to zero, as follows:

$$\frac{dL}{dg} = \frac{d}{dg} \frac{\partial W \pm \sqrt{W^2 - 4QT}}{2Q} \frac{\partial}{\partial \theta} = 0 \quad (25)$$

Assuming $S = 150$ (mm), the angle γ is 40.6464 degrees concerning the X-axis. As a result, Eq. (23) is used to recalculate the mechanism's ideal proportions as a consequence for links 2 and 4, $L_s = 1033.77$ (mm), where L_s is the link's length with assumed S and considering Eqs. (11) and (12) $A_s = 1436.23$ (mm) and $R_s = 1263.66$ (mm) are obtained. As a consequence, after resolving the mechanism's kinematics, we will have the following:

$$\begin{aligned} \mathbf{b}_1 &= [-150 \ 0 \ 0] & \mathbf{a}_1 &= [-383.88 - 150 \ 0 \ R_{S=0} - R_{S=150}] \\ \mathbf{b}_2 &= [0 \ 0 \ 0] & \mathbf{a}_2 &= [0 \ -718.115 \ 0] \\ \mathbf{b}_3 &= [150 \ 0 \ 0] & \mathbf{a}_3 &= [383.88 + 150 \ 0 \ R_{S=0} - R_{S=150}] \\ \mathbf{b}_4 &= [0 \ 300 \ 0] & \mathbf{a}_4 &= [0 \ 718.115 + 300 \ 0] \end{aligned}$$

The third design mode's LDI distribution in the workspace is depicted in Fig. . As depicted in Fig. 9, the lowest and highest LDI values of the mechanism are equal to 1.5×10^{-3} and 3.6×10^{-3} , respectively. The GDI value for the third mode is equal to 2.5×10^{-3} .

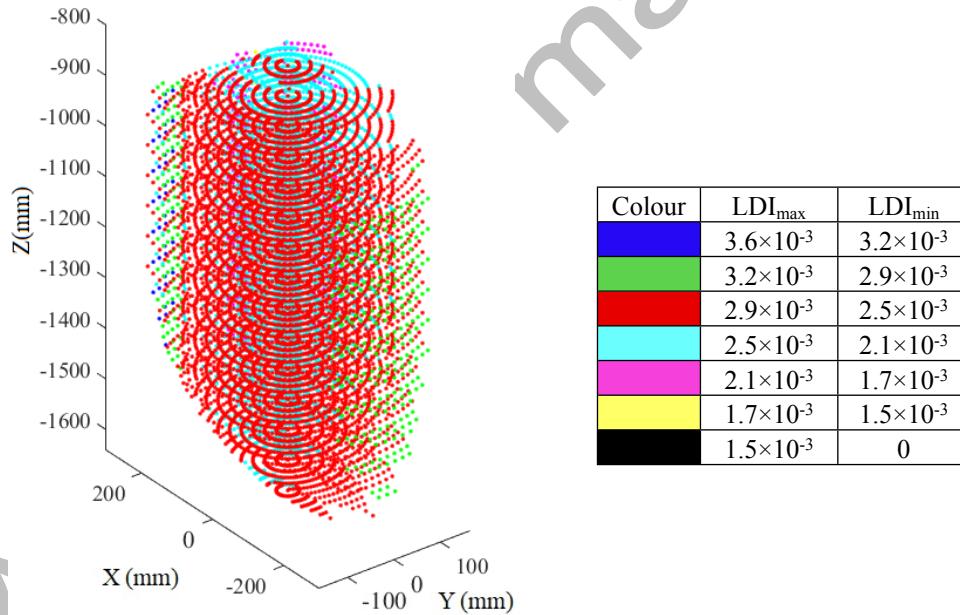


Fig. 19. LDI distribution throughout the mechanism workspace under the third design scenario.

The third design scenario, regarding GDI and LDI, is the most fabulous fusion of two 2-DoF mechanisms to produce a 3T1R 4-DoF parallel-based machine tool, according to the studied

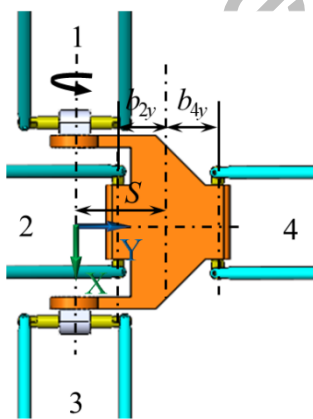
design modes. This section also examines the impact of the distance between links 2 and 4, i.e., b_{2y} and b_{4y} , on the end factor's transverse dimension along the Y axis.

