International Journal of Mathematical Education in Science and Technology

ISSN: (Print) (Online) Journal homepage: www.tandfonline.com/journals/tmes20

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To cite this article: G. R. Fulford (2024) Mathematical modelling using scenarios, case studies and projects in early undergraduate classes, International Journal of Mathematical Education in Science and Technology, 55:2, 468-479, DOI: 10.1080/0020739X.2023.2244490

To link to this article: https://doi.org/10.1080/0020739X.2023.2244490

Published online: 12 Oct 2023.

INTERNATIONAL JOURNAL OF MATHEMATICAL EDUCATION IN SCIENCE AND TECHNOLOGY
2024, VOL. 55, NO. 2, 468–479
https://doi.org/10.1080/0020739X.2023.2244490

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Mathematical modelling using scenarios, case studies and projects in early undergraduate classes

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ABSTRACT

Mathematical modelling has great potential to motivate students towards studying mathematics. This article discusses several different approaches to integrating research work with a second-year undergraduate, mathematical modelling subject. I found sourcing papers from the areas of epidemiology and ecology to be a fruitful source area, particularly models involving only two or three coupled differential equations. These models were amenable to students as well as interesting and relevant to students because they came from real research papers. I will describe the use of scenarios and case studies in lectures, and group projects for assessment. The scenarios and case studies were published in a textbook that I wrote. Scenarios, case studies and projects provided an opportunity to expose students to some novel applications of differential equations. One example is developed here as a *Classroom Note*: modelling the dynastic cycles in Chinese history.

ARTICLE HISTORY
Received 15 March 2023

KEYWORDS
Mathematical modelling; dynastic cycles; projects; case studies

1. Introduction

This article describes the use of scenarios, case studies and projects in a mathematical modelling subject at Queensland University of Technology. Each of these played a role in increasing student interest by demonstrating to them how mathematics is used in real-world situations.

I first taught a subject called mathematical modelling in 2004 at Queensland University of Technology. The subject was offered to students in second year, but sometimes some students took it in first year and sometimes third-year students would undertake this subject as an elective. The subject assumed a first-year knowledge of differential equations, such as separating variables, first-order linear and second-order constant coefficient differential equations. The main focus was on mathematical modelling; how to formulate models and how to interpret solutions back in terms of the application area. An emphasis was given to models that involved coupled differential equations. But, later in the subject, an introduction to partial differential equations was given.

A feature of the subject was that models were explored numerically before analytically. The idea was that features of the model would be identified before being generalised with

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Table 1. A sample of four scenarios used in Barnes and Fulford (2015). Given is the title of the scenario, the area of modelling it comes from, and its main reference.

Title	Area	Reference
Pacific rats colonise New Zealand	Exponential decay	Anderson (1996)
Nile Perch catastrophe	Predator–prey	Quammon (1997)
Protection of lerps and nymphs	Predator–prey	Loyn et al. (1983)
Fish and chips explode	Spontaneous ignition	Smedley and Wake (1987)

analytic work. The sort of analytic work undertaken for coupled ordinary differential equations (ODE) models was finding equilibrium solutions, analysing the stability of these equilibrium solutions and phase-plane analysis.

Another feature of the subject was the use of scenarios, case studies and group projects. These were introduced formally in 2009. Scenarios were links to phenomena that were consistent with the models. Case studies were self-contained works based on research papers and these were condensed into a 1-hour lesson to explain to students. Group projects were for assessment; these were also based on research papers that took up 2 weeks of class time. In the following, I discuss each of these in detail and I give one example of a case study in the form of a classroom note. A textbook, Barnes and Fulford (2015), was published that was based on this subject and its predecessors. The textbook contained the scenarios and case studies from the subject but not the projects.

2. Scenarios

Scenarios, as defined here, are short examples of how some of the phenomena uncovered in models apply to real-world situations, but do not have any mathematics in them. They are short excerpts from popular articles that can form a small part of a lecture. The purpose of scenarios is to connect a model's behaviour with some real-world event. However, the scenarios do not describe actual models. Scenarios are useful if the instructor does not want to devote a whole lecture to a case study but still wants to show a connection of the model behaviour with something interesting in the real world. A selection of four scenarios that were published in Barnes and Fulford (2015) is shown in Table 1.

For example, the scenario on 'Protection of lerps and nymphs' referred to a model of competing species where the outcome of the model was that one of the two species always tended to extinction. The scenario discussed the complexity of ecological relationships is an example of the value of interspecies territoriality.

