An Investigation into supporting the teaching of calculus-based senior mathematics in Queensland

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2 Statement on the Contribution of Others

This thesis is composed of my original work, and to the best of my knowledge does not contain any material previously published by any other person outside those acknowledged in the text. The contents of this thesis only include the work I commenced after I was accepted as a candidate of this higher degree and is not part of any award or another program at any other institution. I undertook the research described and presented in the thesis under the supervision of Doctors Philemon Chigeza and Subhashni Taylor who provided academic guidance, professional mentoring, assisted in data analysis and reviewed every stage of the research.

This thesis is by publication meaning a version of most chapters have been published in peer reviewed journals or are under review for publication. The table below provides information on the contribution of the author and advisory panel to articles under review or already published.

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Chapter Number	Title of publication	Nature of contribution	Extend of contribution in %	Publication status and Journal
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		ideas and editorial review		
7	Teachers' perceptions on how content sequencing enhances planning, teaching and learning of senior secondary mathematics.	David developed original idea, completed literature review, data analysis and authored the article. Philemon & Subhashni data analysis, checked coherence of ideas and editorial review	David 80% Philemon 10% Subhashni 10%	Published Eurasia Journal of Mathematics, Science and Technology Education.
8	Concept maps as a resource to enhance teaching and learning of mathematics at senior secondary level.	David developed original idea, completed literature review, data analysis and authored the article. Philemon & Subhashni data analysis, checked coherence of ideas and editorial review	David 80% Philemon 10% Subhashni 10%	Published International journal of innovation in science and mathematics education (IJISME)
9	Procedural flowcharts can enhance Senior Secondary Mathematics.	David developed original idea, completed literature review, data analysis and authored the article. Philemon & Subhashni data analysis, checked	David 80% Philemon 10% Subhashni 10%	Published Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3-7, 2022

		coherence of ideas and editorial review		
10	Procedural flowcharts can enhance mathematics students' problem-solving skills.	David developed original idea, completed literature review, data analysis and authored the article. Philemon & Subhashni checked coherence of ideas and editorial review	David 80% Philemon 10% Subhashni 10%	(Under review). Mathematics Education Research Journal (MERJ)

3 Declaration of Ethics

The study was conducted with the ethics approval from Department of Education, Queensland: Reference number: 550/27/2383 and James Cook University Human Research Ethics Committee: Approval number: H8201.

4 Publications during the course of the study

4.1 Peer reviewed Journal articles

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Chinofunga, M.D., Chigeza, P., Taylor, S. (2022). Procedural Flowcharts can Enhance Senior Secondary Mathematics. Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3-7, pp 130-137. https://merga.net.au/common/Uploaded%20files/Annual%20Conference%20Proceedings/2022%20Annual%20Conference%20Proceedings/Research%20Papers/Chinofunga%20RP%20MERGA44%202022.pdf

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2023). Teachers' perceptions on how content sequencing enhance planning, teaching and learning of senior secondary mathematics. *Eurasia Journal of Mathematics, Science and Technology Education. https://doi.org/10.29333/ejmste/13108*

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4.2 Under Review

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2023). Procedural flowcharts can enhance mathematics students' problem-solving skills. *Mathematics Education Research Journal (MERJ).*

4.3 Short communications

Chinofunga, M.D., Chigeza, P., Taylor, S. (2022). Concept Maps as a Resource for Teaching and Learning of Mathematics. Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3-7), p 584. https://merga.net.au/common/Uploaded%20files/Annual%20Conference%20Proceedings/2022%20Annual%20Conference%20Proceedings/Short%20Comms/Chinofunga%20SC%20MERGA44%202022.pdf

4.4 Conference presentations

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2020) Senior high school Mathematics subjects in Queensland: Are options largely pre-determined. Hosted by: Centre of Educational Research – University of Liverpool; United Kingdom - 08 July 2020.

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2021). Resources to enhance effective mathematics teaching and learning. Annual Regional Queensland Association of Mathematics Teachers; QAMT hosted by St Mary's College; Cairns - 28 May 2021

Chinofunga, M. D., Chigeza, P., & Taylor, S. (2021). Uptake of calculus-based mathematics in the new Queensland curricula. Australia Association for Researchers in Education (AARE), Melbourne – 29 November-03 December 2021.

4.5 Media Articles

Chinofunga David. (January 16, 2022). Hard numbers: calculus study declines among students who could benefit. Campusmorningmail. https://campusmorningmail.com.au/news/hard-numbers-calculus-study-declines-among-students-who-could-benefit/

Chinofunga David. (April 27, 2022). Lessons from Ukraine on the importance of planning – opinion. Education Review. https://www.educationreview.com.au/2022/04/lessons-from-ukraine-on-the-importance-of-planning-opinion/

Chinofunga David. (August, 14, 2022). Maths learning: plan to build on what students know. Campusmorningmail. https://campusmorningmail.com.au/news/maths-learning-plan-to-build-on-what-students-know/

5 Abstract

Mathematics is central to science, technology, engineering and mathematics (STEM), which is a core agenda for many governments (Siemon, 2021). In Australia, calculus-based mathematics subjects are a prerequisite for STEM university courses. In Queensland, better and more diverse career opportunities are available to high school students who graduate with Mathematical Methods or Specialist Mathematics. However, the decline in student participation and the high dropout rate in calculus-based mathematics options in Australia (and consequently Queensland) has been a cause for concern for researchers and policy makers. The Australian Mathematical Sciences Institute [AMSI] (2022) report called for urgent action to address the declining trends. This doctoral study investigated trends in student enrolment in calculus-based mathematics in Queensland and mathematics teachers' perceptions on planning and teaching resources that can support student participation.

This study was conceptualised within a constructivist epistemology and has four phases. In Phase One, the study used quantitative data from the Queensland Curriculum and Assessment Authority (QCAA) to investigate the trends in enrolment of Year 11 and Year 12 students in calculus-based mathematics. The study went on to investigate (1) the effects of socio-economic indices for areas (SEIFA) from the Australian Bureau of Statistics (ABS); (2) the schools' index of community socio-educational advantage (ICSEA) values from the Australian Curriculum, Assessment and Reporting Authority (ACARA), and (3) the effect of schools' transfer ratings from the Queensland Department of Education (DoE) on student enrolment in calculus-based mathematics. In Phase Two, the study investigated and developed a planning framework and associated pedagogical resources (procedural flowcharts and concept maps) to sequence content from the Australian Mathematics Curriculum (Years 7 to 10) to the Senior Queensland Mathematical Curriculum (Years 11 to 12) [QCAA, 2018], using the Mathematical Methods Unit 1 on Functions that is taught in Year 11. In Phase Three, the study investigated teachers' perceptions on the effectiveness of the planning framework and associated pedagogical resources in mathematics teaching. Sixteen purposefully sampled senior high school mathematics teachers participated in workshops and completed surveys that included Likert scaled and open-ended questions. Eight of these teachers were available for in-depth interviews. The quantitative data from the Likert scale items was analysed using descriptive statistics in Excel and thematic analysis

was used to analyse the qualitative data. In Phase Four, the study investigated how procedural flowcharts could support problem solving in the Mathematical Methods subject. Data for this final phase was obtained from an in-depth follow-up interview with a teacher and from student-generated artefacts. Analysis of the data was informed by the stages of problem solving (QCAA, 2018).

There are four main findings from this study. First, the trends analysis showed a high dropout rate in calculus-based mathematics options as students progressed into their initial course/s of study. The trends also showed that the SEIFA indices, the ICSEA indices and the schools' transfer ratings correlated positively with the student dropout rate. Second, the study developed a framework on content sequencing on how prior knowledge can be linked to new knowledge during mathematics planning. The step-by-step systematic sequencing of mathematics content using the framework can promote interlinking, coherence and spiralling of concepts between the *Australian Mathematics Curriculum* (Years 7 to 10) and the recently introduced Senior Queensland Mathematical Curriculum: Mathematical Methods. The study identified that, depending on the level of assumed prior knowledge and the skills that students can recall and apply, teachers can start teaching from any level of the sequenced content. Third, the study revealed that the use of the pedagogical resources developed during the study (procedural flowcharts and concept maps) can support students in mathematics because of their visual nature. In particular, the pedagogical resources can be used to represent key procedural and conceptual mathematics knowledge. Additionally, procedural flowcharts can support student-centred teaching of mathematics procedures while concept maps can support the interconnection of mathematical concepts, consolidation and assessment of mathematics knowledge. Fourth, the study revealed that procedural flowcharts can support mathematics problem solving through organising and communicating the proposed problem solutions.

One major outcome from this study is the development of the planning framework on content sequencing and associated pedagogical resources (procedural flowcharts and concept maps). The study suggests that using the framework on content sequencing can play an important role in planning and teaching new mathematical knowledge by building on prior mathematical knowledge. In the same way, procedural flowcharts and concepts maps can play a significant role in representing mathematical knowledge that can support teaching of mathematics. The study suggests that these tools can be adapted to all mathematics subjects and levels, can help identify relationships between lower-level and upper-level topics, concepts and skills, can link the two levels and can provide opportunities of building and organising mathematical knowledge in familiar and unfamiliar contexts.

This study emphasises that to address the high dropout rate and declining enrolment and to promote participation in calculus-based mathematics, system-wide professional learning is imperative to support teachers with content sequencing that can foster effective teaching of mathematics. The content sequencing can be developed from prior knowledge and provide gradual knowledge development as students build from what they already know. Thus, the study advocates the use of concept maps and procedural flowcharts as visual representations of mathematics conceptual and procedural knowledge and recommends the use of procedural flowcharts to support problem-solving in mathematics.

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Chapter 1: Introduction and Background

1.1 Chapter Introduction

Mathematics is central to Science Technology Engineering and Mathematics (STEM), which is a core agenda for most governments (Siemon, 2021). The ability to apply mathematics in real-life situations and make predictions is vital in STEM (Office of the Chief Scientist, 2012). Mathematics plays a central role in innovation, scientific, technological, economic and social knowledge development (Watt et al., 2017), making modern life heavily dependent on mathematics (Australian Academy of Science [AAS], 2016). Furthermore, mathematics is an enabler of innovation, scientific and technological development, all of which are considered prosperity drivers and central to jobs of the future (Black et al, 2021; Watt et al., 2017). In Australia, "innovation and digital technologies have the potential to increase Australia's productivity and raise GDP by $136 billion in 2034, and create close to 540,000 jobs" (PwC, 2013, p. 13); hence, mathematics is pivotal in reshaping the future (Chubb, 2012). Australia needs graduates with advanced mathematics skills to promote science, innovation, engineering, data synthesis and technology if it is to remain competitive in the global scenario, and this study uses Queensland as a representative case.

Enhancing students' participation and achievement in advanced or calculus-based mathematics in schools is a focus of most governments all over the world (Noyes & Adkins, 2016; Treacy et al., 2020). As the Australian Council of Deans of Science (2006) noted, "Calculus-based mathematics school graduates are essential for a strong science, research and innovation capacity. The statistics at hand indicate that enrolment numbers are shrinking in these areas and students are instead electing to take elementary mathematics" (p. 2). Similarly, Australia's former Chief Scientist, Professor Ian Chubb, has expressed concern about the lack of students studying higher levels of mathematics in the last two years of high school (*The Guardian*, 2014). In a report by the Education Council (2018), another former Australian Chief Scientist, Dr Alan Finkel, went further, pointing out that more students are choosing low-level mathematics at upper secondary school, but opt for STEM courses at university, although they would need advanced mathematics skills to graduate in such degrees. This was supported by the Australian

Mathematical Sciences Institute's [AMSI](2021) 2020 report, which confirmed a shift by students from calculus-based mathematics subjects to basic mathematics. Importantly, another report by the AMSI (2022) raised an alarm that participation rates in calculus-based mathematics have reached a critical point and called for action to be taken. These reports also suggest that teachers can support students' participation in calculus-based mathematics if those teachers are supported and use research-informed resources, which is the focus of this study.

1.2 Background to the Project

The structure of high school mathematics across Australia offers two diverging pathways, one calculus-based and the other not (Maltas & Prescott, 2014). The senior mathematics subjects in Queensland schools between 2008 and 2019 comprised Mathematics B and C as calculus-based options and Mathematics A and Prevocational Mathematics as non-calculus options. When the new curriculum was introduced in 2019, Mathematical Methods and Specialist Mathematics were offered as calculus-based options and General and Essential Mathematics as non-calculus options. Calculus-based options are regarded as advanced mathematics options because they play an enabling role for STEM careers, especially at tertiary level (Adelman et al., 2003; Carnevale et al., 2011; Long et al., 2012; Rasmussen et al., 2011). Importantly, they offer broader and more diverse career opportunities for high school graduates. Thus, students' participation in these options have implications for their future prosperity at both the personal and societal levels. To bring to light these implications in depth, this study investigated trends in students' participation in calculus-based mathematics in Queensland. It also investigated mathematics teachers' perceptions of planning, teaching and learning resources that could support students' participation in calculus-based mathematics.

Phase One of the study investigated trends in students' participation in calculus-based mathematics in Queensland, in both the phased-out and recently introduced curricula. Trends are important as they can be considered the most basic indicator of educational progress. "In competing economies, they are often used as an index of educational strength and they are a strong predictor of the future educational achievement of a country" (Wilson & Mack, 2014, p. 35). Importantly, trends in students' participation can help evaluate progress and inform policy makers considering the huge expense involved in delivering education (Kelly, 2013). There has been significant but conflicting media coverage on student participation trends in advanced mathematics in Australia, which shows the public's interest in the subject (Kennedy et al., 2014). However, the state of Queensland has been lagging behind other states such as New South Wales in terms of the literature on such trends (Jaremus et al., 2018). When the central role of advanced mathematics as enabler of several disciplines and key to success in STEM courses at university means that participation in it can be an indicator of national educational progress and workforce projections, student participation trends can be used to identify areas that need improvement.

In Phase Two of this study, a planning framework and associated pedagogical resources (procedural flowcharts and concept maps) were investigated and developed for content sequencing from the Australian Mathematics Curriculum (Years 7 to 10) to the Senior Queensland Mathematical Curriculum (Years 11 to 12) [QCAA, 2018] focusing on the Mathematical Methods Unit 1 on Functions that are taught in Year 11. Several mathematics subjects are on offer at senior secondary level in Queensland; however, this study focused on Mathematical Methods, which is a calculus-based subject. In Phase Three, the study investigated mathematics teachers' perceptions of the effectiveness of the planning framework and associated pedagogical resources. In Phase Four, the final phase, the study investigated how procedural flowcharts can support problem-solving skills in mathematics. Mathematics teachers are curriculum deliverers and thus their perceptions after applying planning, teaching and learning resources during teaching and learning to support students' participation is of significant importance. "The primary focus of the classroom teacher is on the planning, preparation and teaching of programs to achieve specific student outcomes. The classroom teacher engages in critical reflection and inquiry in order to improve knowledge and skills to effectively engage students and improve their learning" (Victoria Department of Education and Training, 2017, p. 4). Similarly, according to the Australian Institute for Teaching and School Leadership (AITSL), teachers are expected to plan for and deliver effective teaching and learning (2011). The mathematics teachers' feedback after using the resources developed in this study represents their views, observations and experience during teaching and learning.

1.3 Research Focus and Questions

This study was conceptualised according to a constructivist epistemology. Constructivists believe that new knowledge is attained and tested when people purposefully interact to exchange ideas, beliefs, views, skills and experiences (Garbett, 2011; Taber, 2019). In fact, "knowledge is attained when people come together to exchange ideas, articulate their problems from their own perspectives, and construct meanings that make sense to them" (Gordon, 2008, p. 324). During knowledge development, learners compare the new ideas presented to them with their prior experience and ideas and during this process they may either reject the new knowledge if there is a contradiction or update their previous knowledge (Tomljenovic & Vorkapic, 2020). The filtering of new insights through the lens of prior experience helps to make sense of what is presented and thus makes learning an active process (Garbett, 2011). Therefore, the purposive and active interaction between the researcher and senior school mathematics teachers and the sharing of experiences, views, beliefs, observations and ideas were key in developing and evaluating the planning and teaching resources in this study.

The focus of this study was not only to provide insight into the trends in students' participation in calculus-based mathematics in Queensland but also to develop resources that could be used by teachers to support the teaching and learning of the subject.

Therefore, the overarching research questions were:

1. What are the trends in Queensland senior students' enrolment in calculus-based mathematics subjects?
2. What pedagogical resources support the planning, teaching and learning of Mathematical Methods for Queensland senior students?

As stated earlier, this study was divided into four phases. Phase One of the study investigated the enrolment trends of Year 11 and 12 students in calculus-based mathematics. The research question addressed in this phase was: What are the trends in Queensland senior students' enrolment in calculus-based mathematics subjects? An additional study was undertaken in this phase to provide further insights into the impact of other external factors such as socio-economic status and school location on student enrolment in calculus-based mathematics. The sub-question addressed in the additional study was: What is the relationship between students' enrolment in calculus-based mathematics in the new Queensland curriculum and school level indicators such as socio-economic status, school location and transfer rating?

Phase Two of the study investigated and developed a planning framework and associated pedagogical resources (procedural flowcharts and concept maps) for content sequencing from the Australian Mathematics Curriculum (Years 7 to 10) to the Senior Queensland Mathematical Curriculum (Years 11 to 12) [QCAA, 2018], using the Mathematical Methods Unit 1 on Functions that is taught in Year 11. This phase involved interacting with teachers to exchange ideas, experiences and collaboratively trial the teaching and learning resources. The sub-questions addressed in Phase Two were:

- What framework for content sequencing can support transition from junior to senior mathematics?
- What teaching and learning resources can support students' participation in senior mathematics?

Phase Three of the study investigated mathematics teachers' perceptions of the effectiveness of the planning framework and associated pedagogical resources that were developed in Phase Two. The sub-questions addressed in this phase were:

- What are teachers' perceptions of a planning framework on content sequencing for the teaching and learning of mathematics?
- What are senior secondary teachers' perceptions of how concept maps support the teaching and learning of mathematics at senior secondary school?
- What are senior secondary teachers' perceptions of how procedural flowcharts support teaching and learning of procedural fluency in the Mathematical Methods subject?

Phase Four of the study investigated use of procedural flowcharts in supporting problem-solving in mathematics. The questions addressed in this phase were:

- What are teachers' perceptions of how procedural flowcharts support students' problem-solving skills in the Mathematical Methods subject?

This study focuses on supporting the teaching and learning of mathematics. "Mathematics teaching and learning practices range from practising essential mathematical routines to developing procedural fluency, through to investigating scenarios, modelling the real world, solving problems and explaining reasoning" (QCAA, 2018 p. 1). Importantly, the study aims to support teachers with resources to develop students' mathematics proficiencies which are understanding, fluency, problem solving and reasoning (ACARA, 2010). Supporting students in making connections between related concepts and progressively applying the familiar to develop new ideas is key in developing mathematical understanding (ACARA, 2010; QCAA, 2018). Moreover, when teachers provide students with the opportunity to develop the capacity to select appropriate procedures, and carry them out flexibly, accurately and efficiently, it can support fluency (ACARA, 2010; QCAA, 2018). Problem solving as a critical 21st century skill can be supported through enabling students to plan a solution to a problem through interpreting, formulating, modelling and investigating problem situations, and communicating solutions effectively (ACARA, 2010; QCAA, 2018). Furthermore, students are supported to develop reasoning skills when they explain their thinking, deduce and justify strategies that thy have used and conclusions that they have reached during problem solving. Similarly, QCAA identified building new knowledge from prior knowledge and the ability to represent mathematical knowledge from one form to another as vital for mathematical teaching and learning, which is also a major focus of this study.

The teachers' perceptions in this study are their observations, experiences and opinions after using the resources developed that were in the study. The extent and depth of the support that the resources offered teachers and students during teaching and learning of mathematics at senior secondary was key in understanding the significance of the resources and evaluating them. The perceptions sought from senior mathematics teachers in this study were centred on how the resources supported the teaching and learning environment and promoted students' development of mathematics proficiencies. In summary, the perceptions allowed teachers to provide in-depth feedback on how they used the resources that were developed in this study to support students' mathematical development during teaching and learning of mathematics at senior secondary level.

1.4 Significance of the Study

This study investigated trends in Queensland students' enrolment in calculus-based mathematics subjects. The findings are significant because they can be used to evaluate different programs, plan for the future and inform policy makers and interested groups. Identification of the latest trends allows comparisons to be made between Queensland and other states or jurisdictions in Australia. The main findings from the trends analysis are the high dropout rate and continued decline in students' enrolment in calculus-based mathematics.

This study makes a significant contribution to mathematics education through the development of a planning framework and associated pedagogical tools. The framework links junior to senior level mathematics content, with an emphasis on building new knowledge from prior knowledge. In Australia, the states and territories are responsible for developing the senior curriculum (Year 11 to 12) while the junior curriculum is developed by the federal government. Thus, the framework provides a crucial tool for linking the two curricula. In Queensland there are currently limited resources that teachers can use to link them and to develop a scope and sequence during planning. The framework is not limited to calculus-based mathematics subjects but can be applied to any other mathematics subjects to articulate the hierarchical nature of the discipline.

Lastly, the study developed and explored pedagogical resources (concept maps and procedural flowcharts) that can support the teaching and learning of calculus-based mathematics. Concept maps were developed and explored to support the development of webs of concepts that link prior concepts to new concepts, thus emphasising the importance of developing new knowledge from prior knowledge. Furthermore, using concept maps in this way also support students' conceptual knowledge, teacher consolidation and assessment of students' mathematical knowledge. Procedural flowcharts were also developed to support students' mathematics procedural fluency and these serendipitously played an important additional role in supporting mathematics problem-solving. Similar to the framework described above, these resources are not limited to use in calculus-based mathematics only but may be applied to mathematics in general.

1.5 Structure of the Thesis

This thesis comprises 12 chapters. Chapter 2 reviews trends and factors influencing students' enrolment in calculus-based mathematics, planning for mathematics teaching and learning, mathematical representations and how representations support problem solving. Furthermore, the chapter reviews constructivism as a learning theory in mathematics education and argues that although constructivism is strongly encouraged in mathematics teaching and learning, cognitivism and behaviourism can also enrich the learning environment. The chapter goes on to discuss these approaches and their impact on instruction at the senior secondary level. These three learning theories are discussed in Chapter 2 in order to explore areas where the resources can complement different instructional models.

Chapter 3 outlines the theoretical framework for the study along with the methodology, research design, data collection and analysis procedures. Chapter 4 analyses the trends in Year 11 and Year 12 students' enrolment in mathematics subjects from 2010 to 2019, using data from the Queensland Curriculum and Assessment Authority (QCAA). This analysis focuses on the phased-out Queensland senior certificate mathematics subjects and was published in the journal *PRISM: Casting New Light on Learning, Theory and Practice*, 2022. The Chapter 5 analyses the impact of social and economic factors on trends in students' enrolment in calculus-based mathematics in the new Queensland Certificate in Education (QCE) from 2019 to 2020. This work was published as a research paper in the conference proceedings of the *International Conference of Education in Mathematics, Science and Technology, Antalya; Turkey 24 - 27 March 2022*.

Chapter 6 argues for effective sequencing of mathematics content from junior (Years 7 to 10) to senior-level (Years 11 and 12) concepts and introduces a framework for (re)conceptualising and sequencing the mathematics content. Unit 1 on Functions in the Mathematical Methods subject is used as an example to demonstrate the framework. This chapter was published in the *Eurasia Journal of Mathematics, Science and Technology Education*, 2022.

Chapter 7 investigates teachers' perceptions of the utility of the content sequencing framework developed in the previous chapter after they had used the framework in the planning, teaching and learning of mathematics in Queensland, Australia. This chapter was published in the *Eurasia Journal of Mathematics, Science and Technology Education (EJMSTE)*.

The following two chapters focus on teachers' perceptions of the pedagogical resources that were developed as part of this research, namely, the concept maps and the procedural flowcharts. Chapter 8 explores teachers' perceptions on the utility of concept maps as a resource to link junior (Years 7 to 10) concepts to senior (Year 11 and 12) concepts and how they support the teaching and learning of conceptual knowledge in senior secondary mathematics. This chapter has been published in the *International journal of innovation in science and mathematics education (IJISME).*

Chapter 9 discusses mathematics teachers' perceptions of the utility of procedural flowcharts for developing procedural fluency and supporting student-centred teaching and learning of mathematics. This chapter was published as a paper in the *conference proceedings for MERGA 44, 2022.*

Chapter 10 examines how procedural flowcharts can support problem-solving in Mathematical Methods which is a senior secondary calculus-based mathematics subject. This chapter is under review in the *Mathematics Education Research Journal (MERJ).*

Chapter 11 is a synthesis of the overall findings, starting with the analysis of student enrolment trends in calculus-based mathematics and thus highlights the main research problem. The rest of the discussion focuses on teachers' perceptions on how the pedagogical resources that were developed in this study can support teaching and learning of calculus-based mathematics to address the high dropout rates and declining enrolments in this discipline. Finally, Chapter 12 presents the conclusions along with the implications for the teaching of calculus-based mathematics.

Chapter 2: Literature Review

2.1 Introduction

This chapter begins with a brief discussion of the importance of calculus-based mathematics in senior high school and its role in future career opportunities. It then goes on to review the trends and factors that influence students' enrolment in calculus-based mathematics. A discussion of the research on recommendations to support student enrolment in calculus-based mathematics follows. The role of the mathematics teacher emerges as highly influential in student participation. An effective mathematics teacher not only possesses requisite subject content knowledge but also needs a thorough understanding of methods and skills to effectively deliver the content, known as pedagogical content knowledge. Furthermore, pedagogy that is collaboratively planned and underpinned by appropriate learning theories has important implications for mathematics teaching and learning. This leads into a review of the role of collaborative planning and learning theories in the teaching and learning of mathematics. Finally, specific pedagogical resources such as mathematical representations and their role in supporting problem solving are explored.

2.2 Importance of Senior Secondary Advanced (Calculus-based) Mathematics

High school advanced mathematics has been labelled as a critical filter of future opportunities (Watt et al., 2017). Across the world student enrolment in advanced mathematics have been a focus of researchers (Kennedy, 2014; Hine, 2019; Hodgen, 2010; Noyes & Adkins, 2016) as the benefits of enrolling in the subjects go beyond personal prosperity (Adkins & Noyes, 2016; Gijsbers et al., 2020). In Australia and Queensland, in particular, advanced mathematics subjects are the preferred prerequisites for high impact mathematics intensive programs such as engineering and medicine.

Furthermore, advanced mathematics plays an important role in the economic growth, research and innovation, and the general competitiveness of a country. Advanced mathematics subjects are generally those that enable students to further their participation in Science, Technology, Engineering, and Mathematics (STEM) fields (Wilkie & Tan, 2019) as well as develop students' logical thinking and reasoning skills which are 21st century skills (Attridge & Inglis, 2013; O'Meara et al., 2023). Wolf (2002) notes that studying advanced mathematics at senior secondary level positively influences future earnings. Advanced mathematics offers interdisciplinary skills needed to be successful in other mathematics related courses (Kennedy et al 2014; Ker, 2013). Moreover, its key enabling role in STEM helps to develop a scientifically literate workforce (Chinnappan et al., 2007; Maass et al., 2019).

In Australia, advanced level mathematics subjects can be referred to as calculus-based mathematics subjects and are now regarded as an 'endangered species' (Maltas and Prescott, 2014) because many senior students are avoiding them (Wienk & O'Connor 2020). Determining trends in students' participation in calculus-based mathematics at senior secondary levels in Australia has been a focus for researchers for a long time (see Malone et al., 1993; Dekkers & Malone, 2000; Forgasz, 2006; Kennedy, 2014; Hine, 2019; Jennings, 2022). McPhan et al., (2008) suggest that these trends can inform a country on its preparedness to supply the Science, Technology, Engineering and Mathematics sectors with students possessing the necessary prerequisites and can be used to evaluate the future economic competitiveness of Australia in the technologically advancing world. Although different factors that influence student enrolment and participation in advanced mathematics have been identified by researchers (Hine, 2019; Kirkham et al., 2020; McPhan et al., 2008), limited resources have been developed for teachers to support student participation. Thus, investigating the participation trends in calculus-based mathematics can provide valuable insight into what factors influence the trends as well as identify areas that need intervention.

2.3 Factors that influence student participation in calculus-based mathematics at senior secondary.

Researchers have identified an assortment of personal, educational, social, economic and demographical reasons that impact student participation in calculus-based mathematics. Generally, students are influenced by low levels of perceived competence in mathematics (Nagy et al., 2010; Kirkham et al., 2020; Sikora & Pitt 2019); students' dissatisfaction with Mathematics (Hine, 2019); prior experiences of mathematics (Li 2019; Ng, 2021; Fullarton & Ainley 2000; McPhan et al. 2008) perceived level of difficulty of the subject (Hine, 2019), parents, siblings and teacher influence (Jennings, 2022; Kirkham et al., 2020) and the excessive amount of time that the subject requires in order to succeed (Jaremus et al. 2019; Kirkham et al., 2020; McPhan et al. 2008).

In addition, most Australian universities have removed Mathematics as a prerequisite for many courses and instead offer bridging courses resulting in students regarding calculus-based mathematics as unnecessary as they can catch up after senior secondary school (Hine 2023). Importantly, students aim to achieve a high Australian Tertiary Admission Rank (ATAR) score to be accepted in highly sought-after courses so avoiding calculus-based mathematics gives them a better chance to achieve a high score (Kirkham, 2020). Thus, students with good prior results in calculus-based mathematics choose to opt out of these subjects because they perceive them as possible threats to achieving a high ATAR at senior secondary school. Furthermore, many students pursue some undergraduate programs without the requisite mathematical knowledge required to be successful in the courses (Nicholas et al., 2015). However, students who enter university to pursue mathematics intensive courses without required mathematics knowledge and skills have a low attainment rate (Jennings, 2011).

Research indicates that the most common reason that students opt out of advanced mathematics is the perceived level of difficulty of the subjects (see Brown et al., 2008; Hine, 2019; Kirkham et al, 2020; McPhan et al., 2008; O'Meara et al, 2020). Students hold the perception that there is a level when mathematics starts to be difficult, for example as they transition from junior to senior secondary school (Brown et al., 2008). Similarly, students feel calculus-based mathematics is too challenging and the knowledge and skills they have is not adequate as they perceive themselves not to be "really smart" (Hine 2019, 2023). However, a recent study by Jennings (2022) noted that students in Queensland do not consider their ability and skills as a factor in choosing a mathematics subject at senior secondary level but rather the usefulness of the subject post-secondary. This is important because students choosing subjects based on future interests may result in diverse ability groups of students in the different mathematics options which might put pressure on the teacher on how to best engage the students during teaching.

Students' perceptions point to the notion that if they did not understand some junior mathematics concepts well, then they would not understand senior advanced mathematics, hence there was no need to pursue such options (Brown et al., 2008). Additionally, lack of an engaging and challenging pedagogy may lead to students choosing not to pursue mathematics subjects perceived as difficult (Goss, 2010). Some students also identified that they experienced poor teaching of mathematics and as a result they lost interest in the subject Easey (2019). Easey went further to note that the teaching of students by out-of-field teachers at junior level might contribute to students not choosing advanced mathematics at senior secondary level. Whilst Brown et al., (2008) posit that students feel they need to have obtained high grades at junior level for them to do well in advanced mathematics at senior level. Conversely, Mujtaba and colleagues (2014) find no relationship between prior attainment and the decision to pursue senior advanced mathematics. This finding is also true for Queensland (Jennings, 2022). Thus, senior Advanced Mathematics classes consist of students with diverse mathematics experiences which may influence their perception of the subject.

The economic and social variability of a nation is significantly influenced by the extent to which students from diverse backgrounds (that include educational, economic, linguistic, cultural, racial) are empowered to sustain their aspirations in mathematics (Ng, 2019). Ng (2019) emphasised that in Australia students from socially, economically and educationally disadvantaged backgrounds are overrepresented among those who obtain poor results in national and international benchmark assessments. Similarly, schools in regional and remote areas are highly impacted by the decline in student enrolment in calculus-based mathematics compared to metropolitan schools (Lynos et al., 2006). They went further to note that schools in regional areas are twice as likely, and those in remote areas are six times as likely as their metropolitan colleagues to report high annual staff turnover rates in mathematics. Furthermore, most schools in non-metropolitan areas resort to composite classes because of shortage of teachers, for example, combining Year 11 and 12 Mathematical Methods classes. Lynos and colleagues posit that the majority of teachers of Mathematics outside metropolitan areas indicated a significantly higher unmet need for teaching and learning resources that can cater for student diversity and ability levels. Consequently, at university level, students from rural and remote areas are underrepresented in mathematics intensive programs (Thomson & De Bortoli, 2008). The following section considers some recommendations to support student enrolment and participation in calculus-based mathematics.

2.4 Recommendations to support student enrolment and participation in calculus-based mathematics

To increase student enrolment, research has recommended providing incentives to students in the form of bonus points (Hine, 2023; O'Meara et al., 2020, 2023) and engaging with mathematics extra-curricular activities (Mujtaba et al., 2010). Other researchers suggest that universities should have unambiguous prerequisites for courses that are mathematics intensive (Maltas & Prescott, 2014; McPhan et al., 2008) and seek new and effective ways to deliver information of the importance of Advanced Mathematics at an earlier age (Kaleva et al., 2019). Of key importance to my study is a call by Mujtaba and colleagues (2014) for support to be provided to in-service teachers to meet the demand for high quality mathematics teachers if participation in Advanced Mathematics is to be boosted. This is because access to mathematics resources for teachers has important implications for their classroom practice which in turn can influence student participation (Mujtaba et al., 2010).

Research shows that there is no greater influence on student participation and achievement than the teacher (Stronge, 2013). Although there are many factors that can affect student achievement - choices, learning, attitudes and beliefs - an effective teacher is the greatest asset in making a positive difference (Hattie, 2012). For example, Hattie argues that an expert teacher can identify the best and most effective ways in which to represent the subject they teach. Teaching effectiveness is the single most important school related factor influencing student engagement, experiences, and achievement (Hattie 2013; Leigh, 2010; Rivkin et al., 2005; Rowe, 2003). To clarify, it is what teachers know and do that has the greatest influence on students' learning and achievement (Hattie, 2003). "The nature of classroom mathematics teaching significantly affects the nature and level of students' participation" (Hiebert & Grouws, 2007 p. 371). In mathematics, teachers can be effective by focusing on building upon what students know, employing different forms of representations, making connections, building procedural fluency and fostering communication (Sullivan, 2011).

Similarly, developing teacher capacity in teaching mathematics is identified as one of the key strategies to increase student participation in Advanced Mathematics. Hine (2018) posits that there is a need to develop teaching and learning practices that support the mathematics curriculum. Quality teachers supported by appropriate resources are a significant factor that can assist in the teaching of mathematics and thus have a positive impact on student achievement (Hoyles, 2009). Ngu (2019) notes that a significant number of socially and economically disadvantaged students are motivated to pursue Advanced Mathematics and it is important for teachers to develop pedagogical practices to sustain the students' motivation and aspirations to learn advanced mathematics. Maltas and Prescott (2014) recommend the development of teachers' resources that they can use to support student engagement in calculus-based mathematics if Australia is to increase enrolment in the subjects. Outside of the Australian context, a review conducted by Smith (2017) of the status of mathematics education at senior secondary level in England, recommends the need to support senior mathematics teachers in their teaching through professional development and research informed resources.

Lynos and colleagues (2006) also suggest that research informed resources and strategies can support teachers in schools outside metropolitan areas thus having a positive impact on student participation in Advanced Mathematics. Importantly, Murray (2011) notes that students emphasised mathematics teaching should concentrate more on ways that can help students understand mathematics at every level. Additionally, the focus should not only be linked to boosting enrolment but retaining students in the subjects.

Retention of students in calculus-based mathematics should be a focus for all stakeholders (Rasmussen & Ellis, 2013). "Instructional variables such as actively engaging students, having students explain their reasoning, etc. may make a difference in retaining STEM majors" Rasmussen & Ellis, 2013 p. 463). Students who are less engaged in mathematics at lower levels are more likely to drop out from Advanced Mathematics subjects at upper levels (Ellis et al., 2014). There is a high chance of dropping out of the subject when students are not confident that the skills that they obtained from lower levels are adequate for them to engage meaningfully with content at the higher level (Rasmussen & Ellis, 2013). It is important for teachers to build new knowledge from prior knowledge to support students' understanding of mathematics. Therefore, sequencing concepts in a manner that allows students to use their prior knowledge to make sense of new knowledge has important implications for mathematics teaching and learning. Collaborative planning can support teachers in such sequencing. The following section discusses the role of planning in mathematics teaching.

2.5 Role of planning in mathematics teaching.

The planning and teaching of mathematics is complex as teachers are faced with ever-increasing demands to cater for students' cognitive diversity, and supporting students' engagement and understanding (Davidson, 2019; Sullivan et al; 2013). Teachers consider planning as the core of teaching (Akyuz et al., 2013), therefore planning is a fundamental step in the mathematics teaching cycle (Davidson, 2019). However, planning is a broad activity and much of the focus has been on the time teachers spend preparing and designing activities for students (Superfine, 2008). But when teachers know and plan what they hope students will learn, they are better placed to support them in the learning process (Sullivan et al., 2012). The sequence of knowledge and skills fostered by a teacher during planning influences student engagement and learning in mathematics (Kilpatrick et a., 2001). However, there is limited research on how to support teachers during planning in developing mathematics content sequencing that can support teaching and student engagement (Roche et al., 2014).

Prior research on content sequencing indicates that this process has many benefits in teaching and learning of mathematics. Identifying prerequisite concepts that underpin new knowledge supports instruction planning (Panasuk et al., 2002). To clarify, sequencing of content in a unit is more than ordering content but is informed by the relationships and connections between the concepts, and the deeper understanding that the sequence allows students to access (Howard & Hill, 2020).

> "The curriculum in many subjects is dependent on a deliberate approach to the sequencing of concepts because one concept often relies on the understanding of what has come previously and what will come next. Effective sequencing

> provides a way of embellishing and unifying what may otherwise seem like disconnected fragments of knowledge" (Howard & Hill, 2020, p. 3).

They went further to posit that sequencing content requires a systematic, streamlined approach to explicitly demonstrate connections between what has been learnt and what is to come next so that these connections strengthen students' cognitive architecture, rather than act as an extraneous distraction. Given the above, content sequencing during planning plays an important role in mathematics teaching.

Mathematics teachers are responsible for selecting and sequencing mathematics tasks, responding to students' misconceptions, catering for the cognitive diversity of students, engaging students, using different representations of mathematics in their teaching and determining learning progressions. Therefore, it is critically important to support teachers in the planning and teaching process if effective teaching is to be realised (Galant, 2013; Roche et al., 2014). "Learning progressions mean an evidence-based sequence of key concepts in mathematics, supported by suggested approaches to learning and teaching that are tailored to different stages of the sequence" (Callingham et al., 2021, p. 334). Teaching that starts from prior knowledge can address preconceptions which might interfere with learning new content (Hodgen et al., 2018; McGoven & Tall, 2010). When students are constructing new knowledge, the form in which prerequisite concepts are presented affect how the new knowledge is constructed (Panasuk et al., 2002). Importantly, Australian mathematics teachers are expected to plan mathematics sequences that promote student engagement, flexibility, creativity and problem solving to develop deeper understanding (Davidson, 2019). Davidson went further to note that mathematics teachers in Australia are expected to plan (and teach) mathematical sequences and experiences that encourage students to think flexibly and creatively about concepts to allow them to develop "big picture" thinking. Thus, content sequencing is a key first step when planning for mathematics teaching. Additionally, use of pedagogical resources to support student engagement, flexibility, creativity and problem solving is essential for deep understanding.

Mathematics teaching should focus on providing students with opportunities to engage, reflect and demonstrate understanding. Teaching that allows students to reflect, build upon, transform and restructure their prior knowledge support the development of mathematics competences (Donovan & Bransford, 2005). Emphasising meaningful relationships between concepts and prompting students to search for connections provide better opportunities for student understanding and achievement (Panasuk et al., 2002). Teachers should plan for time and support for students to make mathematical connections (Davidson, 2019). In fact, when teaching for understanding, students should be supported and given time to explore, make connections, build meaning and understanding (Black, 2007; Davidson, 2019). Similarly, the ability to recognise interconnections between mathematics concepts and to develop different representations of mathematics concepts demonstrates a deeper level of mathematics understanding (Galant, 2013). During teaching for understanding, visual representations can be used as tools for manipulation and communication and conceptual understanding of mathematical ideas (Zazkis & Liljedahl, 2004). Similarly, research indicates that teachers who develop and organise content knowledge in an integrated manner are positioned to be expert teachers (Hattie, 2012). Additionally, mathematics teachers have to appreciate the importance of learning theories in informing practice in educational settings. This is discussed at greater length in following section.

2.6 Importance of Learning Theories in Mathematics Education.

Learning theories may contribute immensely to the current learning environment through offering solutions (Ertmer & Newby, 2013). Knowledge of learning theories allows practitioners to understand and know when to apply these theories to encourage student participation (Ertmer & Newby, 2013; Garbett, 2011). Theories of learning and instruction in mathematics education are tools for either transmitting knowledge to learners or directing them to construct their own knowledge (Cobb, 1988). However, calls for more student participation opportunities during learning have been amplified (NCTM, 2000) if learning outcomes are to improve (Eronen & Kärnä, 2018), if students are to appreciate the value, relevance and importance of mathematics (Riegle-Crumb et al., 2019) and to enjoy learning mathematics (Noyes, 2012). Thus, the contribution of learning theories to shaping classroom practice and supporting students' participation is at the centre of effective teaching and learning of mathematics. Constructivism has emerged as the most advocated learning theory in mathematics education because it is student-centred (Confrey & Kazak, 2006).

2.7 Constructivism

Constructivism emerged as a learning theory when scholars started realising the limits of the notion that "knowledge is independent of the knower" and began advocating for problem posing and interconnection of ideas (Glasersfeld, 1995). Bruner, Dewey, Piaget and Vygotsky are credited as the main scholars who laid the foundation of constructivism (Kumar, 2006; Bada & Olusegun; 2015). The pillars of constructivism include Piaget's (1953) views on how learners construct knowledge, Bruner's (1973) cognitive structures, Dewey's inquiry learning (real-world problems) and Vygotsky's social view of acquiring learning (Brau, 2018; Perkins 1992). Although there are many different types of constructivism, it can be broadly divided into mainly two; cognitive (individual) and social constructivism (Powell & Kalina, 2009, Stewart, 2021).

2.7.1 Cognitive and Social Constructivism

Constructivism broadly focuses on how knowledge is constructed or how people acquire knowledge and learn. Specifically, cognitive constructivism was developed by Piaget (1953) who focused on individual construction of knowledge through a personal process. According to Piaget, the knowledge people interact with is added to schemas of prior knowledge wherein learners construct knowledge only in their minds (Alanazi, 2016; Stewart, 2021). The schemas are developed through the process of assimilation and accommodation (Powell & Kalina, 2009). Piaget (1953) noted that humans cannot be given information, which they immediately understand and use; instead, humans must construct their own individual knowledge.

In contrast, social constructivism is when knowledge is constructed through interaction in a social setting (Powell & Kalina, 2009). Vygotsky is regarded as one of the main proponents of social constructivism when he questioned Piaget's views that focused mainly on cognitive development as an individual and not as a collaborative process (Martinez, 2010). Vygotsky went from focusing on the internal processes of learning that Piaget focused on to include external forces such as society and the environment (Alazani, 2016). Social constructivists believe reality is constructed through interaction in a social setting, and knowledge is socially and culturally constructed as people interact with each other and the environment they live in, while learning is a social process that occurs when learners engage in social activities (Kim, 2001). What makes constructivism unique and appealing is the understanding that students have a greater contribution in the learning process compared to teacher-centred learning theories.

2.7.2 Learning informed by constructivism

Constructivists view learning as a process of constructing new knowledge from learners' (or students') beliefs, skills and prior experience; as a result, learners are simultaneously creators of knowledge (Garbett, 2011; Bruning et al., 2004). Constructivism is generally regarded as a theory of learning or meaning making, that emphasises individuals creating new knowledge on the basis of the interaction between prior knowledge, beliefs, ideas and any new knowledge with which they come into contact (Richardson, 2003). This means that, learners' beliefs, skills, experiences and attitudes are an important factor in their learning (Agarkar, 2019), as they test the viability of anything new presented to them (Bodner et al., 2001).

The knowledge learners have about any subject (phenomenon) of interest is determined by their experiences (Ertmer & Newby, 2013). Learners' prior understanding is central to their understanding of new concepts, while for educators it directs design and implementation of learning instructions (Simon, 1995). Constructivism is a learning theory that is based on the premise that learning is the result of mental construction where new knowledge is examined through what someone already knows (Dennick, 2016). Similarly, in constructivism, knowledge is constructed based on the existing knowledge in learners' minds (Hmelo-Silver et al., 2007). The prior knowledge that each learner holds based on their unique experiences helps them to develop meaning of the world and construct representations, therefore each learner's construct is unique (Begg, 1999). Thus, learners come to learning with knowledge from their prior experience, which forms the foundation of any future learning.

