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ABSTRACT

In this series, we outline a strategy for analyzing electrons and muons in gases in crossed electric and magnetic fields using the straightforward transport equations of momentum-transfer theory, plus empirical arguments. The method, which can be carried through from first principles to provide numerical estimates of quantities of experimental interest, offers a straightforward, physically transparent alternative to "off-the-shelf" simulation packages, such as Magboltz and GEANT. In this first article, we show how swarm data for electrons in helium gas subject to an electric field only can be incorporated into the analysis to generate electron swarm properties in helium gas in crossed electric and magnetic fields and to estimate the Lorentz angle in particular. The subsequent articles in the series analyze muons in crossed fields using similar transport theory, though the absence of muon swarm data requires empiricism of quite a different nature.

I. INTRODUCTION

In this series of three articles, we outline a straightforward strategy for analyzing the behavior of low energy ($\lesssim$ 20 eV) electron and muon "swarms" in gases in crossed electric and magnetic fields, using simple, approximate "fluid" or "momentum-transfer theory" equations,[1–3] supplemented by semi-empirical ideas where appropriate. The method can be carried through from first principles to furnish numerical estimates of all physical quantities of interest and offers a straightforward, physically transparent alternative to both solution of Boltzmann's kinetic equation,[1–3] which can be both mathematically and numerically challenging, and to "off the shelf" packages, such as Magboltz[4,5] and GEANT4.[6] The user is always in control and can readily tailor the analysis presented below to their specific needs. Note that the procedure does not make use of the "equivalent field concept" described elsewhere.[7,8]

In the first paper (Paper I), we use the formalism to estimate the Lorentz angle of electrons in helium gas in crossed E and B fields, after incorporating E-only swarm experimental data directly into the equations. In the following articles (Papers II[22] and III[23]), we analyze muons in gases in crossed electric and magnetic fields. However, unlike electrons, there is no experimental muon swarm data available to supplement the calculations: the required information must, in effect, be assumed, as indeed others[9,10] have done, on the basis of semi-empirical arguments, called "scaling"[10,11] or "aliasing."[12] Using this approach, we establish in Paper II[22] experimental conditions that optimize transverse compression of muon beams, while Paper III[23] looks the question of aliasing for muons in gases in crossed fields under more general circumstances.

Our philosophy though reflects that of Killingbeck[13] in that "if the main aim is to communicate, the best procedure is to use short arguments and simple mathematics."

II. THEORETICAL FRAMEWORK

A. Momentum transfer theory

Momentum transfer theory (MTT) is basically a "fluid" approach, which has its roots in the "equations of change" of Maxwell[14] in 1867, but incorporates the approximation that the average of a function of energy is represented by the same function of the average energy, which enables it to go beyond Maxwell's constant collision frequency model. It is now textbook material[1–3] and provides a straightforward means of calculating the transport properties of charged particles in gases to an accuracy of a few percent or so, compared with the 1% of better accuracy provided by an accurate solution of Boltzmann's equation. While transport processes for various types of charged particles, including ions, muons, electrons, and positrons, have been analyzed using MTT and other approximation methods in electric and magnetic fields, the approach explored here has an additional semi-empirical dimension, which simplifies the analysis even further. Thus, we first outline the conventional use of the MTT equations, which is to calculate the experimentally measurable properties of a swarm, before reversing this role, and showing how they can be used as a vehicle to incorporate experimental swarm data to make further predictions for use in a different experiment.

B. MTT electrical field only

In the absence of a magnetic field, the drift velocity $\mathbf{v}$ is directed along $\mathbf{E}$,

$$\mathbf{v} = K\,\mathbf{E}, \tag{1}$$

where, according to MTT, the mobility coefficient is defined by

$$K \equiv e/[\mu \nu_m(\varepsilon)]. \tag{2}$$

μ and ε are the reduced mass and mean center-of-mass energy of the charged particle and neutral atom or molecule, respectively,

$$\nu_m(\varepsilon) = n_0\sqrt{\frac{2\varepsilon}{\mu}}\sigma_m(\varepsilon) \tag{3}$$

is the collision frequency for momentum-transfer, $\sigma_m(\varepsilon)$ is the momentum-transfer scattering cross section, and n_0 is the number density of the neutral gas.