3. Case studies

Case studies are more detailed models that supplement basic more generic models. However, they usually have specific questions that they attempt to answer and mostly use realistic parameter values. They are often modifications of a more basic model or a real application of a basic model. I found that ecological journals and medical/infectious disease journals provided good sources of case studies for coupled ODE models.

Table 2 is a selection of some of the case studies described in Barnes and Fulford (2015). Most of the case studies in Barnes and Fulford (2015) were based on research papers, but a simplified and summarised version of the model was presented. It needed a whole lesson

Table 2. A selection of case studies from Barnes and Fulford (2015). Given is the title of the case study, the area of modelling it comes from and main reference.

Title	Area	Reference
Single DE population models		
It's a dog's life	Logistic growth	Amaku et al. (2010)
Coupled DE population models		
Rise and fall of civilisations	Predator–prey	Feichtinger et al. (1996)
Bacteria battle in the gut	Competition	Barnes et al. (2007)
Possums threaten New Zealand cows	disease model	Roberts (1992)
Heat and mass transport models		
It's hot and stuffy in the attic	Newton cooling	Sansgiry and Edwards (1996)
Double glazing	Heat conduction	Meterton-Gibbons (1989)
Tumour growth	Diffusion model	Greenspan (1972)

to explain and usually comprised about three or four pages in Barnes and Fulford (2015). The three-page version was written to make it easier for students to understand rather than the original source. The case study always followed a more basic version of the model, for example, the case study 'It's a dog's life' followed the section of the textbook on logistic growth with harvesting and talked about modelling of control of wild dog populations.

The case study on rise and fall of civilisations referred to the periodic dynastic cycles in China's history. It is a novel application of the predator–prey model. In the subject, this case study was taught as a computer lab where students implemented the model in MATLAB. In the following, this case study is represented as a classroom note, with some different variants of the model for students to analyse, compared to what was published in the textbook.

3.1. *Example case study: dynastic cycles in China*

Throughout history populations have continued to grow, but they sometimes go through cycles where the populations fall and then increase again. This phenomena is well known in animal populations where the cycles are driven by predation of one species on another. In human history, China was well known for these cycles.

Some of these cycles are shown in Table 3. Most of the cycles lasted for around 160–250 years with the exception of Eastern Han, Sui and Tang, which were shorter. Combined, the Eastern Han, Sui and Tang dynasties lasted for 240 years, suggesting the factors that cause the growth and decline of the dynasties then may have been due to other factors. The cycles are generally characterised by a long period of growth followed by a relatively short period of decline, where there was warfare and banditry.

Historians speculate on the causes of these cycles. Mathematical modelling can also contribute to the debate. Feichtinger et al. (1996) proposed an explanation for the dynastic cycles in China, where there were only three groups of people, (F)armers, the (R)ulers (i.e. local nobility and their soldiers) and (B)andits. The bandits prey on the farmers while the rulers are predators to both the farmers (through recruitment of soldiers and taxes) and bandits. During times of population decrease, the solders cannot keep the bandit population in check, and this decreases the farmer population. A decrease in the farmer population leads to starvation which then decreases the ruler population and the bandit population. The model is essentially a predator–prey model with prey (farmers), and their predators (bandits) and rulers who are also super-predators since they prey on bandits.

Table 3. Table showing the rise an and fall of dynasties in China. Data obtained from Turchin and Nefedov (2009), Table 10.3.

Dynasty	Period	Duration (growth)	Duration (decline)
Western Han	200CE–40	210	30
Eastern Han	40–220	40	40
Sui	550–630	60	20
Tang	630–770	60	20
Northern Sung	960–1160	120	40
Yuan	1250–1410	100	60
Ming	1410–1650	210	30
Qing	1650–1880	200	30

The simplest predator-prey model is the Lotka–Volterra predator–prey model (1)

$$X' = rX - cXY, \quad Y' = ecXY - mY, \tag{1}$$

where X is the prey population abundance and Y is the predator abundance. Here r is a positive per-capita growth rate of prey, m is a positive per-capita net death rate of predators (in the absence of any prey the predators starve). The term cY is the per-capita death rate due to prey killed by predators, with the positive constant c as a proportionality constant. With this assumption, the higher the number of predators present, the higher the rate of prey killed. The positive constant e represents an efficiency coefficient of turning each single prey into the births of predators. The basic Lotka–Volterra model is not so widely used in ecology since it has several undesirable features. One is that it is neutrally stable, where small perturbations lead to cycles with different amplitude. To overcome this, a saturating term is used for predation terms,