In this section, S , which is the average value between b_{2y} and b_{4y} on the axis, is considered to be 150 millimetres (Fig. 20a). To verify the GDI values in the workspace of the mechanism with a change in the width of the end factor, the assumed $\mathbf{b}_i$ and computed $\mathbf{a}_i$ are as follows:

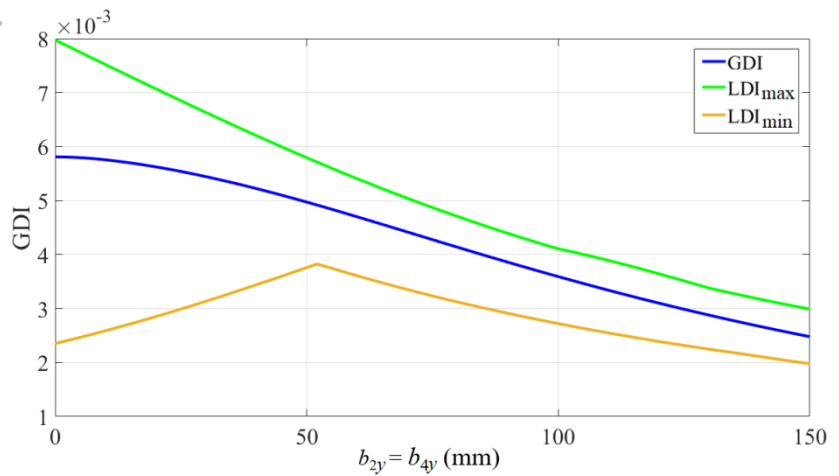
$$\begin{aligned} \mathbf{b}_1 &= [-150 \ 0 \ 0] & \mathbf{a}_1 &= [-383.88 - 150 \ 0 \ R_{S=0} - R_{S=50}] \\ \mathbf{b}_2 &= [0 \ S - b_{2y} \ 0] & \mathbf{a}_2 &= [0 \ -718.155 + (S - b_{2y}) \ 0] \\ \mathbf{b}_3 &= [150 \ 0 \ 0] & \mathbf{a}_3 &= [383.88 + 150 \ 0 \ R_{S=0} - R_{S=50}] \\ \mathbf{b}_4 &= [0 \ S + b_{4y} \ 0] & \mathbf{a}_4 &= [0 \ 718.115 + (S + b_{4y}) \ 0] \end{aligned}$$

The values of A_s , L_s , and R_s have been calculated for the assumed $S = 150$ (mm), similar to the previous case, for links 2 and 4.

Equalizing b_{2y} and b_{4y} Between 0 and 150 millimetres is the range for the disparity between b_{2y} and b_{4y} . Due to links 2 and 4 not colliding, this number cannot be negative. Fig. 20b illustrates the impact of the change in the end factor's width or changes in $b_{2y} = b_{4y}$ on the GDI and the lowest and maximum LDI of the mechanism. According to Fig. b, the GDI of the mechanism declines and becomes lower as the width of the end factor increases. The GDI value at $b_{iy} = 0$, $i = 2, 4$ is 5.8×10^{-3} . Accordingly, we regard b_{iy} as zero for ultimate changes.



(a)



(b)

Fig. 20. The third design scenario: a. connection between the links and the end-effector; b. the effect of change in an end-effector b_{iy} on mechanism GDI.

The impact of the distance between links 2 and 4, and the rotation axis, S , has also been researched, presuming that b_{4y} , $b_{2y} = 0$ are constant (Fig. a). To verify the GDI values in the working area of the mechanism with a change in the end-factor width, the $\mathbf{b}_i$ and $\mathbf{a}_i$ are determined as follows:

$$\begin{aligned} \mathbf{b}_1 &= [-150 \ 0 \ 0] & \mathbf{a}_1 &= [-383.885 - 150 \ 0 \ R_{S=0} - R_S] \\ \mathbf{b}_2 &= [0 \ S \ 0] & \mathbf{a}_2 &= \begin{bmatrix} \dot{\theta}_2 \\ 0 \\ 0 \end{bmatrix} - \frac{1}{2}A_S + S \begin{bmatrix} 0 \\ \dot{\theta}_2 \\ 0 \end{bmatrix} \\ \mathbf{b}_3 &= [150 \ 0 \ 0] & \mathbf{a}_3 &= [383.885 + 150 \ 0 \ R_{S=0} - R_S] \\ \mathbf{b}_4 &= [0 \ S \ 0] & \mathbf{a}_4 &= \begin{bmatrix} \dot{\theta}_4 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2}A_S + S \begin{bmatrix} 0 \\ \dot{\theta}_4 \\ 0 \end{bmatrix} \end{aligned}$$

The values of A_S , L_S , and R_S are calculated for variable values of S using Eq. (25), similar to the previous cases, for links 2 and 4. This investigation assumes $-50 \leq S \leq 50$ (mm). The changes in S values on GDI are not visible beyond the desired range.

The influence of changing the distance between links 2 and 4, as well as the rotational axis on the GDI and the minimum and maximum LDI of the mechanism, has been considered, as depicted in Fig. b. As per the diagram in Fig. b, when the value of S is zero, the GDI of the mechanism reaches its worst possible state, registering a value of zero. This outcome is due to the alignment of the four-link axes in the end factor, causing the mechanism to lack stiffness. By raising this value to $S = 1.5$ (mm), the GDI of the mechanism achieves its maximum value of 0.3658 before beginning to decline after 1.5 millimetres. At $S = 0$ (mm), links 2 and 4 become co-linear, resulting in a singularity. With the minimum distance between links 2 and 4 and links 1 and 3, the maximum LDIs of the two 2-DoF mechanisms are pretty close. Consequently, the maximum value of GDI is obtained. In other words, as the distance S increases, the value of GDI decreases.