In constructivism, learning is about creating knowledge rather than just receiving, understanding and applying it, recalling, thinking about and examining it or just gathering and memorising it (Gordon, 2008). When students are exposed to new knowledge, their minds filter it and create their own meaning based on their prior experience, ideas and attitudes (Agarkar, 2019). When the mind has evaluated the new phenomenon, it may either accept it and alter the existing knowledge or discard it as peripheral or unrelated; thus, learners constantly update their knowledge as they engage with new experiences (Bada & Olusegun, 2015). According to Brau (2018), the central role of learners is to reflect on their prior experiences and consider variables that might limit the assimilation of new knowledge. When they do need to accommodate new knowledge, "this cognitive reconstruction is called *reflective abstraction*, as it involves reflecting the existing cognitive structures to a higher plane of thought and applying these structures to new stimuli" (Faulkenberry & Faulkenbury, 2006, p. 18). As a result, content should be presented in a spiral form so that new knowledge is built upon what learners already know (Bruner, 1973). Thus, constructivism involves applying, testing, reflection, evaluation, drawing conclusions from findings and linking new knowledge with prior knowledge.

Glasersfeld (1995) notes that as learners link new knowledge to prior knowledge, they build conceptual understanding and emphasises that "concepts cannot simply be transferred from teachers to students – they have to be conceived" (p. 2). As a result, drilling students to answer standard questions does not result in competence when responding to unfamiliar questions. Constructivists focus instead on the learners' knowledge construction processes and how knowledge is acquired. Knowledge is constructed not just through remembering facts or perceived universal truths but as a process of sense making (Hein, 1991). In constructivism learners are at the centre of knowledge creation because they are actively involved in the learning process.

Learners are active participants in their learning as they interpret the meaning of new knowledge and reference it to prior experience (Garbett, 2011). Guided by the teacher, learners create knowledge actively, rather than acquiring it passively from the teacher or any other medium (Ertmer & Newby, 2013). Constructivists assert that learners construct knowledge rather than acquire new knowledge; therefore, learning is an active process throughout the learners' experiences and the environment in which they are learning (Alanazi, 2016). Constructivism emphasises active learner participation and engagement with content which in turn promotes attentiveness and effective learning (Hyslop-Margison & Strobel, 2007). Hyslop-Margison and Strobel nonetheless posited that a teacher's role remains critical and unrivalled, as learners depend on their guidance, feedback and support in creating and enabling an environment that promotes knowledge creation. Constructivists emphasise that learners must be active participants in the learning process and a teacher's role involves facilitation of learning (Fernando & Marikar, 2017). Interactions in the learning environment within a constructivist setting is central to knowledge development.

Knowledge, meaning and understanding are developed collaboratively by learners as they interact among themselves and with their environment; thus, learning is also a social process (Kim, 2001). At the same time, Ertmer and Newby (2013) see learners as unique individuals and how they interact among themselves and with learning resources and educators is at the centre of constructivism. Within a social setting, learners have to relate with a problem for them to make sense of it and construct knowledge (Roth, 2000). Roth went further to posit that when learners collaborate, that is, when reality and knowledge are socially constructed, the learning is a social process. Constructivists emphasise that focus should be on learners and on creating collaborative, interactive environments (Alanazi, 2016). However, Ertmer & Newby note that in the social process, guidance can come from the educator and/or capable peers as they collaborate, which results in skills exchange and deeper understanding.

Vygotsky looked at learning as a collaborative process, thus introducing the social aspect of constructivism. The social environment plays an important role in knowledge construction as learners may test each other's knowledge and provide alternative views, thereby questioning the viability of existing knowledge (Thomas et al., 2014). The interaction between learners and an adult or more advanced peers is necessary for knowledge construction and development and it requires the active involvement of all participants (Begg, 1999). Thus, a teacher can intentionally nurture and teach children only in collaboration with them. The process requires the teacher to move ahead of development into what Vygotsky called a zone of proximal development (Howe, 1996).

Vygotsky's zone of proximal development (ZPD) emphasises that the level of understanding that individual learning provides might not match skills that are gained through peer learning or when learning is guided by an adult (Fani & Ghaemi, 2011). ZPD can be defined as "the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978 p. 86). Vygotsky noted that a learner who can understand something with guidance now will be able to do it independently in the future. Thus, learners need guidance to create meaning during the learning process, especially of complex phenomena (Kirchner et al., 2006). Vygotsky emphasised that learners can successfully imitate concepts at their developmental level, but solutions of high-level mathematics problems may not be understood no matter how many times the teacher repeats the solution. Hence, it is important that learners are exposed to problems slightly above their developmental level so that they see the need to work collaboratively or with experts such as teachers in order to solve the problem (Roosevelt, 2008). Consequently, teachers, peers, learning instructions and any other media should guide learners to develop their skills base, not just to consolidate what they already know.

However, some teachers feel that constructivist approach to tasks and activities are difficult to implement as they require more time which is not feasible for teachers faced with pressures such as completing the syllabus and preparing students for assessments (Teong, 2002). From this perspective, constructivism is viewed as vague and having different meaning and interpretation to different people while some scholars have reservations about what it represents (Powell & Kalina, 2009). Constructivism challenges conventional approaches such as behaviourism and cognitivism but it is "notoriously slippery and difficult to pin down" as it means different things to different authors and covers a multitude of differing positions as it lacks a clearly stated set of core claims (Hay, 2016 p. 520). Thus, constructivism has been a popular but contested approach to learning.

Other critics of constructivist approaches say it focuses on promoting group knowledge construction even though education systems promote mostly individual assessments (Alanazi, 2016). Some psychology scholars criticise constructivism because there is a chance a dominant group might control interactions within their social learning environment during collaborative knowledge construction, making other students feel ignored (Gupta, 2011). Gupta went further to posit that dominant learners may end up driving the whole group towards their thinking and overlooking the knowledge construction and experience of others.

Similarly, some critics of the Piagetian concept of constructivism suggest that it focuses mainly on cognitive factors, ignoring other contributing environmental and technological factors (Alanazi, 2016). Piagetian concept of constructivism overlooks important contextual factors in learning environments such as available educational resources, the need to integrate media into learning environments, and the affordance of individual learner thinking (Ackermann, 2001). They emphasise that such resources make a significant contribution to learning and thus should not be ignored. Similarly, the different ways in which learners interpret the world based on their diverse and unique experiences makes instruction less effective because curriculum components might not be commonly constructed (Jonassen, 1991). Moreover, common curricula will be ineffective, inefficient and not applicable if learners are to apply their different thinking with minimum guidance (Carlson, 1992).

In addition, the idea of learning with minimum guidance goes against proponents of structured learning. Kirschner and colleagues (2006) suggest that constructivism promotes a teaching style with minimal guidance for students which they say might result in students feeling "lost and frustrated" (p. 6). They also noted that teaching approaches based on minimal guidance, as practiced through constructivist approaches, ignore empirical studies that have shown that unguided instructions are not effective in learning environments. For example, they pointed out that instruction based on minimal guidance ignores the importance and structure of working memory during learning. However, the emphasis placed on construction of new knowledge from prior experience in constructivism is also supported by the cognitive load theory proposed by Sweller in 1988, one of the critics of constructivism.

2.7.3 Cognitive Load Theory

As students move from a junior to senior level in education, there is an escalation in cognitive demand. The cognitive load theory focuses on prior knowledge playing a central role in lessening the cognitive burden. It emphasises the importance of foundational knowledge in acquiring new knowledge (Sweller et al., 2011). Prior knowledge that is relevant and related to new knowledge makes learning the new knowledge less difficult (Paas & Sweller, 2012). Students who have acquired the necessary schema (foundational knowledge) have a better chance of deriving meaning from new knowledge and can use it as a building block to master a skill (Moreno & Park, 2010), thus learning follows a constructivist approach. Moreover, automation of lower level (foundational knowledge) schemas is critical for developing higher level (new knowledge) schemas (Sweller, 2010). Sweller (2010) went further to note that students who possess the relevant lower-level schemas in their long-term memory can learn and retain new knowledge effectively. Therefore, students who are highly skilled and can readily learn new knowledge have acquired enormous stores of schematic knowledge in their memory.

The long-term memory and working memory affect the cognitive load. Changes in the long-term memory store, that is, knowledge that has been learnt from others or through problem solving, happens slowly and gradually (Sweller, 2010). The working memory is activated when students are exposed to new information which enable them to transfer available information from long term memory and keep it to support problem solving. However, the working memory has limited capacity when dealing with novel information and does not have the capacity to process more than 4 items (Cowan, 2001). Thus, burdening the working memory can impede learning (Martins & Evans, 2020). When familiar information is involved, few working memory resources are utilized. This freeing up of working memory increases the opportunity to learn and store information in existing schemas in long term memory (Rosenshine, 2009). Thus, the cognitive load theory can complement the main ideas of constructivism. Therefore, if constructivism is to be fully adopted, teachers should be supported with resources to reinforce its implementation.

2.7.4 Constructivism in Mathematics Education

Direct instruction has been the default way of teaching mathematics for a very long period (Kaur, 2019; Faulkenberry & Faulkenberry, 2006). In this case, the educator is regarded as the centre of knowledge and expected to transmit knowledge to students during the lesson (Ampadu & Danso, 2018). Lecturing is still considered applicable and viable in some mathematics classes, but from the 1970s there has been a consensus that constructivism offers students a better opportunity to gain a deeper understanding of the subject, to solve problems and to develop critical thinking (Ampadu & Danso, 2018; Boaler, 2009). Similarly, "critical thinking, problem-solving approach and analytical skills are the most important skills that are developed in the process of mathematics education and are also the cornerstones of sustainability" (Vintere, 2018, p. 6). This is critically important, especially at senior secondary school as students' mathematics subject choices at this level directly influence their future careers.

At senior secondary level, students' prior knowledge from junior levels can provide a foundation for developing better conceptual understanding. A mathematics teacher's planning must be anchored on learners' prior experience which is preceded by a systematically planned teaching sequence with the aim of developing learners who can solve complex problems (Garbett, 2011). This is because the prior knowledge and skills students bring as they interact with new knowledge will determine how successfully they interpret and assimilate the new knowledge (Lambert, 1995). Including prior knowledge in planning also provides an opportunity to correct student misconceptions. Taylor and Kowalski (2014) also advocate that planning by teachers must consider students' prior knowledge and skills because it will allow students to develop new knowledge from junior level concepts, thus enhancing the likelihood of engaging successfully with the learning activities with minimum assistance from the teacher.

Constructivism sees the role of the teacher as a facilitator and moderator rather than a source of facts, rules and principles (Fernando & Marikar, 2017).). Learners engage in activities and/or assessment tasks for the educator to evaluate whether they can apply new knowledge that they have constructed (Garbett, 2011). Constructivist teachers continually assess how an activity is helping learners reach the intended success criteria while continuing to reflect, deepen their understanding and expanding their knowledge (Shah, 2019). Teachers should provide students with opportunities to construct their meaning and interpretations as they engage with the learning (Airasian, & Walsh, 1997). Moreover, teachers should encourage students to explore different methods to solve a problem as they develop new knowledge (Ampadu & Danso, 2018). Ampadu and Danso further posit that teachers should provide minimum guidance and promote students' independent learning. However, a teaching approach that involves moderate teacher involvement but leads to greater learner engagement and understanding requires teacher confidence, content knowledge, experience and resources that help students focus on their learning (Garbett, 2011). Finally, Holmes (2019) emphasises that constructivist teachers both pose questions and prompt learners to ask questions, that they allow alternative explanations or options and they guide students to find their own solutions.

The teaching approach in a constructivist class should foster knowledge creation and making informed decisions about proposed solution(s). Constructivists design learning instruction and strategies that help learners to explore complex phenomena as experts in that field (Ertmer & Newby, 2013). Modelling problems are a good example of this in senior mathematics in Queensland.

> The role of instruction in the constructivist view is to show students how to construct knowledge, to promote collaboration with others to show the multiple perspectives that can be brought to bear on a particular problem, and to arrive at self-chosen positions to which they can commit themselves, while realizing the basis of other views with which they may disagree. (Cunningham, 1991, p. 14)

As architects of learning instruction, teachers have a responsibility to make sure their instruction guides students to create meaning, apply knowledge and experience, evaluate, interact collaboratively and be productive and come up with an acceptable solution.

Problem-solving and inquiry-based activities that encourage students to formulate and test their ideas, come to conclusions and make inferences and then share their knowledge in a collaborative learning environment are ideal for constructivists (Holmes, 2019). Projects and open-ended problem solving fit well with constructivism as teachers can track progress, probe and understand how learners think as teaching and learning progresses (Ahtee et al., 1994). For example, Assessment 1 in QCAA Mathematics general subjects (General, Methods and Specialist) in Years 11 and 12 is an open-ended problem-solving assessment where teachers are required to offer minimum support but allow students to explore and apply prior knowledge. Lesh and Doerr (2003) posited that by applying prior knowledge and real-life experiences to test facts and rules, learners draw conclusions and evaluate solutions that allow deeper understanding and greater participation. Similarly, meaningful learning includes reflecting and systematic linking of concepts from known to unknown, or from the simple familiar to the complex unfamiliar (Muirhead, 2006). Organisation tools such as concept maps, flow charts and other visual aids, including PowerPoint slides to show facts, flow and organisation of ideas, are important for guidance and redirection (Melrose, 2013). Moreover, knowledge representation maps are key in providing educators with mental models of what learners regard as mathematical realities (Thompson, 2013). Thus, constructivism in mathematics education produces a reflective learner who is not only good at applying mathematical facts but also makes informed decisions based on processes and outcomes of the learning process, which is central when sharing ideas during collaborative learning.

Mathematics teachers who are constructivists promote cooperation and collaborative learning because they believe knowledge is socially constructed. The learning of mathematics has moved from “passive and decontextualised absorption of mathematical knowledge and skills … towards the active construction in a community of learners of meaning and understanding on the modelling of reality” (Corte, 2004, p. 280). Other studies have demonstrated that collaborative mathematics learning in small groups produces better outcomes than individual learning, again because, according to Davidson and Kroll (1991) and Schreiber and Valle (2013), knowledge is socially constructed through collaborative activities. Activities in such communities of learning should promote problem solving, reasoning, evaluating and communicating (Goos, 2004). Knowledge sharing within the group results in sharing that will reorganise existing knowledge and make sense of new knowledge, learnt from each other (Plass et al., 2013; Retnowati et al., 2017). However, teachers need to be aware that besides constructivism, there are other learning theories that impact teaching and learning in mathematics (Airasian & Walsh, 1997), and that teaching and learning methods are also informed by learning goals (Hiebert & Grouws, 2007). In other words, other learning theories, such as cognitivism and behaviourism, also contribute to teaching and learning of mathematics.

2.8 Behaviourism

Behaviourists regard learning as a process of reinforcing expected responses to a stimulus (Ertmer & Newby, 2013). Pavlov (classical conditioning), Thorndike (association, reinforcement and incremental growth) and Skinner (instrumental conditioning) are among the founders and proponents of the behaviourist perspective (Stewart, 2012). Pavlov referred to the process of changing behaviour by repeatedly pairing stimuli with conditioning, thus advancing our understanding of learning by association (Stewart, 2021). Experimental work on simple paring of stimuli grew in the United States, deepening the understanding of learning through association, reinforcement and incremental growth in desired outcomes (Thorndike 1898). Stewart (2021) went further to note that by modifying tasks and using a series of positive rewards and negative reinforcers, Skinner demonstrated how behaviours could be shaped and reinforced towards specifically target outcomes.

Behaviourists focus on “learning as a change in behaviour which takes place through connecting actions with outcomes, reacting to feedback and strengthening repeated actions” (Stewart, 2012 p. 4). Behaviourists emphasise that learning occurs by environmental conditioning, connecting actions with outcomes, reacting to feedback and strengthening through repeated action (Stewart, 2021). Skill and drill or practice are associated with behaviourism. Steward (2021) posited that it also stresses the importance of specifying clear learning targets and structuring learning tasks to achieve these. Thus, in behaviourism the teacher is in charge, connecting actions with outcomes, structuring the learning, setting learning targets and making sure there is repeated action until a change in behaviour is witnessed. Consequently, this can be attributed as a weakness as it gives rise to a teacher-centred approach and outcome-based view of learning where the teacher is the owner of knowledge, controller of the learning environment, with students as passive recipients and empty vessels to be filled with knowledge. Stewart (2021) noted that it emphasises rote learning which is effective at achieving results in the short term, but its long-term effectiveness is questionable.

2.8.1 Behaviourism in Mathematics Education

Skill and drill in mathematics education is viewed as a means to develop procedural knowledge that focuses on mastering steps needed to accomplish a goal. It is important to note that "in more advanced levels of mathematics learning, procedural skills can also include higher level cognitive processes, for example focusing on relations between different parts of the procedures or evaluating the effectiveness of a particular procedure for a given task" (Lehtinen et al., 2017, p. 3). Skill and drill are viewed as a way of developing students' fluency in the basic mathematics skills needed for more advanced problems (Klinger, 2009). One of the teacher's key responsibility is to develop and use instructional designs that facilitate step-by-step attainment of increasingly complex competencies and skills (Stewart, 2021). Thus, behaviourism places educators at the centre of the teaching and learning process as they facilitate, determine and control the environment and resources that influence the process.

The teacher explains and demonstrates a concept and the students then practise the skills and techniques to solve the problem, with the teacher now positively reinforcing success and disapproving of failure (Klinger, 2009; Orton, 2004). In behaviourism, clear learning targets should be developed and learning should be structured to promote the competencies and skills needed to meet the targets (Stewart, 2012). Stewart notes that constructivists accept behaviourism as one component of the learning process since for a condition to be associated with a reward it must be incorporated with other concepts in an active process of schemata development and moderation. Klinger posited that in mathematics, cognitivism supplements behaviourism, especially when new concepts are being introduced. Moreover, "behaviourism has generally been proven reliable and effective in facilitating learning that involves discriminations (recalling facts), generalizations (defining and illustrating concepts), associations (applying explanations), and chaining (automatically performing a specified procedure)" (Ertmer & Newby, 2013, p. 49). In behaviourism knowledge is viewed as external and absolute to the learner, teaching as instructional, while in cognitivism the learning process is viewed as the act of internalising knowledge.

2.9 Cognitivism

Cognitivism focuses on internal mental processes as learners acquire knowledge and emphasises knowledge acquisition, processing, storage, retrieval and activation during learning (Clark, 2018; Yilmaz, 2011;-Pritchard, 2014). Internal mental processes such as critical thinking, recalling, recognising, understanding, reasoning and problem solving are the cornerstones of cognitivism (Clark, 2018; Hartsell, 2006). The brain is the centre and processor of all human action, behaviour, memory and it plays a central role in learning (Arponen, 2013; Ertmer & Newby, 2013; Watson & Coulter, 2008). Furthermore, cognitivism places value on ascribing meaning to learners' existing knowledge and linking new knowledge to past experiences (Yilmaz, 2011). Some of the key theorists who have contributed to cognitivism include Piaget (stages of cognitive development), Vygotsky (social cognitive growth) and Gagne (conditions of learning) (Clark, 2018; Yilmaz, 2011). Piaget's (1953) stages of cognitive development laid the foundation for cognitivism, namely, that mental growth develops in stages: motor skills, verbal expressions through mental imagery, abstract concepts, and sequential and logical reasoning (Zhou & Brown, 2015).

The extensive involvement of memory makes cognitivism relevant to learning complex concepts that involve problem solving and reasoning (Ertmer & Newby, 2013; Schunk, 2004). Central to student learning are teacher explanations, demonstrations and matched non-examples, all of which places the teacher at the centre of the learning process (Clark, 2018; Ertmer & Newby, 2013). The teacher's role is that of a coach in charge of the learning process (Ertmer & Newby, 2013; McLeod, 2003). Similarly, observations, practice, coaching, articulation and modelling can help learners acquire knowledge that includes strategies, skills, attitudes and rules (or theorems, as in mathematics) (Schunk, 2004; Yilmaz, 2011). As learners gain experience through active practice, their prior knowledge is modified or updated (Simmons & Watson, 2014).

In cognitivism, learners are active participants in the learning process because knowledge needs to be encoded, transformed, rehearsed, recalled and restructured (Ertmer & Newby, 2013). According to Clark (2018), knowledge consists of units called schema and several of these units form schemata or schemas which are stored in the long-term memory. When students are exposed to new knowledge through reading, observing, learning and experiencing, a new schema is formed, an old, related schema is updated or, if there is a contradiction, the old schema is altered to accommodate the new knowledge (Clark, 2018; Gillani, 2003). Schemas include misconceptions and inaccuracies and provide a foundation on which to map expected results when new knowledge is introduced (Clark, 2018; Gillani, 2003).

The main criticism of cognitive psychology which forms the foundation of cognitivism is that it is abstract and not directly observable (Garnham, 2019). Moreover, it ignores other reasons for behaviour such as environmental and social reasons (Stewart, 2021). Thus, it fails to account for environmental, biological or genetic influences on cognitive function. Importantly, Garnham (2019) posited that internal processes are not measurable and instead a behaviour or external feature that is believed to be associated with the internal process is measured without conclusive evidence of the connection.

2.9.1 Cognitivism in Mathematics Education

Teaching that considers prior knowledge does not only increase the chance of learners understanding by providing a step-by-step building of concepts; it may also provide opportunities of correcting students' misconceptions. "Students who are accelerated in their mathematics studies harbor misconceptions or knowledge in isolation that may make future connectivity with advanced mathematics problematic" (Bell, 2017). Being a teacher-centred approach, cognitivism demands that educators organise instructions in such a way that learners can connect new information with prior knowledge to make the learning meaningful and effective (Clark, 2018). Prior knowledge is critical for comparison purposes when learners are exposed to new knowledge and it provides some form of mental scaffolding (Ertmer & Newby, 2013; Yilmaz, 2011). However, a teacher's assumptions that students have prior knowledge of a linked concept might disadvantage those learners with only a limited schema to compare with stored memory. This is because hierarchical relationships in learning material help link prior experience to new knowledge (Ertmer & Newby, 2013). The stages of cognitive development are hierachical; thus, they play a significant role in the teaching of mathematics, which is a hierachical subject.

The stages of cognitive development play a central role in mathematics teaching and learning especially at senior secondary school. Mathematics education has benefited from understanding stages of cognitive development because new knowledge on how children learn can support teaching (Ghazi et al., 2014). At senior secondary level, students are expected to be at the formal operational stage where they can form hypotheses, build their own mathematical understanding, evaluate options and use abstract concepts to represent a mathematical thought (Ojose, 2008). Learners are expected at this stage to solve problems involving application in real life, justifications, generalisations and mathematical inferences (Anderson, 1990). Brain & Mukherji (2005) go further to explain that at the formal operational stage, conceptual words or symbols can replace the physical representation of objects. Teachers can benefit from understanding what this stage entails as they prepare instruction.

Lessons based on cognitivist principles should include probing questions to gain learners' attention, a clear statement of lesson objectives, stimulation of prior knowledge, well-organised content presentation, practice, clear instructions for learners, corrective feedback, assessment and relating content to real-world problems (Clark, 2018; Ertmer & Newby, 2013; Yilmaz, 2011). A cognitivist educator's main role is to drive the learning process. Teachers have to break down information to make it easier for learners to accommodate new information with their existing schema. Furthermore, clear instructions and timely corrective feedback that guide mental processes can maximise learning (Clark, 2018; Ertmer & Newby, 2013). Breaking down complex problems into smaller parts can help students comprehend the problem better. Similarly, observing educators modelling complex skills can help learners acquire those skills, while practice can help teachers give corrective feedback and provide a variety of information sources that expand knowledge and inform and motivate learners (Schunk, 2004). Although practice is important, cognitivists put more value on observation (Schunk, 2004). And as much as observation can help students identify and acquire mathematical skills, practice is equally important as it provides an opportunity for feedback that will eliminate misconceptions and result in deeper understanding.

2.10 Relationship between Learning Theories in Mathematics Teaching and Learning

Mathematics education aims to promote the development of mathematical knowledge that goes beyond just recalling mathematical facts and imitation (Thompson, 1985). Thompson goes further, saying, "Constructing mathematical knowledge is the creation of relationships, and creating relationships is the hallmark of mathematical problem solving" (p. 3). Learning is effective when it is linked to real-life problems that create meaningful context and insight for learners (Ertmer & Newby, 2013).

Effective learning must include students actively developing knowledge with the assistance of resources that are not limited to an educator (Ertmer & Newby, 2013; Stewart, 2012). Both constructivism and cognitivism acknowledge that students must be active in the learning process. Moreover, the importance of prior knowledge in developing new knowledge is key in fostering the active participation of students. This is highlighted by Stewart's (2012) work, which posited that learners do not come to the learning process empty for the teacher to just fill them up with knowledge.

Tasks that involve mathematical facts, rules and their simple applications are more suited to behaviourism and cognitivism, while those that demand advanced level mathematical abilities, for example, investigative problem-solving tasks, to a large extent require a constructivist mindset (Ertmer & Newby, 2013; Klinger, 2009). Jonassen (1991a, cited by Ertmer & Newby (2013)) classified knowledge acquisition into three stages: introductory, advanced and expert, with constructivism more applicable in the advanced stage. The introductory level of new concepts in most mathematics lessons is the stage of acquiring mathematical facts, rules and basic skills, which is better suited to cognitivism and behaviourism (Klinger, 2009; Stewart, 2012). The expert level is when students have gained the knowledge to apply concepts in complex unfamiliar or unstructured problems, to validate and evaluate solutions and make generalisations. Constructivism may be more ideal at the stage when learners would have mastered concepts so they can apply concepts to real-world problems, such as problem-solving tasks that include extracting the mathematical meaning from word problems and applying concepts to new situations. It is important to note that pedagogy that is underpinned by appropriate learning theories has important implications for mathematics teaching and learning. Pedagogical resources such as mathematical representations and their role in supporting problem solving are explored in the next section.

2.11 Visual representations in mathematics

A number of research studies explore the idea of representation in mathematics education (e.g., Arcavi, 2003; Zazkis & Liljedahl, 2004; Stylianou, 2010). Teaching and learning that include visual representations of mathematics concepts result in improved performance in mathematics and lower cognitive load for students than learning without visual representations (Yung & Paas, 2015). Clearly, the significance of visual representations in mathematics is emphasised by Arcavi (2003):

> "Mathematics, as a human and cultural creation dealing with objects and entities quite different from physical phenomena (like planets or blood cells), relies heavily (possibly much more than mathematicians would be willing to admit) on visualisation in its different forms and at different levels, far beyond the obviously visual field of geometry, and spatial visualisation" (pp. 216-217).

To mathematicians, diagrams may be most beneficial for exploration of unfamiliar problems (Pantziara et al, 2009), and can be part of the creative process when used to develop novel diagrammatic representations (Zahner & Corter, 2010). In fact, "Structured diagrams are thought to be more comprehensible than just words, and a clearer way to illustrate understanding of complex topics." (Davies, 2011 p. 279). Although mathematicians have been aware of the value of visual presentations as tools for teaching and as heuristics for mathematical discovery, visual representation remains underutilised in both the theory and practice of mathematics (Barwise & Etchemendy, 1991). Importantly, creating, interpreting and reflecting on visual representations should be encouraged in mathematics and mathematics education as it promotes the development of previously unknown ideas and advances understanding (Arcavi, 2003; Zimmermann & Cunningham, 1991).

Significant research (e.g., Diezmann & English, 2001; Friedlander & Tabach, 2001; Lamon, 2001) identifies representation in mathematics as a tool for thinking, gaining insight and demonstrating understanding. Understanding can be demonstrated through the ability to develop or apply various representations and identifying the appropriate representation for a problem situation (Zazkis & Liljedahl, 2004). Visual representations have been found to be effective for supporting students' learning of content knowledge compared to text-based activities (Rau, 2017). Of importance is that this effectiveness is particularly observed if students have low prior knowledge (Mayer and Feldon 2014). Visual representation is recognised as a powerful teaching and learning tool as it can be used to focus on relevant information, promote relational and logical understanding, and support knowledge construction (Ainsworth & VanLabeke, 2004; Barmby et al., 2007; Yung & Paas, 2015), leading to deeper understanding. Similarly, visual solutions to a problem may enable engagement with concepts and meanings which could have been overlooked if a non-visual solution had been offered (Arcavi, 2003).

Visual representations offer many opportunities for a rich mathematics classroom experience. Availability and awareness of shared representations in a mathematics class create a social backdrop for mathematical discourse (Zazkis & Liljedahl, 2004), which can deepen mathematics understanding through reflection, meaning making and exchange of knowledge. Also, visual representations may play a central role in inspiring a complete solution, beyond the merely procedural (Arcavi, 2003). When students include the appropriate type of visual representation during problem solving, higher rates of solution success are observed compared to when they do not include representations (Zahner & Corter, 2010). In addition, visual representations may function as a guiding tool for situations in which students may be uncertain about how to proceed as they solve a problem (Arcavi, 2003). Undoubtedly, visual representations are a useful communication tool in mathematics (Rodriguez et al., 2020), thus they are key in promoting success in the field (Paoletti et al., 2022). Visual representations can structure and modify mathematical activities in class as they can influence the course and focus of a lesson and activities (David & Tomaz, 2012).

Importantly visual representations are tools that teachers can introduce to support students' learning in mathematics. Researchers (Ainsworth, 2006; Cobb, 2003; Dreher & Kuntze, 2015; Kaput et al., 2008) emphasise that representations are an essential and delicate issue for mathematics teaching and learning as it is a vehicle for capturing mathematics concepts. "The use of a variety of representations in a flexible manner has the potential of making the learning of mathematics more meaningful and effective" (Stylianou, 2010, p. 327). In fact, teachers should be aware that students learn more through multiple representations when they are provided with an opportunity to self-explain the relationship between the different representations (Rau et al., 2009) thus providing a multi-faceted concept image (Dreher & Kuntze, 2015). When visual representations are developed by students, they become an assessment tool to gain insight into students' thinking, reasoning and understanding (Stylianou et al., 2000). The representation that a teacher chooses to use in class can impact classroom discussion and facilitate students' attention to particular mathematics connections and concepts (Stein et al., 2008). Contrastingly, some research found that visual presentation serves a peripheral and limited role in teacher instruction, because teachers have limited knowledge of their role and how best to use the different forms (Dreher & Kuntze, 2015; van Garderen et al., 2018; Stylianou, 2010; Sullivan et al., 2019).

Representational competencies (knowledge about how visual representation represent information about the content) are required for visual representations to be effective during teaching and learning (Rau, 2017). "Teachers' conception of representation as a process and a mathematical practice appears to be less developed, and, as a result, representations may have a peripheral role in their instruction as well" (Stylianou, 2010, p. 325). However, every time students are introduced to a new representation; they must learn how it is used and interpreted (Rau, 2017). Therefore, visual representations should be fostered explicitly so as not to impede learning (Dreher, 2012; Renkl et al., 2013). Research identifies multiple benefits for students' learning; however, representations may fail to support students' learning if they are not used in the "right" way (Rau & Matthews, 2017). For example, they have to clearly represent the information they intend to convey, otherwise they will confuse students. With the increased push to include mathematics representations in class, this may place tremendous pressure on teachers as there is little evidence that necessary support is in place to implement such a move (Stein et al., 2008; Stylianou, 2010). Thus, teachers may have limited expertise in the use of visual representations when teaching mathematics (Izsák, & Sherin, 2003). Research (see Ball 1993; Dreher & Kuntze, 2015) notes a lack of awareness amongst mathematics teachers that the use of visual representations can support the development of mathematical thinking, hence deeper understanding of mathematical knowledge. Although, research demonstrates several benefits of including representation in mathematics education, teachers still find it challenging as how to incorporate them in the curriculum is not well articulated (Stylianou, 2010). Teachers might have a narrow view that representation is a tool for a selected few students, therefore it is the responsibility of those who support or prepare teachers to demonstrate and support the expansion of teachers' use of representation across the mathematics curriculum (Morris, 2008; Smith et al., 2009; Stylianou, 2010). This study will develop visual representation tools and support teachers on how they can be incorporated in teaching and learning of mathematics including problem solving.

2.12 Problem solving and Visual Representation.

Problem solving is seen as a key and significant aspect of mathematics and mathematics education. It permeates mathematics curricula across the world resulting in calls for the teaching of problem solving as well as the teaching of mathematics through problem solving growing louder (Liljedah et al., 2016). However, there is no agreed definition of problem solving (see English & Gainsburg, 2016). This study will use Hegedus's (2013) definition, which stipulates that: "problem solving is not just about solving a specific problem, which has a specific answer or application in the real world, but rather it is an investigation that might have multiple approaches and where students can make multiple observations" (p. 89). Importantly, the process should allow construction of meaning in open-ended, non-procedural tasks which will have been carefully developed to have mathematical purpose (Hegedus, 2013; Mamona-Downs & Downs, 2013). The problem-solving process is a dialogue between the prior knowledge the problem solver possesses, the tentative plan of solving the problem and other relevant thoughts and facts (Schoenfeld, 1983). Research in problem solving provides deeper understanding on the subject and offers insight into directions for future research.

For decades, research in mathematics problem solving, including special issues from leading mathematics education journals (see, *Educational Studies in Mathematics,* (Vol. 83, no. 2013); *The Mathematics Enthusiast,* (Vol. 10, nos. 1–2); *ZDM*, (Vol. 39, nos. 5–6)), have offered significant insights but struggled to produce well-articulated guidelines for educational practice (English & Gainsburg, 2016). This could possibly be the reason why mathematics teachers' efforts to improve students' problem-solving skills have not produced the desired results (Anderson, 2014; English & Gainsburg, 2016). Despite Polya's (1945) list of steps and strategies being so valuable in successful problem solving, there appear to be limited success when translated into the classroom environment (English & Gainsburg, 2016). English and Gainsburg went further to posit that one of the issues to be addressed is how to support problem-solving competency. However, use of visual representation as a tool that can support problem solving is well documented (see Krawec, 2014; Stylianou, 2008). This study will explore how visual representations such as procedural flowcharts can help to build problem-solving competency.

Research on how visual representations support mathematics discovery and structural thinking in problem solving has come a long way (see Hadamard, 1945; Krutetskii, 1976; Polya, 1957). Visual representations can be used as a tool to capture mathematics relations and processes (van Garderen et al., 2021) and used in many cognitive tasks such as problem solving, reasoning, and decision making (Zhang, 1997). Indeed, representations can be modes of communicating during concepts exploration and problem solving (Roth & McGinn, 1998). Likewise, visual representations can be a powerful way of presenting the solution to a problem, including self-monitoring on how the problem is being solved (Kingsdorf & Krawec, 2014; Krawec, 2014). Using visualisations created by teachers or students in mathematics can support students' problem-solving abilities (Csíkos et al., 2012). Furthermore, visual representations can be used to facilitate different subtasks during problem solving, for example, as a tool to facilitate exploration of concepts (Stylianou, 2008; Stylianou & Silver, 2004). They can be used to illustrate the problem-solving process and to create connections among concepts (Stylianou, 2010).

2.13 Chapter Conclusion

The literature reviewed in this section shows that students are influenced by intrinsic and extrinsic factors when enrolling in calculus-based mathematics. The effectiveness of mathematics teachers is one of the main contributing factor that influences enrolment, participation and achievement in calculus-based mathematics. Importantly, in this chapter, literature has identified the scope of resources that can be used to support teachers in the teaching and learning of mathematics. Understanding of learning theories that are used by mathematics teachers is critical so as to explore ways the resources can be more effective in supporting teaching and learning.

Teaching and learning of mathematics are informed by learning theories with a focus on maximising the impact of the teacher and other resources in influencing the learning process. Importantly, it considers how students develop mathematical knowledge and how they can represent that knowledge for educators to determine changes. Undoubtedly, teachers might benefit with increased access to resources that are underpinned by such theories. Moreover, for teachers to guide learning they need to be supported by resources that can promote student-centred learning and the gradual development of new knowledge from prior knowledge. Such resources can facilitate active participation of students in the learning process, both individually and collaboratively. On the contrary, lack of such resources may have a negative impact on student participation in mathematics, leading students to view mathematics as a difficult subject, with unrelated concepts and uninteresting calculations that need to be committed to memory. In this study, constructivism informed the conceptual framework.

When senior high school mathematics teachers are exposed to new knowledge, they can relate it to their own experiences, ideas and skills in teaching, evaluate it and then discard it, accept it or modify their practice, thus constructing new knowledge. The active participation of teachers in developing, applying and evaluating resources though workshops, surveys and interviews brought different perceptions together to guide this study.

Chapter 3: Methodology

3.1 Chapter Introduction

This chapter outlines the theoretical position that frames the research questions and the methodology used to address them. It then describes the specific research methods used within this methodology. The primary focus of this study was to investigate, develop and explore pedagogical resources (framework on content sequencing, concept maps and procedural flowcharts) for mathematics teachers that will support students' participation in calculus-based mathematics, drawing examples from the topic of functions. This researcher holds the view that teaching and learning play an important role in influencing participation in calculus-based mathematics and that teachers as classroom practitioners need to be supported through research-informed resources to be more effective in delivery. Since their input to and perceptions of such resources is central to their effectiveness and possible adoption, teachers, inferring from their experience, beliefs and skills, were at the centre of this study no how new knowledge can be constructed. Choosing the appropriate research methods consequently played a significant role in deepening knowledge and validating findings.

3.2 Theoretical Framework

This study was conceptualised within a constructivist epistemology. The researcher holds a constructivist view that individuals construct knowledge when they purposefully interact, share and reflect on beliefs and experiences and several other researchers, such as Cahyono (2018), Cobb (1994) and Mita et al. (2017), have used constructivism to underpin their research in mathematics education. Constructivists believe that knowledge is constructed by the learner, not just transmitted from the educator to the learner (Narayan et al., 2013). From a constructivist viewpoint, learning is about creating knowledge, not just receiving, about understanding and applying, not just recalling, and about thinking and examining, rather than just gathering and memorising (Gordon, 2008). This study used constructivism as a theoretical framework because mathematics teachers as participants were involved in constructing knowledge that informed their daily practice of teaching. Involvement of mathematics teachers provided them with the opportunity to be actively involved in drawing from their experience and creating new knowledge.

Constructivists strongly believe that during learning, individuals are expected to be actively involved, and not passive recipients of knowledge (Lew, 2010). As Jenkins (2000, p. 601) states, "If there is common ground among constructivists of different persuasion it presumably lies in a commitment to the idea that the development of understanding requires active engagement on the part of the learner." As a result, the teacher's role is that of a facilitator or organiser who indirectly encourages and manages learners to "research, discover, and make conclusions" (Tomljenovic & Tatalovic Vorkapic, 2020, p. 15). Active participation of senior mathematics teachers in this study brought diverse experiences, which then facilitated the development of teaching and learning resources. Mathematics teachers were given the opportunity to apply the pedagogical resources and then evaluate the resources as active research participants. The researcher was a facilitator of meetings and workshops, working with mathematics teachers as practitioners on strategies to support teaching and learning of mathematics. In summary, participants were actively constructing new knowledge using their relevant existing knowledge.

Constructivists view learning as a process of building new knowledge from beliefs, ideas, skills and prior experience (Garbett, 2011; Bruning et al., 2004; Taber, 2019). In constructivism, it is the learners who are the creators of knowledge, as they make sense of new knowledge using their existing knowledge and cognitions (Taber, 2019). Constructivism is "an approach to learning that holds that people actively construct or make their own knowledge and that reality is determined by the experiences of the learner" (Elliott et al., 2000, p. 256). During the process of learning, "new insights are compared with previous experiences and ideas, whereby old beliefs may be altered, or new information may be dismissed as irrelevant" (Tomljenovic & Tatalovic Vorkapic, 2020, p. 15). Hence, learners are active participants in their learning as they interpret the meaning of new knowledge and reference it to what they already know (Garbett, 2011). In other words, the mind filters everything that it is exposed to and creates its own meaning which results in new knowledge being developed. The researcher holds the view that mathematics teachers' beliefs, experiences, perceptions and skills are resources that can be used to create and evaluate new knowledge. Exposing mathematics teachers to new ideas and resources provided them with the opportunity to compare them with previous ideas, resources, beliefs and experiences which resulted in them rejecting or adopting new ideas. Constructivism in this study provided the basis to actively involve teachers to interpret meaning of the new knowledge and how best the pedagogical resources could be utilised to benefit teaching and learning processes.

Interactions between this researcher and teachers around the sharing of opinions and experiences and evaluating a program provided an opportunity to construct new knowledge. Constructivists emphasise that "knowledge is socially constructed through interaction of the researcher with research participants" (Tavakol & Sandars, 2014, p. 747). When individuals interact within a social setting, they have an opportunity to generate knowledge (Kim, 2001). Hence, "the individual mind becomes collective mind through social phenomena such as relationships, participations, negotiations, and sharing" (Belbase, 2011, p. 3). Collaboration among individuals with varied experiences provide the opportunity to co-create new knowledge (Powell & Kalina, 2009). The coming together of the researcher and senior mathematics teachers on different platforms that included a mathematics teachers' conference, workshops and check-in sessions created a social engagement that facilitated an exchange of knowledge. Constructivism emphasises the importance of co-creating new knowledge and this co-creation of knowledge informed this research. Participants brought their different experiences and qualifications which constructivism identifies as a strength in supporting knowledge development. Therefore, the active interaction between the researcher and senior mathematics teachers and the sharing of experiences, beliefs and ideas was crucial in developing and evaluating the planning and teaching resources in this study.

3.3 Methodology

It is important to clarify the use of the terms 'methodology' and 'methods' before providing a description of the methodology used in this study. Research methodology incorporates all the steps involved in the study starting with research design, data collection and analysis as well as the social, ethical and political viewpoints that the researcher brings to the study. According to (Kothari, 2004, pp. 7-8), "research methods may be understood as all those methods or techniques that are used for conduction of research while research methodology is a way to systematically solve the research problem". During this study the methodology explained the logic behind the method or technique and contextualised it to the study. In this case, research methods are part of the overall research methodology of this study.

The research methodology directs and guides activities involved in the research. Importantly,

> "A research methodology is like a strategy encompassing principles, processes, procedures and techniques to seek a solution to an identified research problem. In some sense, the methodology provides an architecture for the entire research exercise that determines the research methods to be applied in a given research exercise, developed to proceed from an understanding of the research question(s) and oriented towards providing direction and guidance to the whole effort to seek the answer(s) to the question(s)" (Mukherjee, 2020, p. 20).

Furthermore, the research methodology provides directions for designing and executing evidence-based research that include quantitative and qualitative data (Acharyya & Bhattacharya, 2020). Therefore, knowledge of research techniques and procedures and where they can be best applied plays an important role in formulating an effective research methodology considering there are two basic research approaches in educational research: quantitative and qualitative.

Quantitative research is used on a phenomenon that can be expressed in numeric data. It is influenced by the positivist paradigm that suggests reality is concrete or singular and must be independent to the opinion and influence of the researcher (Tavakol & Sandars, 2014). Moreover, quantitative research is ideal for use when the sample or population size is significantly large (Kaplan, 2004). It involves testing relationships between variables to determine patterns and correlations with the primary purpose of explaining and evaluating (Leavy, 2017). Undertaking statistical calculations on numeric data to draw conclusions that answer a research question is central to quantitative research (Habib et al., 2014). Therefore, it was ideal to use quantitative research to investigate trends in students' participation in Queensland as enrolment data is numerical and covers all students in years 11 or 12 state-wide. Moreover, correlation between different factors that may affect enrolment can also be investigated. "The fundamental goal of quantitative research is to make a convincing argument based on numerical data in response to a research question" (Hjalmarson & Moskal, 2018, p. 179). Research that involves collection of numeric data mostly requires quantitative analysis. However, if non-numeric data is involved, qualitative analysis is more appropriate.

Qualitative research is more concerned with the individual's personal experiences of the problem under study. It aligns with post-positivist paradigm or constructivist beliefs that several individually formulated realities exist, and knowledge and participants cannot be separated. It is defined as

> "the study of the nature of phenomena's quality, different manifestations, the context in which they appear or the perspectives from which they can be perceived without involving their range, frequency and place in an objectively determined chain of cause and effect" (Philipsen & Vernooij-Dassen, 2007 p. 5).

Similarly, qualitative research is the collection, analysis, and interpretation of data using observation and what participants say through interviews (Habib et al., 2014). In fact, "the actual words of people in the study, offer many different perspectives on the study topic and provide a complex picture of the situation" (Creswell, 2014, p. 535). Qualitative research is used to unpack and explore meaning people attribute to activities, situations, events, or artefacts (Leavy, 2014). Moreover, it focuses upon drawing meaning from the experiences and opinions of participants (Cohen et al., 2011). Therefore, qualitative research was used in this study for teachers to share their individual experiences, opinions, and context to gain a deeper understanding of the phenomenon. It is used to explore and explain people's subjective experiences and meaning-making processes and acquiring a detailed and in-depth understanding which is ideal even with a small sample (Leavy, 2017).

Quantitative and qualitative designs have their own weaknesses. The main limitation of quantitative research is its lack of detail and context that might help to provide deeper understanding of the phenomenon (Griffin & Museus, 2011; Johnson & Onwuegbuzie, 2004). Similarly, a key limitation of qualitative research is that it might be influenced by the researcher and because personal experiences differ, the findings may not be generalisable to other contexts (Griffin & Museus, 2011). Importantly, the weaknesses of both quantitative and qualitative research can be offset by the strengths of both as words (from the qualitative approach) can add meaning to numbers (from the quantitative approach) and numbers can add clarity to words (Johnson & Onwuegbuzie, 2004).