$$cXY \to \frac{cX}{b+X}Y.$$

This modification accounts for the predator being limited in their acquisition of prey, i.e. when they have eaten their fill, they no longer hunt prey. So cX would be the rate of prey killed if there was no saturation, but for large numbers of prey, X, the rate eaten by each predator is a constant rate. This is known as a Holling type II response in ecology. It can be written in a different form that interprets the parameters as attack rates and prey handling time, see Barnes and Fulford (2015) or Begon et al. (2006) for more details. Another modification is to use logistic growth for the prey growth, $rX \to rX(1 - X/K)$ where K is the carrying capacity of the whole population. This results in a predator–prey model where small changes in initial conditions tend back to the same cycle.

Feichtinger et al. (1996) proposed a system of three nonlinear differential equations to model the dynastic cycles in China. Reproducing their model, I let $F(t)$ denote the number of farmers, $R(t)$ the number of the ruling class (rulers and soldiers) and $B(t)$ the number of bandits.

$$F' = rF\left(1 - \frac{F}{K}\right) - a\frac{FB}{b+F} - hFR,$$
$$B' = -mB + ea\frac{FB}{b+F} - c\frac{BR}{d+B},$$

$$R' = -gR + fa\frac{FB}{b+F}. \tag{2}$$

Here there are several parameters r, K, a, e, h, b, d, m, c, f and g all of which are positive constants for the model to make sense.

The predation term of bandits on farmers is $aFB/(b+F)$ in place of aFB from the Lotka–Volterra model, so the model has a saturating predation rate. Similarly there is a saturating predation rate for the predation of the ruling class on bandits. The predation of the ruling class on the farmers hFR is not saturated. Although not explained in the original paper (Feichtinger et al., 1996), presumably this is because the ruling class will continue to tax farmers and recruit soldiers at the same per-capita rate, no matter how large the ruling class is, so no saturating effect is needed.

Barnes and Fulford (2015) used the following parameter values:

$$r = 1.0, \quad K = 1.0, \quad m = 0.4, \quad g = 0.009, \tag{3}$$

with parameters for the predation coefficients,

$$a = 1, \quad h = 0.1, \quad c = 0.4, \quad e = 1.2, \quad f = 0.1, \tag{4}$$

and parameters for the search times in the predation terms,

$$b = 0.17, \quad d = 0.42. \tag{5}$$

Feichtinger et al. (1996) carried out a full scaling of the equations, but then didn't discuss where they obtained their non-dimensional parameter values.

In Barnes and Fulford (2015), the graph of the numerical solution mistakenly has years as the horizontal axis whereas the time variable had been scaled $t \to rt$. The parameter values used by Barnes and Fulford (2015) were chosen arbitrarily but consistent with the dimensionless parameters used by Feichtinger et al. (1996). Instead of scaling the time it is more transparent for students to use units of years. A suitable, realistic value of r is $r = 0.02$. This would correspond to an average life expectancy of $1/r = 50$ years. With this change, the numerical solution is shown in Figure 1. In this figure, the plots show each of the populations as a fraction of the farmer carrying capacity K. This was done since a suitable value of K was not available.

With these parameter values, population cycles occur roughly every 200 years. Furthermore, the farmer population grows steadily over about 200 years while the bandit population stays small. But it grows slowly until it undergoes explosive growth as the farmer population declines dramatically. The ruler numbers also decrease gradually while the farmer population is growing, but when the bandit population starts to grow the soldier population increases, which controls the bandit population. This allows a new cycle to begin. The periods of rapid decline of the farmer population and rapid growth of bandits and soldiers in Figure 1 was about 30–40 years. This was consistent with the short length of the periods of instability and decline in Table 3.

Is this what happens in real historical populations? Reality is no doubt more complex, but the model shows that a simple explanation of three interacting population groups might be a plausible theoretical explanation for the main drivers of the dynastic cycles.