It is emphasized that in what follows the mobility coefficient is to be regarded as a function of mean energy, i.e.,

$$K = K(\varepsilon), \tag{4}$$

rather than a function of electric and magnetic field.

The link between drift velocity and mean energy is provided by the generalized Wannier relation[15–17]

$$\varepsilon = \frac{3}{2}kT_0 + \frac{1}{2}m_0 v^2 - \Omega(\varepsilon), \tag{5}$$

where $\Omega(\varepsilon)$ is a term accounting for inelastic processes, whose explicit expression in terms of inelastic cross sections $\sigma_I(\varepsilon)$ can be found elsewhere.[3,16,17] The simultaneous equations (1) and (5) may be solved, if desired,[16,17] to yield v and ε, and hence K and v, as functions of E/n_0, for given cross sections $\sigma_m(\varepsilon)$ and $\sigma_I(\varepsilon)$.

This is the conventional approach which, however, we do not follow in this article. Instead, we adopt a different perspective, in effect semi-empirical, in which v vs E/n_0 data obtained directly from swarm experiments are used in conjunction with the equations of MTT to make further predictions for crossed electric and magnetic fields. Furthermore, in this approach, Eq. (5) is viewed, for present purposes, as simply establishing that a relationship between mean energy and average velocity exists, or equivalently, $\varepsilon = \varepsilon(v)$, but without the need to establish just what that relationship is.

This leads us to the key point in the strategy: for reasons which will be made clear, instead of considering mobility to be a function of mean energy, as in (4), we may choose to consider it as a function of average velocity, that is,

$$K = K(v). \tag{6}$$

To take a simple example, consider electrons undergoing elastic collisions with the atoms of a cold gas, governed by a cross section σ_m which is a constant, independent of energy. Then, if we were proceeding conventionally, we would use Eqs. (1) and (5), with $\Omega = 0$, $T_0 = 0$, to find $v \sim E^{\frac{1}{2}}$, and hence $K \sim E^{-\frac{1}{2}}$, or equivalently,

$$K(v) \sim v^{-1}. \tag{7}$$

However, the procedure described below employs neither Eq. (5) nor does it require input of cross sections, elastic or inelastic, model or real. Rather, it furnishes the required relationship (6) using only swarm experimental data. In addition, it offers a direct path from E-only experimental swarm data to allow analysis of the more general situation where both E- and B-fields are present. Nevertheless, model (7) proves useful in illustrating the method.

C. Crossed electric and magnetic fields

As is well known,[3,9] application of a magnetic field $\mathbf{B}$ at right angles to $\mathbf{E}$ results in the drift velocity $\mathbf{v}$ having two components, along the $\mathbf{E}$ and $\mathbf{E} \times \mathbf{B}$ directions, respectively. These two components may then be expressed in terms of magnitude v of the drift velocity, and the *Lorentz angle* φ between $\mathbf{v}$ and $\mathbf{E}$, as given by the simultaneous solution of the following equations:

$$v = \frac{E}{B}\sin\varphi \tag{8}$$

and

$$\tan\varphi = K\,B. \tag{9}$$

Here, K is the same *function* of ε as for the E-only case, as defined by Eq. (2), but ε corresponds to the more general case where both electric and magnetic fields are present. Moreover, Wannier relation (5) has the *same mathematical form* in this more general case. Similarly, the *functional* relationships, $\varepsilon = \varepsilon(v)$ and $K = K(v)$ derived for the E-only case, also apply the more general case where both E and B are present. However, the actual *value* of v corresponds to the drift velocity in the more general case.

These observations are pivotal in establishing the procedure for evaluating the Lorentz angle using E-only swarm experimental data.