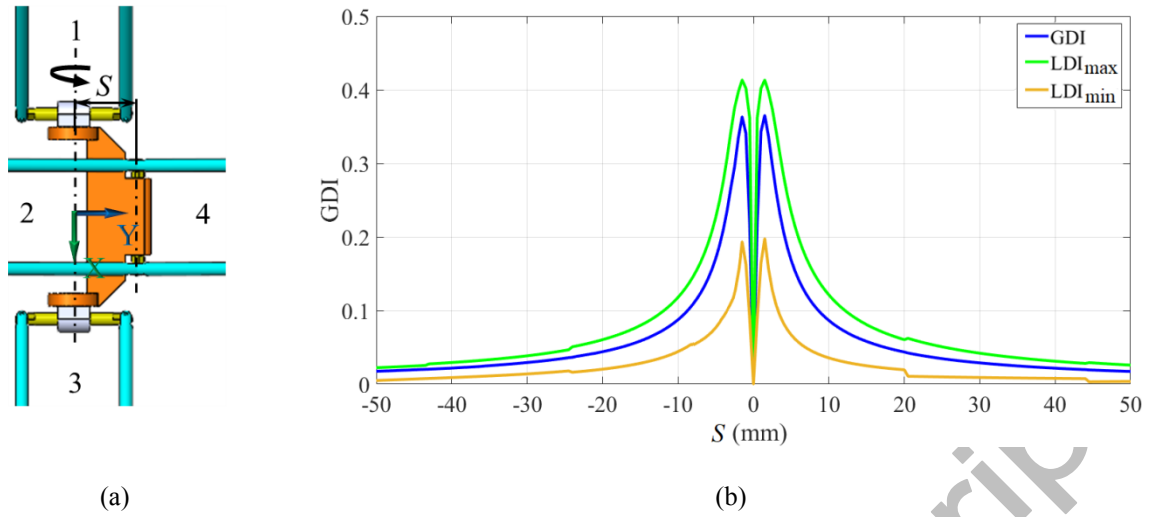


Fig. 21. The third design scenario: a. connection between the links and the end-effector; b. the effect of change in the distance between links 2 and 4, and the rotational axis when $b_{4y}, b_{2y} = 0$.

The height of the intersection of links 2 and 4, i.e. b_{2z} and b_{4z} , from the end factor along the Z axis, has also been considered here in this paper (Fig. 6a). To verify the GDI values in the workspace of the mechanism with a change in the height of the connection point of links 2 and 4 to the end factor, the assumed $\mathbf{b}_i$ and computed $\mathbf{a}_i$ are as follows:

$$\begin{aligned} \mathbf{b}_1 &= [-150 \ 0 \ 0] \\ \mathbf{b}_2 &= [0 \ 1.5 \ b_{2z}] \\ \mathbf{b}_3 &= [150 \ 0 \ 0] \\ \mathbf{b}_4 &= [0 \ 1.5 \ b_{4z}] \end{aligned} \quad \begin{aligned} \mathbf{a}_1 &= [-383.885 - 150 \ 0 \ R_{S=0} - R_{S=1.5}] \\ \mathbf{a}_2 &= \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} - \frac{1}{2} A_{S=1.5} + 1.5 \begin{bmatrix} \dot{b}_{2z} \\ 0 \\ 0 \end{bmatrix} \\ \mathbf{a}_3 &= [383.885 + 150 \ 0 \ R_{S=0} - R_{S=1.5}] \\ \mathbf{a}_4 &= \begin{bmatrix} \dot{\theta} \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2} A_{S=1.5} + 1.5 \begin{bmatrix} \dot{b}_{4z} \\ 0 \\ 0 \end{bmatrix} \end{aligned}$$

The values of A_s , L_s , and R_s are calculated for $S = 1.5$ (mm), similar to the previous cases, for links 2 and 4.

In this investigation, $0 \leq (b_{2z} = b_{4z}) \leq 50$ (mm) is assumed. The effect of change in the vertical distance between the links 2 and 4, and the rotational axis on the GDI of the mechanism, has been considered when b_{4y} and $b_{2y} = 0$ and $S = 1.5$ (mm), as depicted in Fig. 6b. As per depicted in Fig. 6b, as the value of b_{iz} approaches zero, the maximum GDI value is attained.

Fig. 6b shows that changes in the position of links 2 and 4 in the Z-direction relative to links 1 and 3 decrease the mechanism's GDI.