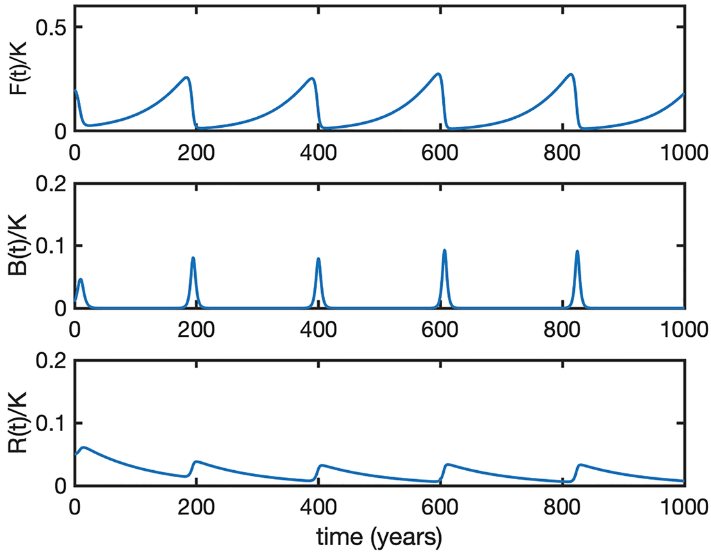

Figure 1. Numerical solution of the full dynastic cycle model (2), using parameter values given in model (2), using parameters as defined in (3), (4) and (5), but with r changed from $r = 1$ to $r = 0.02$ to give a realistic life expectancy.

Some student investigations could be made by looking at some simplified versions of the model. A variant of the simplified model is to retain the logistic growth term for the farmers but not include predation saturation. This means the farmers' growth rate would be suppressed as their population grows near what the land can support. So the growth rate reduces due to lack of food. The modified version of the model is

$$\begin{aligned} F' &= rF(1 - F/K) - aFB - hFR, \\ B' &= -mB + eaFB - cBR, \\ R' &= -gR + faFB. \end{aligned} \tag{6}$$

The result of simulating this model is shown in Figure 2 with the same parameter values used in Figure 1.

For the model with logistic growth of farmers, but no predation saturation, shown in Figure 2, there is now a decreasing amplitude of oscillation in all three populations. This shows us that the farmer population is the main driver of the oscillations in the bandit and ruler populations.

As a further exercise, students could investigate a further simplified version of the model corresponding to no predation saturation and without logistic growth of farmers, as shown by the following coupled system:

$$\begin{aligned} F' &= rF - aFB - hFR \\ B' &= -mB + eaFB - cBR, \\ R' &= -gR + faFB. \end{aligned} \tag{7}$$

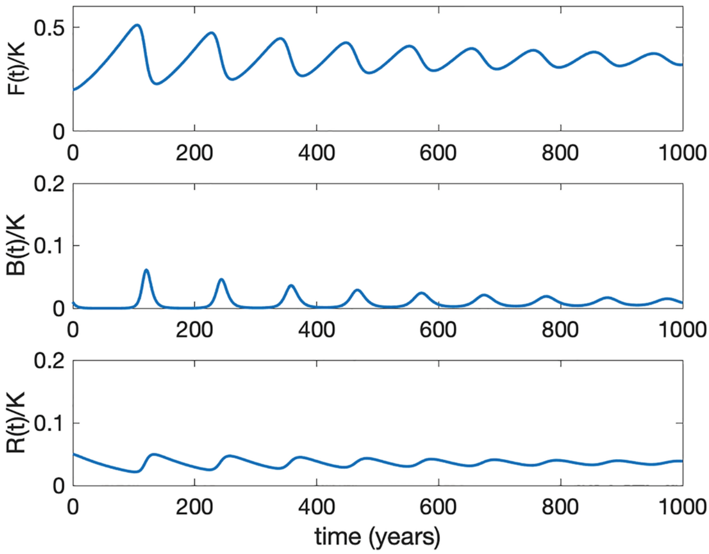

Figure 2. Numerical solution of the reduced dynastic cycle model (6), with no predation saturation but includes logistic growth of farmers. This uses the same parameter values given in Equations (3) and (4), with r changed to $r = 0.02$.