D. Modified equations

Although it is not the normal practice for electrons, we choose to follow the convention for analyzing ion swarms (see Papers II[22] and III[23] of this series) and work with *reduced mobility*

$$\mathcal{K} = \frac{n_0}{n_L}K, \tag{10}$$

rather than K, where $n_L = 2.69 \times 10^{25}\ \mathrm{m}^{-3}$ (Loschmidt's number) is the number density of an ideal gas under standard conditions of temperature and pressure. Like K, the reduced mobility is also to be considered as a function of v, i.e.,

$$\mathcal{K} = \mathcal{K}(v). \tag{11}$$

Then, Eqs. (1) and (9) become, respectively,

$$v = 2.69 \times 10^4\ \mathcal{K}\ (E/n_0)_{Td}\quad \mathrm{V/m} \tag{12}$$

and

$$\tan\varphi = \frac{n_L}{n_0}\mathcal{K}(v)\ B, \tag{13}$$

where $(E/n_0)_{Td}$ is the reduced electric field in units of Townsend ($1\ Td = 10^{-21}\ \mathrm{V\ m}^2$).

With these formulas in place, we can now outline the procedure for calculating Lorentz angle and drift velocity for charged particles in crossed electric and magnetic fields from swarm drift velocity data for electric fields only.

III. CALCULATING LORENTZ ANGLE

The requirement is to solve the two simultaneous equations (8) and (13) for v and φ, for (e, He) for specified electric and magnetic fields E and B, respectively, in this case,

$$E = 10^5\ V/m,\ B = 5\ \text{tesla}, \tag{14}$$

with gas properties (designated by a subscript "0") $T_0 = 12$ K and $p_0 = 8$ mbar. The corresponding gas number density n_0 follows from the ideal gas equation of state

$$\frac{n_0}{n_L} = 0.182. \tag{15}$$

Note that the *reduced electric field* corresponding to (14) and (15) is

$$E/n_0 = 20.4\ \ \mathrm{Td}. \tag{16}$$

Note also that the concept of an *effective electric field* $E_{eff} = E\cos\varphi$ is sometimes considered useful[7,8] in making a connection between an E-only situation and the more general case where both E and B fields are present. The reduced effective electric field corresponding to (16) is

$$(E/n_0)_{eff} = 20.4\ \cos\varphi\ \ \mathrm{Td} \tag{17}$$

but the right-hand side cannot be evaluated since the Lorentz angle φ is unknown. We shall return to this point subsequently after calculating φ in the manner outlined below.

First, we illustrate the solution of the problem for a simple case (effectively, the model discussed in Sec. II B), before tackling the general problem. In both cases, the first step is to find the function $\mathcal{K}(v)$ from E-only swarm data.

A. Simplified calculation

Table I of Crompton *et al.*,[18] which shows measured drift velocities of electrons in helium for E/n_0 up to 3.64 Td, may be used along with Eq. (12) to generate the following dataset shown in Table I.

It is to be emphasized that the value of electric field shown in this table bears no relation to the value of the electric field in the $\mathbf{E} \times \mathbf{B}$ experiment. Inspection of the second and third columns indicates that to a good approximation,

$$v\ \ \mathcal{K} \approx constant = 0.9\ \mathrm{m}^3/\mathrm{V/s}^2, \tag{18}$$

or, in other words,

$$\mathcal{K}(v) \approx \frac{0.9}{v}\ \ \mathrm{m}^2/\mathrm{V/s}, \tag{19}$$

as in the simple model case discussed briefly in Sec. II B.

For the parameters given by Eqs. (14) and (15), Eqs. (13) and (8) become

$$\tan\varphi = 27.5\ \ \mathcal{K}(v) \tag{20}$$

and

$$v = 2 \times 10^4\ \ \sin\varphi, \tag{21}$$

respectively. Substituting for $\mathcal{K}(v)$ from (19) in Eq. (20), and then eliminating v using Eq. (21), gives

$$\tan\varphi = \frac{1.24}{\sin\varphi}, \tag{22}$$

the solution of which is

$$\varphi \approx 56^\circ. \tag{23}$$

TABLE I. The drift velocity of electrons in helium taken from experimental data[18] (first two columns) is converted to reduced mobility $\mathcal{K}$ (third column). The fourth column shows the product $v \cdot \mathcal{K}$ indicating that to a good approximation $v \cdot \mathcal{K} \approx$ *constant*.