Mixed methods approach involves both quantitative and qualitative research. A mixed methods approach provides the platform to optimise the opportunities offered by quantitative and qualitative research as well as address the limitations of both (Johnson et al., 2007). It is used mostly where use of either the quantitative or qualitative approach will not be sufficient to gain a deeper understanding of the problem (Creswell, 2014), as it integrates the two (Creswell & Zhang, 2009). More, importantly qualitative data can be used to support or validate findings from quantitative data (Fetters et al., 2013). It is important to note that mixed methods research is:

> "the type of research in which a researcher or team of researchers combine elements of qualitative and quantitative research approaches (e.g., use of qualitative and quantitative viewpoints, data collection, analysis, inference techniques) for the broad purposes of breadth and depth of understanding and corroboration" (Johnson et al., 2007, p. 123).

Therefore, this study adopted a pragmatic approach as it spanned the middle ground between quantitative and qualitative research because both numeric and textual data were collected and analysed to address the same problem (Griffin & Museus, 2011).

Quantitative and qualitative research have the potential to provide concrete analysis; however, individually they have limitations especially in intersectional research analysis (Griffin & Museus, 2011). Despite the limitations, "qualitative methods are, for the most part, intended to achieve depth of understanding while quantitative methods are intended to achieve breadth of understanding" (Palinkas et al., 2013, p. 534). A mixed methods approach was adopted in this study to allow the generation of valuable data from senior mathematics teachers on the teaching of calculus-based mathematics options. The study positioned itself on exploiting the strength of both qualitative and quantitative research approaches. The approach aligns with Rocco and colleagues (2003) who noted that research is more robust when it mixes research paradigms, as a fuller understanding of the phenomenon can be gained. Similarly, Creswell and Plano (2011) posited that mixed methods enables a greater degree of understanding to be formulated than if a single approach were adopted to specific studies. Thus, mixed methods approach is ideal for the purposes of breadth and depth of understanding and corroboration (Greene, 2007). The collection of both quantitative and qualitative data can provide a more complete picture and better understanding of the research questions compared with using either one of the methods alone (Guetterman et al., 2015).

A mixed methods approach is generally appropriate when the purpose is to describe, explain or evaluate phenomenon (Leavy, 2014). In this study senior mathematics teachers as participants explained their views and experiences as well as evaluated the pedagogical resources developed in the study. Importantly the mixed methods approach allows triangulation of data, meaning “collecting and converging or integrating different kinds of data bearing on the same phenomenon” (Creswell, 2014, p. 536). Triangulation provides opportunities for convergence and corroboration of results that are derived from different research methods which enhances validity of data (Creswell & Plano, 2018).

Similarly, mixed methods help to deepen and broaden the understanding of the phenomenon under study, hence providing opportunities for future research (McKim, 2017; O’Cathain et al., 2010). The study’s focus on a relatively less researched area of teaching of calculus-based mathematics at senior secondary level provides insight into what teachers view as important in the delivery of this subject at this level. Therefore, the methodology in this study employed both qualitative methods, involving semi-structured and in-depth interviews, and quantitative methods such as survey instruments. These data collection methods were the most appropriate for addressing the research questions outlined below and thus the methodology of this study is a mixed methodology.

3.4 Research Design

A research design explains how a study seeks answers to the research questions. It is a conceptual layout that guides how the research will be conducted (Dubey & Kothari, 2022; Kothari, 2004). A research design is a plan that recognises the processes and procedures to be followed during data collection and analysis (Creswell, 2014; Habib et al., 2014). This study comprised four phases to address the overarching research questions, outlined below:

1. What are the trends in Queensland senior students' enrolment in calculus-based mathematics subjects?
2. What pedagogical resources support the planning, teaching and learning of Mathematical Methods for Queensland senior students?

3.4.1 Phase One

The research question addressed in this phase was:

- What are the trends in Queensland senior students' enrolment in mathematics subjects?

The intention of Phase One was to investigate Queensland senior students' mathematics enrolment in different mathematics curricula options from 2010 to 2020.
Until the end of 2018, the mathematics options at senior level in Queensland were Mathematics A, B, C and Prevocational. In 2019, these were changed to Essential, General, Methods and Specialist Mathematics. This new curriculum brought some changes to the mathematics options that were offered as well as to the assessment policy because Queensland introduced an external examination at the end of Year 12. An additional study was undertaken in this phase to further understand trends in enrolment and some of the external factors that influence students' participation in calculus-based mathematics under the new curriculum. The sub-question addressed by this additional study was: What is the relationship between students' enrolment in calculus-based mathematics in the new Queensland curriculum and school level indicators such as socio-economic status, school location and transfer rating?

Quantitative methods were used to analyse data from the Queensland Curriculum and Assessment Authority (QCAA) to identify trends in student enrolment in different mathematics options. This method was most appropriate because "quantitative research identifies a research problem based on trends in the field or on the need to explain why something occurs" (Creswell, 2014, p. 13). Permission was sought from the QCAA and consent was given to use publicly available data on students' mathematics enrolment at senior secondary level from 2010 to 2020. Likewise, quantitative data from the Australian Bureau of Statistics' Socio-Economic Index for Areas (SEIFA), Schools Index of Community Socio-Economic Advantage (ICSEA) and schools transfer ratings were used to determine their influence on students' enrolment in calculus-based mathematics options. Data was analysed using Excel suite and Statistical Package for the Social Sciences (SPSS). SPSS was used to analyse correlation between the different factors that affect enrolment in calculus-based mathematics. Excel can be used to analyse data in quantitative research and SPSS is ideal for developing comparative graphs especially on trends (Davis & Davis, 2016; Kolluri et al., 2016).

3.4.2 Phase Two

The sub-questions addressed in Phase Two were:

- What framework for content sequencing can support linking of concepts from junior to senior mathematics?
- What teaching and learning resources can support students' participation in senior mathematics?

The aim of this phase was to develop pedagogical resources that could support planning, teaching and learning of calculus-based mathematics with a special focus on functions in mathematical methods. Indeed, pedagogical decisions and resources teachers use during teaching can play an important role in influencing student participation and achievement (Little, 2020; Witterholt et al., 2016). Furthermore, pedagogical resources can be used to; support teacher capacity, build concepts from prerequisites and experimenting, supporting deeper understanding of concepts (Larson & Murray, 2008). Thus, this phase will focus on developing, and identifying pedagogical resources that can support mathematics teachers in the delivery of calculus-based mathematics.

Planning is key to effective teaching and learning as it can be used to link resources to the content and the teaching method. The study therefore argued first for content sequencing in the planning and teaching of mathematics to aid transition from junior to senior mathematics. A coherent sequencing of content supports meaningful reflection on concepts and the nature of mathematics (Conner et al., 2011), which is important for mathematics teachers and students. To this end, the literature was synthesised to develop an original mathematics planning framework on content sequencing. Synthesis involves pulling various sources together into some kind of harmony so that the sources combine clearly and coherently with your own (Clevenger, 2011, p. 1). This provided the opportunity to build on previous findings, integrate existing findings and identify gaps (Grant & Booth, 2009). Creating a learning environment in which students' participation is anchored on creating skills and knowledge based on prior experience is one of the most effective pillars of a robust and effective teaching methodology (Hailikari et al., 2008). The planning framework for this study emphasised the importance of prior knowledge and the hierarchical and spiral nature of mathematics and mathematics teaching respectively.

This phase laid the foundation of a framework on how content sequencing in schools play a significant role in effective linking of prior knowledge to new knowledge. A framework specifies the relationships between the constructs within a phenomenon (Johnson & Morgan, 2016) and advocates planning that focuses more on how new knowledge is developed from relevant concepts that students have been exposed to in previous levels. The relevant prior concepts should be clearly linked to the new knowledge using the framework on content sequencing.

Second, the development of tools that can be used by teachers to promote procedural and conceptual knowledge in mathematics was equally important. Such tools would play a critical role in supporting the teaching and learning of mathematics. The involvement and input of teachers in the development of the pedagogical resources is important because the sharing of their experience, skills and ideas makes them active participants in the process. Teachers as architects of planning, teaching and learning were active participants in this study.

The regional Association of Mathematics Teachers conference for the Cairns region held on the 28th of May 2021 presented an opportunity to share the initial draft of the pedagogical resources with the region's mathematics teachers. The resources included associated examples from the Functions section in Unit 1 to demonstrate how the resources were applied in Mathematical Methods. The researcher conducted an interactive presentation with mathematics teachers at this conference and teachers provided feedback verbally and through survey questions. The survey consisted of Likert scale items and open-ended questions. A more detailed description of these instruments is provided in the data collection section below. The teachers' feedback and contributions were used to improve the pedagogical resources. The planning tool also provided teachers with an opportunity to brainstorm concept development, thus deepening their understanding. The understanding that complex unfamiliar questions are developed from simple familiar concepts was enhanced, demystifying mathematics by demonstrating the importance of every level in learning the subject. Vocabulary development was also prioritised in the planning tool to keep pace with the development of the students' mathematical knowledge.

The outcomes of Phase Two included development of a framework on content sequencing and associated pedagogical resources to support teaching and learning of mathematics. It was important to evaluate the framework and pedagogical resources with a sample of senior mathematics teachers in Queensland; this was undertaken Phase Three, which is described below.

3.4.3 Phase Three

The sub-questions addressed in Phase Three were:

- What are teachers' perceptions of a planning framework on content sequencing for the teaching and learning of mathematics?
- What are senior secondary teachers' perceptions on how concept maps support the teaching and learning of mathematics at senior secondary school?
- What are teachers' perceptions on how procedural flowcharts support teaching and learning of procedural fluency in the Mathematical Methods subject?

This third phase of the study generally focused on evaluating the framework and pedagogical resources that were developed in Phase Two with a sample of senior mathematics teachers in Queensland, as described above. It used both quantitative and qualitative research to enable triangulation of results from both types of data and increase the validity of the findings (Yin, 2009). The inclusion criteria for participant selection were teachers currently teaching or who had taught mathematics, especially calculus-based options at senior high school level, that is, Years 11 and 12 in Queensland. Purposive sampling was used to select the 16 participants. Purposive sampling involves identifying and selecting knowledgeable participants or those who have experienced the phenomenon of interest and are available and open to share their experiences and opinions (Bernard, 2011; Cresswell & Plano, 2011). Purposive sampling was used because participants are selected by virtue of their capacity to provide richly- textured information about the phenomenon being investigated (Patton, 2002; Vasileiou et al., 2018). Purposive sampling has shown greater efficiency compared to random sampling in research that involve qualitative data (van Rijnsoever et al., 2017). The collection of both quantitative (Likert-scale survey items) and qualitative data (open-ended surveys questions and interviews) from the teachers after they had engaged with the framework and pedagogical resources provided a more complete picture and better understanding of results compared with using either one of the methods alone (Creswell, 2015).

Table 3.1 Demographic Information for Participants

Participant	Gender	Data collection method	Qualifications	Schooling system	Number of years teaching mathematics
1	Female	Survey and Interview	Masters	Public	31
2	Female	Survey and Interview	Bachelors	Private	10
3	Female	Survey and Interview	Bachelors	Public	25
4	Female	Survey and Interview	Bachelors	Public	17
5	Male	Survey and Interview	Bachelors	Private	19
6	Male	Survey and Interview	Bachelors	Public	11
7	Female	Survey and Interview	Masters	Public	15

8	Male	Survey and Interview	Masters	Public	19
9	Female	Survey	Bachelors	Public	11
10	Female	Survey	Bachelors	Public	15
11	Male	Survey	Bachelors	Public	7
12	Male	Survey	Bachelors	Public	9
13	Male	Survey	Bachelors	Private	10
14	Female	Survey	Bachelors	Public	13
15	Male	Survey	Masters	Public	5
16	Female	Survey	Bachelors	Private	11

A half hour video presentation was developed to articulate the framework and pedagogical resources (concept maps and procedural flowcharts) with associated examples. This video presentation was used during a workshop with teachers to train them in how to apply the framework and pedagogical resources in their classroom practice.

3.4.3.1 The Presentation to participants

The video presentation started by explaining to participants that the research developed resources that can support mathematics teachers during planning for content sequencing and representation of mathematics knowledge. The participants were invited to apply the resources in their teaching and learning for a full school term then share their opinions, feedback and experiences on the level of support the resources offered in their practice. Firstly, the presentation focused on the framework on content sequencing by unpacking its four pillars:

1. What exactly do students need to know and be able to do in this unit?
2. What prerequisites, conceptual understanding and skills are necessary for students to effectively learn new knowledge?
3. How do the concepts identified as prior knowledge link with new knowledge?
4. What do we expect students to retain?

A demonstration was given on how the framework on content sequencing is used to sequence mathematics content using an extract from a section of the QCAA Mathematical Methods syllabus document. The extract is on Functions.

In this sub-topic, students will:

- understand the concept of a relation as a mapping between sets, a graph and as a rule or a formula that defines one variable quantity in terms of another
- recognise the distinction between functions and relations and use the vertical line test to determine whether a relation is a function.
- use function notation, domain and range, and independent and dependent variables.
- examine transformations of the graphs of $f(x)$, including dilations and reflections, and the graphs of $y=af(x)$ and $y=f(bx)$, translations, and the graphs of $y=f(x+c)$ and $y=f(x)+d$; $a,b,c,d\in R$
- recognise and use piece-wise functions as a combination of multiple sub-functions with restricted domains.
- identify contexts suitable for modelling piece-wise functions and use them to solve practical problems (taxation, taxis, the changing velocity of a parachutist).

(QCAA, 2018 p. 20)

During the presentation the presenter went through the processes advocated by the framework on content sequencing in addressing the first pillar: identifying key words then combine related key words to determine main conceptual connections. For the second pillar teachers were shown how to develop a concept break-down table by identifying the synonym of key words, defining key words, identifying prior knowledge of concepts through backward mapping and identifying conceptual connections. An example of a concept break-down table developed from the section on functions in Table 1 was shared with the teachers.

Table 3.2: Concept break-down table shared with participants during video presentation

Keyword	**Definition of keywords (were possible)**	**Assumed prior knowledge concept linked to keyword**	**Vocabulary transition**	**How assumed prior knowledge link with new knowledge**
Relations (Sets, domain, range, independent and dependent variables, rule, functions, mapping, piecewise, vertical line, graph and restricted domain)	Domain -set of all the first (x) coordinates of ordered pairs- independent variable. Range – set of all second (y) coordinates of ordered pairs- dependent variable. -a relation defines the relationship bet ween sets of values of ordered pairs	Cartesian Plane, ordered pairs, sets, tables of values of graphs, inequalities, linear and non-linear equations and graphs.	-x-values that satisfy a graph – Domain. -y-values that satisfy a graph – Range. -inequalities – restricted domain -Combination of linear and non-linear equations is piecewise. -Ordered pairs – Relations.	-In ordered pairs the set all x (first) coordinates represent the domain (independent variable) and the set of y (second) coordinates is the Range (dependent variable). A vertical line is a line parallel to the y-axis. (Yr 7 & 8). The relationship between the x and y is the rule, formula, equation or mapping, arrow diagrams. Represent linear and non-linear

				equations graphically after using general substitution (start Yr7) to create a table of values. Represent quadratic equations graphically (Yr 9 &10). Inequalities solutions makes a statement true.
Transformations (Dilation, reflection and translation)	**Transformation-** Changing a shape using turn, flip, slide and resize.	Flip, slide, resize	Flip- Reflection Slide- Translation Resize- Dilation	Rules of translation- translating horizontally or vertically. Reflection about the x and y axis (Yr 7). Enlargement and reduction as a form of dilation (Yr 9).

The third pillar focused on identifying essential concepts through synthesising concepts under the keywords' column in the concept break-down table. In this example the essential concepts were functions, relations and transformations. The fourth and last pillar developed the sequence guided by conceptual connections, prior concepts and hierarchical nature of mathematics. Using the three guiding principles, a hierarchical table for the identified content on a section on functions in Table 2 was shared with the participants.

Table 3.3: Hierarchical table shared with participants during video presentation

Domain & Range	**Relations (mapping and graphing)**	**Transformations**
Cartesian plane Ordered pairs Sets Table of values Domain and range Inequalities Restricted domain	Rule (general substitution into linear and non-linear equations) Independent and dependent variable. Sketch graphs from tables of values Vertical line test Piecewise functions	Flip- reflection Slide- translation Resize – dilation combinations

Using all the pillars of the framework on content sequencing the final sequence for teaching the content under consideration was presented as follows:

- Cartesian Plane
- Ordered pairs.
- Sets
- General substitution
- Relations, rule, mapping- linear and non-linear functions
- Tables of values
- Domain and range
- Inequalities
- Restrict domain.
- Graph linear and non-linear
- Vertical line test
- Piecewise

- Flip (reflection), slide (translation), resize (dilation) and combination of transformations.

Participants were encouraged to use the framework on content sequencing collaboratively with colleagues who were teaching the same year level as well as across different levels so as to also gain feedback from them. In concluding this section of the presentation, the researcher identified key concepts at junior level which are critical for the teaching and learning of functions in Mathematical Methods at senior secondary level.

The second section of the presentation focused on representations of mathematical knowledge focusing mainly on conceptual knowledge, procedural knowledge and fluency. Firstly, the researcher introduced procedural flowcharts as a resource that can be used to represent mathematics procedures. The flexibility of procedural flowcharts was illustrated through the representation of more than one procedure on a single flowchart. Their ability to guide decision making during problem solving and communicating the solution to the problem was highlighted and participants were encouraged to explore how they could adopt the flowcharts more broadly in their practice. Importantly the researcher discussed the different ways that procedural flowcharts could be used:

- Procedural flowcharts could be used as a resource by teachers as they outlined the procedure to solve a problem.
- Students could use them to communicate the procedure as they solved a problem.

In summary the researcher explained to teachers that this resource was mainly developed to support teaching and learning of procedural knowledge and developing procedural fluency. An example of a procedural flowchart in Figure 9 (see Chapter 9) was shared with the participants.

In the second part of the presentation, the researcher started by emphasising that concept maps can be used to create a web of connections, thus leading to conceptual understanding. However, in this study the main focus was to use concept maps in linking junior concepts to senior concepts and showing how prior knowledge connects or supports the construction of new knowledge. The researcher referred participants to the important role of the content sequencing framework in identifying prior concepts in developing new concepts. Using the results from the section on functions, the researcher emphasised the importance of junior concepts in teaching functions at senior level. For example, the Cartesian Plane, and flip, slide and resize are the basis of transformations in functions. The researcher also emphasised that when concept maps are used in this way, they can be used to introduce a topic, knowledge construction, assessment or as a consolidation resource. The researcher encouraged participants to explore how this resource could support their practice. The concept map in Figure 7 (see Chapter 8) was shared with the participants. Finally, the researcher informed the participants that after the implementation period, they would be asked to respond to survey questions and an interview that will provide them with the opportunity to share their feedback.

Teachers were given a full term to use the resources before data collection began. They were then asked to assess how the planning framework and pedagogical resources with a focus on Functions in Mathematical Methods had worked. The teachers were also asked to reflect on possible improvements to both the planning framework and the pedagogical resources.

3.4.4 Phase Four

The question addressed in Phase Four was:

- What are teachers' perceptions of how procedural flowcharts support students' problem-solving skills in the Mathematical Methods subject?

This phase involved an in-depth follow up interview with a teacher who had applied flowcharts in a problem-solving task. Semi-structured in-depth interviews focus participants' attention on the phenomenon being investigated as they elicit data from participants' experiences and the relationship of the experiences with existing constructs within the area of focus (Galletta & Cross, 2013). Thus, this study used an in-depth interview to elicit a mathematics teacher's experiences and observations when applying procedural flowcharts to teaching and learning of calculus-based mathematics. Artefacts from the students involved in the problem-solving task were also collected. In this study, students' artefacts provided insight into how procedural flowcharts supported their problem solving in the task. Importantly, students generated artefacts can be linked to knowledge, beliefs, and logic expected within the domain (Risan, 2020). Figure 3.1 below illustrates the four phases in the research design.

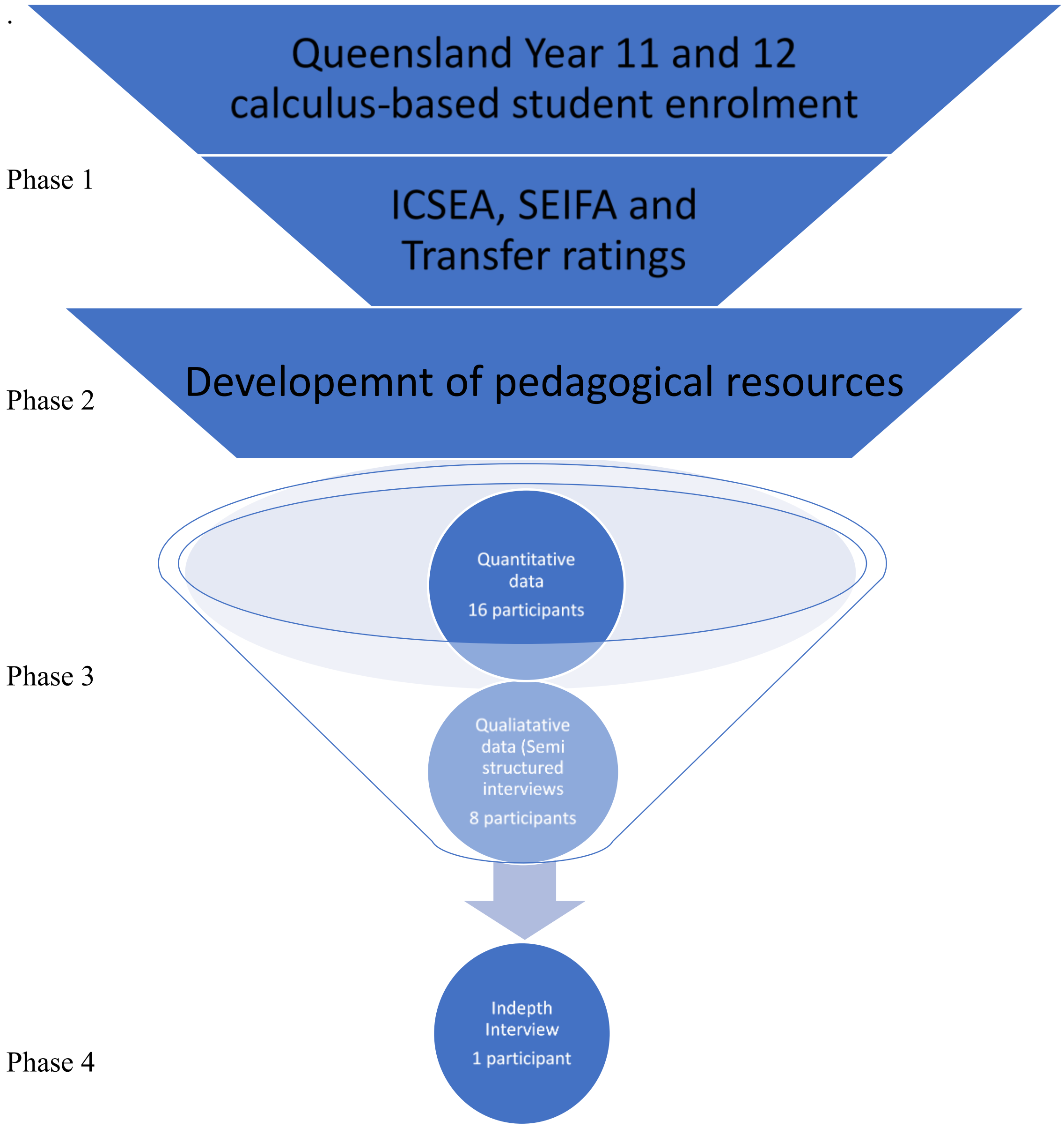

Figure 3.1: Phases in the research design

In summary, Table 3.2 provide an overview of the phases involved in the study, the study timeline and data source used to address the research questions. The research phases coincided with COVID 19, hence physical access to schools and face to face contact with teachers was limited.

Table 3.4: Research phase timeline, questions and data source

Phase	Research Question	Data Source
1 School Terms 2 & 3 2020	What are the trends in Queensland senior students' enrolment in mathematics subjects?	• Quantitative (QCAA enrolments data)
	What is the relationship between students' enrolment in calculus-based mathematics in the new Queensland curriculum and school level indicators such as socio-economic status, school location and transfer rating?	• Quantitative (QCAA enrolments data, ICSEA values, SEIFA index and school transfer ratings).
2 School Term 4 2020 to term 2 2021	What framework for content sequencing can aid linking of concepts from junior to senior mathematics?	• Literature synthesis
	What teaching and learning resources can support students' participation in senior mathematics?	• Literature synthesis
3 School Term 3 and 4 2021	What are teachers' perceptions of a planning framework on content sequencing for the teaching and learning of mathematics?	• Quantitative (Likert Scale items) • Qualitative Data (Open ended questions and semi structured interviews).
	What are senior secondary teachers' perceptions on how concept maps support the teaching and learning of mathematics at senior secondary school?	• Quantitative (Likert Scale items) • Qualitative Data (Open ended questions, semi structured interviews and artefacts).
	What are teachers' perceptions on how flowcharts support teaching and learning of procedural fluency in the Mathematical Methods subject?	• Quantitative (Likert Scale items) • Qualitative Data (Open ended questions and semi structured interviews and artefacts).
4 School Term 1 and 2 2022	What are teachers' perceptions of how procedural flowcharts support students' problem-solving skills in the Mathematical Methods subject?	• Qualitative data (semi structured interviews, in-depth interview and students' artefacts).

3.4.5 Ethics

Initially, James Cook University Human Research Ethics Committee conditionally approved the researcher's ethics application on the condition that approval was gained from the Queensland Department of Education. Since the research covered more than one site (school), approval was required from the state Department of Education. The COVID-19 pandemic presented new challenges as the Department of Education wanted a clear outline on how the research could be done safely. To limit exposure, the researcher proposed to present to participants remotely and follow latest Queensland Health guidelines for face-to-face interviews. Ethical approval was gained from the Department of Education, Queensland: Reference number: 550/27/2383. Following this, James Cook University Human Research Ethics approval was also gained: Approval number: H8201.

To ensure confidentiality and privacy of all respondents, several measures were undertaken to make participants aware of the nature of the study and their rights. The principals of identified schools and their senior secondary mathematics teachers as participants were provided with detailed information regarding the purpose of the study, methods of data collection, rights to privacy, confidentiality and the ability to withdraw at any point during the research. As the study evolved, an ethics amend from the university was obtained so as to include students' artefacts in the study. Detailed information about the study, and students' rights to terminate consent on use of their artefacts at any point in the study was made available. Information was shared through information sheets and consent forms that were prepared for the various participants involved in the study. Contact details of the researcher, advisory panel members and university ethics committee were made available on the documents in case participants had concerns they wanted to raise.

3.5 Research Tools

Research has indicated that identification of student prior knowledge (Bringula et al., 2016; Fyfe et al., 2012), professional collaboration among teachers (Boyle & Kaiser, 2017; Fernandez & Cannon, 2005) and social learning groups (Ashman & Gillies, 2003; García-Carrión & Díez-Palomar, 2015) support students' participation. Creating a learning environment in which students' participation is anchored on creating skills and knowledge based on prior experience is one of the most effective pillars of a robust and effective teaching methodology (Hailikari et al., 2008). The planning framework and pedagogical resources with associated examples developed in Phase Two are research instruments that were shared with the senior mathematics teachers participating in this study.

The introduction of summative evaluation in 2020 (QCAA, 2018) in Queensland's senior secondary schools brought challenges and pressures both to teachers and students, as students had to retain their mathematical knowledge for longer. The external examination covers two years of learning, with most of the questions based on concepts taught in Further Calculus (Unit 3) and Further Functions and Statistics (Unit 4). Thus, strategies that provide opportunities for students' independent learning, skills development and deeper conceptual understanding are required to support their participation in these subjects. The framework developed in Phase Two proposed the use of visual tools as a way of helping students attain the necessary skills and comprehend conceptual connections faster, based on the understanding that procedural flowcharts are critical in promoting fluency and skills development. Above all, they promote students' independent or self-paced learning. Similarly, concept maps help students visualise their mathematical knowledge and clearly demonstrate conceptual connections. Importantly, procedural and conceptual knowledge depend on students' prior skills, knowledge and mathematical facts. Therefore, development of procedural and conceptual knowledge depends on the sequencing of concepts.

The aims of the framework and associated pedagogical resources that were developed in this study were:

- to emphasise the value of content sequencing during planning
- to highlight the importance of concept maps in concept development in mathematics
- to highlight the importance of prior knowledge in students' participation and minimisation of misconceptions
- to draw attention to the importance of flowcharts in teaching procedural knowledge and fluency

- to propose ways of checking for understanding in a mathematics class that are learner centred.
- to explore how procedural flowcharts support problem solving in Mathematical Methods subject.

3.6 Data Collection Methods

Surveys and semi structured interviews were used to collect data from timetabled senior mathematics teachers to evaluate the effectiveness of the planning framework and pedagogical resources in teaching and learning of mathematics. Data collection was done after the teachers had spent a term utilising the resources. The importance of research instruments such as surveys is that it can add more detailed information about the research problem under consideration (Habib et al., 2014). The surveys took approximately 20 minutes to complete. Surveys provide opportunities to collect responses from each participant which helps in identifying different viewpoints or experiences (Gürbüz, 2017). Moreover, surveys are used with the aim of determining the attitudes, beliefs, opinions, and expectations of participants (Kelley-Quon, 2018). In this study, surveys were used to determine the opinions, viewpoints, and experiences of senior mathematics teachers in the teaching of calculus-based mathematics. The surveys were sent via email, which has been identified as a better way of providing participants with more time to answer the questions carefully and minimises the researcher's influence on participants hence allowing more accurate data to be obtained (İslamoğlu & Alnıaçık, 2014).

The survey instruments comprised of open and closed response items.

> "Quantitative approaches use more closed-ended approaches in which the researcher identifies set response categories, whereas qualitative approaches use more open-ended approaches in which the inquirer asks general questions of participants, and the participants shape the response possibilities" (Creswell, 2014, p. 19).

The study used open response questions because they require participants to develop their own response while closed response questions provide participants with the opportunity to select from the responses provided. Open responses provided participants with the opportunity to share their thinking unlike closed response questions.

Importantly, the advantage of using both types of items were that closed response items focused participants' responses to issues important to the study while open response items may provide an opportunity to gain unforeseen responses (Johnson & Morgan, 2016).

The closed response questions were in the form of Likert scale items. Likert items are used to measure participants' attitudes, opinions or beliefs to a particular question or statement (Johnson & Morgan, 2016). The study used a 5-point Likert scale to explore participants' perceptions of pedagogical resources as it can accommodate a "neutral anchor, to allocate equal psychological distance between the neutral category and the adjacent side categories" (Wakita et al., 2012, p. 534). Likert scales are more suitable to use when evaluating an intervention (Sullivan & Artino, 2013). Thus, the study used the Likert scale items for the participants to evaluate the pedagogical resources developed in the study. Importantly, a 5-point scale is regarded as reliable, enough to pick a category fairly fast and provide a good range of choices (Wakita et al., 2012). However, closed response items may limit participants' responses as they are required to read and write or select responses.

Semi structured interviews were also conducted with the senior mathematics teachers. On average the interviews took 15 minutes. Semi-structured interviews were conducted to gain a deeper understanding of how teachers used the framework on content sequencing, concept maps and procedural flowcharts in their teaching of mathematics. The study used semi structured interviews as they are adjustable and adaptable and provide opportunities for the interviewer to ask follow-up questions based on the interviewee's responses (Galletta & Cross, 2013; Kallio et al., 2016). To gain a deeper understanding of teachers' experiences within the study, semi structured interviews "offer a focused structure for the discussion during the interviews but should not be followed strictly" (Kallio et al., 2016, p. 2955), allowing complementarity between interviewer and participant (Galletta & Cross, 2013). This allowed the researcher to prompt questions that allows for further elaboration or follow-up on a participant's response. Similarly, it provides the interviewer with the opportunity to restructure questions and might obtain spontaneous responses as well as obtain supplementary information (Kothari, 2004). In this study, semi-structured interviews were used to provide opportunities for participants to further expand on their answers and to share other important feedback that the researcher might have overlooked. Questions in the research instruments were adapted from Truxaw et al. (2008) and Abdeljaber (2015). The semi structured interview questions were pre-tested with 2 randomly selected teachers to establish flow and clarity of questions.

As senior mathematics teachers were using the pedagogical resources in their teaching, different artefacts were developed. An artefact is a purposeful and intentional object made by humans and is commonly used in critical and qualitative or interpretive research (Czerwinski, 2017). In fact, artefacts can be presented as arguments that embody the response to a research question (Biggs, 2002). Mäkelä (2007) emphasised that "the works created during the research process can be conceived as answers to the posed research questions" (p. 163). Firstly, artefacts were developed during check in sessions with the researcher as consultations and collaboration resulted in jointly developed procedural flowcharts. Secondly some participants also shared artefacts they developed during the implementation stage. Lastly, student developed artefacts were also collected during the teaching and learning using the pedagogical resources.

3.7 Data analysis

Quantitative data collected in phase one from QCAA, ABS, schools ICSEA values and schools transfer rating was analysed using means, frequency counts, percentages, and correlation tests. Descriptive statistical analysis was done using excel suite and inferential statistics using SPSS. Descriptive Statistics (means, frequency counts and percentages) form a major component of all quantitative data analysis when coupled with several graphics' analysis as it summarises raw data from a sample or population (Yellapu, 2018). This study combined descriptive statistics with graphs to offer a comprehensive insight into the data on trends analysis. Yellapu (2018) went further to note that in most cases it is used to break down huge amounts of data into a simpler form or describe the behavior of a sample. The dataset in phase one was large as it was drawn from school enrolments across the state of Queensland and from other institutions. Thus, descriptive statistics provided a general overview of the trends. Importantly, as part of good research practice, it is essential that one report the most appropriate descriptive statistics using a systematic approach to reduce the likelihood of presenting misleading results (Huebner et al., 2016). In summary this study used descriptive statistics because it provided an overview of the general trends (Peace & Hsu, 2018). Descriptive data analysis was done using Excel suite which also provided opportunities to present the data graphically. Calculating descriptive statistics represents a vital first step when conducting research and should always occur before making inferential statistical comparisons (Kaur et al., 2018).

Inferential statistics complements descriptive statistics and involves coming up with a conclusion drawn from the existing data. In this study the Spearman's rank correlation coefficient was used to measure the strength and direction of a monotonic association between a range of variables and students' enrolment. A monotonic association is observed when the value of one variable increases the other value also increases or as one variable increases the other decreases (Sedgwick, 2014). The Spearman's rank correlation coefficient was used to analyse the statistical relationship between ICSEA, SEIFA and transfer ratings on students' enrolment and dropout rate in calculus-based mathematics. Use of SPSS also allowed development of comparative diagrams as it offered diverse resources for complex displays.

During phase 3 both quantitative and qualitative data was collected. The quantitative data was collected using Likert scale survey items. For Likert Scale data, "computing means and standard deviations are considered to be inappropriate, but use of nonparametric statistics is encouraged" (Wu & Leung, 2017, p. 528). This is because, Likert scale data are generally ordinal in nature and are best analysed using modes, frequency, and medians (Stratton, 2018). Therefore, mode and median which are descriptive statistics were used to analyse this data.

Qualitative data constituted open-ended questions in surveys and recordings of interviews with mathematical methods teachers. After transcribing the semi structured interviews, member check was done with participants to verify accuracy of the transcribed scripts. Data analysis of survey open-ended questions and interviews followed a thematic analysis. Thematic analysis aims to identify, investigate, and reveal patterns found in a data set (Braun & Clarke, 2006). In this study, a thematic analysis was used to identify, analyse, and report patterns in the qualitative data. Braun and

Clarke went further to posit that thematic analysis is a fundamental method for any qualitative analysis, and that it provides researchers with core skills that are useful for conducting most forms of qualitative analysis, as most of them are essentially thematic. This process entails a search for themes that are important to the description of the phenomenon and its relation to the study focus (Daly et al., 1997). The study used thematic analysis to identify the themes that best described the qualitative data collected in the study.

Thematic Analysis (TA) is widely used in qualitative research to identify and describe patterns of meaning within data (Braun & Clarke, 2006; Ozuem et al., 2022). It is key for examining the perspectives of different research participants, identifying similarities and differences and generating unpredictable insights (King, 2004). As a foundational method for any qualitative analysis, a TA provides the researcher with critical skills that are useful for conducting different forms of qualitative analysis, as many of them are essentially thematic (Braun & Clarke, 2006). Importantly, a TA offers:

> "Flexibility in terms of research question, sample size and constitution, data collection method, and approaches to meaning generation. It can be used to identify patterns within and across data in relation to participants' lived experience, views and perspectives, and behaviour and practices; 'experiential' research which seeks to understand what participants' think, feel, and do" (Clarke & Braun, 2017, p. 297).

Moreover, another key advantage of a TA is the flexibility of the method to identify constructs (Lawrence, 2012). TA follows an accessible and systematic approach that identifies, analyses, organises, interprets and reports patterns of meaning (themes) (Braun & Clarke, 2006; Clarke & Braun, 2017). Themes are patterns of shared meaning fostered by a core concept and informed by the research questions (Braun & Clarke, 2006, 2019). Through coding the data, a TA develops ideas, meaning and understanding (Ozuem et al., 2022). The researcher plays an active role in coding and theme development following a clear and usable framework for doing TA (Maguire & Delahunt, 2017), shown in Table 3.1.

Theme development is an active process involving the researcher and the qualitative data available (Braun et al., 2022), and is the goal of a TA (Maguire & Delahunt, 2017).

During a TA, data is analysed without engaging pre-existing themes, which makes it ideal for any research that relies primarily upon participants' responses and clarifications (Alhojailan, 2012). Braun and Clarke identified two levels of themes: semantic and latent. Semantic level can be identified within the explicit meanings of the data and the analyst is not looking beyond what participants have provided. However, analysis transcends beyond just what is provided in the data by interpreting and explaining it (Maguire & Delahunt, 2017). Contrastingly, latent level looks beyond what participants said "to identify or examine the underlying ideas, assumptions, and conceptualisations – and ideologies - that are theorised as shaping or informing the semantic content of the data" (Braun & Clarke, 2006, p. 84). A semantic level was adopted for this study because it explored teachers' perceptions on how pedagogical resources developed in the study supported the teaching of calculus-based mathematics.

The emergence of themes can be developed using deductive or inductive analysis. When themes are developed deductively the researcher brings theoretical concepts to the research and when developed inductively the themes emerge from the raw data (Joffe, 2012), thus is data driven (Bonner et al., 2021). The study adopted the inductive approach which produce codes that solely reflective of the contents of the data (Byrne, 2022). Participants shared their perceptions on how the pedagogical resources developed in the study have supported their teaching of mathematics. As participants are practicing mathematics teachers, their opinion after using the pedagogical resources is key in how the resources supported their teaching. Existing research and theory provide a lens for analysing and interpreting the data in TA (Braun & Clarke, 2021). Moreover, the inductive approach ensured that themes were connected strongly to the data and did not use an existing coding frame or the researcher's pre-existing ideas (Braun & Clarke, 2006; Patton, 1990). Importantly, the themes were linked closely to the responses and meanings obtained from participants. The researcher's interpretations and findings should be clearly derived from the data and then inform conclusions and interpretations for confirmability (Tobin & Begley, 2004). Table 3.1 below outlines the phases of thematic analysis as informed by Braun and Clarke (2006, 2019, 2021).

Table 3.5: Phases of Thematic Analysis (Braun & Clarke, 2006, 2019, 2021).

Phase	Description

Data familiarisation and writing familiarisation notes.	Transcribing data, reading and re-reading the data, highlight initial ideas.
Systematic data coding	Coding interesting features of the data systematically across the entire data set, categorise data relevant to each code.
Generating (initial) themes from coded and collected data	Organising codes into potential themes, collect all data relevant to each potential theme.
Developing and reviewing themes.	Checking whether the data supports the themes in relation to the coded extracts and across the data set; generating an initial map of themes.
Refining, defining and renaming themes	Continuous refining of the specifics of each theme, and the overall story the analysis tells, generating clear definitions and names for each theme.
Reporting	Selecting vivid, compelling extract examples, analysis of selected extracts, relating back to the research question and literature.

3.7.1 Data analysis using Thematic Analysis

The six phases of Thematic Analysis proposed by Braun and Clarke (2006, 2019) in Table 3.1 was followed during analysis. In qualitative research, trustworthiness can be achieved if a clearly detailed account of how the data was analysed is available and all assumptions made are included (Nowell et al., 2017). Providing a step-by-step process of analysis is a method of demonstrating transparency of how the researcher formulated the overarching themes from the participants' data (Fereday & Muir-Cochrane, 2006). Indeed, "the analytic process involves immersion in the data, reading, reflecting, questioning, imagining, wondering, writing, retreating, returning" (Braun & Clarke, 2021, p. 332). Although the Thematic Analysis was informed by Braune and Clarke's stages of analysis this study also adopted and referred to Maguire & Delahunt (2017) and Nowell and colleagues (2017) examples of using Braun and Clarke's Thematic Analysis. Importantly, Bree & Gallagher (2016) recommended that Excel can be used in Thematic Analysis as a tool to assist in coding and developing themes because it can identify duplicate entries, can be used to colour code cells and changes can be tracked across different spreadsheets in a workbook. A research team of the principal researcher and two supervisors met every Thursday for three months during this thematic analysis.

3.7.1.1 Phase 1

Data management and understanding the data is key in any credible data analysis. Firstly, the raw data files were issued participants codes as file names. The researcher and supervisors (research team) read and re-read the open-ended questions and the interview transcripts to familiarise themselves with the entire body of data corpus. As the transcripts were read, early impressions were noted. A spreadsheet was created with column headings of open-ended survey questions and rows with participant names. Responses were populated in the spreadsheet for easy navigation through questions and responses. Another spreadsheet was created with column headings of research questions to be addressed by the study.

3.7.1.2 Phase 2

The researcher and supervisors met as a team for the initial coding of the data. From the start of the thematic validity was ensured using theory triangulation. It involves sharing qualitative responses among colleagues at different status positions in the field then comparing findings and conclusions (Guion et al., 2011). Firstly, as a team we coded two participants' open responses and two interviews transcript. As data was being coded the team kept revisiting the research questions to identify each segment of data that captured something interesting about the research questions. The coding process involves identifying and recognising an important moment within the data and encoding it prior to a process of interpretation (Boyatzis, 1998). Moreover, a code is something of interest to the researcher, which they view as of significance in answering the research question (Swain, 2018). Coding involves taking qualitative text data apart to see what they yield before putting the data back together in a meaningful way (Creswell, 2015) "Coding allows the researcher to simplify and focus on specific characteristics of the data" (Nowell et al., 2017, p. 6), with the goal of attaining clarity in organising and interpreting the data (King, 2004). Coding was done with no pre-set codes and line-by-line coding was used as this was mainly an inductive analysis. At the conclusion of step 1, initial ideas on codes were discussed as a team to give team members a background and clarity on the process. Notes were recorded on initial observations about interesting aspects of the data items and emerging impressions. Separately, we went further to code another set of open-ended survey questions and interview transcript. A discussion, comparison and collating of codes followed the initial independent coding. This was done to moderate and modify our coding before we went to code the rest of the data separately. Peer debriefing helped the team to debrief on how their thoughts were evolving as they engaged deeply with the data and the coding process. During this step 2 of coding, codes were copied and pasted under specific research questions in the spreadsheet (see Appendix A).

3.7.1.3 Phase 3

Codes are the building blocks for themes, which are patterns of meaning with a shared core idea (Clarke & Braun, 2017). A code is viewed as something shorter, or basic (Braun & Clarke, 2006), which can be combined or connected to form a much broader understanding referred to as a theme (Fereday & Muir-Cochrane, 2006). A theme is determined through the sound judgement of the researcher which should be applied consistently through the analysis considering that a theme can be judged on whether it is essential to addressing the overall research question (Campbell et al., 2021). The data covered a wide variety of concepts so initially the different concepts that grouped the research questions as ‘conceptual themes’ were utilised to organise the data. The research team examined the codes, checking on their meaning and relationships to determine which ones were underpinned by a central concept. In Excel, codes that shared a core idea from the initial phase that used data from the open-ended responses and interview transcripts were colour coded (see Appendix B). This is supported by King (2004) who suggested that when searching for themes it is best to start with a few codes to help guide analysis. After the independent thematic analysis, the filter function in Excel was used to sort the codes using cell colour. Moreover, Excel provided the opportunity to identify duplicates as codes were collated from the three researchers. Same coloured codes were synthesised to develop a general pattern of meaning, which we referred to as candidate themes (see Appendix C). The code that did not belong to any of the candidate themes were listed under miscellaneous theme for further analysis and review. At this stage, data or codes which do not fit under any of the candidate themes should not be abandoned as without further review during the fourth phase of thematic analysis, it is uncertain whether the themes will hold, be combined, refined, separated, or discarded (Braun & Clarke, 2006). Thus, the sorting and collation approach would bring together all codes under each theme which then would facilitate further analysis and review (Bree et al., 2014). Independent thematic analysis among the team members ended at this stage as codes and candidate themes had been.