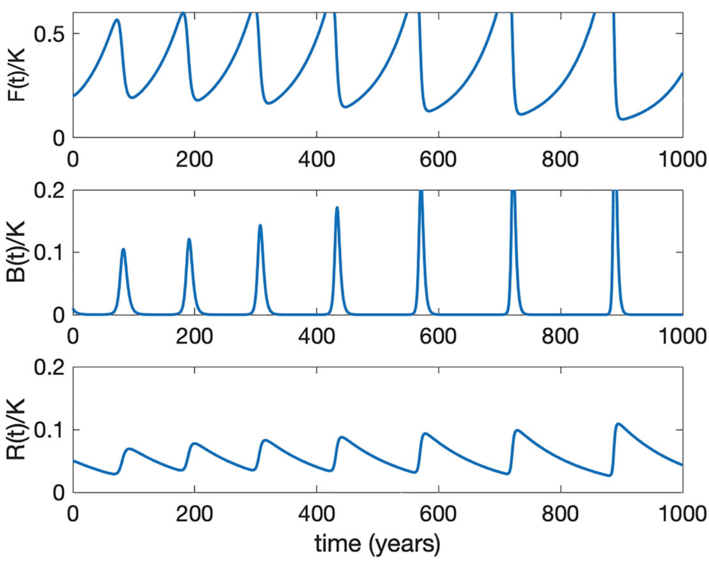

Figure 3. Numerical solution of the reduced dynastic cycle model (7), with no predation saturation terms and no logistic growth of farmers. Parameter values given in Equations (3) and (4), with r changed to $r = 0.02$.

The numerical simulation of this simplified model is given in Figure 3 with the same parameter values used in Figure 1.

In this simplified model, it is clear now that r is the constant per-capita growth rate of farmers, and m, and g are the per-capita death rates of bandits and soldiers in the absence of farmers. Then aFB is the predation term of bandits on farmers. This term also appears in the bandit differential equation for bandits as a growth term, so it is the rate of increase of bandits due to the killing farmers (and stealing their food). Similarly, the cBR term represents predation of soldiers on bandits. There is a positive term in the soldiers equation corresponding to a positive growth rate due to predation of soldiers on bandits.

In Figure 3, the cycles are now growing in amplitude and the period is still only around 100 years. The model with both logistic growth of farmers and predation saturation does seem to be necessary go get stable cycles with a period that is similar to many of the dynasties found in China's history.

It is unlikely to find historical data on the population sizes for the three populations to fit the parameter values, but one possibility could be to use least squares with the data in Turchin (2003, Figures 8.3, 8.6). However, given the number of parameters to fit this might be difficult to do. This data also shows that irregular cycles occurred historically. Chaotic solutions might be possible since there are three coupled nonlinear differential equations. Piccardi and Feichtinger (2002) investigate this and demonstrate chaotic behaviour exists. Searching parameter space for further chaotic solutions might make a suitable investigation for students as an extension of this paper.

These two variations make useful exercises for students since they give them opportunities to compare different models and develop richer interpretations, even without data. Other possibilities for exercises include varying an individual parameter value and observing and commenting on the effects. There are also more routine exercises such as deriving expressions for equilibrium population and examining their stability.

4. Group projects

Projects were used as a form of assessment. Students worked in small groups (usually three, sometimes four students). In these, students could choose from a list of research papers, and then they needed to provide a report on it. The report could involve:

- reproducing some of the results in the paper,
- exploring some different aspects of the model (e.g. producing different graphs),
- making some simple extension of the model.

Generally, the projects were more work than a single student could accomplish, so groups were used. Each group chose a different project. An important aspect of the group projects was that 2 weeks of class time was set aside (in the middle of the semester) devoted exclusively to the projects. Group meetings were held during normal lesson time. Students were required to keep minutes which detailed the actions of each member until the next meeting. It was also an opportunity for me to circulate amongst the groups and give advice.

Groups of three generally worked best. In one year groups of four were used because there were over 60 students enrolled in the subject. Groups of four (or five) made it easier for some students to not contribute, and this resulted in more friction among group members.

After 2 weeks, the groups worked on a report of about 15 pages detailing their investigations. In the final 2 years that I taught the subject, groups also submitted either a one-page summary of their report or a powerpoint slide for a possible seminar. There wasn't time

Table 4. A sample of group projects together with the references used. Also, the area of the subject matter related to the project is indicated.

Title	Area	Reference
Chimpanzee wars	Battle models	Wilson et al. (2002)
Cholera	Disease models	Codeco (2001)
Unemployment	General compartment	Misra and Singh (2011)
Beetle populations	Discrete populations	Constantino et al. (1998)
Hospital infections	Disease models	Cooper et al. (1999)
Neanderthal extinction	Competition models	Flores (1998)
Chlamydia transmission	Disease models	Regan et al. (2008)
Calicivirus in rabbits	Disease models	Barlow and Keen (1998)
War in Iraq	Battle models	Blank et al. (2008)

for them to give seminars, so the summary or slides were circulated to the other students outside the group so they could see what their peers had achieved.

The main advantages of doing group projects were student obtained (i) a deeper appreciation of what a published research paper was, (ii) a sense of achievement in putting together a professional looking report, (iii) a chance to examine the assumptions of a mathematical model in detail, (iv) a chance to look up further references and (v) sometimes to extend the model in the paper. The disadvantages were (i) the time taken out of the usual lectures, (ii) sometime difficulties in group dynamics not working and (iii) difficulties in being sure that the difficulty levels in different papers were equivalent.