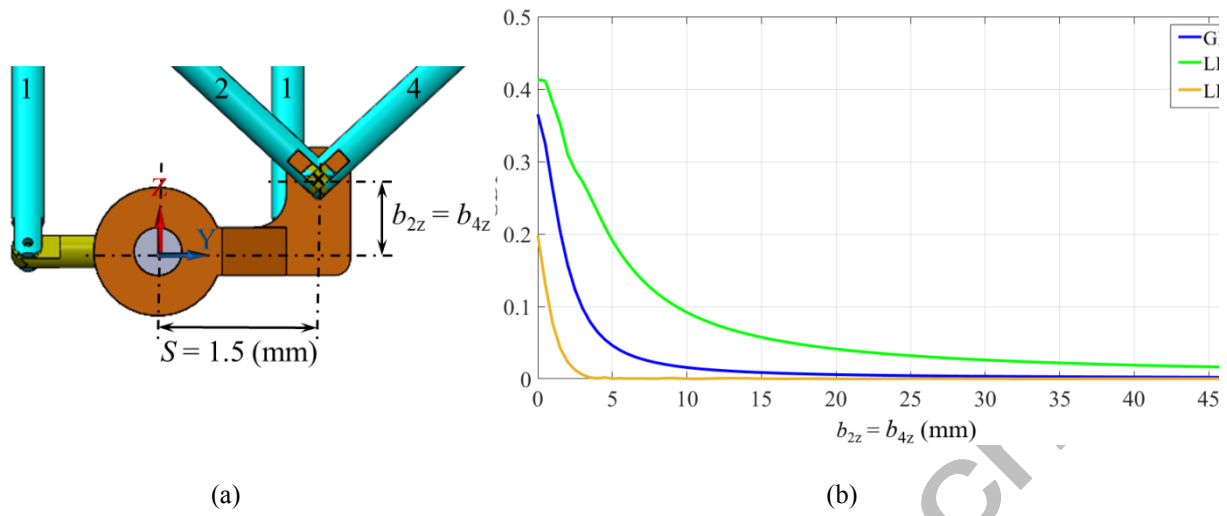


Fig. 6. The third design scenario: a. connection between the links and the end-effector; b. the effect of change in the vertical distance between the links 2 and 4, and the rotational axis when b_{4y} and $b_{2y} = 0$, and $S = 1.5$ (mm).

Investigations into GDI changes in the mechanism caused by variations in the end factor's orientation concerning the rotational axis have also been conducted (Fig. 7b). In this overview, $-30^\circ \leq \beta \leq 30^\circ$ is considered, and Fig. 7b's GDI is provided. It is clear from Fig. 7b that the 0 angle is where the GDI value is the highest. In addition, it can be seen that the GDI of the mechanism diminishes and becomes more uniform within 30 degrees as the distance between links 2 and 4 and the rotation axis is increased. Furthermore, according to Fig. 7b, as the value of S increases, the magnitude of changes in GDI decreases from 30 to 0 degrees. In other words, the peak created in the shorter graph is reduced.

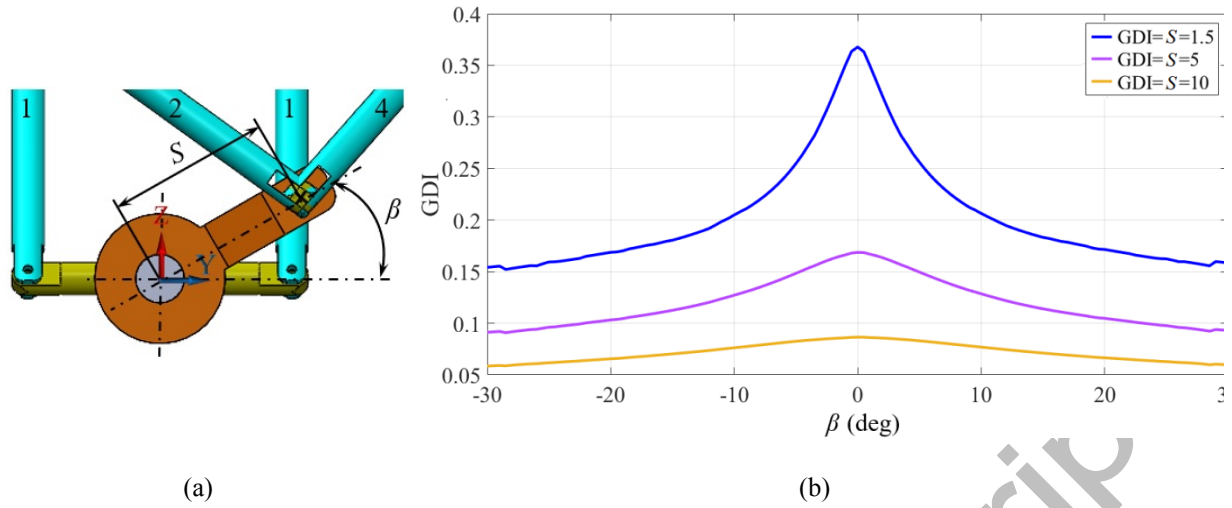


Fig. 7. The third design scenario: a. connection between the links and the end-effector; b. the effect of change in the orientation of the end-effector when $b_{dy}, b_{dy} = 0$, and $S = 1.5, 5$, and 10 (mm).

According to the investigations in this paper, the mechanism dimensions listed in Table 1 are the best possible dimensions that theoretically lead to the best GDI.

Table 1 The dimensions of the mechanism with the maximum possible GDI for the desired workspace of $r = 150$ (mm) and $h = 400$ (mm).

The angle between the two links	$\zeta = 88^\circ$
The position vectors of the i^{th} end-effector connection point in frame $\{\mathbf{P}\}$	$\mathbf{b}_1 = [-150 \ 0 \ 0]$
	$\mathbf{b}_2 = [0 \ 1.5 \ 0]$
	$\mathbf{b}_3 = [150 \ 0 \ 0]$
	$\mathbf{b}_4 = [0 \ 1.5 \ 0]$
The position vectors of the i^{th} guideway in frame $\{\mathbf{O}\}$	$\mathbf{a}_1 = [-533.885 \ 0 \ -3.56]$
	$\mathbf{a}_2 = [0 \ -387.06+1.5 \ 0]$
	$\mathbf{a}_3 = [533.855 \ 0 \ -3.56]$
	$\mathbf{a}_4 = [0 \ 387.06+1.5 \ 0]$
The i^{th} link length	$L_1 = L_3 = 552.63$
	$L_2 = L_4 = 557.19$
The i^{th} guideway length	$R_1 = R_3 = 900.69$
	$R_2 = R_4 = 904.25$
The distance between links 2 and 4 and the rotational axis	$S = 1.5$
The orientation angle of the end-effector	$\beta = 0^\circ$

*All the quantities are given in (mm).