3.7.1.4 Phase 4

This phase focused on reviewing, modification and refinement of the candidate themes identified in stage 3. In this phase, the researcher should conduct a recursive review of the candidate themes in relation to the coded data items and the entire dataset (Braun and Clarke 2012, 2021). The team went back to review the themes and codes independently coded and evaluated the meaning and association within and across themes. We were guided by the questions developed by Braun and Clarke (2012 p. 65) on how to review themes:

- Is this a theme (it could be just a code)?
- If it is a theme, what is the quality of this theme (does it tell me something useful about
- the data set and my research question)?
- What are the boundaries of this theme (what does it include and exclude)?
- Are there enough (meaningful) data to support this theme (is the theme thin or thick)?
- Are the data too diverse and wide ranging (does the theme lack coherence)?

The researcher and supervisors went on to review the relationship of the data and the codes that informed the themes. Moreover, the coded data extracts for each theme was reviewed to check for coherence. Importantly, if the codes form a coherent and meaningful pattern the theme makes a logical argument and may be representative of the data (Nowell et al., 2017). Furthermore, the team also reviewed the themes in relation to the data. This is because Nowell and others posited that themes should provide the most accurate interpretation of the data. Importantly, the focus of this stage is to check inadequacies in the initial coding and themes which may require some changes, for example new codes or subcodes can be developed (King, 2004). As a result, we vetted, reviewed and cross analysed the coded data for each theme and subthemes to ascertain coherence. This also involved going back to the data to make sure participants' voices were reflected. During the review, whenever new themes, old themes were integrated or codes were moved to another theme, a new spreadsheet was created so that if further review was necessary the old data and layout would still be available. Braun and Clarke emphasised that at the end of this phase, researchers should have a good idea of the themes developed, their relationship and the overall story they tell about the data.

3.7.1.5 Phase 5

This phase is considered the final theme refinement stage. Braun and Clarke noted that the aim of the phase was to "...identify the 'essence' of what each theme is about" (p. 92). Each theme or sub-theme should be expressed in relation to the dataset and the research question(s) (Byrne, 2022). As a team the researcher and supervisors discussed and wrote detailed analysis for each candidate theme identifying the main story behind each theme and how each one fit on the overall story about the data through the lens of the research questions. Each meeting focused on one theme and each member was given the opportunity to share their understanding and evaluation and other team members had opportunities to ask questions. Data was read and codes scrutinised and reviewed to ensure credibility. We only moved to the next theme when consensus was reached about the theme names, codes and themes as representational of the data. Finally, in this phase we also linked quotes to final themes reached during the analysis. Illustrating findings with direct quotations from the participants strengthen the face validity and credibility of the research (Bryne, 2022; Patton, 2002; Nowell et al., 2017).

3.7.1.6 Phase 6

This phase is the end point of the research when all themes and subthemes have been finalised. The writeup provided a concise, coherent and logical cogent narrative of the data within and across themes (Braun & Clarke, 2006; Byrne, 2022). Researchers should show the significance of the patterns and their broader meanings, implications and how the findings relate to literature (Braun & Clarke, 2006; Nowell et al., 2017; Starks & Trinidad, 2007). Importantly, more direct quotes from participants were included in the analytical narrative to connect readers with the raw data (King, 2004; Nowell et al., 2017), hence enhancing the validity and merit of the analysis (Braun & Clarke, 2006). The principal researcher was the one responsible for writing all the research reports that emanated from the data. However, all the reports were shared with the supervisors for feedback and validation. Credibility can be obtained through peer debriefing which provide an opportunity for external check on the research process, as well as examining referential adequacy as a means to check preliminary findings and interpretations against the raw data (Lincoln & Guba, 1985). The report of the analytical narrative was shared with participants for them to check if the results represented their responses. Member check in was used to validate participants' responses to a researcher's transcription or conclusions about them (Cutcliffe & McKenna, 2002). Furthermore, part of the results were presented at the Mathematics Education Research Group of Australasia (MERGA) conference for peer feedback. Conference presentations of the researcher's data interpretation can allow opportunities for further comment from peers and experienced researchers (Fereday & Muir-Cochrane, 2006).

During phase 4 qualitative data were collected using an in-depth interview and her students' artefacts. The stages of problem solving in mathematics (Artigue et al., 2020; Geiger et al., 2021; Polya, 1971; QCAA, 2018) were used in analysing the in-depth interview with the teacher. Interpretation of artefacts overcomes its muteness and gives it a voice and meaning (Mäkelä, 2007). Students' artefacts were analysed using the QCAA's (2018), problem solving and modelling task flowchart (see Appendix A). The stages of problem solving were used in the analysis because the phase was focused on supporting students' problem-solving skills using procedural flowcharts.

3.8 Data Storage

Data were stored using the university data storage protocols. Data was saved offline, replicated three times and saved on different platforms immediately after collection. A data record was created with a link to the master copy used in the active stage of the research and the copy was uploaded onto the university data repository after the data was de-identified. The data will be retained for five years.

3.9 Chapter Conclusion

This chapter has outlined the two main questions this study intended to address, along with the theoretical position that frames them. It has justified the methodology used in the design of the study and detailed the methods within the design that were used for data collection in the four phases involved. Chapter 4 is an analysis of trends in student participation in calculus-based mathematics in Queensland.

Chapter 4: Senior High School Mathematics Subjects in Queensland: Options and Trends of Student Participation

A version of this chapter was published as a research paper in the PRISM Casting New Light on Learning, Theory and Practice. https://openjournals.ljmu.ac.uk/index.php/prism/article/view/446

4.1 Chapter Introduction

Mathematics has been described as a critical filter for future academic and career options and enrolment in Advanced Mathematics subjects in high school paves the way for high-status careers (Watt et al., 2017). Furthermore, Advanced Mathematics is central to the study of many university courses, including science, technology, engineering and mathematics (STEM) courses. “Mathematics is a key science for the future, through its enabling role for science, engineering and technology. This is illustrated by dramatic advances in communications, bioinformatics, the understanding of uncertainty, and dealing with large data sets” (Lemaire, 2003, p. 1). Students need a strong foundation of mathematical skills, especially at secondary school, to make a successful transition from school to studying STEM disciplines at university (Lyakhova & Neate, 2019). Consequently, government programs often target mathematics as one important part of STEM education that will lead to better jobs, innovation, improved economy and greater global leadership (Peters et al., 2017). Importantly, the post COVID-19 economic reboot will require students with advanced mathematics skills as demand for skilled STEM professionals will increase (Vernon, 2020). The important contribution that mathematics makes towards STEM-based careers means it is essential to understand students’ choices in different options that the subject offers, especially options that are prerequisites for STEM courses.

The technology-driven modern world requires a deep understanding of mathematics, hence equipping citizens with advanced mathematics skills becomes a right (Centre for Curriculum Redesign, 2013). Students who take calculus-based or Advanced Mathematics in countries such as Australia, The USA and the UK are better positioned to enrol in STEM-related courses at tertiary level (Carnevale et al., 2011; Lyakhova & Neate, 2019). Advanced mathematical knowledge, skills and understanding of distinct concepts are important for further study in fields where mathematics plays a key enabling role (Maltas & Prescott, 2014). Calculus-based or Advanced Mathematics as prerequisites of tertiary STEM courses have a direct impact on university enrolments and the diverse opportunities students have after high school. Therefore, it is essential to look at the enrolment rates of senior students in different mathematics curricula. Analyses of student enrolment trends in different mathematics options can be confounded by the diverse classifications of mathematics subjects. The following sections discuss how mathematics is classified internationally and the classifications used in Australia. This will be followed by a discussion of global and Australian trends in student enrolment in senior school mathematics, with a final focus on trends in the state of Queensland, which is the context of this study.

4.2 Mathematics Classifications Internationally

Senior high school mathematics curricula differ from country to country. Some countries follow a national curriculum where all students engage with the same mathematics curriculum. In the UK, students who progress to A-level studies and opt for mathematics have the option of obtaining AS (Advanced Subsidiary) qualifications after a year, the full A-level (A2) or Further Mathematics (FM) at the end of two years (Noyes & Adkins, 2016). New core mathematics qualifications were introduced in 2015 as an alternative pathway for students who have passed GCSE mathematics but want to pursue courses that do not demand advanced mathematics (Lee, 2016). Countries that have a national curriculum classify all mathematics options under a common nomenclature. This eliminates complications in defining subject classifications when undertaking analysis of national trends in student enrolment.

Federal countries with autonomous states that determine their own curricula may have no consistent framework for naming mathematics subjects. For example, in the USA, some states allow the education structure to be decided at local level. As nomenclature is not consistent between state jurisdictions, compiling data into nationally consistent and coherent information is problematic. For example, subjects with very similar course content can have different titles and possibly be classified as belonging to different learning areas. The National Centre for Education Statistics [NCES], (2007), cited in

Rasmussen et al. (2011), classified different mathematics subjects in order of complexity, as follows: Algebra I or Plane Geometry, Algebra II, Algebra III/Trigonometry or Analytical Geometry, Pre-calculus, Calculus and Advanced Placement (AP). Clarity around the categorisation of mathematics options offered in different states is an important prerequisite for an informative analysis of student enrolment trends in this subject, as the criteria used for categorisation can be contested.

4.3 Mathematics Classification in Australia

In Australia, senior school curricula are the responsibility of states and territories. This means that the classification and scope of the mathematics subjects can be different from one jurisdiction to another. In addition, researchers in Australia have differing views on the way that mathematics subjects ought to be classified. Some take into consideration only the *opportunities* that the subject offers *post-secondary school*, while others use only subject *content* as the basis for their classification. This prompted some scholars to meet in 2004, when they resolved that the categorisation of subjects and compilation of enrolment data be listed alongside each other (Barrington & Brown, 2014). Table 4.1 shows the different classifications researchers have since used in analysing mathematics subjects. Mathematics subjects are classified into three categories: basic, elementary or low-level, intermediate, and advanced or high-level (Kennedy et al., 2014). Basic Mathematics covers basic mathematics skills and is not considered for any future educational purposes, intermediate mathematics is considered useful in pursuing courses in which mathematics content is minimal, while Advanced Mathematics is a prerequisite for university courses in which mathematics plays an integral role (Dekkers & Malone, 2000). Thus, entry level (see Table 4.1) is part of elementary mathematics which include mathematics subjects that are considered as a numeracy option for tertiary admission (Kennedy et al., 2014).

Table 4.1: Researchers' Classifications of Australian High School Mathematics Subjects

Dekkers, DeLaeter & Malone Classification (2000).	Barrington & Brown Classification (2004)	Kennedy, Lyons & Quinn Classification (2014)	General Course Content
Low-Level	Elementary	Background	Terminal mathematics courses that are not designed for further tertiary study and do not contribute towards tertiary admissions rankings.
		Entry	Terminal mathematics courses that are not designed for further tertiary study yet do contribute to calculated tertiary admissions ranking.
Intermediate	Intermediate	Intermediate	Mathematics courses that provide a satisfactory knowledge base for tertiary courses requiring minimal mathematics knowledge.
High-Level	Advanced	Advanced	Mathematics courses that provide a specialised knowledge base for tertiary studies in STEM courses or in courses in which mathematics is an integral part.

(Kennedy et al., 2014 p. 36).

The curriculum diversity and options offered in different countries reinforces the idea that mathematics should prepare students for different career choices, highlighting the 'critical filter' tag that has been used to describe the subject (Watt et al., 2017). As countries adjust or change mathematics curricula, their objective should be to increase students' enrolment, especially in advanced or calculus-based options, as these provide students with more diverse and better career opportunities. Increased mathematics choices naturally means that different subjects compete for students. As a result, an analysis of the trends in student choices may shed some light on the distribution of students among subject options.

4.4 International Trends in Student Participation in Mathematics Subjects

Global trends in student enrolment in senior school mathematics indicate that student enrolment in calculus-based mathematics subjects is either declining or has reached a stagnation point. For example, enrolment in Advanced Mathematics in countries such as Germany, Ireland, Netherlands, Russia and Spain is 15% or less of the total student cohort (Hodgen et al., 2010a). Correspondingly, South African enrolment in calculus-based mathematics declined by 16% between 2015 and 2019 (Businesstech, 2020). On the other hand, in Japan, South Korea, New Zealand, Singapore and Taiwan, approximately 31% of upper secondary students chose to study Advanced Mathematics between 2005 and 2010 and these countries had the highest share of students' enrolment in Advanced Mathematics worldwide (Hodgen, 2013; Hodgen et al., 2010b). The USA showed a general increase in Advanced Mathematics enrolments until 2005 when calculus and AP (Advanced Placement) had a combined rate of 23% enrolment, but enrolments stagnated thereafter (Hodgen et al., 2010b; National Science Board, 2018). Following the introduction of Curriculum 2000 in the UK, a steady increase in students opting for mathematics for their A-level was noted between 2006 (7.9%) and 2015 (12.7%) (Hodgen et al., 2010b; Noyes & Adkins, 2016); however, mathematics still remains a minority subject and females are less represented in A2 (Hodgen et al., 2010b).

4.5 Trends in Student Participation in Mathematics Subjects in Australia

Available national trends in Australia focus on Year 12 enrolment statistics from all states and territories and are generally categorised as elementary, intermediate and advanced. Concerns have been raised about student enrolment in intermediate and Advanced Mathematics options. For example, Australia's former Chief Scientist, Professor Ian Chubb, expressed his concerns about the lack of appetite by students to study higher levels of mathematics in Years 11 and 12 (Evershed & Safi, 2014). "Intermediate and, especially Advanced Mathematics students are essential for a strong science, research and innovation capacity. The statistics at hand indicate that enrolment numbers in these areas are shrinking and students are instead electing to take Elementary Mathematics" (Australian Council of Deans of Science, 2006, p. 2).

4.5.1 Elementary/Entry-level/Low-Level Mathematics

In Australia, student enrolment in elementary mathematics has maintained a significant and steady growth in enrolments from 1990 to 2012, with the exception of 2001 (Barrington & Brown, 2014). By 2010, 51% of all mathematics enrolment was in the elementary level, increasing to 52% in 2011, where it stayed until 2015 (Barrington & Evans, 2014; Barrington & Evans, 2016). In 1990, around 51,855 students opted for low-level mathematics but by 2015 about 117, 000 students were enrolled in the subject, a 125.6% increase (Dekkers & Malone, 2000; Barrington & Evans, 2016). Kennedy et al. (2014) also reported an increase in enrolment rates in entry level mathematics between 1994 (38%) and 2012 (49%). The differences in participation rates in elementary mathematics reported by the various researchers can be attributed to the different categorisations used in their analyses. However, it is clear from both trends that there was a significant increase in enrolment in elementary mathematics. Using other criteria, female dominance in elementary mathematics declined between 1990 (56.7%) and 1999 (52.4%) and again to almost parity with male students after 2000 (Dekkers & Malone, 2000; Forgasz, 2006b). The female-to-male ratio of enrolment by 2012 was 11 females to 10 males (Kennedy et al., 2018) and in 2015 the percentage was approximately 51% to 49% in favour of females (Barrington & Evans, 2016). From the different categorisations presented, these trends show that enrolment in elementary mathematics between males and females became fairly balanced from early to mid-2000.

4.5.2 Intermediate Mathematics

Nationally, slight variations in participation rates in intermediate mathematics have been reported by different researchers due to their differing categorisation of this option. For example, according to Barrington & Brown (2014), Barrington and Evans (2014; 2016) and Forgasz (2006a), student enrolment rates in intermediate mathematics declined during the period 1995 (27.3%) to 2015 (19.2%), with the exception of 2002 and 2014. However, Ainley et al. (2008) report slightly different enrolment rates between 2001 (34.7%) and 2007 (30.6%) and Kennedy et al. (2014) report different rates of decline again, between 1994 (38%) and 2012 (27%). However, the findings from the various researchers do evince a similar trend of a steady decline in students' enrolment in intermediate mathematics from the mid-1900s to 2012. These trends also show that males dominated enrolment in intermediate mathematics, although females were not far behind (Kennedy et al., 2014; Barrington & Brown, 2014).

4.5.3 Advanced/High-level Mathematics

Participation rates in Advanced Mathematics declined between 1990 (24%) and 1999 (16%) (Dekkers & Malone, 2000). Kennedy et al. (2014) reported a similar decline between 1994 (16%) and 2012 (9%). The period 2001 to 2007 saw student enrolment numbers in Australia declining from 26,216 to 22,999 respectively (Ainley, 2008). Since 2007, raw enrolment data have been fairly static, between 20,000 to 21,000 until 2012 (Kennedy et al., 2014). One in 10 students in 2013 studied Advanced Mathematics in Year 12 (Mater et al., 2014). With the exception of 2003, 2008 and 2014, enrolment rates between 1995 and 2015 continued to decline until they stabilised at around 9.5% from 2012 (Barrington & Evans, 2014; 2016). Though researchers used different classification categories for mathematics subjects, their findings that Advanced Mathematics enrolment had declined over the last few decades were consistent. Female enrolment rates in Advanced Mathematics also showed a steady decline from 41.1% in 1990 to 38.9% in 1999 (Dekkers & Malone, 2000). The ratio of male and female enrolment in the late 1990s was six females to 10 males, which declined to 14 females to 25 males by 2012 (Kennedy et al., 2014). The trend continued in 2013, when the rate of female enrolment was 6.7% compared to 12.7% of boys (Barrington & Brown, 2014). By 2015, the female participation rate was at 6.9% while the male rate was 12.6% (Barrington & Evans, 2016). Just 6.6% of girls enrolled in Advanced Mathematics in 2013, a 23% decline from 2004 (Mater et al., 2014). Thus, a clear dominance by males characterised enrolment in Advanced Mathematics.

Research indicates that in Australia, calculus-based mathematics is becoming less popular with most students, as indicated by the low number and proportion of Year 12 students studying this option in 2013 compared to 1995 (Barrington & Brown, 2014). In fact, student participation rates in both intermediate and Advanced Mathematics steadily declined to around 19.2 % and 9.6% respectively in 2015. On the other hand, the elementary mathematics enrolment rate has shown a steady increase from 2005 to 2015, stabilising at 52% from 2010. These trends were consistent in the majority of states in Australia, especially for calculus-based subjects, and this decline in enrolment rates in calculus-based mathematics is a cause for concern (Engineers Australia, 2016).

4.6 Trends in Student Participation in Mathematics Subjects in Queensland

The latest literature of trends in students' enrolment in Queensland was part of Ainley et al's (2008) research on national trends in Advanced Mathematics, which was 14 years ago. This is in contrast to states such as New South Wales, where more recent research has been undertaken to analyse trends in student enrolment in mathematics (Jaremus et al., 2018). From 2008 to 2019, Queensland offered Mathematics A, B, C and Prevocational Mathematics, which were replaced by General, Methods, Specialist and Essential Mathematics respectively (Queensland Tertiary Admissions Centre [QTAC], 2018). Mathematics A is considered Elementary Mathematics, Mathematics B is considered Intermediate and Mathematics C is Advanced (Forgasz, 2006b). Mathematics C is a recommended companion subject to Mathematics B and offers more diverse and better career opportunities (Queensland Studies Authority [QSA], 2014). Although Mathematics C provides additional preparation, both Mathematics B and C cater for students interested in university courses with high demands in mathematics, such as science, medicine, mining, engineering, information technology, mathematics, finance, business and economics (QCAA, 2008). This is different from the categories that have been used in previous analyses of enrolment trends nationally, as only Mathematics C was regarded as a prerequisite for such courses. Mathematics A is for students who want to pursue studies and training in courses with moderate demand for mathematics, such as carpentry, plumbing, auto mechanics, tourism, hospitality and administration (QCAA, 2008). Prevocational Mathematics can be classified as background elementary mathematics (Kennedy et al., 2014) as it does not prepare students for any further tertiary studies; hence it is a terminal option. Nor was Prevocational mathematics ever considered in any previous enrolment trends analysis, hence there is no literature on the subject.

The following section includes a discussion of raw data as well as percentages to identify gaps in the literature on Queensland. Raw enrolment numbers of mathematics enrolment in Year 12 increased marginally, from 46,517 in 2000 to 47,465 in 2004, apart from a decline of 694 students in 2001 (Forgasz, 2006b). Between the early 1990s and 2005, there was a significant decline in the proportion of Year 12 students studying Mathematics B and C in Queensland. However, enrolment rates seemed to stabilise by 2013 after a marginal increase (QSA, 2014).

4.6.1 Elementary Mathematics

A marginal increase from 66.0% in 2000 to 67.5% in 2004 characterised students' enrolment in elementary mathematics (Forgasz, 2006b). Raw Year 12 data show that student enrolment increased from 26,298 in 2000 to 27,415 in 2004, which was an increase of 4.2% (Forgasz, 2006b). Females dominated participation year by year from 2000 to 2006 (McPhan et al., 2008). Between 2000 and 2004, male enrolment rates trailed females, increasing by 3.5% compared to 4.9% for females (Forgasz, 2006b). In addition, female enrolment as a proportion of all Year 12 females increased from 67.3% to 69.8%, while male enrolment as a proportion of all Year 12 males was stable at around 65% in the same period (Forgasz, 2006b).

4.6.2 Intermediate Mathematics

A steady decline in enrolment in Intermediate Mathematics (Mathematics B) was witnessed from 1992 to 2008, but this was followed by a steady but marginal increase until 2013 (QSA, 2014). The mean percentage enrolment rate among the Year 12 cohort was 41.5%; however, the Intermediate Mathematics (Mathematics B) participation rate fell by 2.1% for the period 2000-2004 (Forgasz, 2006a, 2006b). From 2000 to 2004, the female participation rate declined by 4.8% while the male enrolment rate increased by 0.3% (Forgasz, 2006a, 2006b). In the same period, both male and female enrolment rates as proportions of their Year 12 gender declined, with the male rate falling from 46.2% to 44.7% and the female rate from 39.3% to 37.0% (Forgasz, 2006b).

4.6.3 Advanced Mathematics

In Advanced Mathematics, a decline in enrolment was witnessed from the early 1990s until 2000 (Ainley et al., 2008), falling from 15.8% in 1991 to 7.8 % in 2007, despite marginal increases in 1995 and 2004 (Ainley et al., 2008). From year 2000 to 2003, the raw data show a decline in enrolment from 3,242 to 3,175; however, a significant increase to 3430 was welcome in 2004 (Forgasz, 2006b). The sum of all Advanced Mathematics enrolments from 2000 to 2004 was only 8% of all Year 12 students (Forgasz, 2006b). However, the increase in enrolment has been credited to the bonus points system Queensland offered in 2008, which incentivised students to enrol (Maltas & Prescott, 2014). The bonus points enabled a student with a pass in Mathematics C to receive two adjustments to boost the selection mark for tertiary courses. Finally, between 2000 and 2004, the female enrolment rate in Advanced Mathematics (5.5%) was slightly lower than male enrolment rate (6%) (Forgasz, 2006b) and the male dominance has not been challenged over that period (Forgasz, 2006b).

No analysis of trends in student enrolment in mathematics options involving Years 11 and 12 enrolment data has been undertaken for Queensland, the last comprehensive study using Year 12 enrolment data having been carried out by Ainley et al. in 2008. This constitutes a significant gap in the literature which this study aims to fill by reporting on an analysis of student enrolment trends for the period 2010 to 2019 in the Sunshine State.

4.7 Study Methods and Results

This study investigated students' options and trends of enrolment in Mathematics A, B, C and Prevocational mathematics between 2010 and 2019 using data from the Queensland Curriculum and Assessment Authority (QCAA). Quantitative methods were applied to analyse trends of student options. Consent to use the data in this study was provided by QCAA. The data covered schools, gender, indigenous or non-indigenous and the number of students in the various different options. Table 4.2 shows the raw data of the student numbers in the year levels from 2010 to 2019.

Table 4.2: Raw data showing student numbers

Year	Year level	Gender		Non-Indigenous	Indigenous	Total
		Male	Female			
2010	11	30571	28856	56866	2561	59427
	12	28534	28143	54666	2011	56677
2011	11	30600	29261	57243	2618	59861
	12	29319	28444	55647	2116	57763
2012	11	30920	29769	57951	2738	60689
	12	29728	29353	56821	2260	59081
2013	11	31770	30560	59278	3052	62330
	12	30378	29724	57724	2378	60102
2014	11	32445	30581	59848	3178	63026
	12	31132	30457	58829	2760	61589
2015	11	32520	30868	60112	3276	63388
	12	31844	30519	59489	2874	62363
2016	11	33314	31556	61389	3481	64870
	12	31964	31116	60005	3075	63080
2017	11	32090	31334	60019	3405	63424
	12	33032	31829	61639	3222	64861
2018	11	24351	21494	43586	2259	45845
	12	31613	31582	60016	3179	63195
2019	11	Introduction of new curriculum				
	12	24247	21868	43912	2203	46115

To perform the analysis, a descriptive quantitative method was employed using Microsoft Excel. Microsoft Excel offers a suite of statistical analysis functions that can be used to run descriptive statistics, to perform several different and useful inferential statistical tests and process data using formulas (Abbott, 2011). Descriptive statistics and graphical representations of data can be useful when making comparisons between sets (Carr, 2008). Descriptive statistics (e.g., calculation of the measures of central tendency such as the mean, mode and median) were used to describe the data using the Microsoft Excel software. According to Aldrich and Rodriguez (2013), multiline graphs can be used to identify trend changes in one or more variables over time. The following section describes the trend changes in (1) average percentage enrolment; (2) schools not offering calculus-based mathematics; (3) gender enrolment in Mathematics A, B and C; (4) Indigenous students enrolment; and (5) dropout rates in Mathematics B and C.

4.7.1 The Average Percentage Enrolment

An analysis of the average percentage of student enrolment in Mathematics A, B, C and Prevocational mathematics between 2010 and 2019 was conducted. The analysis ranked the mathematics enrolment in the different options as follows:

Mathematics A: 42.55% at Year 11 and 43.44% at Year 12

Mathematics B: 30.41% at Year 11 and 29.53% at Year 12

Mathematics C: with 7.82% at Year 11 and 7.62% at Year 12

Prevocational Mathematics (PVM): 19.22% at Year 11 and 19.21% at Year 12

Figure 4.1 below is a graph of student enrolment in all the four mathematics options, namely, Mathematics A, B, C and Prevocational mathematics between 2010 and 2019. The Mathematics A, B, C and Prevocational syllabi terminated at the end of 2019, hence Year 11 data ended in 2018.

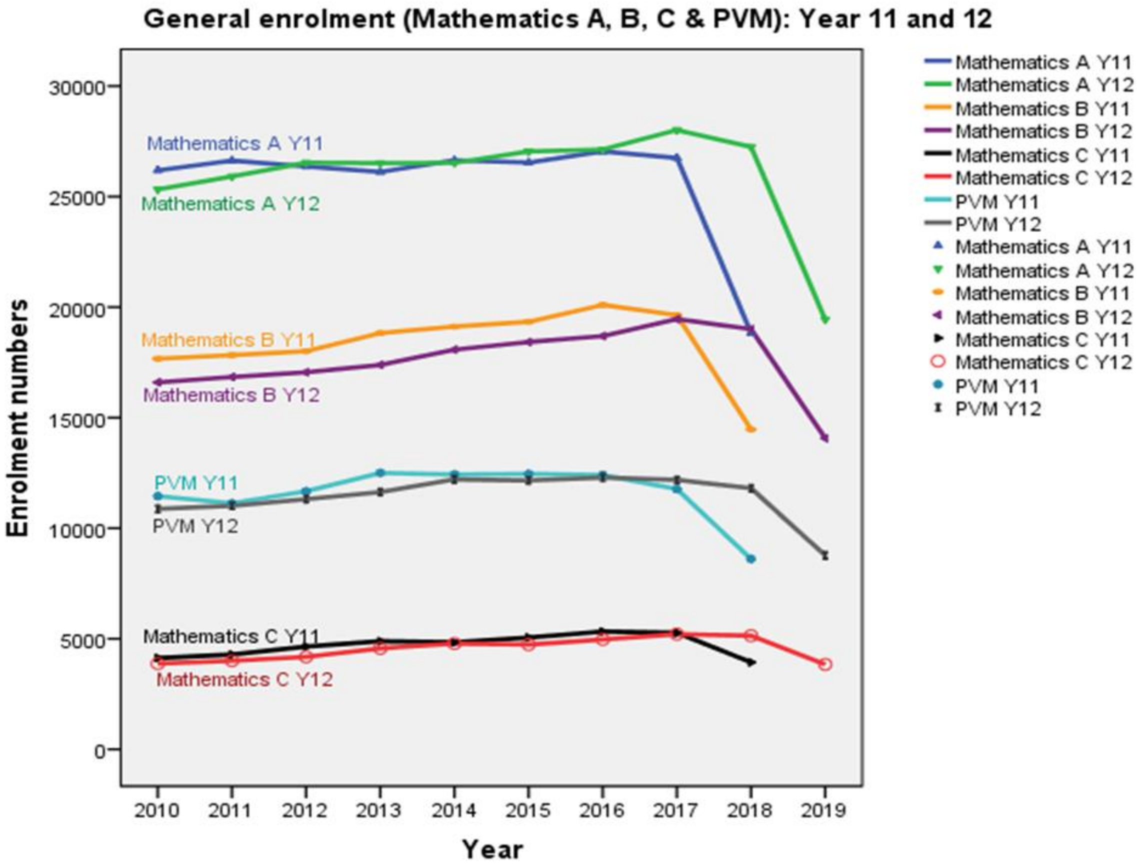

Figure 4.1: Enrolment Summary from 2010 to 2019

4.7.2 Schools with no Students Participating in Calculus-based Mathematics

Figure 4.2 is a graph of the number of schools that did not register any student for Mathematics B and C between 2010 and 2019. The yearly average number of schools that did not have students' enrolment in calculus-based mathematics, that is, Mathematics B, is 13, and Mathematics C is 83. The difference in number between schools offering Mathematics A, B or C gave the number of schools that did not have students' enrolment in calculus-based mathematics.

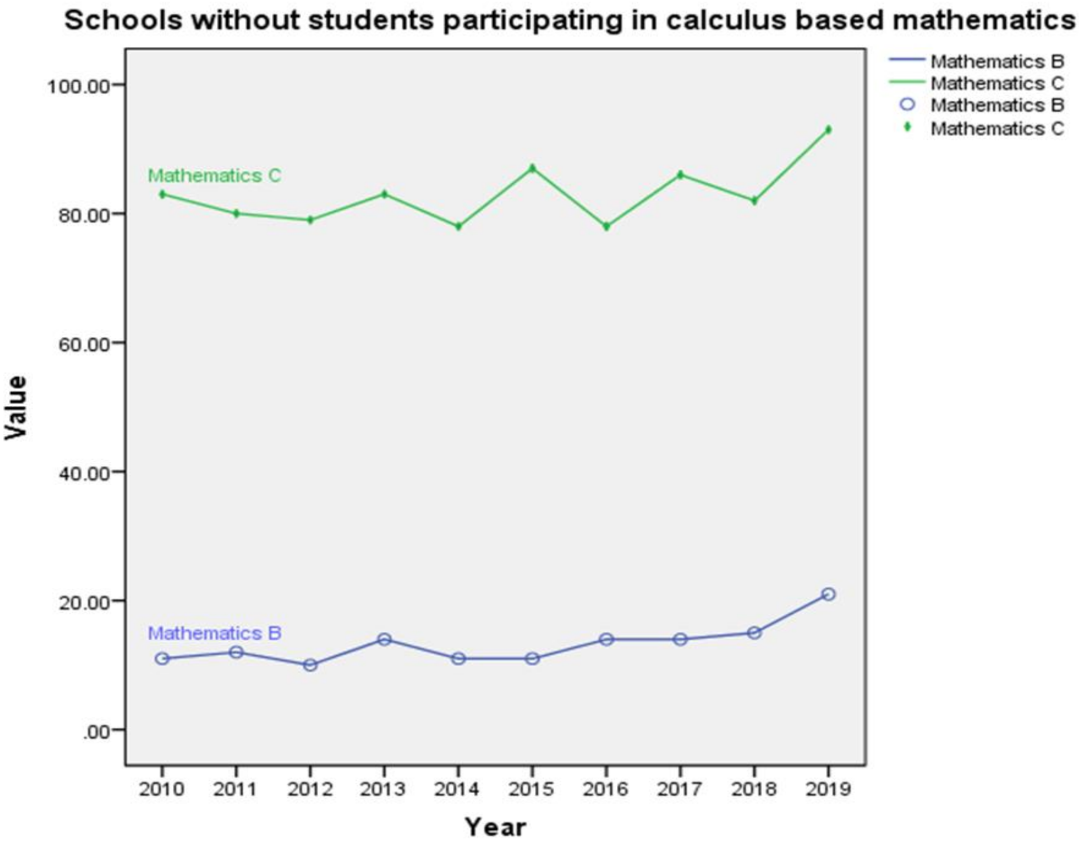

Figure 4.2: Schools with no Students Enrolment in Calculus-based Mathematics from 2010 to 2019

4.7.3 Gender enrolment in Mathematics A, B and C

Table 4.3 shows the average percentage enrolment in Mathematics A, B and C from 2010 to 2019 in gender groups. It also shows the gender distribution in calculus-based Mathematics B and C and non-calculus Mathematics A.

Table 4.3: Average Percentage Gender Enrolment in Mathematics A, B and C from 2010 to 2019

Year Level	Subject	Gender	Average Percentage
11	Mathematics A	Males	46.61
		Females	53.39
	Mathematics B	Males	52.54
		Females	47.46
	Mathematics C	Males	64.71
		Females	35.29
12		Males	46.30

Mathematics A	Females	53.70
Mathematics B	Males	52.37
	Females	47.63
Mathematics C	Males	64.89
	Females	35.11

The results show that male enrolments in calculus-based mathematics was higher than female enrolments. The average percentage of males enrolled in Mathematics B compared to the total Mathematics B enrolment was 52.54% in Year 11 and 52.37% in Year 12. This means for every 13 males enrolled in Mathematics B, there were 12 females. Similarly, in Mathematics C, males constituted 64.71% of the Year 11 cohort and 64.89% in Year 12. For every 13 males enrolled in Mathematics C, there were 7 females. In contrast, females dominated enrolment in the non-calculus option of Mathematics A where, in Year 11, females surpassed males by an average percentage of 6.78%, which increased to 7.4% in Year 12. There was a slight increase in the ratio from every 12 males:13 females in Year 11 to 6 males:7 females in Year 12.

4.7.4 Indigenous Students Enrolment

Figure 4.3 shows trends in Indigenous students' enrolment in the mathematics options. A large number of Indigenous students enrolled in Pre-Vocational Mathematics but only a very small percentage in Mathematics C. Table 4.4 below shows how Indigenous students were distributed among the four options. The percentages were calculated as a total of the state Indigenous student population.

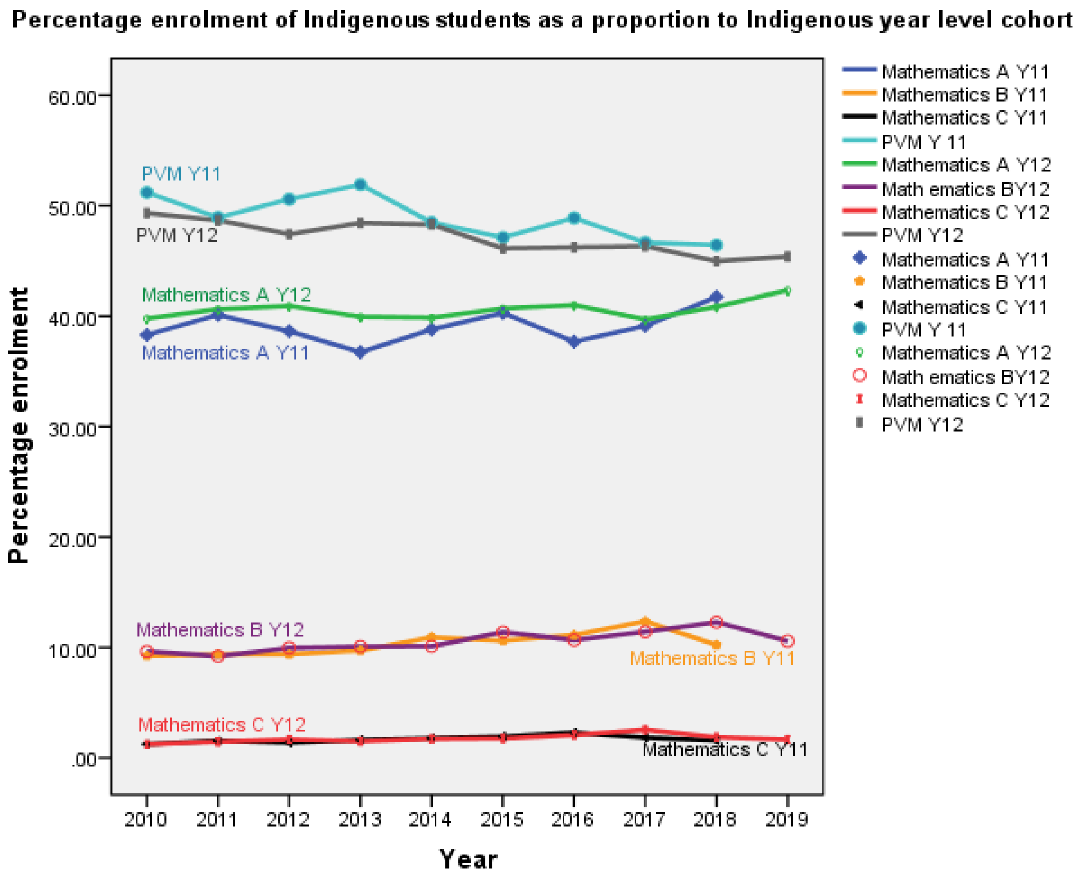

Figure 4.3: Percentage Enrolment in Mathematics of Indigenous Students in Queensland from 2010 to 2019

Table 4.4: Average Percentages of Distribution of Indigenous Students in Mathematics in Queensland from 2010 to 2019

Year 11	Year 12
Mathematics A (39.05%)	Mathematics A (40.58%)
Mathematics B (10.33%)	Mathematics B (10.53%)
Mathematics C (1.70%)	Mathematics C (1.76%)
Prevocational (48.91%)	Prevocational (47.13%)

4.7.5 Dropout Rates in Mathematics B and C

Figure 4.4 shows the dropout rate in Mathematics B and C for all students, while Figure 4.5 shows the dropout rate according to gender. Figure 4.6 shows the percentage drop of Indigenous versus non-indigenous students. Additionally, the trends of students' movement between mathematics subject options can also be determined through data analysis. The availability of both Year 11 and 12 data allowed changes in students' enrolment as they moved from one year to the next to be tracked.

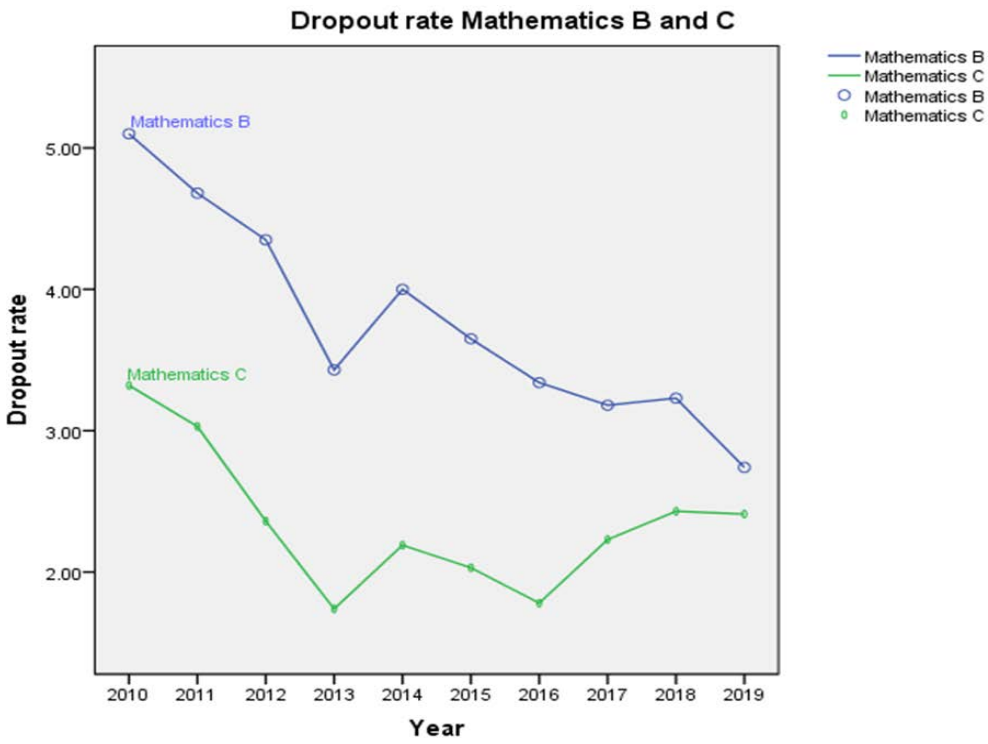

Figure 4.4: Dropout rate in Mathematics B and C for all students from 2010 to 2019

Mathematics B had consistently more students than Mathematics C dropping out as they moved from Year 11 to Year 12. On average, about 688 students dropped from Mathematics B every year compared to 108 students in Mathematics C. This meant that the dropout rate from Mathematics C, although averaging 2.35%, was calculated on a smaller population than for Mathematics B with an average of 3.77%. For the period under consideration, that is, 2010 to 2019, a total of 3372 females and 4582 males dropped out of calculus-based mathematics in Queensland.

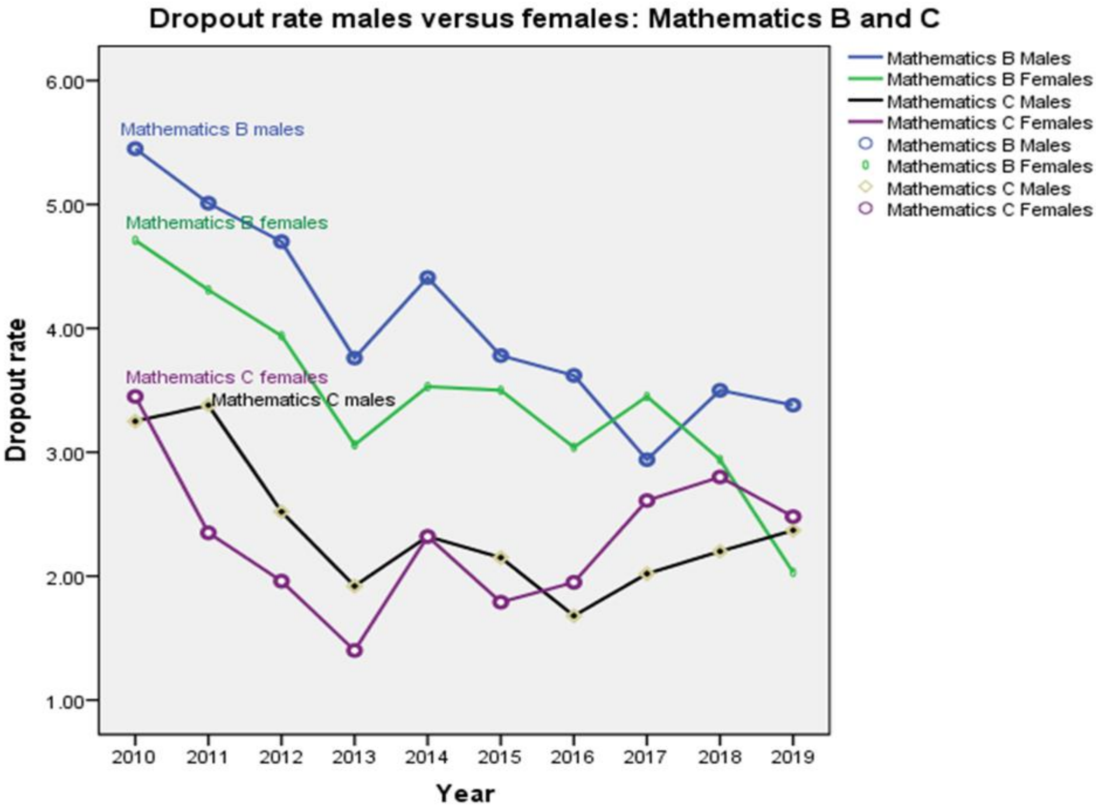

Figure 4.5: Dropout Rate According to Gender from 2010 to 2019

A larger percentage of males than females dropped out of Mathematics B and C, with an average rate of 4.06% in Mathematics B and 2.38% in Mathematics C. By contrasting, females were in the minority in both options but their dropout rate was 3.45% in Mathematics B and 2.32% in Mathematics C.