Table 4 gives a list of some of the projects used in the subject. Most of the projects related to the first half of the subject (population models) since the 2-week period devoted to project work was usually before or just after the second half of the subject lectures had started.

Some of the projects were more successful than others – the reasons are explored below. In 1 year, the number of students doing the subject was large so it was allowed for more than one group to work on the same paper. An important aspect of this was to provide guidance to each group about tackling different aspects of the paper. In fact, the guidance given by the lecturer and tutor was critical for all the projects in helping students to be able to focus on a single achievable aspect of the paper.

Of the projects listed in Table 4, the least successful project was probably the one on Chlamydia transmission. Although students did a good job with it, they needed more guidance than other projects. What made this project difficult for them was that the original paper had ten coupled differential equations and about 20 coefficients. In fact the paper also considered an age-structured model which multiplied the number of differential equations and coefficients by five age groups.

Although solving it numerically (for one age group) was not conceptually more difficult than dealing with two or three differential equations, it took longer to implement and it was much more difficult for students to interpret results. I tried suggesting that a suitable activity for this project was to produce a simplified model with fewer differential equations (I helped them simplify the model to seven coupled ODEs) , but this proved difficult for them to do on their own since it was harder for them to understand the original model. So, for students of second-year level, experience has taught me that limiting the projects to papers with only two or three differential equations (or at most four) is best.

In the project on chimpanzee wars, the model used by Wilson et al. (2002) was the classic Lanchester model for battles.

$$X' = -\alpha Y, \quad Y' = -\beta X$$

Here a and b are positive constants which give the relative fighting strength of individuals in each group.

From the differential equations a phase-plane solution can be easily derived (i.e. by dividing the two equations and eliminating the time variable)

$$\beta(x_0^2 - X^2) = \alpha(y_0^2 - Y^2)$$

This is the classical Lanchester square law. The square law demonstrates the advantage that a fighting force has over their opponents due to numerically superior numbers, modified by the average fighting strength of individuals (implicit in the constants α and β).

Three groups did this project. The groups were guided in different directions, one group followed the paper closely and explained how the Lanchester equation can be varied experimentally. Another group derived the analytical solutions for the number of chimpanzees as functions of time. A third group, who had prior programming experience, developed a stochastic model.

One of the more successful projects was the one on Cholera. One of the reasons was that the group was quite good, and group participants had a variety of skills that allowed them to work effectively on different aspects of the project. The original paper, Codeco (2001), presented models that were still three coupled differential equations, yet modifications of the basic SIR models presented in the subject. The student group managed to look at modifying one of the models to include vaccination since the group found from their research that a vaccine had been developed. This student project eventually became a case study in the latest edition of Barnes and Fulford (2015).

One opportunity that might have been missed in the use of papers as projects was to get students to write their projects as shorter case studies, similar to the textbook, instead of the 15 page report.

Students were surveyed each year the subject ran. Students were generally positive about the subject, particularly commenting on their perceptions of the practicality of the subject. As expected, some students commented they found the subject easy and some found it difficult, which was reflective of the diversity of students (1st year to 3rd year). Comments were mixed about the project; some felt that 20% was too small a percentage for the work they put into it, however, given that it was 2 weeks taken out of a 13 week semester, I thought that 20% was appropriate.

5. Conclusion

All three of scenarios, case studies and projects contributed to students being able to make better connections for differential equations with real-world studies. Scenarios are quick to do, but provide the least strong connections. Projects take some time for students to do and have other benefits such as exposing students to published research papers. Case studies are probably the optimal in terms of the cost benefit of degree of connection versus time spent on topic.

There are good textbooks that focus on teaching differential equations and using mathematical models as motivating examples, e.g. Bryan (2021). However, there is also a benefit in having textbooks that focus on the mathematical modelling skills where the subject matter is organised around grouping similar models together rather than mathematical techniques for solving differential equations. This develops students' model formulation and interpretation skills since they see the similarities among models.

Scenarios, case studies and projects all contribute to students connecting the mathematics with real-world phenomena. I found sourcing papers from the areas of epidemiology and ecology a fruitful area where models involving only two or three coupled differential equations were amenable, interesting and relevant to students because they came from real research papers.

Disclosure statement

No potential conflict of interest was reported by the author.

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