Fig. 8 depicts the distribution of LDI in the desired workspace of $r = 150$ (mm) and $h = 400$ (mm) for the mechanism dimensions listed in Table 1. GDI is 0.3658, and the maximum and

minimum LDI are respectively 0.4132 and 0.1877. The maximum LDI has been illustrated in blue in Fig. 8. It is apparent that this LDI range covers the workspace gamut's central part. It is considered the best distribution of LDI that can avoid singularities within the workspace.

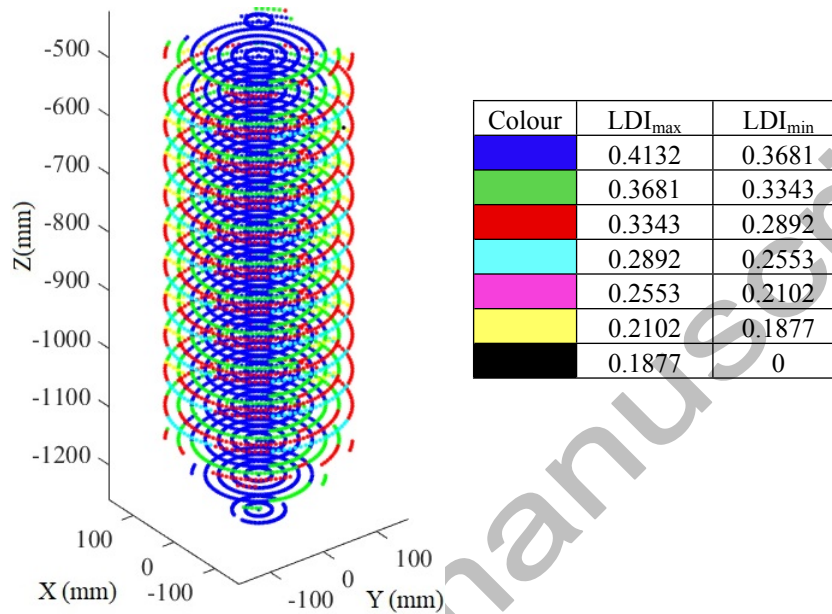


Fig. 8. The distribution of LDI throughout the overall workspace of the mechanism for the mechanism dimensions listed in Table 1 and the desired workspace.

Considering the dimensional feasibility, constructing an end factor with the dimensions of Table 1, which has the highest GDI, is nearly impossible. For the final design of the mechanism, the distance between links 2 and 4 and the rotational axis is considered the possible minimum value to achieve a higher GDI. In this article, the mechanism under study is assumed to be a 4-DoF CNC for machining soft metal parts such as copper and aluminium, and the spindle mechanism with a power of 0.75 (kW) is considered. Therefore, considering the minimum size to hold the spindle and avoid collision between bearings and mechanical equipment and limitations in selecting spindle and bearing components, $S = 50$ (mm) is considered. In the final designed mechanism, the values of b_{2z} , b_{2y} , b_{4y} , and $(b_{2y} - b_{4y})$ are assumed to equal zero.

By altering the value of S from 1.5 to 50 (mm), a significant part of the workspace will shrink by 50 (mm). To counterbalance this situation, the workspace radius for links 2 and 4 can be augmented by 50 (mm). Consequently, the values of A_s , L_s , and R_s for $S = 50$ (mm) have been calculated for links 2 and 4 using Eq. (25), similar to the previous cases. Table 2 provides a mechanism featuring optimal, manufacturable dimensions.

Table 2 The manufacturable final dimensions for the 3T1R parallel-based machine tool with the best GDI for the desired workspace of $r = 150$ (mm) and $h = 400$ (mm)*

The angle between the two links	$\zeta = 88^\circ$
the position vectors of the i^{th} end-effector connection point in frame $\{\mathbf{P}\}$	$\mathbf{b}_1 = [-150 \ 0 \ 0]$
	$\mathbf{b}_2 = [0 \ 50 \ 0]$
	$\mathbf{b}_3 = [150 \ 0 \ 0]$
	$\mathbf{b}_4 = [0 \ 50 \ 0]$
The position vectors of the i^{th} guideway in frame $\{\mathbf{O}\}$	$\mathbf{a}_1 = [-533.885 \ 0 \ -117]$
	$\mathbf{a}_2 = [0 \ -439.677 \ 0]$
	$\mathbf{a}_3 = [533.885 \ 0 \ -117]$
	$\mathbf{a}_4 = [0 \ 539.677 \ 0]$
The i^{th} link length	$L_1 = L_3 = 552.63$
	$L_2 = L_4 = 704.918$
The i^{th} guideway length	$R_1 = R_3 = 900.69$
	$R_2 = R_4 = 1017.7$
The distance between links 2 and 4 and the rotational axis	$S = 50$
The orientation angle of the end-effector	$\beta = 0^\circ$

*All the quantities are given in (mm).

Considering the details listed in Table 2, GDI is 0.0174, and the maximum and minimum LDI are, respectively, 0.0269 and 0.0044. The end factor for the finished mechanism is depicted in Fig. 25a. Links 2 and 4 are constructed arc-shaped to prevent end-effector collision. Fig. 9b also shows the LDI distribution of the mechanism throughout the overall workspace according to the dimensions listed in Table 2 and the desired workspace.