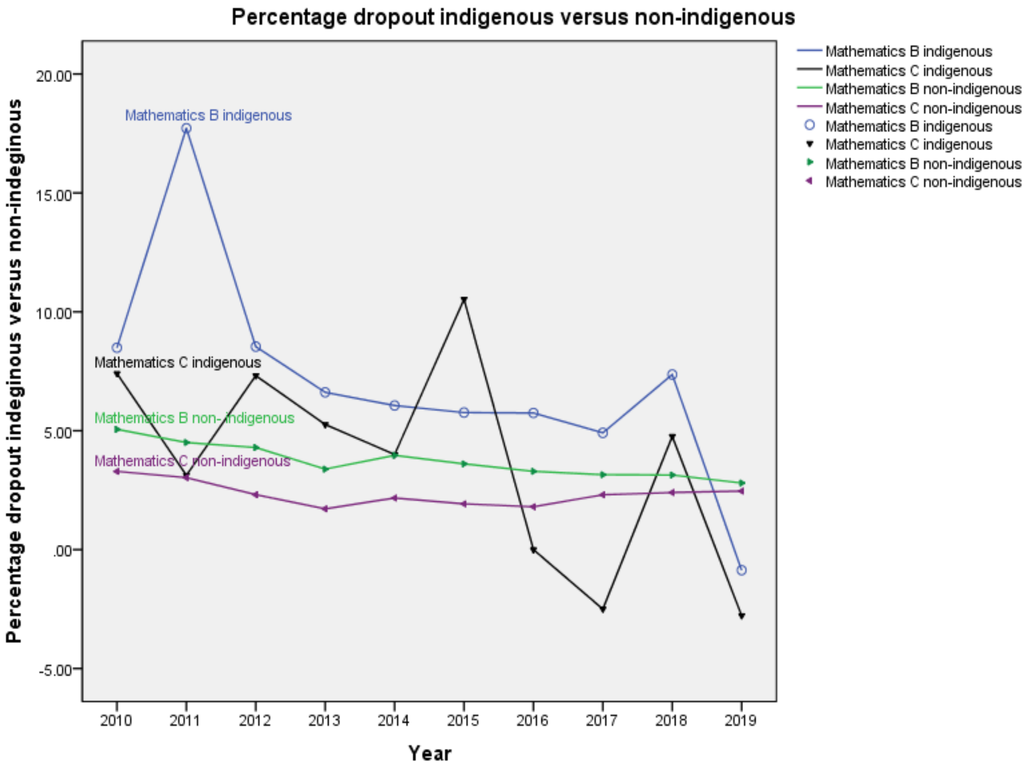

Figure 4.6: Dropout of Indigenous versus Non-Indigenous Students from 2010 to 2019

The average dropout rate for Indigenous students was 7.03% for Mathematics B and 3.71% for Mathematics C. This was despite the fact that Indigenous students comprise a very small percentage of enrolments at any year level. In comparison, non-Indigenous students had an average dropout percentage of 3.72% for Mathematics B and 2.24% for Mathematics C. Raw data show a total of 225 indigenous students dropped out of calculus-based mathematics from 2010 to 2019.

4.8 Discussion

Findings from this research indicate that more male students opted for Mathematics B and C than female students in Years 11 and 12 in Queensland in the years 2010 to 2019. As highlighted in the data, an average of 47.5 % and 35.2 % of all Mathematics B and C respectively in Years 11 and 12 were females. This agrees with the earlier findings of Ainley et al., (2008), Forgasz, (2006a), Forgasz, (2006b) and indicates that fewer females opted for calculus-based mathematics than males for the period 2010 to 2019. The low percentage of females choosing Advanced Mathematics is a concern compared to males. Against that, however, the dropout rates of females from these two subjects in the same period was 3.45% for Mathematics B and 2.31% for Mathematics C, which is less than the dropout rates for males of 4.06% for Mathematics B and 2.38% for Mathematics C. This suggests that the female dropout rate could be further reduced if female enrolment was improved and that sustained improvements in female enrolments could give female students the potential to surpass male students in their numbers in Mathematics B and C. Female students must be encouraged to choose options that offer more STEM opportunities and to perform well in those options. This supports the need for educators to develop strategies that improve female enrolment in these subjects in Year 11. While the results of trends analysis cannot explain why fewer females choose Mathematics B and C, a closer focus on not only academic but also social and cultural factors that support female students' enrolment in Mathematics B and C is essential and this could be a focus of future research.

Mathematics is compulsory in Queensland for all students to achieve a Queensland Certificate of Education (QCE) and students decide on which option to pursue in Years 11 and 12. However, findings from this research indicate that not all schools offer all options. As suggested earlier, the Australian Council of Deans of Science in 2006 found that schools in more remote regions struggled to recruit qualified mathematics teachers. While this study did not focus on this issue, the Australian Mathematical Sciences Institute [AMSI] (2014) noted that this is a particular challenge for Queensland schools. The results in this study exposed a worrying trend as a significant number of schools across Queensland do not have student enrolment in Mathematics B and C, the yearly average being only 13 and 83 schools respectively. This agrees with the AMSI (2014), which suggests that the number of schools able to provide Advanced Mathematics subjects at Years 11 and 12 is steadily declining and with this, the number of students studying Advanced Mathematics. Additionally, the report suggests that shortages in specialised mathematics teachers has meant that around 40 percent of classes are taught without a qualified mathematics teacher. There is need to develop both material and human resources to empower classroom practise, which may help address the decline in student enrolment in Mathematics B and C. However, it is also important to target these resources to schools that currently do not offer some of the mathematics options so that they can do so in the future. Significantly, a declining trend of student enrolment in Advanced Mathematics subjects may also lead to fewer qualified mathematics teachers for the future.

Results from the study showed that Indigenous students opted mostly for Mathematics A and Prevocational mathematics. Although Indigenous students constitute a very small percentage of enrolments at any year level, the average dropout rate is worth noting 7.03% for Mathematics B and 3.71% for Mathematics C. In addition, schools in remote regions have difficulty recruiting qualified mathematics teachers (Australian Council of Deans, 2006), which may impact Indigenous students more than other student groups. The study argues for an urgent focus to redress these trends and imbalances. The data does not include socio-economic status or cultural factors that might also be at play within such settings. Additionally, lack of local STEM career opportunities in remote and regional areas might also play a part in the mathematics subjects students opt. This is because graduating from high school may not necessarily translate to starting a high-status career. It would be worthwhile to conduct further research that investigates the views of indigenous students in remote and regional schools to shed further light on their experience of learning mathematics. However, this is beyond the scope of this study as the aim here is to identify broad trends purely from a statistical viewpoint.

Arresting the dropout rates in calculus-based mathematics can be one way to improve the participation rate in the subjects. Results show that a significant number of students who opted for the calculus-based mathematics subjects in Year 11 dropped the subject and enrolled in non-calculus-based mathematics in Year 12. Mathematics B particularly showed more students than Mathematics C dropping out as they moved from Year 11 to Year 12. Data from 2010 to 2019 also shows that more students opted for Mathematics B or C in Year 11 than in Year 12, which was the opposite for Mathematics A and Prevocational. As suggested by McPhan et al. (2008), schools can arrest the decline and the high dropout rates in calculus-based mathematics if they implement classroom practises that engage students and focus on improving student understanding of important concepts at every level of learning. Arresting this decline becomes imperative to support students' future enrolment in STEM-related careers.

Research is needed to develop teaching and learning strategies that increase student enrolment in calculus-based mathematics subjects. More needs to be done to increase enrolment in calculus-based mathematics to satisfy the demand in STEM-related careers. One way of doing this would be to focus on improving enrolments in

Mathematics B (now called Mathematical Methods in Queensland's new syllabus), which has an average percentage enrolment of 30.41% and offers almost the same opportunities as Mathematics C, now called Specialist Mathematics. It is also important for teachers to engage resources that may increase the chances of students' engagement and success in mathematics which can play an enabling role increasing student enrolment. Thus, Queensland has the potential to have more than 31% of all Year 12 enrolments eligible for the STEM tertiary program and becoming STEM champions.

4.9 Chapter Conclusion

This chapter investigated Years 11 and 12 students' options and trends of enrolment in calculus-based and non-calculus-based mathematics subjects between 2010 and 2019, using data from the Queensland Curriculum and Assessment Authority. It also looked at the central role that mathematics plays as an enabler of STEM-related courses and careers. It found out that the mathematics trends in Year 12 in Queensland are consistent with previous research at national level. Males dominated in Mathematics B and C and fewer female students opted for calculus-based mathematics. Indigenous students opted mostly for Mathematics A and Prevocational mathematics. However, a significant number of schools do not offer calculus-based mathematics options and consequently have no student enrolment in the subject. The study argued for an urgent focus to redress these trends and imbalances and calls for further research that focuses not only on academic factors, but social and cultural factors to support all students' participation in calculus-based mathematics. The next chapter provides an analysis of student participation in calculus-based mathematics using data from the Queensland Curriculum and Assessment Authority (QCAA), Socio-Economic Indexes for Areas (SEIFA) from the Australian Bureau of Statistics (ABS); schools' Index of Community Socio-Educational Advantage (ICSEA) values from the Australian Curriculum, Assessment and Reporting Authority (ACARA); and schools transfer ratings from the Department of Education (DoE).

Chapter 5: Trends in Calculus-Based Mathematics in the New Senior Secondary Queensland Certificate of Education

A version of this chapter was presented at the *International Conference on Education in Mathematics, Science and Technology, Antalya, Turkey*, *March 24-27, 2022.* https://researchonline.jcu.edu.au/76298/

5.1 Chapter Introduction

Mathematics plays a central role in innovation, scientific, technological, economic and social knowledge development (Watt et al., 2017). The sciences digital technologies and innovation in particular are regarded as the economic drivers and main jobs of the future (Black et al., 2021; PwC, 2013), and mathematics is regarded as a significant enabler of these fields (Australian Academy of Science, 2016). In Australia, "innovation and digital technologies have the potential to increase Australia's productivity and raise GDP by $136 billion in 2034, and create close to 540,000 jobs" (PwC, 2013, p. 13), hence mathematics is pivotal in reshaping the future (Chubb, 2012). Australia in general, and Queensland in the context of this research, needs graduates with Advanced Mathematics skills to promote innovation, data synthesis and technology if it is to remain competitive globally.

Indeed, promoting enrolment and achievement in Advanced Mathematics in schools is a focus of most governments all over the world (Noyes & Adkins, 2016; Treacy et al., 2020), because mathematics drives STEM (Shaughnessy, 2013). Similar to other countries such as the United Kingdom, Australia offers bonus points at university entry for students who pass Advanced Mathematics as an incentive to encourage students to study Advanced Mathematics at senior secondary level (Prendergast et al., 2020; Treacy et al., 2020). The distinct advantage of studying Advanced Mathematics in high school is not only to achieve individual goals but because of its recognised value to society.

Developing Advanced Mathematics skills results in high economic value, since "strong mathematical skills are critically important for a thriving and competitive knowledge-based economy" (Adkins & Noyes, 2016, p. 94). Studies have shown that students who pursue Advanced Mathematics are interested in pursuing high-impact jobs (Gijsbers et al., 2020). Indeed, people with advanced mathematics skills progress to earn about 11% more than those without by the time they reach 34 years of age (Adkins & Noyes, 2016). Similarly, choosing Advanced Mathematics is generally regarded as a pathway to high-paying jobs (Light & Rama, 2019). The link between economic development, prosperity and Advanced Mathematics makes mathematics a key transformational focus for governments and understanding trends in students' enrolment in Advanced Mathematics can inform policy makers.

The purpose of this chapter was to determine trends in enrolment in calculus-based mathematics under the new curriculum introduced in Queensland in 2019. The chapter built on Chapter 4 which focused on students' enrolment in calculus-based mathematics in the phased-out curriculum in Queensland (Chinofunga et al., 2021). This chapter expanded the focus further to the relationship between enrolment, dropout rates, SES, school location and teacher mobility and transfer ratings and contributes to the limited literature available on the impact of social and economic factors and school location on enrolment in calculus-based mathematics.

5.2 Importance of Calculus-based Mathematics

Calculus is built on the foundations of the analysis of changing phenomena. Therefore, "calculus is essential for developing an understanding of the physical world" (Queensland Curriculum and Assessment Authority (QCAA), 2018 p.1). Calculus-based mathematics introduces differentiation and integration at high school, which provides students with the opportunity to model quantities that undergo change and a portal for deeper theoretical growth (Maltas & Prescott, 2014). In Queensland, graduates with either Specialist Mathematics and or Mathematical Methods have a pathway to pursue tertiary courses that are mathematics-intensive, such as natural sciences, health sciences and engineering (QCAA, 2018). However, students who opt for Specialist Mathematics also have to study Mathematical Methods but have a distinct advantage at tertiary level as Specialist Mathematics is regarded as more advanced. Thus, studying these subjects is critical as students prepare for careers in a competitive world.

Several scholars have highlighted the importance of Advanced Mathematics in providing better and more diverse career opportunities, (Chinnappan et al., 2008;

Chinofunga et al., 2021; Maltas & Prescott, 2014; Noyes & Adkins, 2017) and facilitating skills for the STEM workforce (Kennedy et al., 2014). Moreover, calculus-based mathematics is critical in "developing students' logical thinking and reasoning abilities" (Prendergast et al., 2020, p. 753). A country's economic status and social wellbeing is enhanced by having a workforce that possess Advanced Mathematics skills as these skills are critical for research, industry and business to thrive (Black et al., 2021). A projected increase in school enrolments of 20.4% by 2026 in Queensland must prompt policy makers to find ways of boosting calculus-based graduates by the same margin (O'Connor & Oam, 2019). Calculus-based mathematics offers distinct advantages for graduates as it supports critical thinking and decision-making which is central to problem solving, thus preparing them for individual growth and flexible but critical career options.

High school calculus-based mathematics increases the chances of entry into highly sought-after courses in higher education (Cogan et al., 2019). Hence,

> Students need a good measure of rigorous, formal mathematics in order to be literate, prepared for whatever career path students choose upon completion of their secondary education whether they choose to enter immediately the work force; to enter a technical, trade or vocational career path, or to continue their formal education at a college or university" (Cogan et al., 2019, p. 531).

Furthermore, studying calculus-based mathematics at senior secondary level enhances the chances of success in STEM courses at tertiary level (Cohen & Kelly, 2020; Gottfried, 2015; Nicholas et al., 2015; Redmond-Sanogo et al., 2016). Research also indicates that students who graduate from high school with Advanced Mathematics subjects do well in health sciences at university with a high-grade average (Ryan et al., 2017). High school graduates with non-calculus options who want to pursue tertiary courses where calculus-based mathematics is a pre-requisite are required to take up bridging or remediation courses (Nicholas et al., 2015; Redmond-Sanogo et al., 2016; Varsavsky, 2010). Undoubtably, the role that calculus-based mathematics plays in STEM tertiary courses cannot be underestimated (Maass et al., 2019).

5.3 Socio-economic Background and Participation in Calculus-based Mathematics

Social and economic background largely determines access to resources. Students from high SES families and/or schools have access to better resources that can provide opportunities for success compared to those from lower socio-economic backgrounds (Bornstein & Bradley, 2014). Consequently, students' enrolment and achievement are significantly influenced by "school characteristics such as location and socio-economic background of the students it serves." (ACARA, 2013 p.1). Additionally, differences in student achievement are often influenced by students' SES (Broer et al., 2019). "In Australia, the magnitude of the socio-economic gap in mathematics achievement at age 10 is about 65% as large as the gap observed among 15-year-olds, and about 58% as large as the gap in numeracy proficiency among 25-29-year-olds" Organisation for Economic Co-operation and Development [OECD, 2018, p. 2]. Consequently, students from low SES are more likely to encounter limited educational opportunities and social inequality (Perry, 2018). Moreover, financial and human capital complemented by resources accessed through networking play an important role in shaping students' choices and beliefs (Bradley & Corwyn, 2002). The better and more diverse opportunities that calculus-based mathematics offer are skewed towards students from high SES families or who go to high SES schools.

Socio-economic factors also influence students' mathematics subject choices and achievement (Valero et al., 2015). Consequently, students from a high SES background are more likely to enrol in and achieve well in mathematics, especially in advanced options, than those from a low SES background (Valero et al., 2015). Moreover, parents of students from high SES background have high expectations and encourage their children to take Advanced Mathematics (Hascoët et al., 2021). In contrast, students from lower SES communities may not interact much with knowledgeable and experienced adults who can act as role models and provide stimulating and motivating experiences, thus limiting the opportunities and options for such students (Bradley & Corwyn, 2002). This is because the immediate social network around students, including parents, teachers, siblings and friends, plays a key role in influencing students' mathematics choices (Kirkham et al., 2019). The critical role that parents and social background play in influencing students' mathematics choices emphasises the importance of school location, school choice and the social network to which a student is exposed.

On average, a student who attends a higher SES school enjoys higher educational outcomes compared to a student from a similar social background who attends a lower SES school (Perry & McConney, 2013, p. 125). This is because high status peers are significantly influential to other peers within a social group (Choukas-Bradley et al., 2015). Schools with a high SES are strongly associated with high academic expectations, competition and achievements (Perry & McConney, 2013), hence students' mathematics choices are influenced by the school environment, which is expected to be highly stimulative, productive and positive (Willms, 2010). Clearly, the interaction between students from different levels of SES in high SES schools provides an opportunity for networking among peers that will boost mathematics achievement, especially for those from a low SES (Perry & McConney, 2013). Hence, school SES plays a critical role in students' mathematics choices regardless of the students' family SES.

A school reflects the demography of the community within its catchment area and those located in communities with low SES have students who are in some way disadvantaged (Hernández, 2014). In fact, "schools that are in the same district, but located in neighbourhoods of differing SES display a large disparity in opportunities and quality of education offered to students" (Hernández, 2014, p1). Students who attend schools in high SES neighbourhoods have access to relevant information and experiences that help them set high expectations and above all better educational resources (Ireneusz, 2020; Pritchett, 2001). Schools in affluent areas have better physical and material resources that differentiate them from other schools. As Broer et al. (2019) said, differences in educational opportunities are influenced by accessibility to well-resourced schools. Similarly, "It is not just the relative wealth of parents that holds large numbers of bright kids back: it is postcode inequality too. What part of the country a child grows up in has a real impact on their life chances" (Nick Clegg, former leader of the UK Liberal Democrats, 2016). In contrast, students from low SES areas who attend high SES schools score 86 points higher than their counterparts in low SES schools (OECD, 2018). Students from low SES families and communities have limited options to pursue because of the social and financial capital that is needed to attend reputable and well-resourced schools.

Student enrolment and achievement in Advanced Mathematics is linked to school resources that include discipline-trained teachers and family social economic status (Chiu, 2010). Importantly, it is the mathematics teachers' expertise in *teaching* the subject and making it more engaging and understandable to students that plays a critical role in student enrolment and participation in calculus-based mathematics (Kirkham et al., 2019). As Chinnappan (2008) says, "the likelihood of a student pursuing further studies in mathematics would be influenced by their experiences in mathematics classes at secondary school" (p. 33). For example, past mathematics achievement directly influences students' attitude towards mathematics (Birgin et al., 2010; Hascoët et al., 2021; Sikora et al., 2019). Clearly, "attitudes concerning mathematics show significant impact on one's decisions about the amount and nature of mathematics one will study in the future" (Recber et al., 2017). As a result, students' choice of schools influences the mathematics options they select (Sikora et al., 2019). Students from low SES families have limited options in terms of school choices as they are more likely to enrol in schools within their communities.

The location of a school is a major factor in the resources and opportunities that school can offer, not least in how it contributes to its teacher mobility and transfer rating. Queensland state schools are allocated transfer ratings from 1 to 7 depending on their remoteness, access to and level of amenities in the area, the complexity of the school environment and staffing requirements (Department of Education [DoE], 2019). Remoteness is determined by distance from Brisbane or Toowoomba or any coastal city of more than 8000 people (DoE, 2019). In fact, school transfer ratings are the basis of the transfer points teachers accrue (Department of Education, 2020). Therefore, "teachers who elect to work for longer periods in schools of rating 3 to 7 increase their prospects of securing a transfer to a preferred location where they choose to return, while schools benefit from the greater stability and stronger community integration." (DoE, 2020, p. 5). Teachers who are attached to a school for a longer period perform better than those who have a short stint at the school and this pattern is more apparent in disadvantaged schools (Hanushek & Rivkin, 2010). Teachers at a school with a rating of 7 are due for transfer after two years while others are expected to serve three years at a school to qualify (DoE, 2020). However, any other personal, social, professional circumstances and transfers from a school with a lower rating to one with a higher rating may also lead to approved transfers (DoE, 2020). The higher the school transfer rating, the more transfer points teachers accrue, which may result in unintended consequences of high teacher turnover in such schools.

High teacher turnover in schools is also a key factor in hindering quality education and better options for students in disadvantaged communities (Barbieri et al., 2011). Teachers may target schools with high transfer ratings because they “are simply waiting to move on to a desired location, putting low effort into their current work duties and disregarding any longer-term plans for their students” (Barbieri et al., 2011, p. 1430). Therefore, a substantial number of teachers tend to be more effective and more focused on delivery after a voluntary transfer (Jackson, 2013). Contrastingly, teachers who teach students who are keen to engage or are high achievers are less likely to transfer (Boyd et al., 2011). This means that teachers in low transfer-rated schools may serve longer in a school, which in turn provides stability, consistency and confidence for students to enrol in calculus-based mathematics, if other factors like socio-economic disadvantages are minimised.

5.4 Socio-economic Measures in the Study

A significant number of researchers (Anastasiou et al., 2020; Avan & Kirkwood, 2010; Broer et al., 2019) have linked family and neighbourhood socio-economic status (SES) with educational outcomes. SES differences mainly involve accessing material (financial, assets) and social (community networking, neighbourhood) resources that impact the wellbeing and development of individuals, families and neighbourhoods (Bornstein & Bradley, 2014; Bradley & Corwyn, 2002). However, obtaining individual family SES data is very difficult considering the sensitivity of the subject to society (Broer et al., 2019). Nevertheless, the SES of an area can be determined using the Socio-Economic Index for Areas (SEIFA), which indicates the relative advantage and disadvantage of a neighbourhood (Australian Bureau of Statistics [ABS], 2018b). This study sought to determine the correlation between the school districts’ SEIFA indices, schools’ Index of Community Socio-educational Advantage (ICSEA), teacher mobility and transfer ratings with students’ dropout rates in calculus-based mathematics subjects in Queensland state schools.

The Australian Bureau of Statistics census data can be used to infer important school information such as relative advantage and disadvantage of a neighbourhood (Gibson & Asthana, 2000). The SEIFA index is developed after every census, the current index being based on the 2016 census. The data includes SES index in percentiles and name of area. This data was correlated with school data obtained from the QCAA, which included the name of the district, the postcode, and enrolment per unit. The period under study was of particular interest because Queensland changed to a new senior curriculum in 2019 and the first external examination was in 2020. Importantly, the analysis would help determine the impact of school postcodes and SES on enrolment in calculus-based mathematics.

The SEIFA value was used to better understand the relationship between socio-economic advantage and disadvantage and social and educational outcomes (ABS, 2018a). The ABS broadly defines "relative socio-economic advantage and disadvantage in terms of people's access to material and social resources, and their ability to participate in society" (2018a, p. 6). While the percentile value on the SEIFA index is meant to indicate where each area sits in terms of SES within Australia as a whole (ABS, 2018a), this study focused only on Queensland. The socio-economic status of an area is mainly attributed to the collective income, education, employment and occupation of people in a neighbourhood (ABS, 2018a). Thus, a low score on the index indicates a high proportion of relatively disadvantaged people in an area (ABS, 2018a, p. 6). This index was used comparatively in the trend analysis in this study.

To better understand the impact of socio-economic factors in relation to different schools and their location, the Australian Curriculum, Assessment and Reporting Authority (ACARA) developed an Index of Community Socio-educational Advantage (ICSEA). ICSEA values are developed using students' family background data, location of school and demography of indigenous and non-indigenous students (ACARA, 2013). It enables "comparisons between schools based on the level of educational advantage or disadvantage that students bring to their academic studies." (ACARA, 2013, p. 1). Similarly, it can be used as a measure of socio-economic advantage in education (Callingham, 2017). The ICSEA values range from 500, representing schools with students from hugely underprivileged educational backgrounds, to 1,300 for schools with students from very highly privileged educational backgrounds, and they have a benchmark average of 1,000 (ACARA, 2013). This study analysed trends in enrolment in calculus-based mathematics using ICSEA values of all Queensland government secondary schools to investigate if school location and socio-economic background plays a role in students' enrolment in the subjects.

5.5 Study Methods and Results

The study used data from a range of institutions (ABS, ACARA, DoE, QCAA) to investigate the impact of social and economic factors on enrolment in calculus-based mathematics. Quantitative methods were used to analyse trends from within and across data sets to establish a comprehensive picture of how socioeconomic status and school location affect enrolment. QCAA provided consent for the use of its data, which included school name, subject name, postal code and enrolment per unit. Each school and district were matched to their relevant SEIFA index (ABS), ICSEA value (ACARA) and transfer points (DoE). Statistical Package for the Social Sciences (SPSS) was used for inferential statistics as it involves coming up with conclusions drawn from the existing data. The Spearman's rank correlation coefficient was used to measure the strength and direction of a monotonic association (Sedgwick, 2014) between a range of variables (ICSEA, SEIFA and transfer ratings) and students' enrolment across the state of Queensland. The association was also tested on the variables and dropout rate. A monotonic association is observed when there is dependence on variable changes among two variables (Sedgwick, 2014).

Similarly, descriptive quantitative methods were applied to analyse trends using the Microsoft Excel suite of functions because it "provides a comprehensive approach to quantitative data analysis" (Johri, 2020, p. 4). Microsoft Excel is especially ideal for descriptive quantitative statistical analysis and data management through its use of functions and data organisation tools (Rubin & Abrams, 2015). Measures of central tendency such as mean (average) and mode, together with Excel in-built functions (e.g IF, COUNTIF, LOOKUP, graphics), were used to determine trends in students' participation. Specifically, the data analysis explored (i) students' enrolment and dropout rates per district, (ii) school location SEIFA index and students' enrolment, (iii) school ICSEA value and students' enrolment and (iv) transfer ratings and students' enrolment. The next section describes the data analysis using the SEIFA index, ICSEA value, school transfer rating and student enrolment to determine trends in students' enrolment in calculus-based mathematics.

5.5.1 Students' Enrolment and Dropout Rates per QCAA district

First, an analysis was carried out of the average percentages of student enrolment in Mathematical Methods and Specialist Mathematics in state schools per QCAA district between 2019 and 2020. Distance education schools were considered separately because their catchment area can span more than one district. In both Tables 1 and 2, enrolment in Unit 1 was considered for 2019 because it is the first unit students engage with in Year 11. Similarly, Unit 4 enrolment was considered in 2020 because it is the last unit before students sit for the external examination, hence it indicates the number of students who completed Year 12 calculus-based mathematics.

Table 5.1: Mathematical Methods Enrolment per QCAA District, 2019 to 2020

QCAA District	Unit 1 Enrolment 2019	Unit 4 Enrolment 2020	Dropout	% dropout
Brisbane-Ipswich	563	405	158	28.1
Brisbane Central	950	718	232	24.4
Brisbane East	619	405	214	34.6
Brisbane North	829	513	316	38.1
Brisbane South	625	334	291	46.6
Cairns	451	267	184	40.8
Gold Coast	731	457	274	37.5
Mackay	247	134	113	45.7
Rockhampton	337	179	158	46.9
Sunshine Coast	661	390	271	41.0
Toowoomba	388	223	165	42.5
Townsville	364	209	155	42.6
Wide Bay	354	208	146	41.2
Distance education	88	53	35	39.8
Total	7,207	4,495	2,712	

Table 5.1 shows the raw data on enrolment and dropout rates in Mathematical Methods in state schools per district at the beginning of Year 11 in 2019. The data shows that 7,207 state school students opted for Mathematical Methods in 2019 but those still enrolled for Unit 4 in Year 12 in 2020 numbered 4,495, representing a percentage dropout rate of 37.6%. This means that the total number of students in state secondary schools who opted out of Mathematical Methods from the start of Year 11 to the end of

Year 12 was 2,712. That is, for every 14 students who chose this subject, 5 did not complete it. Brisbane Central and Ipswich were the only districts with a less than 30% dropout rate, while Brisbane South, Mackay and Rockhampton were over 45%. Due to the high dropout in Brisbane South, Mackay and Rockhampton districts, for every 20 students who chose Mathematical Methods in Year 11, about 9 of the students had dropped out by the end of Year 12.

Table 5.2: Specialist Mathematics Enrolment per QCAA District 2019- 2020

QCAA District	Unit 1 Enrolment 2019	Unit 4 Enrolment 2020	Dropout	% Dropout
Brisbane-Ipswich	113	88	25	22.1
Brisbane Central	330	280	50	15.2
Brisbane East	168	131	37	22.0
Brisbane North	225	170	55	24.4
Brisbane South	196	139	57	29.1
Cairns	101	67	34	33.7
Gold Coast	191	147	44	23.0
Mackay	33	23	10	30.3
Rockhampton	91	59	32	35.2
Sunshine Coast	191	141	50	26.2
Toowoomba	99	60	39	39.4
Townsville	68	49	19	27.9
Wide Bay	100	77	23	23.0
Distance education	55	34	21	38.2
Totals	1,961	1,465	496	

Table 5.2 shows the raw data on enrolments and dropout rates in Specialist Mathematics in state schools per district from 2019 to 2020. The total number of students who opted to study Specialist Mathematics in Year 11 at the beginning of 2019 was 1,961 (Table 5.2), but only 1,465 enrolled for Unit 4; that is, 496 students, or 25.3%, dropped out. Thus, for every 20 students who opted for Specialist Mathematics in Year 11,15 continued until the end of Year 12. Cairns, Mackay, Rockhampton and Toowoomba districts had greater than 30% dropout rates. Similarly, distance education schools had a 38% dropout rate, the highest of all the jurisdictions under consideration. Brisbane Central remained the district with the lowest percentage dropout rate (15.2%) followed by Brisbane East and Brisbane- Ipswich at 22%. Mackay contributed the smallest percentage of 2.28% of students studying calculus-based mathematics among all the districts.

An analysis of the number of schools offering calculus-based mathematics in each district was also done. Figure 5.1 shows the distribution and number of schools offering Mathematical Methods and Specialist Mathematics in the 13 districts.

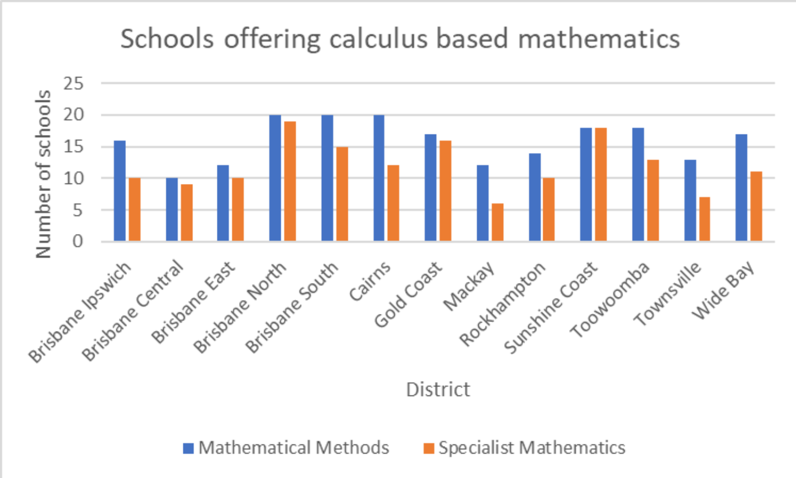

Figure 5.1: Schools Offering Calculus-based Mathematics per District 2019-2020

An analysis of Figure 5.1 and Tables 5.1 and 5.2 gives a deeper understanding of enrolment and the number of schools offering the options per district. Figure 5.1 shows Brisbane Central having only 10 and 9 schools respectively offering Mathematical Methods and Specialist Mathematics, but the enrolment in this district was the highest in Queensland. It also had the lowest percentage dropout rate (24.4%) than any other district. Contrastingly, Brisbane East and Mackay districts each had 12 schools offering Mathematical Methods, but Brisbane East had almost three times the enrolment of Mackay and the dropout rate was significantly lower. This was also true if a comparison is made between the Sunshine Coast and Toowoomba, Brisbane North and Cairns districts in Mathematical Methods. The number of schools offering Mathematical Methods and Specialist Mathematics in Mackay, Brisbane Ipswich, Brisbane South, Cairns, Mackay, Rockhampton, Toowoomba, Townsville and Wide Bay was significantly different.

5.5.2 School location SEIFA index and student enrolment

The first aspect was to investigate if the relationship between; enrolments in Mathematical Methods and SEIFA, and dropout and SEIFA, was statistically significant or not. Correlation analysis in the form of Spearman correlation coefficient was used to examine the nature and strength of the relationship under the following hypothesis.

H_0: There is no statistically significant relationship between enrolments/dropout and SEIFA.

Versus

H_1: There is a statistically significant relationship between enrolments/dropout rate and SEIFA.

Results in Table 5.3 were obtained from the analysis of the relationship between SEIFA values and enrolment.

Table 5.3: Spearman's rho correlation coefficient SEIFA, enrolment and dropout

			Enrolments	SEIFA Index
Spearman's rho	Enrolments	Correlation Coefficient	1.000	.335**
		Sig. (2-tailed)	.	.001
		N	88	88
	SEIFA Index	Correlation Coefficient	.335**	1.000
		Sig. (2-tailed)	.001	.
		N	88	88
			Dropout	SEIFA Index
Spearmen's rho	Dropout	Correlation Coefficient	1.000	.341**
		Sig. (2-tailed)	.	.000
		N	201	201
	SEIFA Index	Correlation Coefficient	.341**	1.000
		Sig. (2-tailed)	.000	.
		N	201	201

**. Correlation is significant at the 0.01 level (2-tailed).

The data from Table 5.3 indicate that there is a weak positive link between; enrolments and the SEIFA Index, and dropout and SEIFA index with a Spearman's correlation coefficient value of 0.335 and 0.341 respectively. Additionally, the probability value of 0.000, which is smaller than the threshold value of 0.01 for both, makes the connection statistically significant at 1%. The alternative hypothesis, according to which there is a statistically significant correlation between; enrolments and the SEIFA Index, and dropout and SEIFA is supported by these findings. Even though the correlation is modest, it indicates that enrolments improve with increases in the SEIFA Index.

In addition, an analysis of student enrolment in Mathematical Methods and Specialist Mathematics and school location based on their SEIFA index was undertaken on 203 schools. This excluded 4 distance education schools because their location had no influence on students' enrolment. Since the SEIFA data was presented as percentiles, 50% and upwards was considered as the upper half and thus designated areas with economic advantage while below 50% was considered as areas that were economically disadvantaged. Although there were 115 schools with students enrolled in Mathematical Methods in the lower half, they constituted only 39.8% of the Unit 1 Mathematical Methods cohort. Schools below the 50% percentile had an average percentage dropout rate of 42%, while those above the 50% economic advantage percentile had a dropout rate of 34.7%. Similarly, in Specialist Mathematics, the group in the 50% economic advantage percentile had a dropout rate of 24% compared to 26.6% in the economic disadvantage percentile. Although 49.7% (76 out of 153) of schools were considered to have economic advantage, they contributed 63.1% of all students who studied Specialist Mathematics in Unit 1.

5.5.3 School ICSEA value and student enrolment

A correlation analysis in the form of Spearman correlation coefficient was used to examine the nature and strength of the relationship between; enrolments and ICSEA values, ICSEA and dropout. Results of the analysis of the relationship are shown in Table 5.4.

Table 5.4: Spearman's rho correlation coefficient ICSEA, enrolment and dropout

			Enrolments	ICSEA
Spearman's rho	Enrolments	Correlation Coefficient	1.000	.613**
		Sig. (2-tailed)	.	.000
		N	39	39
	ICSEA	Correlation Coefficient	.613**	1.000
		Sig. (2-tailed)	.000	.
		N	39	39

			Dropout	ICSEA
Spearman's rho	Dropout	Correlation Coefficient	1.000	.496**
		Sig. (2-tailed)	.	.000
		N	201	199
	ICSEA	Correlation Coefficient	.496**	1.000
		Sig. (2-tailed)	.000	.
		N	199	199

**. Correlation is significant at the 0.01 level (2-tailed).

The results in Table 5.4 results show a Spearman's correlation coefficient value of 0.613 which suggests that there is a strong positive relationship between enrolments and ICSEA. These results suggest a statistically significant relationship between enrolments and ICSEA thus as ICSEA values increase enrolment also increase. Similarly, the results suggest that there is a weak positive relationship between ICSEA and dropout rate as supported by the correlation value of 0.496. The probability value of 0.000 which is less than the threshold of 0.01, implies that the relationship is statistically significant at the 1% significant level. According to the findings, as ICSEA increases, dropout rate also increases.

Similarly, descriptive statistical analysis of student enrolment in Mathematical Methods and Specialist Mathematics according to school ICSEA index was undertaken and the results indicated that dropout rates were influenced by school ICSEA index. Schools with an ICSEA value of more than 1,100 had a dropout rate of 27%, those between 1,000 and 1,100 had a dropout rate of 29.2% and those with an ICSEA value of less than 1,000 had a dropout rate of 43.4% in Mathematical Methods. The trend was the same in Specialist Mathematics in schools with an ICSEA value of 1,000 and above having a dropout rate of 20.3% compared to 29.2% of schools with a value less than 1,000.

5.5.4 School transfer ratings and student enrolment

Spearman's correlation coefficient was used to determine the relationship between enrolment and transfer ratings and also dropout and transfer ratings. Results of the analysis are shown in Table 5.5.

Table 5.5: Spearman's rho correlation coefficient Transfer rating and enrolments.

			Transfer Rating	Enrolments
Spearman's rho	Transfer Ratings	Correlation Coefficient	1.000	-.505**
		Sig. (2-tailed)	.	.000
		N	207	202
	Year 11 enrolments	Correlation Coefficient	-.505**	1.000
		Sig. (2-tailed)	.000	.
		N	202	202

**. Correlation is significant at the 0.01 level (2-tailed).

Results show a weak negative correlation between enrolment and transfer rating as evidenced by the correlation value of -0.505. The relationship is statistically significant at 1% significant level since the p-value of 0.000 is less than the chosen threshold value of 0.01. These results support the null hypothesis, which states that there is a statistically significant negative relationship between enrolments and Transfer ratings. Thus, as the transfer ratings decrease enrolment rises.

Lastly, an analysis of school transfer ratings and student enrolment in Mathematical Methods and Specialist Mathematics was undertaken. In 2019, at the end of Unit 1, there were 106 state secondary schools with transfer ratings of 1 and these schools had an enrolment of 4,919 students in Mathematical Methods. There were 101 schools with transfer ratings of 2 and above, but they had only 2288 students enrolled in the same option. Only 31.7% of all students who studied Unit 1 of Mathematical Methods were enrolled in schools with transfer ratings of 2 and above. Hence, the total enrolment of all the other schools with a transfer rating of 2 and above was less than half of those with a transfer rating of 1. Despite schools with a transfer rating of more than 2 enrolling 68.3% of all students studying Mathematical Methods, 1691 (34.4%) students dropped out of the subject from schools with a transfer rating of 1, compared to 1,021 from schools with a transfer rating of 2 and above. In fact, 54.5% of the enrolled students in schools with transfer ratings of 7 dropped out. Figure 5.2 shows the dropout rates in relation to school transfer ratings.

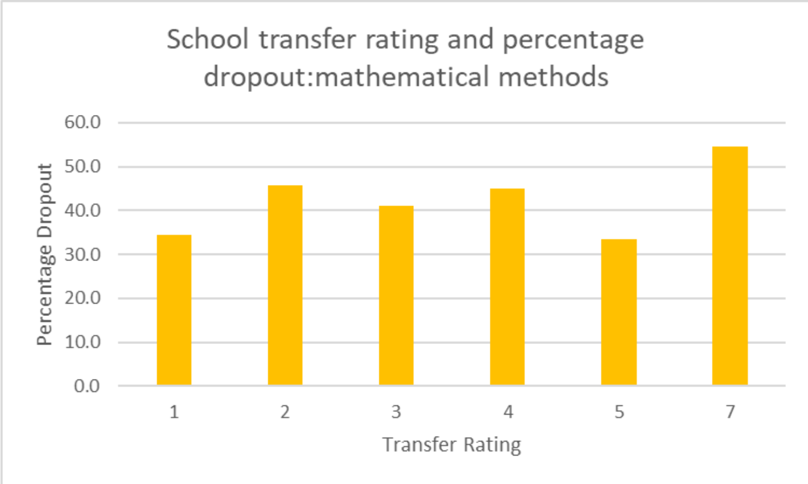

Figure 5.2: Dropout Rates and School Transfer Ratings 2019-2020

Fewer students in schools with a transfer rating above 1 chose to study Specialist Mathematics. Out of 156 schools with students studying the subject, only 61 had a transfer rating of 2 and above. This means less than half of schools with higher transfer ratings offer Specialist mathematics as compared to those with a rating of 1. In addition, only some schools with transfer ratings from 1 to 5 had any students who enrolled in Specialist Mathematics, as shown in Figure 5.3.

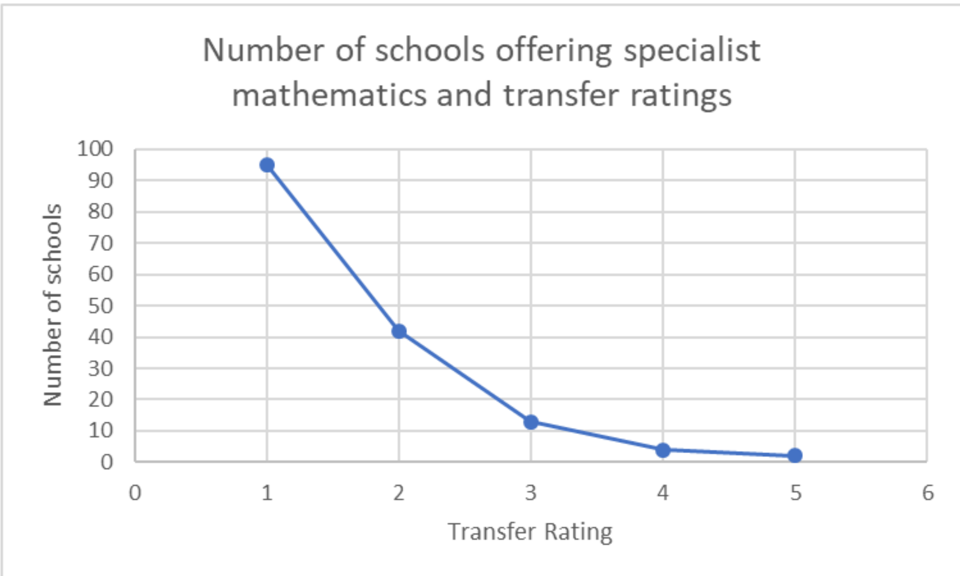

Figure 5.3: Schools Offering Specialist Mathematics and their Transfer Ratings 2019-2020

5.6 Discussion

Education systems all over the world aim to support and nurture students to reach their goals when they have chosen a career path. Hence, minimising dropout rates in calculus-based mathematics is fundamental. Undoubtedly, dropping out has "considerable social and economic implications," (Goss & Andren, 2014), especially considering the importance of calculus-based mathematics as a key enabler of STEM courses at tertiary institutions (Maltas & Prescott, 2014). In this study, enrolment in calculus-based mathematics in all districts showed a high dropout rate. In fact, for Mathematical Methods (about equivalent to the Mathematics B subject previously offered), the dropout rate in state schools of 37.6% from 2019 to 2020 was 10 times the average dropout rate of 3.77% of all Queensland secondary schools from 2010 to 2019 (Chinofunga et al., 2021). Similarly, in Specialist Mathematics (about equivalent to the Mathematics C subject previously offered), the dropout rate of 25.3% from 2019 to 2020 was more than 10 times the average dropout rate of 2.35% of all secondary schools in the state from 2010 to 2019 (Chinofunga et al., 2021). The substantial increase in student dropout rates from calculus-based mathematics between 2019 and 2020 is alarming, in that it indicates that students who initially showed interest and opted studying these subjects found it hard to continue.

Importantly, students who drop out from calculus-based mathematics options are not the same as students who choose to avoid the options at senior secondary school (Hine & Mathematics Education Research Group of Australasia, 2017). These are students with an initial genuine interest in calculus-based mathematics options as they think about and prepare for these options in Year 10 and then enrol in Year 11. If we are to increase the number of students enrolment in calculus-based mathematics, then the focus should start from retaining students who drop out. Disrupting this trend would reverse the enrolment and enrolment numbers in these subjects which have been shown as tumbling across Australia (Kennedy et al., 2014; Maltas & Prescott, 2014). Teachers as facilitators of learning can help to retain these students through effective planning and teaching that support student participation and engagement, thus increase the chances of success. Calculus-based mathematics teachers' planning must focus on enhancing students' confidence and their relationship with their chosen subject (Grundén, 2020) through providing a coherent and spiral sequencing of mathematical concepts that are anchored on student's prior knowledge and interest to support student participation and achievement (ACARA, 2015). In other words, effective mathematics teaching must support the connection of prior knowledge to new knowledge, and build, interconnect and expand knowledge and skills from familiar to unfamiliar contexts, (Novak, 2010;

Stoll et al., 2012) increasing opportunities for success. Such an approach is likely to arrest the dropping enrolment in calculus-based mathematics subjects. This approach might call for new and innovative research focused on supporting mathematics content sequencing and ways of promoting mathematics knowledge development at all levels in the school curriculum to stop the decline in enrolment in calculus-based mathematics.