E/n_0 (Td)	v (10^3 m s^{-1})	$\mathcal{K}$ (m^{2}V^{-1}s^{-1}) Equation (12)	$v \cdot \mathcal{K}$ (m^{3}V^{-1}s^{-2})
0.212	2.21	0.387	0.86
0.273	2.52	0.344	0.87
0.364	2.93	0.300	0.88
0.455	3.28	0.268	0.88
0.607	3.78	0.232	0.88
0.759	4.23	0.207	0.88
0.910	4.63	0.189	0.88
1.214	5.33	0.163	0.87
1.517	5.97	0.147	0.87
1.820	6.55	0.134	0.88
2.124	7.07	0.124	0.88
2.43	7.75	0.116	0.88
2.73	8.07	0.110	0.89
3.03	8.57	0.105	0.90
3.64	9.47	0.097	0.92

TABLE II. The drift velocity of electrons in helium for higher electric fields than Table I, taken from experimental data[19–21] (first two columns), is converted to reduced mobility $\mathcal{K}$ (3 column), generating Lorentz angle data φ (columns 4 and 5), and estimates v of drift speed (6 column) for the crossed field experiment under consideration.

E/n_0 (Td)	v (10^3 m s^{-1})	$\mathcal{K}$ (m^{2}s^{-1}V^{-1}) Equation (12)	$\tan\varphi$ Equations (13) or (20)	φ (deg.)	v (10^3 m s^{-1}) Equations (8) or (21)
4.0	10.0	0.093	2.59	68.9	18.7
5.0	11.5	0.086	2.39	67.3	18.5
6.0	13.1	0.081	2.27	66.2	18.3
7.0	15.0	0.080	2.22	65.8	18.2
8.0	17.5	0.081	2.27	66.2	18.3
10.0	22.0	0.082	2.28	66.3	18.3
15.0	32.6	0.081	2.25	66.1	18.3
20.0	49.2	0.092	2.55	68.6	18.6
25.0	68.1	0.101	2.82	70.5	18.9

Substituting this back into Eq. (21) gives

$$v \approx 16.5 \times 10^3 \ \text{m/s}. \tag{24}$$

It can be seen that the value of v of Eq. (24) lies well above the actual data shown in Table I, on which model Eq. (18) is based. That is, the drift velocity data of Table I have been effectively extrapolated to obtain (23) and (24). Not surprisingly then, these results have a rather large discrepancy (about 20%) as compared with an analysis based upon (i.e. unextrapolated) swarm data, which furnishes a more accurate representation of $\mathcal{K}(v)$.

B. General procedure

(a) First, we employ v vs $(E/n_0)_{Td}$ swarm data based upon Refs. 18–21 and Eq. (12) to create a table of K vs v as before. This is effectively the function $\mathcal{K}(v)$. Using the highest possible values of v obviates the need for extrapolation.

(b) Second, use Eq. (13), along with the value of B for the experiment under investigation, to make a table of the values of $\tan\varphi$ and φ, corresponding to $v(\mathcal{K})$.

(c) Next, use Eq. (8) to make a table of drift velocities v corresponding to various values of φ, using the experimental values of E and B as given by Eq. (14). The data thus generated for (e, He) are shown in Table II.

The solution of the simultaneous equations (13) and (8), or in our specific case Eqs. (20) and (21), respectively, can be found by plotting v for each of columns two and six, respectively, vs ϕ and finding the point of intersection, as shown below.

As shown in Fig. 1 and in the enlarged view in Fig. 2, the two curves intersect at the point

$$v = 18.3 \times 10^3 \ \text{m/s}, \ \varphi = 66.3^\circ, \tag{25}$$

which correspond to the drift velocity and Lorentz angle for electrons in helium in the crossed fields specified by (14). Note that the first three columns of Table II remain the same in all cases, and it is only columns 4–6 that change when different values of E and B are required.

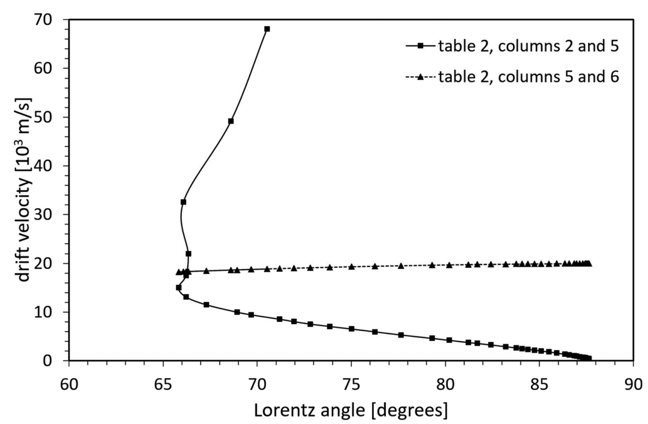

FIG. 1. Intersection of the two curves v vs φ according to Table II, columns two and five, and columns five and six, respectively.