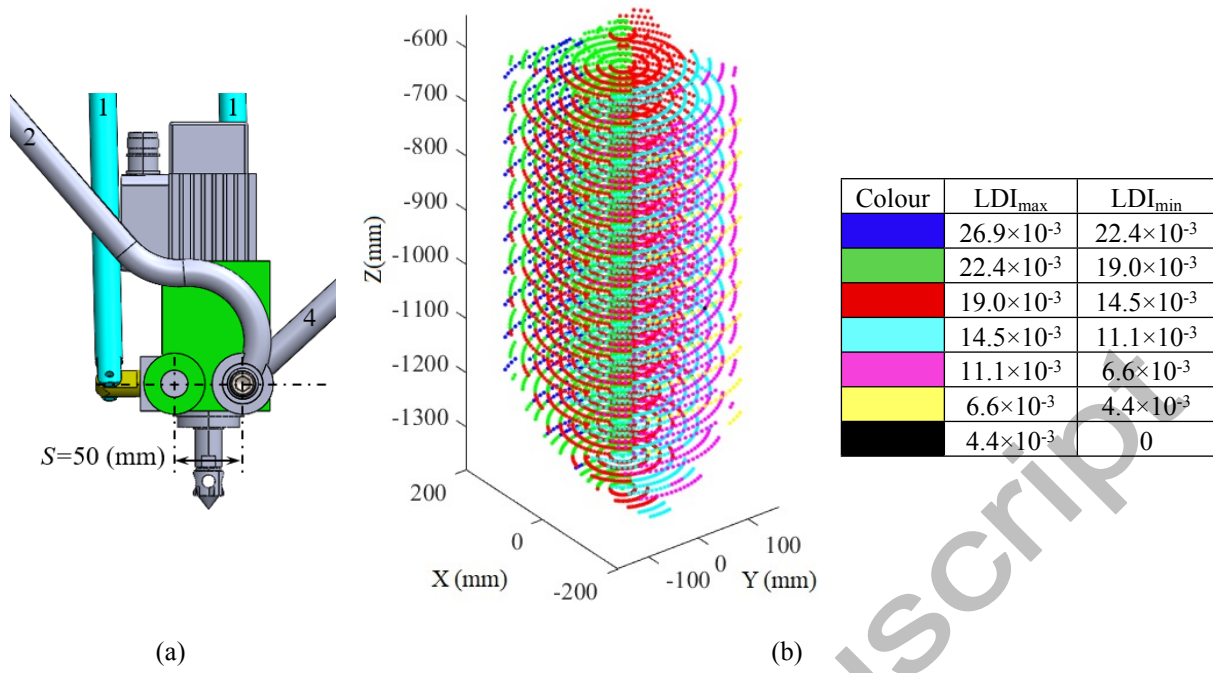


Fig. 9. The final mechanism design based on manufacturing capabilities: a. connection between the links and the end-effector; b. LDI distribution of the mechanism throughout the overall workspace according to the dimensions listed in Table 2 and the desired workspace.

Fig. 10 depicts the distribution of LDI in the desired workspace, specifically the considered cylinder with dimensions of $r = 150$ (mm) and $h = 400$ (mm). According to Fig. 10a, the final design's GDI value and $\beta = 0^\circ$ are 17.5×10^{-3} , while the mechanism's lowest and highest LDI values are 24.1×10^{-3} and 9.8×10^{-3} , respectively. Fig. 10b-c display the LDI distribution in the intended workspace for the final designs with $\beta = 30^\circ$ and $\beta = -30^\circ$. The final design's GDI value and $\beta = 30^\circ$ are equivalent to 16.9×10^{-3} , and the mechanism's minimum and maximum LDI values are 7.8×10^{-3} and 20.3×10^{-3} , respectively. The GDI value and the lowest and highest LDI values of the mechanism were determined to be 16.8×10^{-3} , 7.8×10^{-3} , and 20.2×10^{-3} , respectively, for the final design and $\beta = -30^\circ$.

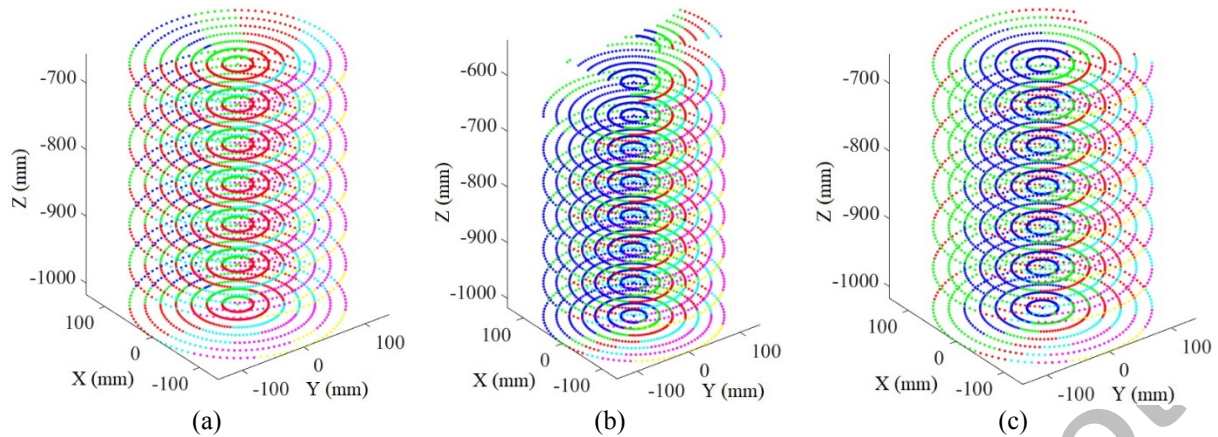


Fig. 10. LDI distribution of the mechanism throughout the overall workspace according to the final design: a. $\beta = 0^\circ$; b. $\beta = 30^\circ$; c. $\beta = -30^\circ$.