The economic advantage or disadvantage of a school location and students who attend a school can be determined by the SEIFA index and ICSEA value. Inferential data analysis using the Spearman's rho correlation coefficient show a strong positive correlation between ICSEA and SEIFA against enrolment. Thus, statewide as the ICSEA and SEIFA values increases the enrolment also increases. Contrastingly, the Spearman rho correlation coefficient show a weak correlation between SEIFA and ICSEA against dropout rate demonstrating that statewide as the SEIFA or ICSEA values increase the dropout rate decreases. This study also showed that, when considering the initial uptake of Mathematical Methods, Brisbane Central district had the highest enrolment. Importantly, all 10 schools in this district had a SEIFA value of more than 92 and an ICSEA value of more than 1,000, demonstrating a high economic advantage enjoyed by the student population. Although it was a district with the least number of schools, it had the highest number of students enrolment in calculus-based mathematics in Queensland. Similarly, the Brisbane North district had 10 school locations out of 20 with a SEIFA value of more than 80 and 5 schools with an ICSEA value of more than 1,000 and it had the second highest enrolment. Contrastingly, the highest SEIFA value of a school location in the Mackay district was 74 and there were only 4 out of 12 schools in areas with values above 50. There were no schools with an ICSEA value of more than 1,000. Likewise, 6 school locations out of 13 in the Rockhampton district had a SEIFA value of more than 50 but less than 72 and there were no schools with an ICSEA value of 1,000 and above. The Wide Bay district had 17 schools offering Mathematical Methods and there was no school location with a SEIFA value above 50 and ICSEA of 1,000 and above. In addition, Townsville and Toowoomba districts had only up to two schools in the top SEIFA index or ICSEA value band, with the rest below average. It was observed that all these districts had low enrolments and a substantial difference between the number of schools offering Mathematical Methods and Specialist Mathematics. This meant that potential students who had the interest and capability to enrol and achieve well in the calculus-based mathematics subject did not have the option of enrolling in these subjects.

A proactive research agenda that supports teachers who teach in low SES areas and less desired schools in relation to the teacher mobility and school transfer ratings must not be limited to financial rewards. The focus should be on planning and pedagogical resources that build a foundation that promotes knowledge and skills development and facilitates independent learning. As argued by some researchers, it is “more meaningful to study what educators can work with to improve students’ participation and achievement” (Valero et al., 2015, p. 288). Thus, proactive research that focuses on planning and developing such pedagogical resources should be a priority. These pedagogical resources would need to include multiple representations, including visuals, as they are easy for students to follow and understand (Raiyn, 2016). Thus exploring how mathematical knowledge (procedures and concepts) can be visually represented can support teaching and learning of mathematics as they promote information processing. This proactive approach may also assist in promoting self-directed learning in students. Importantly, a common framework that can be used by teachers in such schools will help to bring stability to students’ learning because it would provide uniformity in concept development and critical delivery resources.

The economic advantage or disadvantage of a school location can be determined by the SEIFA value. The data analysis in this study showed that schools in the top half of SEIFA indexes of 50 and above contributed more than 60% of all students enrolled in calculus-based mathematics despite accounting for fewer than half of all state schools in Queensland. This is because school location and economic advantage significantly influence the knowledge, skills, experiences and other forms of capital students gain (Ireneusz, 2020). Schools, parents and students located in economically advantaged areas normally have high expectations, as modelled by the community (Pritchett, 2001). Resources offered by schools differ mainly because of SES location (Broer et al., 2019). Considering schools in the top half of SEIFA indexes of 50 and above, the data analysis in this study showed that the dropout rate was less than the lower half of SEIFA indexes, which reinforces the high expectations that schools in such locations foster. It is particularly important to pay special attention to schools with lower ICSEA values. The ICSEA value of a school provides a clearer indication of the economic advantage and disadvantage of students enrolled in that school. The relationship between the average dropout rate and a school's ICSEA value supports Perry and McConney's (2013) findings that schools with highly economically advantaged students are strongly associated with high academic expectations and are competitive, compared to schools with economic disadvantage. Thus, the high expectations and competition in schools with high ICSEA values have a substantial influence on students to continue with the subjects.

One of the most critical resource in any school is teachers. Teachers are attracted to different schools based on a range of considerations. School location and resources are key in attracting and retaining teachers, which is why schools' transfer ratings are mainly based on these factors. Results from the Spearman's rho correlation coefficient show a negative correlation between transfer rating and enrolment. Thus, as transfer ratings decrease which is determined by school location and resources the enrolment increases. In this study, almost 70% of the Mathematical Methods cohort were in the schools that had a transfer rating of 1 and minimal teacher turnover, in other words, schools that had stable and predictable environments. The schools with transfer ratings of 1 also had a significantly lower dropout rate than schools with transfer ratings above 1. A similar trend was witnessed in Specialist Mathematics, where enrolment was biased towards schools with ratings of 1, even if there were fewer of them than those with transfer ratings above 1. Barbieri and colleagues (2011) concluded that teachers in schools with high transfer ratings might not have long term plans to teach in those schools, hence they might be less committed and wait for an opportunity to leave, resulting further in less stable and predictable school environments.

The COVID-19 pandemic impacted education systems in different ways across the world. It might have affected students physically and psychologically and might have influenced to some extent the results obtained in this study. However, Queensland experienced minimum disruptions in 2019 and 2020 and the dropout rate was much higher in 2019, before COVID-19, than in 2020. A total of 3,117 students had dropped out calculus-based subjects by the end of 2019, during which the state experienced no lockdowns or restrictions at all. The introduction of external examinations, which contributed towards 50% of the overall calculus-based subject result, might have had an impact on students' confidence and thus their participation.

5.7 Chapter Conclusion

This paper investigated the numbers of senior secondary students enrolled in calculus-based mathematics subjects between 2019 and 2020 in Queensland state schools from different socio-economic districts. The QCAA data, which included subjects, unit enrolments, school postcodes and districts, was matched to SEIFA index (ABS), ICSEA value (ACARA) and transfer points (DoE). The high overall dropout rate in the new calculus-based mathematics subjects is a concern and the state is consequently losing a large number of students who could have pursued opportunities that are deemed to be jobs of the future. This study showed that socioeconomic factors, school location and transfer rating play a significant role in students' participation in calculus-based mathematics and dropout rates. Specifically, they showed that schools in low socioeconomic locations that enrol students from low SES backgrounds and that have high transfer ratings have both a low uptake in calculus-based options and high dropout rates. Further research is recommended to identify proactive strategies on how mathematics teachers can improve planning and delivery so as to promote participation and achievement and retain more students in calculus-based subjects. Importantly, there is urgent need for research that focuses on developing pedagogical resources that not only build a foundation that promotes knowledge and skills development but facilitates more structured learning for the students, thus, minimising the impact of school location, family SES and teacher turnover. The following chapters focus on the development of pedagogical resources to support the teaching of calculus-based mathematics.

Chapter 6: A Framework for Content Sequencing from the Junior Secondary to the Senior Secondary Mathematics Curriculum

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6.1 Chapter Introduction

According to Roche et al.,

> Given the complexity of mathematics teaching, including addressing curriculum goals, engaging students, catering for the diversity of readiness, connecting mathematics teaching to students' experience, and assessing student learning, to name just a few issues, it is difficult to imagine that teachers of mathematics can perform their role without substantial planning. (Roche et al., 2014, p. 854)

Effective planning provides direction and resources for quality curriculum delivery, particularly in the context of mathematics teaching. Further, planning links curriculum requirements in official curriculum documents and commercial and non-commercial resources to how knowledge is developed in class (Li et al., 2009). This chapter argues for and provides a framework for understanding and engaging in collaborative planning for effective sequencing of mathematics content for the transition from the *Australian Mathematics Curriculum* (Preparatory – Year 10) to the *Senior Queensland Mathematical Curriculum* (Years 11 - 12) [Queensland Curriculum and Assessment Authority (QCAA), 2018]. The Mathematical Methods Unit 1 on Functions that is taught in Year 11 was used as an example to illustrate the framework.

Planning plays a critical role in enacting the curriculum as it involves "activities related to knowing what to teach and how" (Fernandez & Cannon, 2005, p. 485). What and how teachers teach is critical to students' participation and achievement. As Roche and colleagues noted:

> Planning for mathematics teaching is important at all levels from sequencing of content and the structuring of lessons to the selection and preparation of manipulatives and worksheets but despite its centrality to

> curriculum delivery research-based descriptions of the practises of effective mathematics teachers do not emphasise planning. (Roche et al., 2014, p. 854)

Planning by teachers directly influences the quality of learning that students receive (González et al., 2020; Grundén, 2020; Li et al., 2009; Roche et al., 2014). For teachers, "planning is seen as an essential part of their work that has consequences for students' learning as well as work situation – planning can cause stress as well as be a way to reduce stress" (Grundén, 2020, p. 80). In fact, planning should focus on improving students' relationship with mathematics through providing a platform that promotes active engagement (Grundén, 2020). Planning is the foundation that sustains the whole curriculum implementation, as it makes a difference in every aspect of curriculum delivery, and consequently contributes to student participation and achievement as well as determining teaching quality.

An effective mathematics teacher must be an exceptional planner. "Excellent teachers of mathematics plan for coherently organised learning experiences that have the flexibility to allow for spontaneous, self-directed learning" (Australian Association of Mathematics Teachers (AAMT), 2006, p. 4). Australian teachers are expected to plan and teach "mathematical sequences and experiences that encourage students to think flexibly and creatively about concepts to develop 'big picture' thinking" (Davidson, 2019, p. 8). Similarly, the Australian Institute of Teaching and School Leadership (AITSL) (2014) expects teachers to design a teaching and learning sequence using curriculum knowledge, content, students' learning strategies and teaching pedagogies to increase student participation and achievement. This is because, during planning, teachers predict and plan the structure and conditions of the learning space (Conway & Munthe, 2017). Consequently, to ensure that no child is left behind in learning mathematics, planning must be the first port of call.

Supporting current teachers' planning practises can be a starting point (Sullivan et al., 2013). However, ways of improving the current planning in schools must be explored if teaching and learning is to be enhanced (Attard, 2012). "The curriculum that students experience in classrooms is the product of a complex web of decision-making which is shaped, but not determined, by the formal curriculum documentation" (Sullivan et al., 2013, p. 459). Therefore, curriculum planners such as teachers need to be supported on how to select and organise the crux of the curriculum (O'Neill et al., 2014). Mathematics teachers' understanding of the structure of the subject and how best content can be presented for maximum student engagement can be key to effective planning and consequently teaching and learning.

A critical aspect of effective planning is identifying and sequencing content and delivery strategies to optimise acquisition of knowledge, understanding and skills among students (QCAA, 2019). Content sequencing influences student engagement and helps them to develop mathematical knowledge (Kilpatrick, et al., 2001). The "what" of planning informs the "how", thus teacher effectiveness and learner participation and understanding is not only limited to classroom practice, but how the content is planned, sequenced and taught.

6.2 Collaborative Planning

This study draws from intentional collaboration of teachers as defined by the Queensland Department of Education. "Providing time and resources for staff to develop and plan units together was suggested as a way of deepening understanding of the Australian Curriculum" DoE, 2021, p. 7). Nevertheless, how teachers interrelate during collaboration and how they interpret the curriculum has a strong influence on the planning process (Grundén, 2020). Since teachers enact the curriculum, there is a strong correlation between curriculum planning and delivery material (Superfine, 2008). Indeed, the National Council of Teachers of Mathematics (NCTM, 2014 p. 12) states: "Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions." As a result, the level of engagement among teachers during planning influences the quality of the output (Bieda et al., 2020). This chapter will develop a framework on content sequencing that can support teachers on processes to be followed as they plan sequencing of mathematics content. The chapter will advocate for a collaborative approach to planning guided by a proposed framework.

Collaborative planning is not limited only to teachers teaching a year level but all mathematics teachers within or across schools. Many teachers look to each other for support during planning. Thus, school leaders must ensure that collaborative meetings are scheduled for teachers to review and share their experiences and expertise (Clarke et al., 2012). Collaborative planning can present opportunities for teachers to learn from each other, which results in the benefit of students (Gilbert & Gilbert, 2013). Especially, “when whole grade levels are involved, they create a critical mass for changed instruction at all levels; above all teachers serve as support groups for one another in improving practice” (Darling-Hammond & Richardson, 2009 p. 46). Collaborative professional learning brings teachers to work together, resulting in improvements to the whole school system rather than just to the class or grade level (Darling-Hammond et al., 2009). Research also indicates that effective professional learning is a contributing factor in differences in school performance (Darling-Hammond et al., 2009). As Tricoglus (2000) states, professional collaboration improves planning practice and teacher quality as teachers get an opportunity to discuss, share and document important aspects of teaching and learning. Collaboration of mathematics teachers within or across year levels can facilitate learning from each other and improve effectiveness in delivery and resource utilisation.

Mathematics planning must support effective teaching and learning at every year level to ensure students’ success. Many scholars (Kafyulilo, 2013; Konuk, 2018; Lynch, 2017; Schuhl, 2020; Usha, 2010; Voogt et al., 2016) have noted that when mathematics planning is done collaboratively:

- it reminds teachers that all levels/grades play a critical role in developing mathematical knowledge
- it reminds teachers that skills taught at every level/grade are applicable to subsequent levels
- it reinforces the notion that mathematical concepts are interlinked
- teachers develop a sense of ownership of the product
- it enhances teachers’ pedagogical and content knowledge
- it brings consistency across year levels
- it develops individual and team collective teacher efficacy
- it ensures consistent curricular priorities among colleagues
- it ensures students learn identified essential mathematics standards
- it enhances student learning
- teachers realise teaching is a shared responsibility
- it enhances the sense of community and revitalises enthusiasm towards teaching

- teachers might consider issues that they might not have been considered independently

Linking concepts across year levels demonstrates the hierarchical nature of mathematics and shows that every mathematics teacher at different year levels contributes to building students' mathematical knowledge. This is especially important in Australia and Queensland, where the mathematics curriculum transitions from a national curriculum (junior level) to a state curriculum at senior secondary. It also justifies the importance of collaborative planning within the cohort. Furthermore, students grasp that active participation in lower grades contributes towards success in mathematics at higher levels.

6.3 Mathematics Planning in Queensland

Queensland mathematics teachers have a range of resources at their disposal during planning. Apart from the official curriculum documents provided by the QCAA, non-official resources that are commercial or non-commercial in nature, such as textbooks, resources developed by colleagues or mathematics educators' associations and school documents, play an important role in planning, delivery and assessment (Roche et al., 2014; Sullivan et al., 2013). Also, web-based resources have grown in influence and use, especially multimedia video resources like YouTube and Khan Academy, as they are readily available. The diversity of available resources provides dynamic options to teachers as they can be useful in improving the quality of planning, be it individual or collaborative.

Queensland schools and teachers are the drivers of the planning process. Undoubtedly this is important because "curriculum planning is essential for contextualising curriculum content" (QCAA, 2019, p. 1). Thus, different schools can contextualise content according to students' experiences which might not be shared across schools (Demski & Racherbäumer, 2017). Roche's (2014) findings indicate that planning documents produced by teachers within or across schools vary, with some teachers valuing aspects of planning that others do not. Planning templates and samples from the federal Department of Education and QCAA have been developed and distributed to schools. However, it is important for teachers to understand the processes that underpin the planning decisions that have led to the creation of such documents (Roche, 2014). Therefore, a guiding framework is necessary to bring consistency and uniformity to the process of planning. Ultimately, this study proposed that a more relational and contextual planning framework underpinned by constructivism that provides a step-by-step systematic sequencing of curriculum content to promote interlinking, coherence and spiralling of mathematics concepts between lower- level and upper-level topics. Constructivism positions learning as a process of building new knowledge from the learner's prior knowledge, beliefs and skills (Garbett, 2011). Thus, the framework supports planning that fosters the development of new knowledge from prior knowledge.

As part of their planning, Queensland mathematics teachers are required to create a school-specific sequence of content; this is because the official syllabus document is not regarded as a teaching sequence (Roche et al., 2014; QCAA, 2014, p. 8), which in turn suggests that schools must take responsibility for developing "a spiralling and integrated sequence". Clearly, spiral sequencing deepens knowledge through revisiting concepts, building on previous knowledge, creating new knowledge using prior knowledge and dealing with increased conceptual complexity as learning progresses (Harden, 1999). Above all, the manner in which content is structured in the curriculum facilitates how students learn and understand complex phenomena (Bruner, 1977). For example, students are taught fundamental concepts at a lower level of schooling and the concepts are then revisited at a higher level to deepen understanding through application, comprehension and interconnections with other concepts.

Queensland schools classify long-term planning on three levels: (1) whole school curriculum and assessment plan, (2) year-level curriculum and assessment plan and (3) unit overviews (QCAA, 2019). A unit is "a sequence of lessons with a coherent focus, sometimes referred to as a topic sequence" (Roche et al., 2014, p. 854). A whole-school curriculum plan "shows learning sequence within and across the year levels", a year-level plan "outlines the sequence of learning and reflects the development of knowledge, understanding and skills within a level" and a unit overview "links prior and future learning" (QCAA, 2019, p. 3-4). Each level of planning informs the other. Thus, effective planning at all levels has the potential to improve curriculum delivery in Queensland schools.

The *Queensland State Schools Improvement Strategy (2022-2026)* mentions intentional collaboration as an improvement focus on curriculum delivery. It is defined as "the deliberate actions we take to work together, learn together and improve together"

(Department of Education [DoE], 2020, p. 1). Schools have the responsibility to implement the strategy document, thus requiring them to put in place mechanisms for collaboration among teachers. It is common practice in education departments the world over to allocate planning time for teachers as a means of enhancing curriculum delivery and student learning (Li et al., 2009). Queensland teachers are allocated five professional collaboration days, which are not only limited to planning in subject areas but other activities that the profession demands. Professional collaboration days at the beginning of the year provide an opportunity for long-term planning. However, for secondary full-time teachers, an additional 210 minutes a week is also allocated for planning, such as short-term individual planning, preparation, correction and administrative work (Queensland Teachers' Union [QTU], 2020). In addition, schools are encouraged to set time for staff curriculum meetings, which might involve all teachers or a sector.

6.4 Enhancing Student Participation and Understanding through Planning

Focusing planning on how students develop mathematical knowledge, skills and understanding enhances participation, as teaching becomes student centred (Grundén, 2020). Therefore, planning should be informed by hypothesising students' current level of understanding and how to develop it further (Simon, 1995). It is important during planning for teachers to be mindful of students' abilities and learning needs, the goal being for all students to participate and engage optimally (Attard, 2012). As a result, planning that focuses on student learning indirectly develops teachers' pedagogy, content knowledge and practice (Darling-Hammond & Richardson, 2009; Garet et al., 2001; Smith 2007). Because student-focused planning anticipates the learning process, it also supports student understanding.

In enacting the curriculum, teachers have the responsibility to identify key topics and provide students with the opportunity to deepen their understanding of such topics (ACARA, 2009). As the QCAA emphasised, "To support the development of complexity and independence of student learning, when planning units of work for a course of study, teachers should consider a range of designing opportunities together with the sequencing, content and interrelatedness of teaching strategies and learning experiences" (2013, p. 1). Content that is coherently planned provides students with an opportunity to deepen their mathematical knowledge, understanding and skills if they understand the fundamental concepts.

Planning for student understanding focuses on how students develop mathematical knowledge. Procedural knowledge, conceptual knowledge and procedural flexibility are critical for students to develop their mathematical knowledge and competency (Rittle-Johnson, 2017). Procedural knowledge is defined as knowledge of sequences of steps or operations, mathematical rules and facts that can be used to solve problems (Crooks & Alibali, 2014; Rittle-Johnson et al., 2015). Conceptual knowledge is the "comprehension of mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p. 5). Procedural flexibility involves knowledge, the use of varied procedures and the robust application of these to a variety of conditions (Rittle-Johnson & Star, 2007). Conceptual knowledge also plays an important role in flexible problem solving because understanding the conceptual foundations of a procedure will lead to generalisations when confronted with new but related problems. The relationship between conceptual and procedural knowledge is bi-directional as they both support the development of the other. However, both rely on students' prior knowledge as a foundation on which to build.

Planning that builds on prerequisites helps a teacher to identify gaps in student understanding that are likely to be encountered in class (Reys et al., 2020). A significant number of teachers administer diagnostic tests and studies support the practice as they may stimulate interest in learning and decode forthcoming lessons (John et al., 2013). At the same time, diagnostic tests help teachers to gain understanding of students' prior knowledge, understanding and skills since in most cases students may be at different levels. However, checking prior knowledge is insufficient on its own as teachers must also ensure that the planning provides every student with the opportunity to acquire the knowledge that is critical to engage with new knowledge meaningfully. When gaps in student knowledge are identified, the teacher can start and build from the concepts identified as prerequisites. Gaps in prior knowledge and skills impede students' understanding of new knowledge (Hailikari, 2008). A comprehensive sequence of learning provides flexibility in a class because students can start from varying levels of competence. For this reason, and in the sequencing of content, an ideal framework must develop a system of linking concepts and determine procedures that are involved in solving problems within a concept.

6.5 Content Sequencing in Unit 1 on Functions in the Mathematical Methods Subject

Once students have finished Year 10 or reached the age of 16, they have the option to remain in school or seek vocational traineeships. Students who choose to proceed to senior secondary are expected to engage with a mathematics option of their choice. In Queensland, students who plan on pursuing Advanced Mathematics are encouraged to engage with the 10A curriculum for gifted students at Year 10. However, students who choose to pursue the general Year 10 curriculum can still enrol in Advanced Mathematics in senior school. The mathematics curriculum from primary school to Year 10 is governed by the Australian curriculum while the Queensland curriculum, which is developed by the QCAA, is followed at senior secondary level. This chapter describes how the Australian mathematics curriculum (P-10) and the QCAA Mathematical Methods curriculum documents were used to develop examples on how to apply the proposed framework.

For the purpose of this study, prior knowledge will be defined as prerequisite concepts from lower levels that interlink with concepts at upper levels. Assumed prior knowledge is identified from the Australian Curriculum (P-10) that students have engaged with before entering senior secondary school. New knowledge is outlined in the Mathematical Methods syllabus. "To make decisions about the mathematical content in the planning process, teachers reflect and have considerations in relation to students' abilities and their prior knowledge" (Grundén, 2020, p. 78). Correspondingly, prior knowledge is important in developing quality programs and sequencing as it demonstrates continuity and reinforces the importance of fundamental concepts and structure of mathematics (Reys et al., 2020). The hierarchical nature of mathematics must be the basis of effective planning and classroom practice.

Learning in mathematics is sequential, which means basic concepts presented in lower levels must be mastered to enhance the chances of understanding new knowledge (Brosvic & Epstein, 2007). Similarly, Hailikari and Nevgi (2010, pp. 2082-2083) emphasise, "Concepts presented in the introductory courses are usually needed throughout the academic career and should provide building blocks for more advanced courses in the same subject." During planning, teachers have the responsibility of identifying relationships between lower-level and upper-level topics, concepts, and skills, linking the two levels and providing students with the opportunity to build from the familiar to the unfamiliar.

Creating a framework to support and improve existing planning practices is of critical importance (Superfine, 2008; Sullivan, 2012; 2013). Not only does a framework provide transparency, accountability and evaluation of the process by stakeholders (O'Neill et al., 2014), but frameworks that are flexible can accommodate adjustments during implementation (Grundén, 2020). The proposed framework in Figure 6.1 will provide a step-by-step systematic sequencing of curriculum content to promote interlinking, coherence and spiralling of concepts. This will cater for mathematical methods students at every level of their mathematics journey in Unit 1 of Year 11. Depending on the level of assumed prior knowledge and skills students can recall and apply, teachers can start teaching from any level of sequenced content. The framework can be adapted to all mathematics options and levels, although for the purposes of this study, Queensland Mathematical methods Unit 1 were considered.

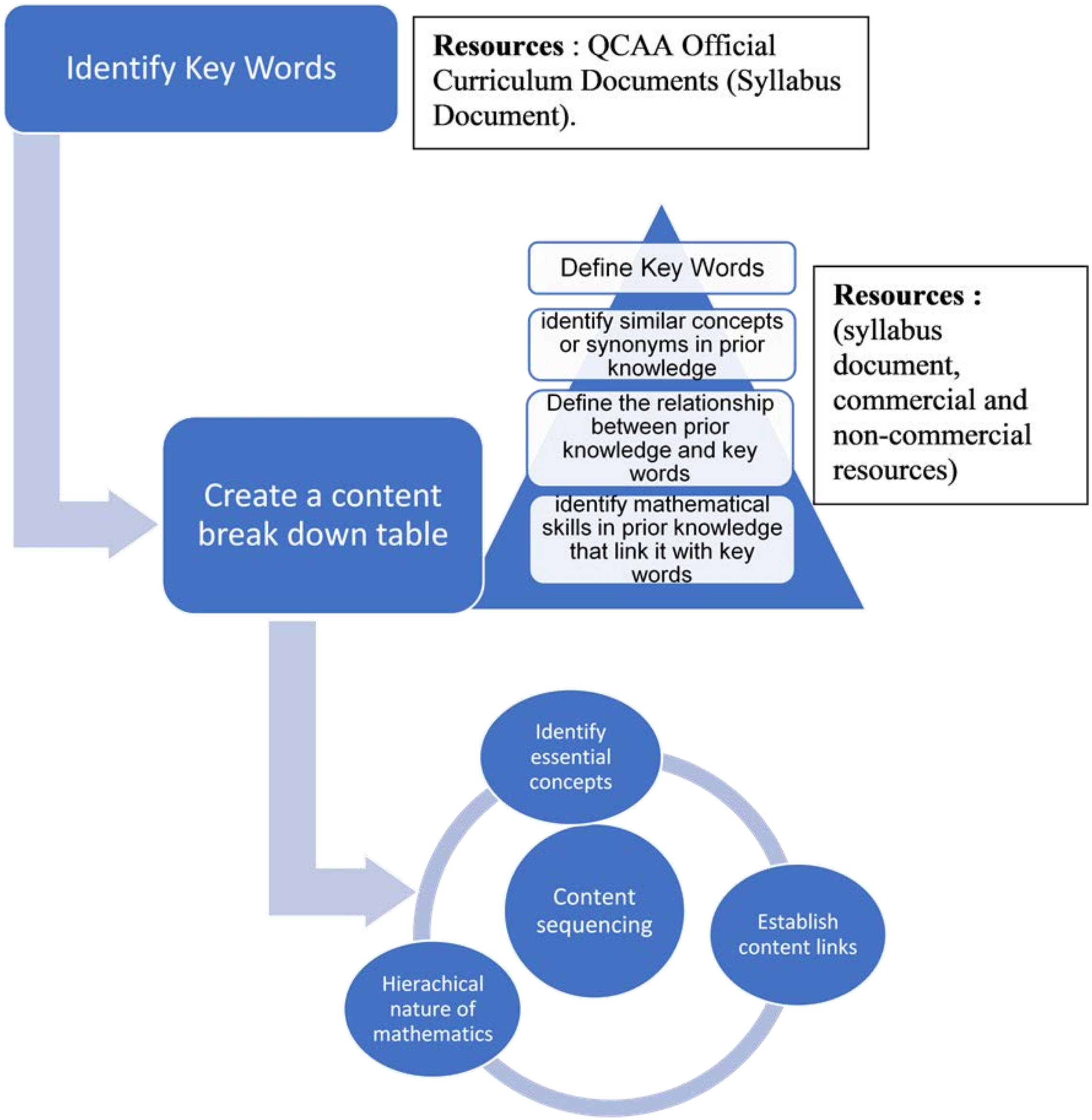

Figure 6.1: Diagramatic Representation of a Framework on Content Sequencing

The foundation of the framework is coherence of content so that students can construct new knowledge from assumed prior knowledge. Scholars (Schuhl, 2020; Usha, 2010) have argued that for content coherence to be mastered, mathematics teachers should be guided by the following questions during collaborative planning:

1. What exactly do students need to know and be able to do in this unit?
2. What prerequisite conceptual understanding and skills fluency are required for all students to effectively learn new knowledge?
3. How do the concepts identified as prior knowledge link with new knowledge?
4. What do we expect students to retain?

Tackling these questions collaboratively provides equity and consistency to students' learning experiences from one teacher/class/level to the next (Schuhl, 2020). As a result, "student learning improves because your entire team is working to ensure each student learns the organised mathematics content from one concept to the next" (Schuhl, 2020, p. 13). The four questions guide the collaborative framework on concept sequencing being applied to Unit 1 of the Mathematical Methods option discussed below.

6.5.1 Mathematical Methods Unit 1 Functions and Graphs (QCAA, 2018, p 20-21)

Unit 1

Firstly, identify key words from the syllabus document:

Functions

In this sub-topic, students will:

• understand the concept of a relation as a mapping between sets, a graph and as a rule or a formula that defines one variable quantity in terms of another.

• recognise the distinction between functions and relations and use the vertical line test to determine whether a relation is a function.

• use function notation, domain and range, and independent and dependent variables.

• examine transformations of the graphs of $f(x)$, including dilations and reflections, and the graphs of $y=f(x)$ and $y=f(bx)$, translations, and the graphs of $y=f(x+c)$ and $y=f(x)+d$; $a,b,c,d\in R$.

• recognise and use piece-wise functions as a combination of multiple sub-functions with restricted domains.

• identify contexts suitable for modelling piece-wise functions and use them to solve practical problems (taxation, taxis, the changing velocity of a parachutist).

Review of quadratic relationships

Recognise and determine features of the graphs of $y = x^2, y = ax^2 + bx + c, y = a(x-b)^2 + c$, and $y = a(x-b)(x-c)$, including their parabolic nature, turning points, axes of symmetry and intercepts.

Inverse proportions

In this sub-topic, students will:

• examine examples of inverse proportion

• recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{(x-b)}$, including their hyperbolic shapes, their intercepts, their asymptotes and behaviour as $x \to \infty$ and $x \to -\infty$.

Powers and polynomials

In this sub-topic, students will:

• identify the coefficients and the degree of a polynomial

• expand quadratic and cubic polynomials from factors

• recognise and determine features of the graphs of $y = x^3, y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$

• use the factor theorem to factorise cubic polynomials in cases where a linear factor is easily obtained.

• solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained.

• recognise and determine features of the graphs $y = a(x-b)^4 + c$, including shape and behaviour.

• solve equations involving combinations of the functions above, using technology where appropriate.

Graphs of relations

In this sub-topic, students will:

• recognise and determine features of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, centres and radii

• recognise and determine features of the graph of $y^2 = x$, including its parabolic shape and axis of symmetry.

Exponential Functions 1

Indices and the index laws

In this sub-topic, students will:

• recall indices (including negative and fractional indices) and the index laws

• convert radicals to and from fractional indices

• understand and use scientific notation

6.6 Applying the Framework to Functions and Graphs

6.6.1 Importance of Keywords

The *Oxford Advanced Learner's Dictionary* (2000) defines a keyword (noun) as a main idea or concept that is very important in a particular context. Keywords "provide significant clues for the main points about the sentence" (Li et al., 2020, p. 8196). Therefore, a keyword is one that is essential to the meaning of a sentence. Definitions of some keywords help in identifying prerequisites of the concept as they provide more detail about the key word. For example:

Question 1: What exactly do students need to know and be able to do in this unit?

Key words in the syllabus highlight critical skills and concepts as well as link prerequisites to new concepts. When they are closely analysed by teachers, different concepts not directly mentioned in the syllabus will emerge as prerequisites. An example of a definition that can directly link to prerequisites is the definition of a relation. A relation is a set of ordered pairs (Evans et al., 2019). Ordered pairs are points on a Cartesian plane that are represented in the form (x, y). The definition helps to realise the importance of a Cartesian plane in understanding relations and any other concepts related to them. In the ordered pairs we derive the Domain and Range. It is critical to ensure that every student understands a Cartesian plane and can identify all $x \, and \, y$ values that satisfy a graph represented on the plane. How x values will be manipulated to give corresponding y values is called mapping.

Key words that are repeated or mean the same can be combined or expanded under one unifying name.

Examples:

- Shapes and intercepts, asymptotes and shapes, behaviour and features, centre and radii can all be features of graphs.

- Coefficients, variables and formula can fall under algebra.
- Factors, factor theorem, factorise linear and non-linear functions (linear, quadratic and cubic) can fall under factorisation.
- Mapping, domain, range, sets, independent and dependent variable come under relations
- Index laws, negative and fractional indices fall under indices.
- Translation, reflection and dilation fall under transformations.
- Solving linear quadratic and simultaneous equations fall under solving equations.

6.6.2 Curriculum Mapping of Concepts

Curriculum mapping is a critical tool used to display the comprehensive coherence of the curriculum (Levin & Suhayda, 2018), investigate the degree of how concepts in a curriculum are interlinked (Vashe et al., 2020) and improve communication among teachers on content, skills and teaching and learning (Koppang, 2004). Curriculum mapping promotes long-term planning as it reflects topics or content, concepts to be covered and skills both new and old to be mastered in a specific period (Koppang, 2004). The investigation of content connectedness enables educators to identify gaps that might be addressed during teaching to help students gain a deeper understanding (Vashe et al., 2020). While curriculum mapping involves creating visual representation of linked displays, it is not limited to a diagrammatic linking of curriculum content but also to structure and assessments, which are beyond the scope of this study.

Mapping provides visual displays, which are quick to understand and easy to compare. “Mapping is a visual representation of information and can be in the form of tables, flow charts or textual information” (Ervin et al., 2013, p. 310). Undoubtedly, diagrams or visual displays enhance explanatory power (Peterson et al., 2021). Tables and scope and sequence charts provide a visual representation of knowledge. “Graphical displays are more effective than text for communicating complex content because processing displays can be less demanding than processing text” (Ioanna, 2002, p. 262). Concept breakdown tables and flowcharts will be used in this study to present a diagrammatic representation of how content is broken down and sequenced to realise coherent planning.

The tables and flowcharts can also be used to demonstrate how content develops from familiar to complex unfamiliar, that is, from prior knowledge to new knowledge. Therefore, "a careful examination of such a chart reveals how the sequence of activities related to a particular unit is organised in a spiral approach, giving students repeated opportunities to develop and broaden concepts" (Reys et al., 2020, p. 55). Spiralling involves building from assumed prior knowledge or from what is known and then navigating through to complex phenomena.

Mapping a unit plays an important role in providing a visual representation of knowledge. It provides resources to visualise how concepts are developed from foundational principles to new or future developments, hence exposing the complications involved in learning (Wilson et al., 2016). In this instance, a breakdown table formulated from the syllabus document became a starting point. Collaborative mapping of mathematical concepts bring together teachers' knowledge and understanding of the topic or concepts under consideration. Done collaboratively, the exercise will provide an opportunity for teachers to have better insight on how prior knowledge can link with new knowledge.

A range of researchers (Gurupur et al., 2015; Novak, 2010; Reina, 2018) have identified the following advantages of mapping:

- it breaks down concepts and link them to develop high cognitive skills
- it lays the foundation of how concepts will be developed.
- teachers share content knowledge as the map is being developed.
- it develops deeper conceptual understanding
- it showcases the importance of prior knowledge
- teachers become better prepared to teach
- other planning documents like unit plans and term planners can use it as a foundation
- it gives teachers an opportunity to interrogate the syllabus
- it expands the knowledge and scope of key concepts, which enhance teaching and learning
- pictorial representation of knowledge is easy to understand and adjust when need arises.
- it helps create connection activities or tasks as a new concept is being introduced.

6.6.2.1 Concept Breakdown Table

The concept breakdown table was instrumental in addressing the following questions:

Question 2: What prerequisite conceptual understanding and skills fluency are required for all students to effectively learn new knowledge?

Question 3: How do the concepts identified as prior knowledge link with new knowledge?

Concept breakdown tables explore how the key words link to prior knowledge. They include defining key words, identifying similar assumed prior knowledge concepts and linking assumed prior knowledge to new knowledge. This aspect of the proposed framework is necessary because mathematical language is content specific (Harmon et al., 2005). In addition, it is important to note that mathematics terminology increases in complexity as students progress from lower to higher levels of school. "Students who lack the formal language of mathematics have difficulties reasoning and communicating about mathematics" (Ben-Hur, 2006, p. 67). In fact, mathematical language has been identified as a hindrance to students as they engage with new concepts (Schuhl, 2020). Including mathematical vocabulary in the proposed framework demonstrates how language changes as concepts develop and reinforces the importance of terminology in enhancing teaching and learning.

For example, at Year 9 and Year 10 levels, students learn about quadratic expressions and equations which are key in understanding parabolas. Likewise ordered pairs on a Cartesian plane in Year 7 is a mapping of $x\ onto\ y$. The concept breakdown tables can be made available to students to dissuade their view of mathematics "as a series of unrelated procedures and techniques that have to be committed to memory" (Swan, 2006, p. 162). Their views are influenced by how they are taught and consequently how they learn (Wong et al., 2001). Therefore, the planning process undertaken by teachers has a strong impact on how students are taught. Lack of content coherence will promote students' memorisation of procedures if concepts are taught in isolation. Mathematics has a highly connected web of concepts and skills; therefore, these must be firmly consolidated to provide a basis for new learning (Australia Academy of Science, 2015, p. 17). Above all, concept breakdown tables provide "a clear line-of-sight for the development of students' cognitive skills across year levels" (Department of Education (DoE), 2021 p. 23).

Thus, a concept breakdown table will influence students' views on mathematics as it will demonstrate that mathematical concepts are interconnected and hierarchical and therefore that procedures and skills are transferable. Table 6.1 shows the relationship between assumed prior knowledge and new knowledge for Unit 1 of the Mathematical Methods option in Year 11.

Table 6.1: Concept Breakdown Table: Linking junior concepts with senior Mathematical Methods concepts for Unit 1: Functions

Keywords (QCAA mathematical methods Unit 1)	**Definition of keys words where applicable**	**Assumed prior knowledge of similar concept (Australian Curriculum)**	**Link between assumed prior knowledge from Australian Curriculum and key words**
Relations	Ordered pairs	Cartesian plane, ordered pairs	On ordered pairs the set of x (first) coordinates represent the domain which is also an independent variable and the set of y (second) coordinates is the Range which is also a dependent variable. A vertical line is a line parallel to the y-axis (Years 7 & 8). The relationship between the $x \text{ and } y$ is the rule, formula, equation or mapping, arrow diagrams.
Transformations (reflection, translation & dilation)	Changing a shape using: turn, flip, slide, or resize.	Flip, slide and enlargement	Rules of translation- translating horizontally or vertically. Reflection about the $x \text{ and } y$ axis (Yr 7). Enlargement and reduction as a form of dilation (Yr 9).
Piece-wise	Combination of multiple sub functions	Combining linear and non-linear equations and graphs	Distinguish linear and non-linear using highest powers of variables (degree). Represent linear and non-linear equations graphically (Years 9 & 10).
Inverse Proportion	When one value increases and the other decreases	Direct proportion	For direct proportion Increase in one variable result in an increase in another variable (Year 9) which is opposite for inverse proportion.
Features of the graphs	Characteristics of graphs	Linear and non-linear graphs	Calculate intercepts, increasing and decreasing graphs. Distinguish between linear and non-linear graphs

(including quartic)			comparing shapes. Graph quadratic equations, identify intercepts and turning points (Yeasr 9 – 10A).
Algebra	Rules to manipulate symbols		Identify coefficients (Year 7), group and simplify like terms (Year 7), general substitution (Years 7-9), making one variable a subject of formula (Years 9-10A).
Expand	Multiply factors	Distributive law	Removing brackets using distributive laws (Years 8-10A).
Factorisation	Express as a product of several factors	Factors	Factorise algebraic (Years 9 &10A) and quadratic expressions (Year 10). Factor theorem and remainder theorem to find factors of polynomials (Year 10A).
Solve equations	Find solutions in a balanced system through algebraic manipulation.	-Linear equations -Quadratic equations (factorisation, quadratic formulae, completing the square & graphically) Simultaneous equation (substitution and elimination)	Solve linear equations (Years 7 &8). Solve quadratic equation using quadratic equations (Year 9), factorisation, and completing the square (Years 10 &10A). Completing the square can also be used to standardise a quadratic function and the equation of a circle to determine coordinates of centre and radius. Solve simultaneous equation (Year 10A) Equations show the relationship between variables (mapping) (Years 7-10A).
Indices	Power or superscript	Exponents	Write surds in indicial notation, index laws, negative indices, fractional indices and solve simple indicial equations (Years 8-10A).
Scientific notation	When a number between 1 and 10 is multiplied by a power of 10		Expressing numbers to scientific notation (Year 9).

The next question after the concept breakdown table should emphasise identification of the important concepts that must be learnt to prepare students.

Question 4: What do we expect students to retain?

Essential Concepts represent the most critical content from the content domains – the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas (NCTM, 2018, p. 11). They are the big ideas in a unit (Schuhl, 2020), the ideas that play an important role in building students' mathematical conceptual understanding. However, Sullivan et al., (2012) noticed that during planning, teachers are less clear when asked to articulate the important ideas in a topic. Mapping concepts helps identify the essential concepts that students must retain.

6.6.3 Determining essential concepts

Scholars Ervin et al. (2013) and Harden (2001) emphasised the need to create main conceptual conceptions by synthesising concepts that are interlinked. The main concepts are identified below:

Relations – number/ algebra/graphs
Transformations (Reflection, Translation & Enlargement) – algebra/ graphs
Combination of multiple sub functions - graphs/algebra
Inverse proportion – algebra/graphs
Features of graphs - graphs
Algebra - algebra
Expand - algebra
Factorisation - algebra
Solve equations – relations/algebra
Indices – number/algebra
Scientific notation – number

Creating a table such as Table 6.2, with the main concepts identified in the conceptual connections and all the other concepts students must learn listed under the corresponding main concept will help teachers check if some concepts have been left out. It also provides an opportunity to further link, expand or collapse the main concepts.

Table 6.2: Grouping concepts under main concepts

1. Numbers	2. Relations	3. Algebra	4. Graphs
-indices -scientific notations -relations	-relations - solve equations	-relations -algebra - combination of multiple functions -inverse proportion -algebra -expand -factorisation Solve equations	-transformation of graphs -relations -combination of multiple functions -features of graph -inverse proportion

Table 6.2 shows that different concepts can be repeated in a range of main concepts. Hence the table can be condensed to identify only the essential concepts that students must retain. For example, "relations" are found under all four main concepts, hence the need to have relations as one of the main concepts is eliminated. Additionally, in the Australian Currriculum, "mathematics", "numbers" and "algebra" have a linked relationship and thus can be combined into one concept. In another example, graphs have different features and characteristics, for example, "if the $x\ variable$ in a hyperbola $y = \frac{1}{x}$ is increased to a very big value (approaches positive infinity), the value of y approaches zero." Consequently, different types of graphs can be renamed as characteristics and features of graphs. Thus, the essential concepts can be distilled down to "numbers", "algebra" and characteristics and features of graphs.

6.6.4 Content Sequencing

The main conceptual connections identified in this unit on Functions were "number", "relations", "algebra" and "graphs". Using the main conceptual connections (instead of the essential concepts, which may be too broad) will ensure all concepts to be taught are included. For example domain, range and rule are all part of the definition of relations. It is important to include all the assumed prior knowledge from the concept breakdown table in their hierachical order to show the structure of knowledge development. "Mathematics is a hierarchical subject, where new learning builds on earlier learning in a highly connected way" (Australian Academy of Science, 2015, p. 17). The hierachical nature of mathematics means concepts increase in complexity as they develop hence assummed prior knowledge must generally follow levels of hierachy to new knowledge as shown in

Figure 6.2. This is important to develop a logical cohension of content (topics) that build on each other as teaching and learning progresses.

negative numbers
fractions
surds
indices (numbers only)
scientific notations
Cartesian Plane
sets
independent and dependent variable
domain and range
general substitution into linear and nonlinear relationships(mapping)
identify coefficients
grouping and simplify like terms
subject of the formulae
distributive law
factorisation of linear and quadratics expressions
direct proportion
solve linear and quadratic equations
simultaneous equations
indicial equations
factorise cubic functions (Remainder and Factor Theorem)
transformation of graphs
linear, quadratic functions and their inverse
piecewise-defined functions
hyperbolic functions
cubic functions
quartic functions
circles (graph, recognize characteristics and features)

Figure 6.2: Sequenced Content using the Framework.

6.7 How the Planning Framework Influences Effective Teaching of Mathematics

Teachers have a responsibility to ensure that mathematics learning is effective. Mathematics teachers are expected to unpack subject matter, sequence content, provide students with an opportunity to connect prior knowledge to new knowledge and gradually release support for students (Stoll et al., 2012). Similarly, effective teaching and learning require students to have suitable, relevant and applicable prior knowledge and new knowledge that interconnects and can be expanded to other concepts as well as allow students to link concepts (Novak, 2010). The framework on content sequencing emphasizes the identification of prerequisites needed for students to access senior level concepts which can help teachers in addressing identified gaps in students' prior knowledge.