A separate calculation using Magboltz(v7.1)[5] gives $v = 18.3 \times 10^3$ m^2/s and $\varphi = 66.0^\circ$. Our present results (25) agree with these values to within the 1% accuracy cited for Magboltz.[4]

Having found the value of Lorentz angle, we may now return to Eq. (17) and calculate the reduced effective electric field as

$$(E/n_0)_{eff} = 8.3 \ \text{Td}, \tag{26}$$

which corresponds (approximately) to the fifth data point in Table II. This establishes the consistency between the method outlined above and the role of the equivalent field as establishing a conceptual bridge between E-only case and the more general situation where both E and B are present. Its usefulness as a more practical tool will be discussed in Paper II[22] in connection with muons in gases.

Finally, we note that while the value of the model discussed in Sec. III A lies mainly in the fact that it is amenable to an analytical solution, it may also be used as a benchmark for the above general

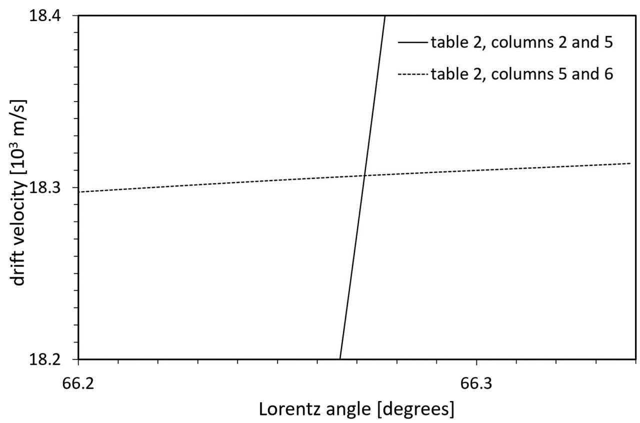

FIG. 2. Enlarged image detail of the intersection point of the two curves v vs φ of Table II, columns 2 and 5, and columns 5 and 6, respectively.

procedure. Thus, by expanding Table I to the form of Table II, using appropriately extrapolated drift velocity data, curves similar to those shown in Fig. 1 can be generated. The values of v and φ obtained from their intersection agree well with the analytic values shown in Eqs. (23) and (24), respectively. While the consistency of the two approaches for the model is thereby established, the model results themselves have a discrepancy of around 20% as compared with Eq. (25), which have been derived on the basis of unextrapolated data.

IV. CONCLUSION

In this article, we have demonstrated how swarm data for electrons in helium from E only experiments can be processed with the aid of MTT (momentum transfer theory) to obtain drift speed and Lorentz angle crossed E and B fields. The method is characterized by both its simplicity and generality and has been benchmarked in two ways: (a) for internal consistency using a model and (b) for numerical accuracy against an independent calculation using Magboltz. The calculations presented here are carried out rigorously from first principles using MTT and do not rely on the "equivalent field concept" or any other assumption. In Paper II, the procedure is modified somewhat and used to provide parameters that optimize compression of a muon beam in helium in crossed fields. The need for swarm data and analysis covers a number of fields, ranging from plasma technology to beam physics. Theory underpins the success of these applications, and the new procedure presented here offers potential users a more utilitarian alternative to more conventional approaches, such as simulation and solution of Boltzmann's equation.

AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

Author Contributions

Malte Hildebrandt: Conceptualization (equal); Data curation (equal); Formal analysis (supporting); Investigation (equal); Methodology (equal); Visualization (lead); Writing – original draft (equal); Writing – review & editing (lead). **Robert E. Robson**: Conceptualization (lead); Data curation (equal); Formal analysis (lead); Investigation (lead); Methodology (equal); Visualization (equal); Writing – original draft (lead); Writing – review & editing (equal).

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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