4. Design Scenarios in brief

In scenarios one and two of the design, other mechanism parameters have not been examined due to singularity occurring in the workspace. In the third design scenario, different influential parameters in optimizing the dimensions of the mechanism for the desired workspace have been investigated and evaluated because singularity does not occur within the workspace. Table 3 summarizes the scenarios studied in this section.

Table 3 Design scenarios in brief.

Design scenarios	Guideways and the connections between links and the end effector.	Singularity	Figure	GCI	LDI _{max}	LDI _{min}
Scenario 1	Two mechanisms are aligned orthogonally, and the rotational joint is positioned between the first and third joints in the positive and negative orientations of the X-axis to form the rotational axis.	Singularity occurs along the axis perpendicular to the axis of rotational joint.	Fig. 15b	6.1×10^{-4}	7.1×10^{-3}	4.2×10^{-20}
Scenario 2	Guideways 1 and 3 are positioned diagonally, and the rotational axis between the joints 1 and 3 extends in the positive and negative Y-axis directions, as well as the negative and positive X-axis directions, respectively.	Singularity is diagonal, and the instability of the mechanism will be in the direction normal to the rotational axis.	Fig. 16b	4.2×10^{-4}	2.8×10^{-3}	2.3×10^{-35}
Scenario 3	The rotational axis is situated between joints 1 and 3, outside the end-effector plane, aligned with the X-axis, with a coordinate of zero. Links 2 and 4 are likewise oriented along the positive or negative Y-direction.	For $S = 1.5$ (mm) This scenario is non-singular.	Fig. 24	0.3658	0.4132	0.1877
		For $S = 50$ (mm) This scenario is non-singular.	Fig. 25	0.0174	26.9×10^{-3}	4.4×10^{-3}
			Fig. 26			

The details for the considered and obtained design parameters for each design scenario have also been given in Appendix.

VII. CONCLUSION

A novel geometric method for dimensionally synthesizing the 3T1R parallel-based machine tool with a desired workspace has been presented in this paper. At first, the paper briefly examined the inverse kinematics of the mechanism and the best possible design for the 2-DoF mechanism with the desired workspace using global and local dexterity indices. A discussion of the 3T1R parallel-based machine tool, obtained from merging two mechanisms with 2-DoF, followed. The various conditions of the individual parts of the mechanism were discussed, and improved dimensions for the 3T1R parallel-based machine tool were established to achieve the desired workspace. The best global dexterity index and local dexterity index range were considered to improve the design of 3T1R parallel-based machine tool parts for the desired workspace. A 3T1R parallel-based machine tool with the desired workspace and the best global dexterity index and local dexterity index range is designed using the improved dimensions. Further, the final improved mechanism design was developed considering the manufacturing possibilities. The best possible dimensions for manufacturing the 3T1R manipulator with the desired workspace and the best global and local dexterity indices were introduced. The reported design method can be applied to a wide range of parallel-based machine tool s, which can be designed via merging two or more manipulators.

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APPENDIX

The details for the obtained/considered design parameters for each design scenario:

No.	Parameter	Definition	Obtained/considered value
1	ζ_{opt}	The angle between the end-effector joints	The optimized value for this angle is considered to be 88 degrees
2	r	The radius of the desired workspace	In this article, $r = 150$ (mm) is considered
3	h	The height of the desired workspace	In this article, $h = 400$ (mm) is considered
4	B	The distance between the end-effector joints	The distance between two links for spindle placement has been considered, with a fixed value of $B = 300$ (mm) for links 1 and 3 throughout the entire article. After examination, links 2 and 4 considered $B = 0$ (mm) according to the Fig. 18b diagram
5	S	The distance between the end effector center and the rotational axis along the Y-axis (Eq. 1)	$S = 0$ (mm) has been considered for scenarios one and two For the third scenario $S = 1.5$ (mm). In the case of the design based on manufacturing capabilities, $S = 50$ (mm) has been considered
6	γ	The angle between the link and the image is parallel to the horizon and is reflected on the intersected surface	For scenarios 1 and 2 where $S = 0$ (mm), $\gamma = 32.25^\circ$ is obtained based on Eq. 20 For scenario three, with $S = 1.5$ (mm), $\gamma = 32.3^\circ$ is calculated according to Eq. 25 for links 2 and 4. For links 1 and 3, $\gamma = 32.25^\circ$ is obtained according to Eq. 20. Simoultanously with $S = 50$ (mm) γ is 34.22° for links 2 and 4, and for links 1 and 3, γ is 32.25°
7	L	Link length	For scenarios 1 and 2 where $S = 0$ (mm), $L = 552.63$ (mm) is obtained based on Eq. 19 For scenario three, with $S = 1.5$ (mm), and $\gamma = 32.3^\circ$, $L = 557.19$ (mm) is obtained according to Eq. 23 for links 2 and 4. For links 1 and 3 with $\gamma = 32.25^\circ$, $L = 552.63$ (mm) is obtained according to Eq. 19. Simoultanously with $S = 50$ (mm) and $\gamma = 34.22^\circ$, $L = 704.88$ (mm) is calculated for links 2 and 4, and for links 1 and 3 with $\gamma = 32.25^\circ$, L is 552.63 (mm)
8	A	The distance between two opposite guides	For scenarios 1 and 2 where $S = 0$ (mm), $A = 767.77 + B$ (mm) is obtained based on Eq. 11