The hierarchical nature of mathematics and spiral sequencing of concepts across levels make senior level mathematics teaching and learning highly dependent on junior level mathematical understanding. The amount and quality of prior mathematics knowledge a student possesses determines how that student builds new mathematical knowledge (Schneider et al., 2011). It is a prerequisite for successful achievement of learning outcomes (Achmetli et al., 2019). High levels of understanding of prior knowledge helps students identify different methods of solving a mathematical problem and choosing the most efficient one (Newton et al., 2020). The connection of critical and relevant prior knowledge and corresponding new knowledge, as emphasised in the concept breakdown tables, is critical in supporting effective teaching and learning.

Students have a better chance of participating and achieving in mathematics when links are developed between what students already know and new concepts (Australian Curriculum, & Assessment and Reporting Authority [ACARA], 2018; QCAA, 2018). For example, the Cartesian plane, creating a table of values of linear and non-linear relationships may support students' understanding of independent and dependent variables, domain and range and mapping of functions and relations. To illustrate this, when students are asked to create a table of values for a linear relationship at Year 8 level, they substitute $x - values$ in the given relationship to obtain corresponding $y - values.$ Importantly teachers can emphasise that the $y - value$ obtained is dependent on the $x - value$ substituted, thus defining independent and dependent variables. Knowledge of the Cartesian plane is vital when representing the relationship graphically. Importantly all the $x - values$ in the table of values of the linear relationship satisfy the graph, hence defining the domain of the graph, since domain is a "set of all the first coordinates of the ordered pairs in a relation (Evans et al., 2018, p. 215). Correspondingly, the $y - values$ of the table of contents will define the range of the linear relationship. However, restricting a domain involves considering only a smaller portion of a domain. Inequality solutions when displayed on a number line can also be used to indicate only the part that satisfies the solution. Similarly, restricting a domain is considering only the $x - values$ that satisfy a given condition in a relation or a function, hence inequalities might be prior knowledge that support students' understanding of restricting a domain. In addition, inequalities can also help build foundational knowledge for piece-wise functions as piece-wise functions have "different rules for different subsets of the domain" (Evans et al., 2018, p. 231). Thus, a piece-wise function has the domain divided into different sections which can be defined by inequalities. Knowledge of linear and non-linear relationships at Year 9 level can facilitate students' understanding of different rules for different sections of a piece-wise function. For example: to sketch the graph of $f(x) = \begin{cases} x^2 + 1 & x \geq 0 \\ 1 - x & x \leq 0 \end{cases}$, students apply the knowledge from linear and quadratic graphs.

Tables of values are not limited to linear relationships but can also be extended to non-linear relationships that include parabolas, hyperbolas, exponential graphs and logarithmic graphs, to mention just a few. It follows that as students are creating their tables of values, they are mapping an independent variable to a dependent variable. At Year 8 level, the linear relationship is the rule or formula for mapping the variables. Grouping all $x - values$ in one set and all $y - values$ in another set, then using arrows to match all corresponding ordered pairs, will demonstrate an arrow diagram. Different relationships shown from arrow diagrams will allow the teacher to introduce conditions for a relationship to be defined as a function or not. Similarly, when linear and non-linear relationships are represented diagrammatically from the tables of values on the Cartesian plane, students can be asked to use the vertical line test to determine if the relationships are for functions or not. Different ways of determining if relationships are functions or not will support flexibility and deeper understanding of the concept.

From junior secondary level (Years 7-10), students are expected to represent relationships graphically. The relationship between the rule of the relationship and the shape of the graph must be emphasised. In fact, "the likelihood of information being maintained in memory increases when students' brains are prepared in advance to 'catch' the new input" (McTighe & Willis, 2019, p. 99). To develop mastery of features and shapes of graphs in Year 11, prior knowledge on features and shapes of graphs from lower levels is significant. For example, linear relationships are represented by straight lines while quadratic relationships are represented by a concave shape. Features and shapes can also include turning points that are expected to be covered in Year 9 when non-linear graphs are introduced. Other points, such as intercepts and tables of values, can also be important when emphasising the zeros on intercepts. Most of the graphs in Year 11 are also in the Year 10A curriculum, hence it is important for teachers to start by recapping the assumed prior knowledge. Furthermore, when teaching and learning in mathematics start from prior knowledge, it not only facilitates the retention of ideas but also deepens mathematical knowledge by integrating the ideas and creating effective mathematical meaning (Kilpatrick, 2001). Indeed, "the most significant variable in learning something new is prior knowledge" (McTighe & Willis, 2019, p. 99). Thus, students with high cognition of prior knowledge are better positioned to use both procedural and conceptual learning effectively and efficiently (Newton et al., 2020). In fact, mathematical understanding is enhanced when students are presented with the opportunity to adapt or reflect on their prior experience and knowledge and make connections between concepts, resulting in a gradual development of new knowledge (ACARA, 2018; Lowrie et al., 2018). Similarly, effective teaching involves "activating prior knowledge by making explicit connections to new learning" (DoE, 2021 p. 14). Therefore, teaching that is informed by starting with the familiar then progressing to unfamiliar concepts can promote student participation, knowledge building and understanding.

6.8 Chapter Conclusion

The planning framework can reinvigorate the pedagogical dialogue as classroom teachers collaboratively plan to deliver effective teaching of mathematics. To reiterate, a central premise of this chapter is the development of a framework on sequencing of mathematics content that can support the linking of junior mathematics (Years 7 to 10) content to the senior mathematics (Years 11 and 12) content in Queensland. The potential implementation of this planning framework can mean that the hierarchical nature of mathematics and spiral sequencing of concepts across levels can be articulated more explicitly. The identification and linking of critical and relevant prior knowledge and corresponding new content knowledge, as emphasised by the pillars of the framework, can support gradual development of mathematical knowledge during teaching and learning. However, there are potential limitations when implementing this framework, which focuses mainly on the spiral sequencing of mathematics concepts across levels. The limitations might include a lesser focus on catering for individual student needs, diversity of readiness and connecting mathematics teaching to students' diverse everyday experiences.

This chapter provides the basis of supporting collaborative planning for effective sequencing of mathematics content between lower-level and upper-level topics and across different level mathematics subjects and proposes a step-by-step systematic sequencing of mathematics content to promote interlinking, coherence and spiralling of concepts between the *Australian Curriculum* (Prep – Year 10): *Mathematics* and the *Senior Queensland Mathematical Curriculum*: Mathematical Methods Unit. It has identified that depending on the level of assumed prior knowledge and skills students recall and apply, teachers can start teaching from any level of the sequenced content.

This chapter suggests that the framework can be adapted to all mathematics subjects and levels; it can help identify relationships between lower-level and upper-level topics, concepts and skills and it can link the two levels and provide students with the opportunity to build their mathematical knowledge from the familiar to unfamiliar contexts. The aim is to encourage further research, dialogue and professional development to (re)conceptualise collaborative planning for effective sequencing of mathematics content. The next chapter outlines teachers' perceptions on the importance of content sequencing in teaching and learning of mathematics.

Chapter 7: Teachers' Perceptions of the effectiveness of a Planning Framework on Content Sequencing for the Teaching and Learning of Mathematics.

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7.1 Chapter Introduction

The Australian Mathematical Sciences Institute (AMSI) director Professor Tim Marchant warns that year 12 students studying Advanced Mathematics in Australia has dropped by 10 per cent for the first time, mathematics enrolments have dropped to an alarming level and that action must be taken now (AMSI, 2022). With enrolment rates in Advanced Mathematics at senior secondary level declining in most western countries, that include the United Kingdom (Noyes & Adkins, 2016; Watt, 2007) and especially Australia (Bita & Dodd, 2022; Kennedy et al., 2014), planning for effective teaching and learning of mathematics needs renewed focus. Importantly, how teachers plan informs teaching and learning which influences participation and achievement (Australian Institute for Teaching and School Leadership [AITSL], (2014). Moreover, the sequence of concepts and tasks teachers develop during planning are informed by several preparatory actions and is central to teaching and learning (Sullivan et al., 2013). Therefore, teachers' views on how content sequencing can inform teaching and learning of mathematics can assist planning at senior secondary level and support student participation and achievement.

Planning is an instrument for effective teaching and learning of mathematics which focuses on "how pupils learn mathematics; the structure of the mathematics curriculum; the specific content, skills and concepts you are teaching; the prior knowledge of the pupils; ways of teaching mathematics" (Jones & Edwards, 2017, p. 70). Planning informed by sequencing from fundamental to more complex content enhances teaching and learning (Fautley & Savage, 2014). However, limited research is available on how sequencing mathematics content and tasks inform the teaching and learning of mathematics (Sullivan et al., 2013). This chapter seeks to explore teachers' perceptions on how mathematics content sequencing, a key pillar of mathematics planning, can inform teaching of senior mathematics with the view to supporting students' participation and achievement.

Mathematics is hierarchical in nature (Nakamura, 2014). This means that sequential development of concepts fosters deeper mathematical understanding (Newton et al., 2020). In Japan and Thailand, the use of 'Bansho' which emphasises making use of board space to sequence learning from prior knowledge has been hailed as an effective teaching and learning strategy (Kuehnert et al., 2018). Importantly, significant research (Duncan et al., 2007; Geary et al., 2013; Pagani et al., 2010; Schneider et al., 2011; Watts et al., 2014) indicates that prior mathematical knowledge supports high achievement at upper grades. Similarly, creating a learning environment in which students' participation is anchored on creating skills and knowledge based on prior experience is one of the most effective pillars of a robust and effective teaching methodology (Ealy, 2018; Hailikari et al., 2008). Content sequencing by teachers maximises their ability to set clear goals for the teaching and learning program (Smith et al., 2020). Therefore, sequencing of content supports teaching and learning and content sequencing is key when planning for effective teaching and learning of mathematics as delivery should reflect planning. This article investigates teachers' perceptions of how a framework (Chinofunga et al., 2022) on content sequencing from junior prior mathematics knowledge (years 7 to 10) to senior new mathematical knowledge (years 11 to 12) supports teaching and learning of mathematics.

7.2 Mathematics Planning

Planning sets the foundation and path for teaching and learning. Mathematics planning involves "imagining a learning trajectory" through sequencing content to be taught "in an order that is likely to lead learners to develop further" (Mousley et al., 2007, p. 466). Likewise, effective planning promotes development of coherent content and experiences that facilitate self-paced learning [Australian Association of Mathematics Teachers (AAMT, 2006)]. However, planning is currently influenced by official curriculum documents which sometimes act as a textbook (Remillard, 2005). In Australia, secondary teachers mainly use commercial publications such as textbooks for their yearly, termly and unit planning (Sullivan et al., 2012). However, the quality of textbooks has always been questioned as limited options are available that can support linking of concepts to promote opportunities for gradual development of content (Mithans & Grmek, 2020). Drawing from China, planning focuses on the process of reviewing existing knowledge and linking it to new knowledge, meaning investigate current knowledge then transfer to new context (Jin, 2012). Jin went further to note the planning that has significantly contributed to student learning and success. In addition, China obtained the best results in the 2018 PISA, under the 15-year category in mathematics (Organisation for Economic Co-operation and Development [OECD], 2019). Therefore, teachers as curriculum implementers are best placed to evaluate if the framework on content sequencing can support linking prerequisite knowledge to unfamiliar contexts during planning.

During planning, hypothesising how students will engage with sequenced content helps teachers choose the most effective teaching and learning instruction and activities that will be used during lesson planning (Mousley et al., 2007; Simon, 1995). When mathematics planning is done collaboratively, it builds teacher capacity through knowledge sharing and demonstrates that every mathematics teacher at different year levels contributes to building students' mathematical knowledge (Davidson, 2019). Content sequencing informs mathematics lesson planning and sequencing, which is particularly beneficial to teachers if done collaboratively.

Collaborative planning provides teachers with an opportunity to share knowledge and learn from each other (Gilbert & Gilbert, 2013). "If teachers spend time collaborating and providing critical feedback on their tasks with a goal of conceptual understanding, then their students have a better chance of developing mathematical understanding and increase interest in mathematics" (Boyle & Kaiser, 2017, p. 406). This echoes the National Council of Mathematics Teachers [NCMT] (2014), which says that teachers need a deep understanding of the mathematics that their students have to learn and this will help them to collaboratively determine a suitable progression of how concepts should develop to new knowledge. Similarly, Schuhl et al. (2020) say that collaborative mathematics planning increases the chances of uniformity in students learning expectations across grade level or school because colleagues decide what students should be taught and key concepts and skills to retain enhancing students learning. Hence collaborative planning can be used to support teacher efficacy and teaching and learning.

Teachers are heavily involved in mathematics planning at school level in many countries. Official curriculum documents and in most cases centrally approved or endorsed resources such as textbooks are provided. However, teachers in most countries have the responsibility of sequencing content (Davidson, 2019) as well as contextualise official commercial (eg textbooks) or non-commercial (syllabus) documents to suit their classroom dynamics (Remillard, 2005). In China, while planning is heavily influenced by official nationally approved textbooks and curriculum and instructional materials, teachers still have to contextualise content to suit the needs of their students (Li et al, 2009). Similarly, in the United States of America, states develop the curriculum and provide suggested sequencing but mathematics teachers during planning decide on how content is sequenced and enacted in a classroom (Remillard, 2005). In Australia, Queensland mathematics teachers have the responsibility to sequence content during planning.

The Australian curriculum, developed by the federal government, sets the national curriculum from preparatory to year 10 (P-10) while each state or territory determines its own senior secondary curriculum (Years 11 to 12). Long term planning such as teaching and learning plans or unit planning involve sequencing and contextualising content to students' needs and learning experiences as schools' dynamics differ (Roche et al., 2014). Most curriculum bodies provide templates and exemplars that teachers can use as reference material during planning (Grundén, 2020). The framework on content sequencing, linking junior to senior content developed by Chinofunga and colleagues (2022), links the nationally designed Australian curriculum (prior knowledge) to state developed senior mathematics curriculum (new knowledge). The focus of the research described in this chapter was to evaluate mathematics teachers' perceptions on how the framework supports the teaching and learning of mathematics especially at senior level. The framework emphasis on linking foundational concepts identified at junior level to concepts to be developed at senior level to promote the gradual and deeper understanding of mathematics to reduce students' cognitive overload.

7.3 Framework on Content Sequencing from Junior to Senior Mathematics

The framework on content sequencing in Figure 6.1 outlined in the last chapter, was developed to provide consistency and a broad understanding on how mathematics content can be sequenced from prior to new knowledge. The key objective was to promote collaborative planning among teachers through linking mathematics concepts from the national curriculum (P-10) to concepts at senior secondary (Years 11-12). In Queensland, at senior secondary level students are required to choose mathematics subjects between calculus-based and non-calculus-based options. Mathematical Methods and Specialist Mathematics are calculus-based options. Some students who previously achieved good results in junior secondary school (Years 7-10) found themselves struggling to comprehend concepts in calculus-based subjects at senior secondary level (Bennett, 2019). Therefore, the framework on content sequencing demonstrates that prior knowledge (from junior secondary mathematics) is critical in developing new knowledge (senior secondary mathematics concepts).

Constructivists believe learners are active participants in their learning as they interpret the meaning of new knowledge and reference it to what they already know (Garbett, 2011). As a result, the chapter was conceptualised within a constructivist epistemology. Similarly, there is emphasis that "knowledge is socially constructed through interaction of the researcher with research participants", as they share experiences (Tavakol & Sandars, 2014, p. 747). Therefore, the active interaction between the researcher and senior mathematics teachers and the sharing of experiences, beliefs and ideas played a vital role in evaluating the framework on content sequencing.

The framework "provides a step-by step systematic sequencing of curriculum content to promote interlinking, coherence and spiralling of mathematics concepts between lower-level and upper level topics" (Chinofunga et al., 2022, p. 3). It ensures that prior knowledge is central when mapping mathematics content from junior secondary to senior secondary level. Thus, the emphasis is on developing new knowledge from prior experiences. Such a framework is designed based on the constructivist view that students learn by making sense of what is presented to them through the lenses of their prior knowledge and skills (Hu et al., 2011; Taber, 2019). Constructivism has been credited with reshaping the teaching and learning of mathematics over the years despite advocacy from traditional rote learning (Hu et al., 2011; Mallamaci, 2018; Simon, 1995; Stemhagen, 2016). Content sequencing helps to reduce the cognitive load of the official curriculum and make it familiar through linking new knowledge to prior knowledge. Hence evaluating teachers' perceptions on how the framework supports teaching and learning of mathematics is key in realising a critical part of mathematics planning and delivery of the lessons.

The content sequencing framework informs the process of sequencing mathematics concepts from familiar to unfamiliar concepts as described by Chinofunga and colleagues (2022). The framework is based on four elements as described below:

The first element identifies and defines key words and their synonyms from the subject matter provided in the syllabus and is central to identifying skills and prerequisites of new knowledge. Keywords "provide significant clues for the main points about the sentence" (Li et al., 2020, p. 8196) in the content descriptions in the official curriculum documents. Similarly, key words give meaning to a sentence as dominant sentences are composed by important keywords (Domínguez et al., 2016; Wang, 2012). Importantly, by identifying key words teachers can identify the main concepts related to subject content provided in official curriculum documents (Chinofunga et al., 2020). The second element details how the prior skills and concepts link with new knowledge in the subject content and is central to content sequencing. Importantly, for deeper understanding students are expected to link mathematical concepts (Novak, 2010). Therefore, backward mapping using a concept break down table is critical in this process as it provides the opportunity to clearly link prior knowledge to new knowledge which enhances teaching and learning of mathematics (Queensland Curriculum and Assessment Authority [QCAA], 2018). The third element identifies essential concepts. These are concepts and skills that students are expected to retain at the end when the teaching and learning process is complete and this is done by grouping new knowledge and prerequisites into main concepts. Essential concepts are the key ideas in a unit (Schuhl, 2020), that enhances conceptual understanding (Hansen, 2011) and are to be retained long after the teaching and learning process (National Council of Teachers of Mathematics [NCTM], 2014). The fourth and final element will follow the hierarchical nature of the identified main concepts and the sub-concepts under each main concept. "Mathematics is a hierarchical subject, where new learning builds on earlier learning in a highly connected way" (Australian Academy of Science, 2015, p. 17). Therefore, the framework takes into consideration the fact that mathematics concepts build in complexity as more teaching and learning take place.

7.4 Methods

This study followed a mixed-methods approach (see Chapter 3). Mixed methods involve the use of quantitative and qualitative data in order to better understand the research problem because it builds on the strength of both types of data (Creswell, 2014). Importantly, a mixed-methods approach also provides the opportunity to converge or integrate data in a study (Fetters et al., 2013) and helps to deepen (qualitative) and broaden (quantitative) the understanding of the phenomenon under study, hence providing opportunities for future research (McKim, 2017; Palinkas et al., 2013).

This chapter focused on the following research question:

- What are teachers' perceptions of a planning framework on content sequencing for the teaching and learning of mathematics?

Purposive sampling was used to select 16 high school mathematics teachers in Queensland. Purposive sampling involves identifying and selecting knowledgeable participants or those who have experienced the phenomenon of interest and who are available and open to share their experiences and opinions (Bernard, 2011). The inclusion criteria were teachers who were currently teaching or who had taught mathematics, especially calculus-based options at senior high school level (Years 11 and 12) in Queensland. Ethical approval was gained from the Department of Education, Queensland: Reference number: 550/27/2383 and James Cook University Human Research Ethics Committee: Approval number: H8201.

Sixteen (16) research participants watched a 10-minute video where they participated on how the framework on content sequencing could be used in planning for teaching and learning of mathematics (see Chapter 3). This was the most convenient way due to COVID 19 restrictions and time constraints among participants distributed across Queensland. The mathematics content used in the presentation and exercise was drawn from Unit 1 in Mathematical Methods, with functions as a focus. The participants were given a full term (10 weeks) to apply the framework in their planning sessions before data collection began.

7.4.1 Data Collection and Analysis

Data collection was conducted through a survey and semi-structured interviews. The survey was made up of six five-point Likert scale items and five open-ended questions. The researcher and participants had follow-up and check-in sessions fortnightly via Zoom. The sessions were used to check on progress and challenges and if participants needed support or more information as they were applying the framework. Semi-structured interviews were conducted with eight of the 16 participants who were available. This provided opportunities for the interviewer to ask follow-up questions based on the interviewee's responses (Galletta & Cross, 2013; Kallio et al., 2016). Each interview took approximately 25 minutes.

Quantitative data from the 5-point Likert scale survey was collated and the initial results tabulated. The mode and median responses for each question were determined. This was because Likert data are generally ordinal in nature and are best analysed using modes and medians (Stratton, 2018). Thereafter, a table of questions and percentage responses was created to summarise the results. Data analysis of the open-ended questions and interviews followed a thematic analysis (see Chapter 3). Thematic analysis aims to identify, investigate and reveal patterns found in a data set (Braun & Clarke, 2006). To ensure validity, the study used theory triangulation, which involved sharing qualitative responses among colleagues at different status positions in the field then comparing findings and conclusions (Guion et al., 2011). Coding was independently undertaken by the researcher on the open-ended survey responses and interview transcripts. This included independent initial identification of themes and data related to the themes, collaboratively reviewing findings, revising and discussing themes (see Appendix A, B and C).

7.5 Results

The survey data collected using the five-point Likert scale were analysed and the findings are presented in Table 7.1.

Table 7.1: Likert Scale responses showing Participants Perceptions of how the Framework on Content Sequencing Support Teaching and Learning of Mathematics

Question	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree
1. Content sequencing as outlined in the framework is a critical component of mathematics planning and teaching as it provides a clear link between relevant and significant assumed prior knowledge and corresponding new knowledge.	(14) 87.5%	(2) 12.5%	0 0.0%	0 0.0%	0 0.0%
2. Content sequencing as outlined in the framework places assumed prior knowledge, skills and conceptual connections at the centre of mathematics knowledge development.	13 81.3%	3 18.8%	0 0.0%	0 0.0%	0 0.0%
3. Content sequencing as outlined in the framework helps identify key concepts in a unit and hypothesising effective delivery methods.	13 81.3%	3 18.8%	0 0.0%	0 0.0%	0 0.0%
4. Collaborative content sequencing as outlined in the framework reinforces teachers' responsibility of effective teaching of mathematics concepts at every level.	13 81.3%	3 18.8%	0 0.0%	0 0.0%	0 0.0%
5. Collaborative content sequencing as outlined in the framework fosters a common agenda of focusing on how students develop mathematical knowledge.	13 81.3%	3 18.8%	0 0.0%	0 0.0%	0 0.0%

6. Collaborative content sequencing as outlined in the framework makes mathematics teaching a collective responsibility as students understanding and participation at higher levels depend on lower levels.	14 87.5%	0 0.0%	2 12.5%	0 0.0%	0 0.0%

All participants strongly agreed or agreed that the collaborative content sequencing as outlined in the framework supports teaching and learning of mathematics. In fact, at least 13 which is 81.3% of participants strongly agreed that content sequencing informed by the framework linked development of new knowledge to prior knowledge. Likewise, at least 13 which is 81.3% of participants strongly agreed that the framework highlighted the hierarchical nature of mathematics through collaborative content sequencing and mapping of concepts. The majority of participants strongly agreed with all the Likert scale items. This was further demonstrated by the mode and median of all items being 5 or strongly agree. The study strongly supported the importance of the framework on content sequencing in enhancing teaching and learning of mathematics. It further underpinned the significance of collaboration during content sequencing in fostering mathematics teaching and learning and knowledge development and cohesion within and across levels.

The data from the open-ended survey questions and semi-structured interview questions were analysed and the following themes agreed upon as capturing the views of the participants on:

- the utility of content sequencing framework in creating an environment that promotes development of new knowledge from prior knowledge.
- the utility of the framework on content sequencing in articulating the hierarchical nature of mathematics

7.5.1 Theme 1: The utility of Content Sequencing Framework in Creating an Environment that Promotes Development of New Knowledge from Prior Knowledge.

The general observations from participants in the open-ended survey questions showed that participants agreed that content sequencing as guided by the framework supports the development of new concepts from prior knowledge. Participants noted that the framework on content sequencing emphasised:

- sequencing content appropriately and logically to support student understanding
- identifying skills needed to engage with new knowledge
- linking prior knowledge to new concepts in the unit
- breaking down concepts to determine fundamental concepts students need to understand or access new concepts
- identifying key concepts in the new unit and sequencing them in a logical way that links prior knowledge and builds on to new knowledge, thus develop new knowledge in small steps.
- building from concrete to abstract

These results demonstrated the importance of the framework on content sequencing in fostering how new and unfamiliar mathematics knowledge is developed from prior and familiar knowledge. Semi structured interviews supported the general observations but went further to include participants' perceptions on the four elements of the framework.

Semi structured interviews provided more detail on participants' views on element 2 of the framework. This aspect of the framework emphasised the importance of linking prior to new knowledge. The central role of prior knowledge in teaching and learning of mathematics was noted by participant 5 when he provided an example "*if you're doing measurement and geometry, make sure that the kids are good in numbers field, that number has to come before measurement.*" Thus, this provides students with an opportunity to participate and engage in the learning if they understand prior knowledge. The participant is emphasising the importance of including in the planning and teaching, relevant and necessary prior knowledge to aid students' understanding of new knowledge. Participant 8 said

> "*The proposed framework is very important because it provide guidelines and steps to follow when we are planning... Expectations across each level are now uniform and teacher empowerment in different ways for example developing unit plans is being achieved"*

The participant identified consistency in planning across levels as something that can be achieved by using the framework. The participant went further to say*, "we really did not take content sequencing as so important until we become part of research participants but it's a weakness we are prepared to correct as we have realised it is very important for our students to develop their knowledge gradually from known to unknown."* The extract demonstrates that in some cases teachers might not have appreciated the importance of content sequencing, but the framework might highlight the benefits it brings to effective planning and teaching. Participant 1 summed the content sequencing framework by saying "*cut down an awful lot of time that we spend doing sequencing*" and pointed out that "*there is no document that I know of that links the current senior syllabus back to the knowledge that students need to know at P-10*." Therefore, the framework on content sequencing provides the basis of linking junior to senior curriculum.

Interestingly participants also highlighted how the framework on content sequencing helps to contextualise learning for different students depending on their capabilities. Participant 4 emphasised that "*how we use sequenced content varies, depends on your local context and also conceptual and procedural connections between subject matters."* She went further to share her experience in two different schools when she said "*my second school, this is a more rural school, and students, their prior knowledge, has been observed, not as solid as in an urban school, so therefore content sequencing is helpful."* The participant is highlighting the key role content sequencing might play in a differentiated class. Participant 7 noted the importance of framework on content sequencing in a class when she said *"I, myself personally feel that is best practice, that is an amazing opportunity to really customize for children."* She went further to say *"We, we keep forgetting that every class has a specific group dynamic, every school has a specific context."* Importantly participant 3 noted that the framework could support teaching and learning after identifying "*student ability level and their prior knowledge to see where we need to start."* Therefore, students operating at different levels of prior knowledge can significantly benefit as teachers have a pathway to follow which is informed by planning. The different elements proposed in the framework play a significant role in content sequencing.

Participants also highlighted the importance of element 1 of the framework. This element emphasised the importance of identifying and defining key words and their synonyms from the subject content as central to identifying skills and prerequisites of new knowledge. Participant 6 appreciated the importance of key words in identifying prior knowledge when he said "*Get those keywords that you talked about from the whole thing then from there, go back to your content, try to see, which are the key concepts that I need to cover for students to understand new concepts.*" Participant 2 went further to also include a benefit of identifying prior concepts when she said "*identify prior concepts that you do need to teach for each particular topic using key words, this makes you think about the students' needs and what they already know*." Similarly, participant 8 pointed out that "*key words help identify prior knowledge then fundamental and essential concepts that students have to master*." The participants emphasised how key words can provide a deeper insight into concepts. Identifying key words helps identify prior skills and concepts that are fundamental to develop new knowledge, however it is also important to explore and link the identified concepts to new knowledge.

Participants highlighted the importance of linking prior skills and concepts with new knowledge using a concept break down table as central to content sequencing. Participant 3 appreciated the framework by saying "*the framework actually enhance content sequencing starting from prior experience through to at level content, and in fact I was keen to develop a content break down table when I saw it*." This was supported by participant 8 when he said *"It highlights the importance of content sequencing as it is central to any planning and demonstrate to teachers the importance of prior knowledge as demonstrated in the content breakdown table."* Importantly the participant went further to say,

> "*Not that teachers are not aware of the importance of prior knowledge but this goes deeper by including much more prior knowledge in our planning as in the content breakdown table so that our students can even correct prior knowledge misconceptions and increase their chances of understanding new knowledge with this clear and defined link*."

Participant 4 had a similar view when she said "*building connection between prior experience and new knowledge using backward mapping in the content breakdown table is very important in systematically developing students' mathematics* *understanding.*" Participants appreciated that linking of prior knowledge to new knowledge using a concept breakdown table provides more detail during planning on how new knowledge will be developed. The concept-break-down table plays a significant role in clearly defining how prior knowledge links with new knowledge. During check-in with participants, we collaboratively developed a concept break down table in Figure 7.1, for a section on Introduction to differentiation in Unit 2 of Mathematical Methods.

Unit 2: Introduction to differential calculus				
Year 7	**Year 8**	**Year 9**	**Year 10**	**Year 11**
Number & Algebra : -rate, distance–time graph (travel graph), speed, gradient (and slope), variable			**Year 10 Science** (Distance, speed and time- calculate & graph). Rates of chemical reactions-eg calculate and graph average concentration versus time	explore average and instantaneous rate of change in a variety of practical contexts
Fractions- dividing a numerator of 1 by denominators of integers from 1 and keeping on increasing the denominator to very big numbers till the answer converge to zero **(limit as denominator turns to infinity).**		factorisation (including factor and remainder theorem) and simplification - necessary before applying limit.		use a numerical technique to estimate a limit or an average rate of change
ordered pairs & Cartesiane Plane		Calculate the gradient of a line segment (interval) on the Cartesian plane. **Gradient(m)= (change in y)/(change in x) or rise/run** - *(change in y is dy and change in x is dx)*	Solve problems involving linear equations, including determining gradient.	examine the behaviour of the difference quotient (f(x+h)-f(x))/h as *h*→0 as an informal introduction to the concept of a limit
	Simplify algebraic expressions. General substitution. Factoring out common pronumeral.		Substitute values into formulas to determine an unknown eg determine f(2). (substituting a value into a given function). Factorisation.	differentiate simple power functions and polynomial functions from first principles
		Find the gradient of a line segment (interval) on the Cartesian plane. Gradient(m)= (change in y)/(change in x) or rise/run - *(change in y is dy and change in x is dx)*. **Importantly- Equation of a straight line - y = mx +c**		interpret the derivative as the instantaneous rate of change
			Solve problems involving linear equations, including those derived from formulas. **Modify equation of line to y-b = m(x-a) where (a,b) is a point that lie on the straight line.**	interpret the derivative as the gradient of a tangent line of the graph y = f(x).
		linear and non-linear relationships	Apply understanding of a range of polynomials and describe the features of these curves from their equation and contextualise problems. Year 10 Science- Chemical Reactions.	examine examples of variable rates of change of non-linear functions
Operations with Directed numbers. Ordered pairs & cartesiane Plane		Find the gradient between two points. Identify the gradient given the equation of line- gradient is the derivative of line.		establish the derivative formula of positive integers
	linear relationships	linear and non-linear relationships	Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions. Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation. **(Helps to understand the derivative of a quatic is cubic, cubic is quadratic, quadratic is**	understand the concept of the derivative as a function

			linear-students can match graphs and functions)	
	apply the distributive law to the expansion of algebraic expressions	Calculate gradient		recognise and use properties of the derivatives d/dx (f(x)+g(x))=d/dx f(x)+d/dx g(x)
		Calculate and determine(indentify) gradient of a linear relationship	Apply understanding of polynomials to identify and describe the features of these curves from their equation.	calculate derivatives of power and polynomial function
Number & Algebra : -rate, distance–time graph (travel graph), speed, gradient (and slope), variable		Calculate gradient of two points.	**Year 10 Science** (Distance, speed and time- calculate & graph). Rates of chemical reactions-eg calculate and graph average concentration versus time	determine instantaneous rates of change
	Solve linear equations	gradient and graph linear relationships using y= mx +c	Solve problems involving linear equations, including those derived from formulas	determine the gradient of a tangent and the equation of the tangent
Number & Algebra : -rate, distance–time graph (travel graph), speed, gradient (and slope), variable			Year 10 Science (Distance, speed and time- calculate & graph).	construct and interpret displacement-time graphs, with velocity as the slope of the tangent
	graph linear relationships	Graph simple linear and non-linear relations with and without the use of digital technologies	Describe, interpret and sketch parabolas, hyperbolas, circles and exponential functions. Explore the connection between algebraic and graphical representations of relations such as simple quadratics, circles and exponentials using digital technology as appropriate. Apply understanding of polynomials to sketch a range of curves and describe the features of these curves from their equation. Identify features of different polynomials.	sketch curves associated with power functions and polynomials up to and including degree 4; find stationary points and local and global maxima and minima with and without technology; and examine behaviour as x turns to positive or negative ∞.
		modelling linear relationships	modelling quadratic functions	identify contexts suitable for modelling optimisation problems involving polynomials up to and including degree 4 and power functions on finite interval domains and use models to solve practical problems with and without technology; verify and evaluate the usefulness of the model using qualitative statements and quantitative analysis.

Figure 7.1: Content break down table on Introduction to Differentiation

The content breakdown table shows the prior content from the Australian Curriculum that builds the foundations of the concept of differentiation at senior secondary school. The Year 11 column is the content that students are expected to engage with as new knowledge.

Semi structured interviews also provided positive feedback on element 3 of the framework. This element helps in identifying essential or key concepts which students are expected to return to as they develop their conceptual understanding. Participant 6 noted that the framework on content sequencing helped to identify, "*exactly the concepts that are very relevant and essential to teach…. which are the key concepts that I need to cover*." Participant 2 supported the view when she said "*very good in terms of identifying what are the key concepts*." Participant 5 gave an example of essential concepts when he said "*depend on the matrix of the 3 big ideas algebra, geometry and number. If you're doing measurement and geometry, you make sure kids are good in numbers field."* The participant observed that foundational concepts may be key in developing higher order concepts and need to be included during planning to support teaching and learning. The key or essential concepts help build students' mathematics knowledge as they are the concepts that students need to retain or use as a foundation for conceptual understanding.

7.5.2 Theme 2: The Utility of the Framework on Content Sequencing in Articulating the Hierarchical Nature of Mathematics

Generally, results showed that participants agreed that collaborative content sequencing during planning illuminated the hierarchical nature of mathematics. From the open-ended survey results, participants' responses emphasised the following:

- hierarchical, spiralling and logical development of concepts
- backward mapping to lower levels
- link-related concepts where one skill from a lower level can easily be transferred to another unit at the level or above.
- the importance of teachers gaining a better understanding on how skills and the content they teach are prerequisites to learning new knowledge at a higher level.

Semi structured interviews showed positive views regarding element 4 of the framework. Participants agreed that use of the framework during collaborative planning articulated the hierarchical nature of mathematics across school levels. Participant 1 noted that,

> "*Collaborative content sequencing places the responsibility on teachers to make sure that their students know how to do this (apply a skill) because it's relevant down the track, whether it's the next topic or two three topics time, you know when a particular skill is important.*"

The participant went further to say,

> *"teachers that never ever taught high level maths to see that okay, what I'm teaching here is really important out there so I better do a really good job. And I really better make sure that my kids are doing or have mastered this because, it's then going to limit or they're going to limit themselves in being able to access higher learning of maths"*

The participant emphasised the responsibility of teachers in determining that students understand junior concepts to be able to engage meaningfully with senior concepts. Participant 8 conclusively said "*everyone is agreeing it remind teachers that mathematics is hierarchical therefore collaborative planning is more beneficial to everyone than individual planning.*" Therefore, applying the framework collaboratively helps to foster the culture of collaboration at all levels and brings to the fore the understanding that mathematical concepts interlink and build on each other.

Participant responses also showed their agreement with the idea that exposure to more learning allowed concepts to develop and deepen for students. Participant 6 demonstrated the hierarchical nature of mathematics articulated by the framework as the basis of teaching and learning when he said,

> "*when you move from one topic to another I always use some of the concepts that they did from previous lessons because if they suddenly jump and feel like there's a sudden jump, there's something that is very different from what they were doing on the previous lesson, it's is a hustle to get them to understand what needs to be done… I'll start with the basis, like the basics of the topic, so that at least I get the understanding of those students*."

The participant went further to say "*you know the concepts that are relevant from other units or levels*." The participant's emphasis was on how the framework promotes linking of concepts to develop a web of knowledge that is coherent and developing in a gradual form. Similarly, participant 2 said the framework *"draw the links between the topics…which one comes first… where we need to go to within that topic."* Participant 3 went on to say *"concepts are presented according to the level they are expected to be taught making content sequencing easy."* Participants appreciated that the framework on content sequencing could provide a foundation for effective planning. Participant 5 considered the broader hierarchy of mathematics when he said *"simple familiar content and build into complexity making sure they know the simple stuff and how to build it into, complex content."* The observation by the participant demonstrates that prior knowledge plays an important role in developing the understanding of complex concepts. Participants' responses show that the framework on content sequencing fosters the identification of prior concepts, development of new knowledge from prior knowledge, identification of key concepts and the hierarchical nature of mathematics.

7.6 Discussion

The purpose of this study was to gain a better understanding of teachers' perceptions on how the framework on content sequencing from junior (Years 7 to 10) to senior level (Years 11 and 12) can support the planning, teaching and learning of mathematics. The results of this research provide supporting evidence that the framework places prior knowledge at the centre of mathematics planning, teaching and learning. All participants in the study agreed that the framework highlighted content sequencing as a critical component of mathematics planning and teaching as it links relevant and significant assumed prior knowledge and corresponding new knowledge. The qualitative data supported this view as participants identified that the framework facilitated the systematic and logical linking of prior knowledge to new knowledge. The findings can add value to current trends in Australia of secondary teachers relying more on commercial publications (Sullivan et al., 2012) which have been found to be limited in explicitly breaking down and linking junior to senior content in mathematics (Mithans, & Grmek, 2020) These results align with previous findings by Hailikari et al. (2008), who posited that linking prior knowledge to new knowledge is key for effective mathematics teaching. The content sequencing framework focuses on including prior knowledge at the planning stage and shows how it contributes to the development of new knowledge (Chinofunga et al., 2022a). These results represent participants' support of the framework as an inherent part of the planning that is key to teaching and learning mathematics.

The identification of key words in the subject matter provided in official curriculum documents played a key role in identifying prior concepts and this is one of the critical processes advocated by the framework. The quantitative results show that at least 14 participants strongly agreed that the framework facilitated the identification of prior knowledge and linked it to new content while the qualitative results provided further evidence that identification of key words in the syllabus was central to the identification of relevant prior knowledge. These results are consistent with the first stage of the framework, which emphasises that identification of key words from content as stated in official curriculum documents assists in identifying prior knowledge (Chinofunga et al., 2022a). These results are also consistent with Li et al.'s (2020) work that emphasised that key words help decode the main focus of a sentence. After using key words in identifying prior knowledge, it is important to present how prior knowledge links with new knowledge in the subject matter.

The use of concept breakdown tables in backward mapping concepts from junior to senior level is also one of the key stages of the framework. Quantitative results from this study show that 13 participants strongly agreed that the framework places assumed prior knowledge, skills and conceptual connections at the centre of mathematics knowledge development. This is important because effective teaching and learning requires students to have relevant prior knowledge to construct new knowledge and allows students to link concepts for deeper understanding (Novak, 2010). Moreover, the open-ended survey results showed that participants agreed with the view that it is important to break down new concepts using prior concepts so that student engagement with new concepts can be supported. The semi-structured interview results also highlighted the importance of concept breakdown tables in this regard and clearly identified the relationship as a gradual way for students to access new knowledge. These results are consistent with other research (QCAA, 2018; Newton et al., 2020) that suggest that a clear definition of the link between prior knowledge and new knowledge supports teaching and learning of mathematics.

A vital feature of the framework is the identification of key or essential concepts that students should retain at the end of the teaching and learning process in order to build conceptual understanding. Both the quantitative and qualitative results provide evidence that the framework contributes to conceptual understanding by facilitating the identification of key or essential concepts. Identification of key or essential concepts is important as it supports Schuhl's (2020) and Hansen's (2011) findings that key concepts are key ideas in a unit and they are the ones that help students build conceptual understanding. Their identification helps teachers to focus on those concepts, which students must retain long after the teaching and learning process (NCTM, 2014). Therefore, the opportunity that the framework offers teachers in identifying the key concepts can support teaching and learning of mathematics.

As stated many times previously, mathematics is a hierarchical subject and reflecting this in mathematics planning, teaching and learning can support understanding. The quantitative data in this study showed that 14 participants strongly agreed that the framework reflected this hierarchical and interconnected nature of mathematics. This was confirmed by the qualitative results, which were consistent with the work of Nakamura (2014) and the Australian Academy of Science (2015) and reinforce that the hierarchical nature of mathematics makes collaborative planning the best way to apply the framework on content sequencing.

The hierarchical nature of mathematics also sets the platform for collaborative content sequencing among teachers. The quantitative results in this study show that at least 13 participants strongly agreed that the framework on content sequencing from junior to senior mathematics emphasised to teachers that understanding senior mathematics depends on how effectively concepts are taught at lower levels. Participants noted that the framework also highlighted that effective teaching of mathematics at junior level is critical for students' participation at senior level. Similarly, the qualitative results support the notion that the framework stresses the interlinking of mathematics content within and across levels, thus supporting Schneider and colleagues (2011) who posited that when students are taught well at junior levels and retain the knowledge, their chances of understanding senior level mathematics is supported. Taken together, the findings indicate that the framework on content sequencing emphasises the hierarchical nature of mathematics as a way mathematics can effectively be planned, taught and learnt.

7.7 Chapter Conclusion

In summary, teachers have a perception that the framework on content sequencing from junior to senior level mathematics can be an effective framework to use in identifying, linking and sequencing mathematics concepts. The results indicate that teachers believe that the stages in the framework can assist them to effectively sequence mathematics content in a way that promotes the gradual development of new knowledge. Moreover, teachers noted that using the framework collaboratively appears to benefit teachers across all levels as the hierarchical nature of mathematics promotes the interconnection and interdependence of mathematics concepts.

Importantly, the chapter provides a framework that teachers can use across schooling levels within a community of practice as they sequence content during planning. The chapter also highlights the importance of content sequencing during planning, teaching and learning. This chapter supports the constructivist view of teaching mathematics that new knowledge is constructed from prior knowledge. Similarly, the chapter advocates for prior knowledge to be included during planning and linked to new knowledge which could contribute towards conceptual understanding.

The chapter used teachers' perceptions as curriculum planners to evaluate the framework on content sequencing from junior to senior concepts in mathematics. Although the present results indicate that the framework on content sequencing can support teaching and learning of mathematics, it is appropriate to recognise that the main limitation of this chapter is the sample size. In terms of future research, it would be useful to extend the current findings by examining the impact of content sequencing using this framework on teacher instruction and student achievement. The next chapter outlines the development of a resource (concept maps) that can supplement the framework on content sequencing in developing conceptual knowledge through linking prior knowledge to new knowledge.

Chapter 8: How can Concept maps as a resource support the teaching and learning of mathematics at senior secondary level.

A version of this chapter has been published in the International Journal of Innovation in Science and Mathematics Education (IJISME).
https://aus01.safelinks.protection.outlook.com/?url=https%3A%2F%2Fdoi.org%2F10.30722%2FIJISME.31.01.003&data=05%7C01%7Cdavid.chinofunga%40my.jcu.edu.au%7Ce83587e109474984bd2f08db7697464e%7C2eba4cf8af764db3bcaf81b5592535ef%7C0%7C0%7C638234164322588590%7CUnknown%7CTWFpbGZsb3d8eyJWIjoiMC4wLjAwMDAiLCJQIjoiV2luMzIiLCJBTiI6Ik1haWwiLCJXVCI6Mn0%3D%7C3000%7C%7C%7C&sdata=qmO8NBWkKKZR9lN%2BxLgojHrgaxoqmVfghIMe2qI7kaU%3D&reserved=0

8.1 Chapter Introduction

Schools are facing challenges in developing students' conceptual knowledge in mathematics (Richland et al., 2012), conceptual knowledge being defined as the knowledge of the interconnection of fundamental concepts in a domain (Schneider & Stern, 2010). According to Richland et al. (2012), students lack the deeper understanding of mathematics that facilitates reasoning, flexibility and generalisations and high school graduates who enter the community college system in USA end up in mathematics bridging courses because they lack conceptual knowledge. Similarly, in Australia, limited conceptual knowledge focus has been identified as the main factor influencing students' participation in mathematics (Smith et al., 2018). However, mathematics teaching and learning can be supported when students learn "with understanding, actively building new knowledge from experience and previous knowledge" (National Council of Teachers of Mathematics [NCTM], 2000, p. 2). Australian teachers believe conceptual knowledge is essential in helping students understand mathematics (Hurrell, 2021) and understanding that conceptual knowledge plays a key role in mathematics knowledge development highlights the importance of interlinking mathematics concepts.