			For scenario three, with $S = 1.5$ (mm), and $L = 557.19$ (mm), $A = 767.77 + B$ (mm) is obtained for links 2 and 4. For links 1 and 3 with $L = 552.63$ (mm), $A = 767.77 + B$ (mm) is obtained according to Eq. 11. Simoultanously with $S = 50$ (mm) and $L = 704.88$ (mm) is $A = 1079.35$ (mm) is calculated for links 2 and 4, and for links 1 and 3 with $L = 552.63$ (mm), $A = 767.77 + B$ (mm) is obtained
			For scenarios 1 and 2 where $S = 0$ (mm), $R = 900.69$ (mm) is obtained based on Eq. 12
9	R	Guideway course: The maximum d that the mechanism can travel to cover the desired workspace height, h	For scenario three, with $S = 1.5$ (mm), and $A = 767.77$ (mm), $R = 904.25$ (mm) is obtained for links 2 and 4. For links 1 and 3 with $A = 767.7$ (mm), $R = 900.69$ (mm) is obtained according to Eq. 12. Simoultanously with $S = 50$ (mm) and $A = 1079.35$ (mm), $R = 1017.7$ (mm) is calculated for links 2 and 4, and for links 1 and 3 with $A = 767.7$ (mm), $R = 900.69$ (mm) is obtained
			Scenario 1: $b_1 = [-150 \ 0 \ 0]$ $b_2 = [0 \ -150 \ 0]$ $b_3 = [150 \ 0 \ 0]$ $b_4 = [0 \ 150 \ 0]$
			Scenario 2: $b_1 = [-150 \ 150 \ 0]$ $b_2 = [0 \ -150 \ 0]$ $b_3 = [150 \ -150 \ 0]$ $b_4 = [0 \ 150 \ 0]$
10	b_i	Distance between the end effector center and the i^{th} joint	Scenario 3, $S = 1.5$ (mm): $b_1 = [-150 \ 0 \ 0]$ $b_2 = [0 \ 1.5 \ 0]$ $b_3 = [150 \ 0 \ 0]$ $b_4 = [0 \ 1.5 \ 0]$
			Scenario 3, $S = 50$ (mm): $b_1 = [-150 \ 0 \ 0]$ $b_2 = [0 \ 50 \ 0]$ $b_3 = [150 \ 0 \ 0]$ $b_4 = [0 \ 50 \ 0]$
			Scenario 1: $a_1 = [-533.885 \ 0 \ 0]$ $a_2 = [0 \ -533.885 \ 0]$ $a_3 = [533.885 \ 0 \ 0]$ $a_4 = [0 \ 533.885 \ 0]$
			Scenario 2: $a_1 = [-533.88 \ 150 \ 0]$ $a_2 = [0 \ -533.885 \ 0]$ $a_3 = [533.885 \ -150 \ 0]$ $a_4 = [0 \ 533.885 \ 0]$
11	a_i	The position vectors of the i^{th} guideway in frame	Scenario 3, $S = 1.5$ (mm): $a_1 = [-533.885 \ 0 \ -3.56]$ $a_2 = [0 \ -385.56 \ 0]$ $a_3 = [533.885 \ 0 \ -3.56]$ $a_4 = [0 \ 388.56 \ 0]$
			Scenario 3, $S = 50$ (mm): $a_1 = [-533.885 \ 0 \ -117]$ $a_2 = [0 \ -439.667 \ 0]$ $a_3 = [533.885 \ 0 \ -117]$ $a_4 = [0 \ 539.677 \ 0]$
12	β	The orientation angle of the end-effector	The maximum and minimum rotation angles of the end effector have been considered as $\pm 30^\circ$
			Scenario 1: $LDI_{\max} = 7.1 \times 10^{-3}$, $LDI_{\min} = 4.2 \times 10^{-20}$
			Scenario 2: $LDI_{\max} = 2.8 \times 10^{-3}$, $LDI_{\min} = 2.3 \times 10^{-35}$
13	LDI	Local dexterity index	Scenario 3, $S = 1.5$ (mm): $LDI_{\max} = 0.4132$, $LDI_{\min} = 0.1877$
			Scenario 3, $S = 50$ (mm): $LDI_{\max} = 26.9 \times 10^{-3}$, $LDI_{\min} = 4.4 \times 10^{-3}$

14	GDI	Global dexterity index	Scenario 1:	$GDI = 4.2 \times 10^{-4}$
			Scenario 2:	$GDI = 6.12 \times 10^{-4}$
			Scenario 3, $S = 1.5$ (mm):	$GDI = 0.3658$
			Scenario 3, $S = 50$ (mm):	$GDI = 0.0174$

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