Conceptual knowledge is a network of concepts that constitute a bigger unit of knowledge (Österman & Bråting, 2019). Complex unfamiliar problems in mathematics mostly require students to make connections of knowledge within or across a domain (QCAA, 2018). Importantly, awareness of the connectedness and coherence of mathematics concepts is often overlooked by mathematics teachers; however, it is an important goal that has reshaped instruction during teaching and learning of mathematics (Novak, 2010). This is because "mathematics is a field of continuous inquiry about new relationships and of proving these relationships." (Bingölbali & Coşkun, 2016, p. 236). Moreover, coherent instruction in mathematics connects prior and fundamental concepts to new knowledge and provides the opportunity to deepen the understanding of complex concepts (Doabler et al., 2012). Importantly, one of the key aims of the Australian Curriculum: Mathematics v9.0 is to help students see the bigger picture and make connections between mathematics concepts (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2023). Mathematical understanding is enhanced when students have the opportunity to adapt or reflect on their experience and knowledge and make connections between prior knowledge and new knowledge, thus gradually developing their own new knowledge, (ACARA, 2013). "Well-constructed knowledge is interconnected, so that when one part of a network of ideas is recalled for use at some future time, the other parts are also recalled" (Sullivan, 2011, p. 6). Thus, this study's focus was on exploring a visual representation that teachers can use to support students' development of mathematical knowledge through the ability to link junior to senior concepts as learning progresses.

Students learn better when exposed to information in visual form (Raiyn, 2016). In fact, students retain visual formats better and longer in their minds, as it is easy to understand and *show* connections (Raiyn, 2016). Similarly, visuals not only provide teachers and students with the opportunity to identify and visualise concepts and procedures but also to realise and illustrate relationships, making recalling easier (Birbili, 2006). Indeed, visuals are "the best tool for making teaching effective and the best dissemination of knowledge" (Shabiralyani, 2015, p. 226). Moreover, they can represent a large amount of information, reducing the time required to go through the information (Raiyn, 2016). As a result, visuals such as concept maps that can link junior concepts (Years 7 to 10) to senior concepts (Years 11 and 12) can support teaching and learning of mathematics.

8.2 Concept Maps

The use of concept maps in teaching conceptual knowledge has been highly recommended. Novak (1990) introduced concept maps in science and mathematics to organise and link concepts. Research on use of concept maps in mathematics has focused mostly on middle school and teacher training level with very limited research at the senior secondary level (Schroeder, Nesbit, Anguiano, & Adesope, 2018). Importantly, a metanalysis by Schroeder and colleagues concluded that research has focused more on explaining the benefits of using concept maps without collecting evidence to support such assertions. This study reports on teachers' perceptions of the benefits of concept maps for mathematics teaching at the senior secondary level.

Concept maps show concepts and how they are connected, thus giving a representation of conceptual understanding. They are a resource that can be used to represent and demonstrate conceptual understanding (Watson, Pelkey, Noyes, & Rodgers, 2016). Moreover, they can be a tool to demonstrate concept cohesion within or across a domain (Hartsell, 2021), which is key to mathematics content sequencing during planning (Chinofunga, Chigeza & Taylor, 2022). In this view, conceptual understanding is represented by concept nodes that are connected by single or bidirectional arrows labelled with verbs to specify the relationship between and among them (Birbili, 2006; Novak, 2010). They can be hierarchical or non-hierarchical in nature as it is the input that determines the shape (Llinas, Macias, & Marquez, 2018).

Concept maps promote higher order thinking (Cañas, Priit, & Aet, 2017), facilitate integration of complex ideas (Beat, 2015) and promote problem solving (Watson et al., 2016). Their ability to provide opportunities to present conceptual interconnections and relationships that include the main concepts and other related prior or sub-concepts promotes critical thinking (Groffman & Wolfe, 2019). Similarly, use of visual representations to show relationships of mathematics concepts encourages critical thinking and enhances teaching and learning of mathematics (Bay-Williams & SanGiovanni, 2021). It follows that, "concept mapping promotes students' understanding of complex constructs and complicated relationships, while stimulating critical analysis and improving critical thinking" (Fonteyn, 2007, p. 200). Furthermore, they enhance the quality of students' learning by facilitating connection of ideas and providing a solid foundation to add and understand new knowledge, which is valuable for problem solving (Kinchin, Möllits, & Reiska, 2019). Being able to break down complex phenomenon into familiar concepts is central in solving complex questions that might require integration of different concepts, which is expected in mathematics at senior secondary (QCAA, 2018). Moreover, linking as much prior knowledge as possible to new knowledge enhances cohesion of concepts and understanding (Mai et al., 2021). At senior secondary level, students' ability to link relevant prior knowledge at junior level to senior level concepts support participation and understanding.

Broadly, concept maps have several benefits to teaching and learning. They are beneficial "in activating students' prior knowledge, identifying misconceptions, focusing discussions, facilitating collaborative learning and as revision and assessment tools" (Kinchin, 2011, p. 183). Concept maps help teachers in focusing students on what they need to learn and the main concepts they need to retain (Hartsell, 2021). They also facilitate a meaningful and consolidated understanding of mathematics, as well as help to show the differences in knowledge and understanding among students (Ho, Harris, Kumar, & Velan, 2018). As a result, they can be used in mathematics formative assessments which to not require students to recall facts and procedures (Bell, 2017). Importantly, they provide an overall picture of the phenomenon in question rather than just focusing on facts (Vasconcelos et al., 2019). Mapping concepts provide opportunities of multiple representation which enhances deeper understanding (Gokalp & Bulut, 2018). Similarly, they can be used to identify students' misconceptions in their conceptual understanding (Watson et al, 2016). Moreover, they enhance integration and clarity of concepts and motivate students to learn (Chiou, 2008). Concept maps support student centred learning by making them active participants in the learning process (Groffman & Wolfe, 2019). Thus, teaching and learning of mathematics that involve concept maps support the interlinking of mathematics concepts (Schroeder et al., 2018).

However, concept maps are also viewed to have several disadvantages in teaching and learning. The relational aim of using concept maps can be a disadvantage as teaching and learning in some cases require arguments and objections to positions (Davies, 2011). Furthermore, Eppler (2006) found that students often feel overwhelmed and demotivated when faced with designing concept maps as they require some expertise to design. Concept maps do not enable easy separation of concepts of critical importance to those of secondary importance (Daley, 2004). Davies suggested that they are not adequate to capture more complex relationships between concepts.

Teachers as classroom practitioners are well placed to evaluate resources. Thus, this study investigated teachers' perceptions of the impact of concept maps that link junior to senior concepts on the teaching and learning of mathematics at senior secondary school. The study addressed the following research question:

> What are senior secondary teachers' perceptions of the impact of concept maps that link junior to senior concepts on the teaching and learning of mathematics at senior secondary school?

8.3 Method

This mixed methods study explored the impact of concepts maps in the teaching and learning of mathematics. The mixed methods approach is ideal because it provides an opportunity for consolidating results from both quantitative and qualitative research methods (Creswell, 2015). Quantitative and qualitative data are analysed and then integrated in order to cross validate findings (Creswell, 2015). The purpose of the study was to investigate secondary mathematics teachers' perspectives on the effectiveness of using concept maps that link junior to senior concepts as tools that mathematics teachers can use in developing students' conceptual knowledge.

Purposive sampling was used to select 16 high school mathematics teachers in Queensland, Australia. The inclusion criteria were teachers who are currently teaching or who have taught mathematics, especially calculus-based options at senior high school level that is Year 11 and 12 in Queensland. Ethical approval was gained from the Department of Education, Queensland: Reference number: 550/27/2383 and James Cook University Human Research Ethics Committee: Approval number: H8201.

Sixteen research participants took part in a 10-minute video presentation where they were provided with information on how concept maps that link junior to senior concepts could be used in teaching and learning of mathematics. The video presentation included how to develop the concept maps and the possible stages where they could be used during teaching and learning. They were given a full school term (10 weeks) to respond after including concept maps that link junior to senior concepts in their teaching. During the implementation period, fortnightly after-hours Microsoft Teams check-in meetings with all participants were organised to check on progress and offer support. When and how to employ such concept maps during teaching and learning was left for teachers to decide considering class dynamics. The concept maps could be teacher developed, student developed and/or class developed depending on the pedagogy employed by the teacher. This provided teachers with the opportunity to be innovative resulting in diverse experiences and perceptions. The concept map tools that link junior to senior concepts were developed using a Content Sequencing framework developed by Chinofunga and colleagues (2022). The mathematics content presented in the concept map presented to teachers was drawn from Unit 1 in Mathematical Methods (Figure 1), with functions as a focus.

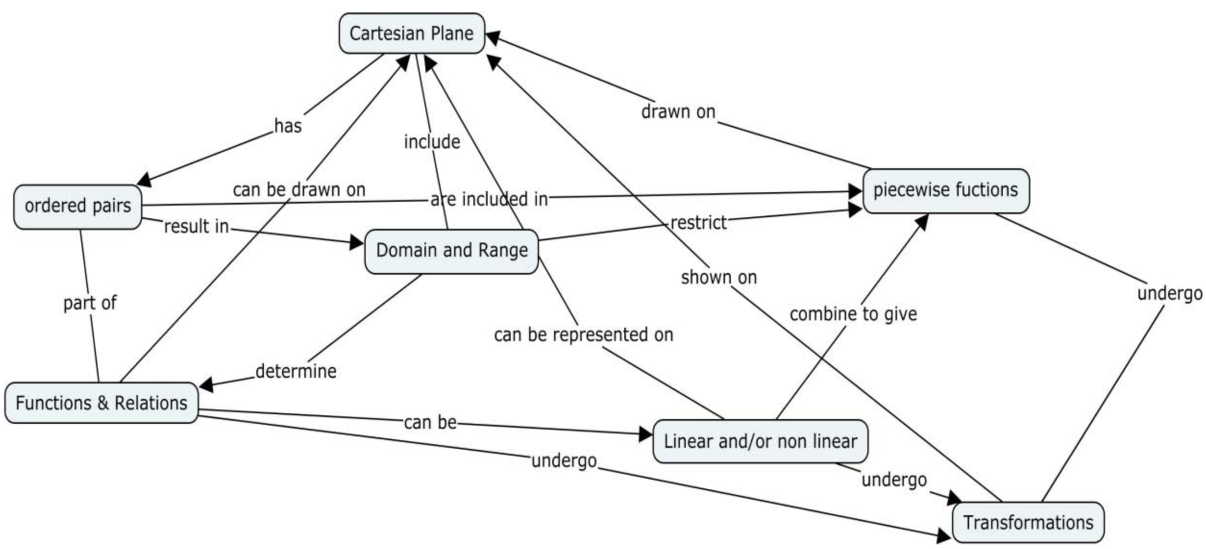

Figure 8.1: Concept map that links junior to senior concepts: Functions

The concept map shown in Figure 8.1 links the concepts on a section of functions in Unit 1 Mathematical Methods. It includes phrases that help explain how the concepts link and the to- and from- arrows show that some connections are bi-directional. Thus, concept maps help to construct meaning and relationship between concepts.

During the designing of concept maps that link junior to senior concepts, the following processes were used: Firstly, a concept of focus at senior mathematics was identified. Novak and Canas (2006) noted that a concept map begins with a concept or phase which represents a focus question that require an answer. Secondly, junior concepts that link to the concept of focus were identified. That is, concepts identified in the concept breakdown table can be used. Thirdly, connecting terms are used to link junior and senior concepts. Davies (2011) posited that connecting terms are used to show relationships between concepts represented.

8.4 **Data Collection**

Data collection were conducted through a survey and semi structured interviews. The survey with a five-point Likert scale and five open-ended questions was shared with the participants. The scaled survey questions required teachers to rate their level of agreement on a scale from 1 to 5 on questions based on use of concept maps that link junior to senior concepts in developing students' mathematics knowledge.

Semi-structured interviews were conducted to gain knowledge of how teachers used the concept maps in their teaching of mathematics. Semi structured interviews are adjustable and adaptable, because they provide opportunities for the interviewer to ask follow-up questions based on the interviewee's responses (Kallio, Pietilä, Johnson & Kangasniemi, 2016). Interviews were conducted with only eight out of the 16 participants who completed the survey due to competing schedules. The interviews ranged about 25 minutes.

8.5 **Data Analysis**

Quantitative data from the 5-point Likert scale was collated in Excel. Rows were allocated to participants and columns to questions. From the initial results tabulation, the mode and median responses for each question were determined. This is because Likert data are generally ordinal in nature and are best analysed using modes and medians (Stratton, 2018). Thereafter a table of questions and percentage responses was created to summarise results.

This study involves two types of qualitative data which are open ended survey questions and semi structured interviews. After transcribing the semi structured interviews, member check was done with two participants to verify accuracy of the transcribed scripts. Data analysis of survey open-ended questions and interviews followed a thematic analysis. Thematic analysis aims to identify, investigate and reveal patterns found in a data set (Braun & Clarke, 2006). To ensure validity the study used theory triangulation. It involves sharing qualitative responses among colleagues at different status positions in the field then comparing findings and conclusions (Guion et al., 2011). Survey open-ended responses and interview transcripts of participants were shared among the principal researcher and his two supervisors for independent analysis. Analysis was informed by the research questions. Coding was independently undertaken by the principal researcher and his two supervisors. This included initial identification of themes and data related to the themes independently. The findings were collaboratively reviewed, and themes were discussed and revised. The following themes were agreed upon which captured the views of participants on:

- the utility of concept maps that link junior to senior concepts in creating an environment that creates awareness of the interconnection of mathematical concepts.
- the utility of concept maps that link junior to senior concepts in creating an environment that supports consolidation and assessment of teaching and learning of mathematics.

Semi- structured interviews gave participants an opportunity to explain their experiences after using such concept maps in teaching and learning of mathematics. Quantitative and qualitative data were integrated to answer the research question. Combining the two data sets may result in validated and well justified findings (Creswell, 2015).

8.6 Results

The teachers' responses suggested that use of concept maps that link junior to senior concepts can enrich mathematics classrooms. Table 1 below represents the Likert scale items that captured the teachers' perceptions on the utility of concept maps in the teaching and learning of mathematics.

Table 8.1: Likert Scale Responses in Percentages

Questions	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree
1. Concept maps help students understand how mathematical concepts are related.	14 88%	2 13%	0 0%	0 0%	0 0%
2. Student or teacher developed concept maps can be used to link prior knowledge to new knowledge.	13 81%	3 19%	0 0%	0 0%	0 0%
3. Concept maps facilitate consolidation of learning.	10 69%	6 31%	0 0%	0 0%	0 0%
4. Concept maps facilitate a visual evaluation of students' learning.	12 75%	4 25%	0 0%	0 0%	0 0%
5. Concept maps give an overview of a topic.	13 81%	3 19%	0 0%	0 0%	0 0%
6. Concept maps help identify key concepts in a topic.	13 81%	3 19%	0 0%	0 0%	0 0%
7. Concept maps promote integration of concepts that deepen mathematical understanding.	10 69%	3 19%	1 6%	1 6%	0 0%
8. The hierarchical nature of mathematics makes concept mapping central to teaching and learning of mathematics.	9 56%	4 25%	3 19%	0 0%	0 0%

The research question is centred on teachers' views on how concept maps that link junior to senior concepts can strengthen the teaching and learning of the interconnection of concepts. The mode and median of all the questions under consideration shows strong agreement. Similarly, all participants agreed or strongly agreed that concept maps support conceptual understanding, facilitate consolidation, are a visual representation of mathematical knowledge, provide overviews and help identify key concepts. Moreover, at least 81% of participants agreed or strongly agreed that concept maps play an important role in enhancing the teaching and learning of mathematics especially through connecting concepts.

8.6.1 Theme 1: The utility of concept maps that link junior to senior concepts in creating an environment that stimulates awareness of the interconnection of mathematical concepts.

This theme focused on the use of concept maps in developing knowledge of the interconnection of mathematics concepts. Results from the survey open-ended questions indicated participants' views on the usefulness of concept maps in enhancing students' knowledge of conceptual connections. Participants identified the following benefits:

- providing concept maps for students helps them to visualise the links between concepts.
- developing concept maps in class helps students make conceptual connections.
- students can also develop their concept maps to represent their own knowledge development.
- concept maps allow students to link prior knowledge or foundational concepts with new knowledge.
- concept maps show students how simple familiar procedures develop into complex problem solving.

These views highlighted the critical role that concept maps can play in developing students' mathematics conceptual knowledge.

The semi-structured interview data further explored feedback from participants on the role of concept maps in the teaching and learning of conceptual knowledge. The value of linking concepts to students' learning was made clear. Participants observed that it helped students value current learning as they realised it was connected to future understanding. For example, Participant 1 combined the importance of visuals and conceptual connection when she said, *"They can see the relevance of what they have learnt in the past and how it links to something you are trying to teach them now and something that you will teach them in the future."* Participant 2 stated, *"So that definitely helps in terms of helping the students make that link between concepts and why they need to actually learn those concepts."* Participant 8 was more specific when he said, "*... have since included concept maps in conceptual teaching and students seem to understand the linking of concepts better."*

In relation to linking prior experience to new knowledge or linking concepts within or across domains, which is key to effective mathematics teaching, Participant 4 stressed that concept maps can show "*connections between prior and current learning, that's one purpose of using a concept map".* The same observation was put forward by Participant 8, who said, *"Concept maps also emphasise the importance of prior knowledge to new content."* It was interesting that the directions of the linking arrows (emphasising that mathematics is spiral and hierarchical in nature and concepts can be integrated) in the concept maps was a key focus. Participant 3 said, *"Conceptual maps actually allow students to have something to hang on and they can go backwards and forwards"* and Participant 8 mentioned *"two-way linking"*. Similarly, Participant 5 noticed the importance of backward arrows when he mentioned *"... forward and backwards arrows that can help your concept map"*.

The use of arrows in facilitating integration of concepts was noted by Participant 2, who said arrows could help in *"showing how they can actually link concepts together and use it, for example, in problem solving where you've got to use multiple concepts at a time."* These indicated that complex problems are a combination of concepts, and a concept map is useful in building that understanding and hence that concepts maps can be very effective in teaching and learning of conceptual knowledge.

8.6.2 Theme 2: The utility of concept maps that link junior to senior concepts in creating an environment that supports consolidation and assessment of mathematics knowledge.

This theme focused on how concept maps can assist in consolidation and assessment of mathematics knowledge, as shown by the following participant responses:

- students can develop concept maps for consolidation of a topic or unit.
- students can be asked to develop a concept map at the end of a lesson, topic or unit as part of assessment.
- uncompleted concept maps can be used as a task for students to fill in the gaps.
- concept maps developed in class can be used to expose misconceptions or common mistakes.

Student-developed concept maps represent their mathematics knowledge and thus can be used as an assessment tool to measure students' understanding. Participant 7 pointed out that concept maps *"... would give me a better way to checklist how each individual student is acquiring knowledge"*. Participant 8 went further when he said he "*used it in a lesson for students to show me how their knowledge has developed."* Participant 2 was more focused on visual learners' representations when she said, *"it helps those visual learners and organising their thoughts"*. These results show that concept maps, when developed by students, can be used to evaluate students' mathematics understanding, hence can be an assessment tool that requires teacher feedback.

The importance of this feedback was emphasised by Participant 7 when she said, *"If kids miss concepts, you will never get them to be able to progress until you go back. By having a map, we know where to go back to and we can trace back until we find the gap."* Thus student-developed concept maps might also open an opportunity to identify students' misconceptions, evaluate their understanding and take corrective action. This was supported by Participant 8 who said, "*Misconceptions can also be identified as students develop concepts that give the teacher the opportunity to reteach or redirect."* Importantly, opportunities for effective consolidation arises at the end of a topic or unit, when teachers take into consideration students' knowledge representations and sum up everything that they have learnt.

Artefacts from teachers and students collected during the course of this study also offer valuable evidence on use of concept maps in teaching and learning. The artefacts provided an insight into how teachers used concept maps as a resource to link prior and new concepts and for consolidation of concepts. Figures 8.2, 8.3 and 8.4 show concept maps from Participants 4 and 8 covering three different concepts in Mathematical Methods. Additionally, Figures 8.5, 8.6 and 8.7 show concept maps developed by students in different topics in Mathematical Methods subject.

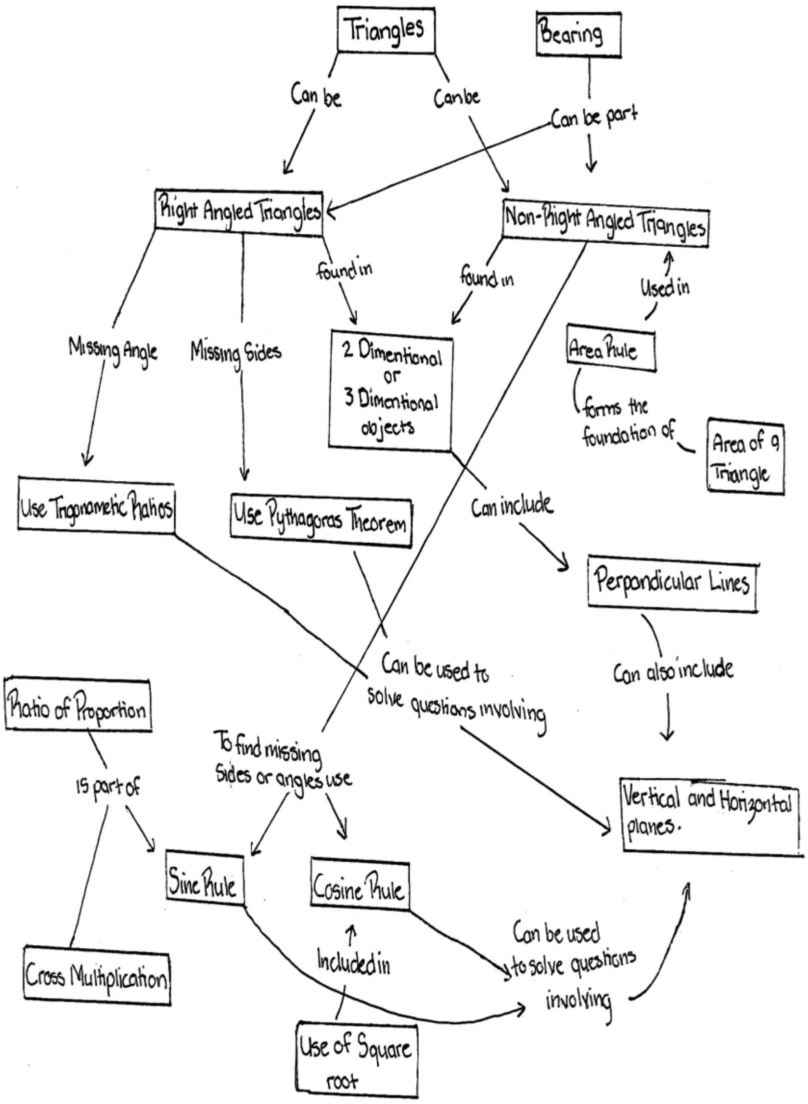

Figure 8.2: Teacher developed concept map on Trigonometry and its applications.

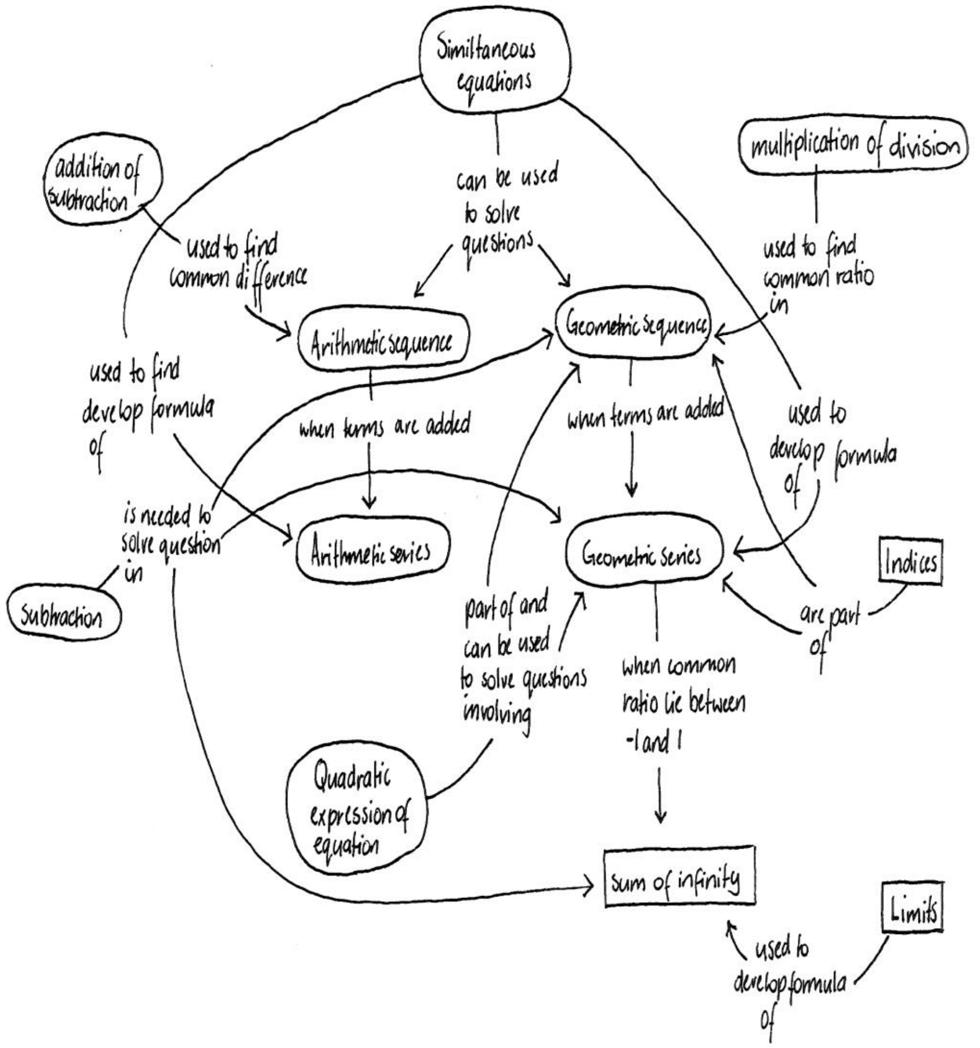

Figure 8.3: Teacher developed concept map on Sequences and Series

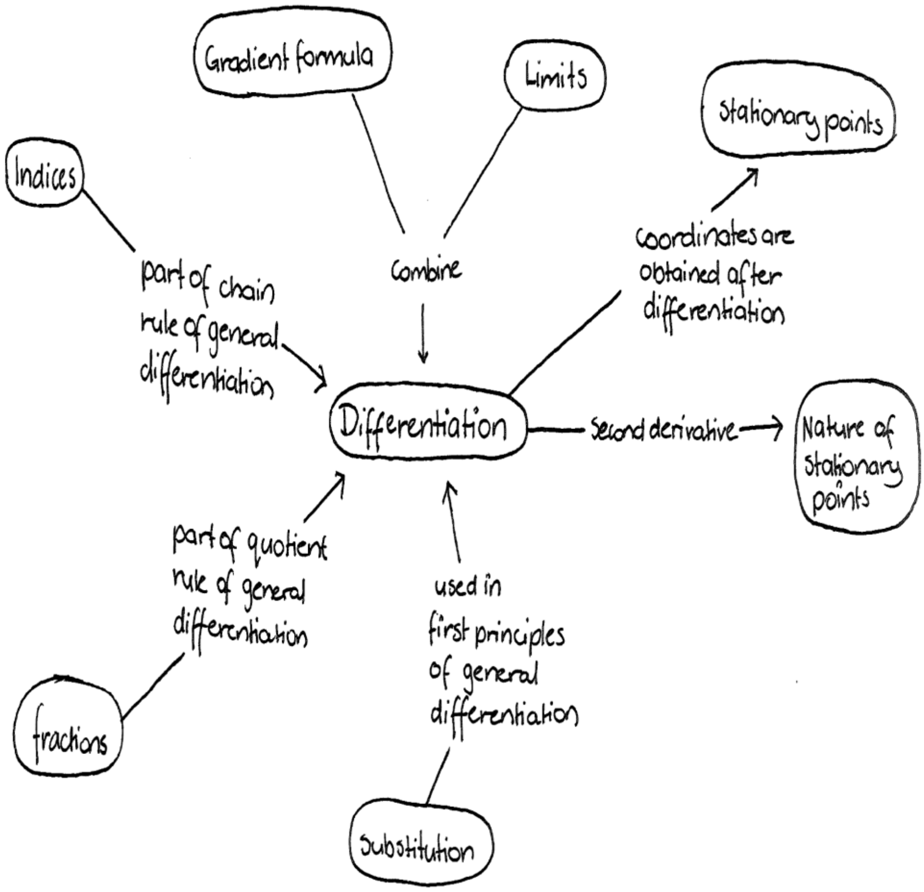

Figure 8.4: Teacher developed concept map on Differentiation.

Figure 8.5: Student developed concept map on Continuous Random Variables.

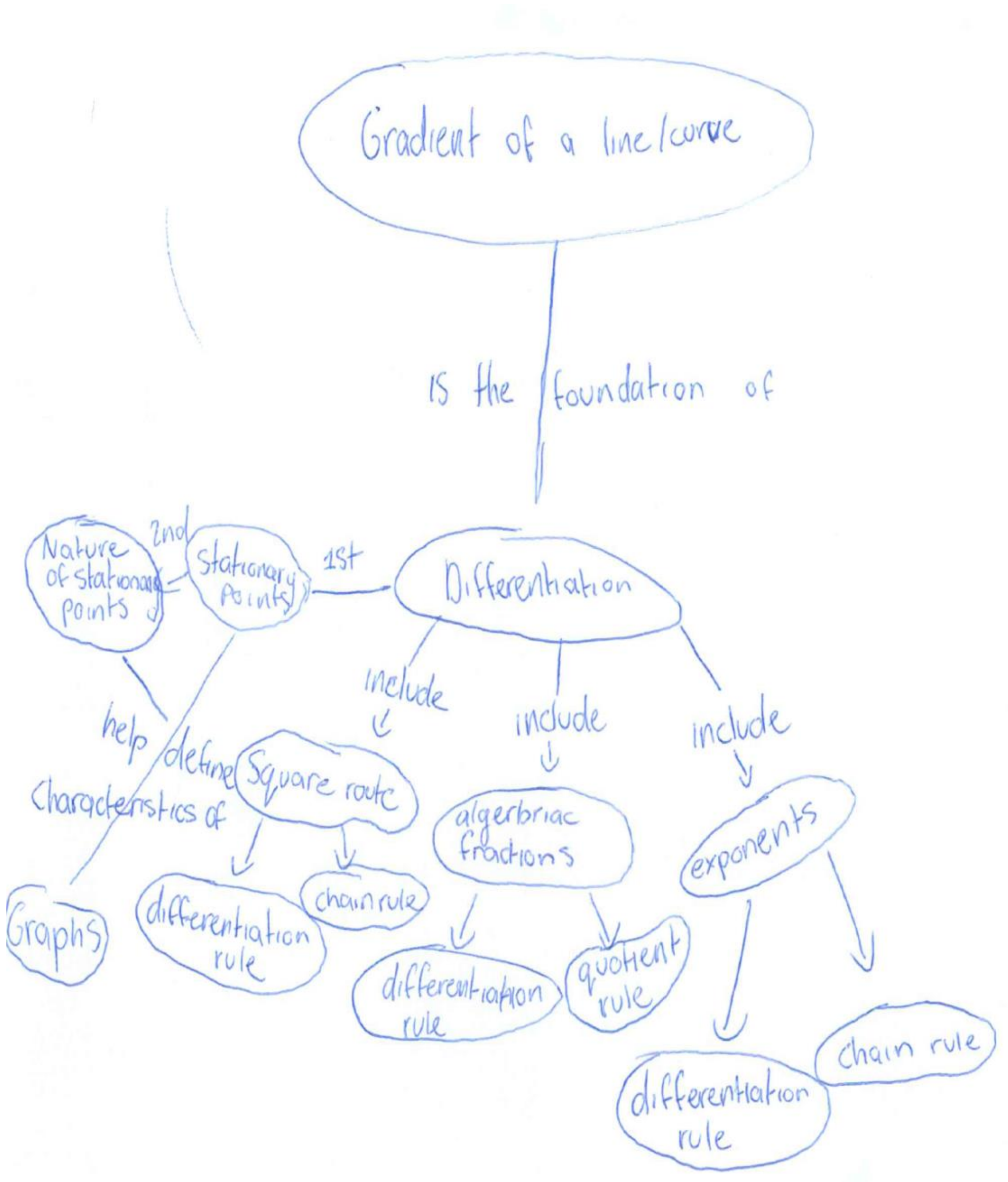

Figure 8.6: Student developed concept map on Differentiation.

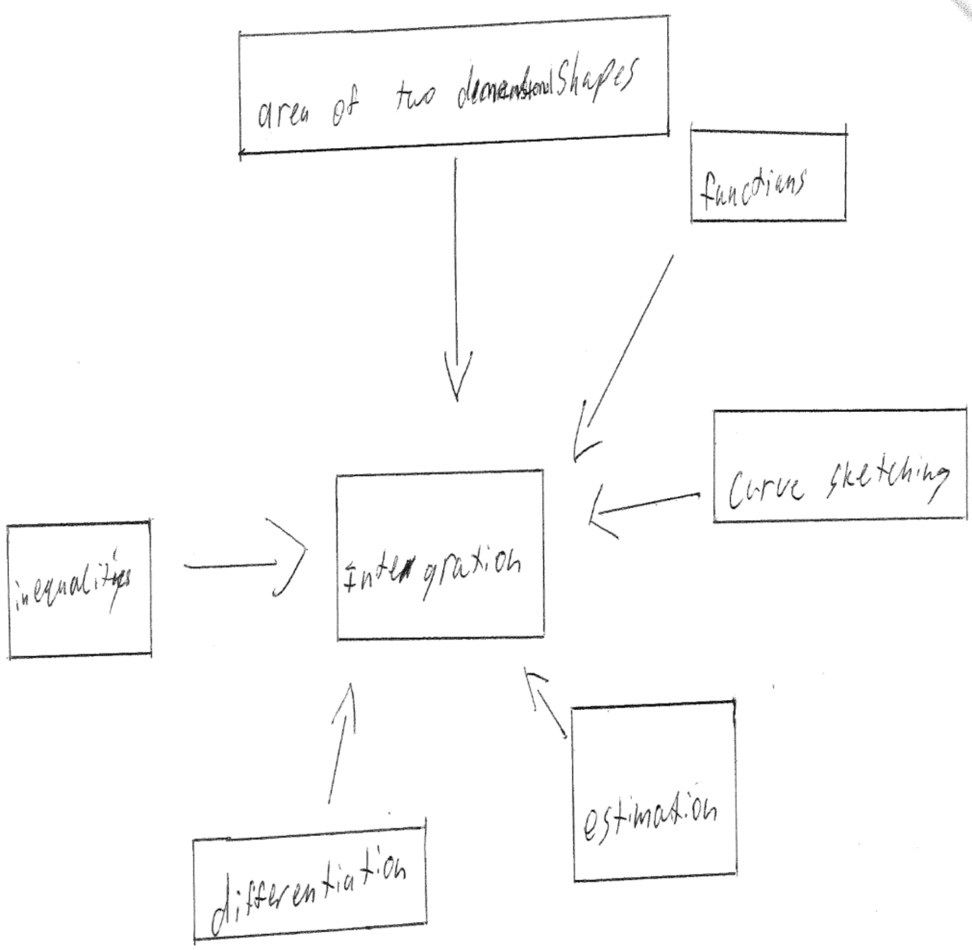

Figure 8.7: Student developed concept map on Integration.

8.7 Discussion

Analysis of both qualitative and quantitative data indicate that teachers have a perception that concept maps that link junior to senior concepts can support the teaching and learning of mathematics at the senior secondary level. In particular, participants' views provide supporting evidence that such concept maps can support students' knowledge of conceptual connections which is critical in making students aware of how mathematics concepts relate to each other. During semi structured interviews, participants noted that students' understanding of mathematics as a web of concepts can be supported by concept maps. Additionally, artefacts from students indicated that students could link several concepts, especially from their prior knowledge to concepts at senior level mathematics. The findings are consistent with previous research by Watson and colleagues (2016) who noted that concept maps enhance conceptual understanding. Thus, promoting the interlinking of mathematics concepts is a more effective teaching and learning strategy in senior school mathematics compared to other instructional models (Novak, 2010). It is because it promotes coherent instructions that foster the development of new knowledge from prior knowledge (fundamental concepts) then provide opportunities to include more complex concepts as teaching and learning progresses (Doabler et al., 2012). These results support the idea that concept maps can also be used to show the cohesion of concepts (Hartsell, 2021), which can promote content sequencing in mathematics. The nature and structure of concept maps, that is the nodes, arrows and linking words, might be key in deepening students' understanding of mathematics.

The uni or bi-directional arrows on concept maps show links between concepts which can support integration and help identify key concepts. Quantitative results show that 88% of participants agreed that concept maps support integration of concepts while all participants agreed that they can be used to identify key concepts. During interviews, participants further emphasised that concept maps provide evidence on how solving a problem may involve several concepts. Similarly, during open-ended survey responses, participants noted that concept maps may demonstrate how simple familiar concepts integrate to complex unfamiliar concepts as concepts integrate. These views support the position that complex problems require students to integrate different concepts (QCAA, 2018). Similarly, Fonteyn (2007) suggest that concept maps may demonstrate how concepts evolve from simple familiar to complex unfamiliar as concepts integrate and relationships get more complex. Artefacts from students and teachers showed links between different concepts which demonstrates that linking a system of foundational concepts may assist in the development of complex concepts in mathematics. Importantly, the findings also indicate that concept maps can help students understand and integrate concepts (Beat, 2015; Kinchin et al., 2019). Concept maps show relationships of mathematics concepts which will help students to understand mathematics as a web of concepts.

The interconnection of concepts is not only important in understanding the nature of mathematics but can also inform how it is effectively taught. Teachers' views from both quantitative and qualitative results show that concept maps can facilitate the linking of prior knowledge to new knowledge. Semi structured interview data provided an in depth understanding as participants explained that the concept maps promoted and enabled connections to be made between junior and senior concepts. This supports the hierarchical nature of mathematics and underscores the importance of content sequencing from prior to new knowledge since this is critical to students' understanding of new concepts. The results provide supporting evidence to Kinchin (2019) who posited that by identifying connections and underlying links between prior knowledge and new knowledge, students have a better chance to learn effectively. This is because mathematics is all about exploring new and existing relationships among concepts (Bingölbali & Coşkun, 2016). Therefore, concept maps that link junior to senior concepts can be a critical resource in supporting the teaching and learning of mathematics as connections between prior knowledge and new knowledge play a key role in comprehension.

The views of participants in this research provide supporting evidence that conceptual maps can be a tool for consolidation and assessment. Consolidating a topic or a unit requires students to have a general understanding of the interlinking of concepts that are involved because topics in a unit or topic are closely connected. The artefacts from students can demonstrate that if students are provided the opportunity to develop concept maps, their concept maps can show what they view as key prior concepts that are fundamental to new knowledge. This can provide a teacher with opportunities to add value by suggesting other concepts students might have overlooked, thus deepening their understanding. Likewise, teacher developed concept maps (artefacts from teacher) provided an overview of the linking words that could be critical in establishing how the concepts are related, which align with Novak's (2010) findings. During semi-structured interviews participants went further to point out that concept maps can help identify gaps in knowledge and also show connections of concepts within a topic or subject matter together. The gaps might indicate misconceptions. These results point to the effectiveness of concept maps in facilitating consolidation, as well as identifying and addressing misconceptions. The results support Kinchin (2011) whose works determined that concept maps can expose students' conceptual misconceptions and also support consolidation. The findings also indicate that concept maps that link junior to senior concepts made by students represent their conceptual understanding and thus can be used as an assessment tool.

Concept maps can be considered a visual representation of students' conceptual understanding. Quantitative results show that all participants agreed that they can be a representation of students' knowledge of the interconnection of concepts. In open-ended survey questions responses, participants noted that students can develop concept maps during or at the end of a learning session or a topic or unit. Interviews with teachers provided an in-depth insight that concept maps can represent students' thoughts. Importantly the student artefacts collected in this study provided an insight into students' conceptual understanding, especially when students identified the prior concepts that linked with new knowledge. Results demonstrate that concept maps developed by students can be used to check for understanding which in turn provides an opportunity for teachers to give feedback. The results align with Ho and colleagues' (2017) work which noted that concept maps represent a visual display of an individual's conceptual understanding. The results are also in line with Bell (2017) who posited that concept maps may be used as an assessment task during formative evaluation to assess knowledge beyond facts and procedures. Furthermore, participants highlighted that incomplete concept maps can be used as assessment pieces for students to complete. Therefore, effective use of concept maps that link junior to senior concepts may play an important role in improving students' participation and achievement in mathematics.

8.8 Implications for practice

Concept maps that link junior to senior concepts can be a resource that teachers can use to support teaching and learning of mathematics at senior secondary school. Linking prior to senior concepts can facilitate gradual development of knowledge and deeper understanding. The linking of junior to senior concepts is critical to teaching and learning as it shows that mathematics is a web of concepts that build on each other. Moreover, it represents continuity, especially in jurisdictions where students choose different mathematics subjects at senior secondary level. Similarly, concept maps are visual representations that research has identified as easy to recall and an effective teaching and learning tool. The use of concept maps at senior secondary level needs to be encouraged and their inclusion in resources such as textbooks and assessments can be further developed.

8.9 Chapter Conclusion

In conclusion, teachers have a perception that concept maps that link junior to senior concepts can play a central role as a key resource of choice in deepening senior secondary students' mathematics knowledge. The results from this chapter can support concept maps as a resource that can create a rich learning environment beyond developing conceptual knowledge in mathematics teaching and learning at senior secondary level. However, the main limitations of this study are that a small number of participants was used, and senior secondary students' views and experiences as key stakeholders were not solicitated. Furthermore, this study did not include evidence of impact on students' learning outcomes. Despite these limitations, the present study has contributed important insights into our understanding of the role of concept maps in the teaching and learning of mathematics at senior secondary level. We hope this study will stimulate further investigation on the importance and role of visuals in mathematics teaching and learning especially at senior secondary level. The next chapter outlines the development of another resource (procedural flowcharts) which can support the teaching and learning of mathematics.

Chapter 9: Role of Procedural Flowcharts in Teaching and Learning of Senior Secondary Mathematics.

A version of this chapter was published in N. Fitzallen, C. Murphy, & V. Hatisaru (Eds.), *Mathematical Confluences and Journeys* (Proceedings of the 44th Annual Conference of the Mathematics Education Research Group of Australasia, July 3-7, 2022), pp 130-137.
https://merga.net.au/common/Uploaded%20files/Annual%20Conference%20Proceedings/2022%20Annual%20Conference%20Proceedings/Research%20Papers/Chinofunga%20RP%20MERGA44%202022.pdf

9.1 Chapter Introduction

When the senior secondary Mathematical Methods subject was introduced by the Queensland Curriculum and Assessment Authority (QCAA) in 2019, parents raised the issue that students who obtained 'As' at junior level were getting lower grades at senior level (Bennett, 2019). In Queensland, trends have shown a decline in student participation and achievement in calculus-based senior secondary mathematics options (Chinofunga et al; 2022a, b) and international trends have shown a similar decline in participation in most countries (Hodgen et al., 2010a, b). Researchers have pointed to pedagogy and classroom practices that are disengaging (Tytler, et al., 2008) as one of the causes of the decline in participation and achievement in advanced mathematics subjects. Additionally, students' limited procedural fluency has been highlighted as one of the causes that is limiting their understanding of mathematics ideas and solving mathematics problems (Kilpatrick et al., 2001), hence affecting participation and achievement.

9.2 Procedural Fluency

Procedural knowledge is a part of procedural fluency in mathematics education and is defined as knowledge of procedures and steps to a solution (Braithwaite & Sprague, 2021). Procedural fluency is more than just being able to perform a procedure as it involves conception of the problem, choosing the appropriate method and adaptability in applying the chosen method (Bay-Williams, 2020). It also involves "using procedures efficiently, flexibly, and accurately" (Bay-Williams et al., 2022, p. 178). Bay-Williams and San Giovanni (2021) define "efficiency" as selecting the best method and using it to solve a mathematics problem within a set time and "accurately" as using a procedure correctly, while "flexibility" is conceptualised as knowing more than one procedure and being able to modify procedures when solving a mathematics problem (Star, 2005). "To support flexibility, teaching standards in numerous countries recommend that students be introduced to multiple procedures early in instruction and be encouraged to compare the procedures" (Rittle-Johnson et al., 2012, p. 437). Students demonstrate procedural fluency when they exhibit flexibility in using a skill, obtain the correct solution and can effectively communicate the method used (McClure, 2014). Procedural knowledge is therefore part of procedural fluency and teachers are expected to help students build procedural fluency, using different strategies and teaching styles to do so.

Teachers in Queensland use explicit teaching approaches to help students execute procedures accurately and to select the optimal method to solve a problem while practice brings flexibility and efficiency. In this approach, teachers demonstrate the skill, then guide students' practice and finally provide the opportunity for unprompted practice (Archer & Hughes, 2010). Thus, after the students have been taught explicitly a method to solve a mathematics problem, they must be given an opportunity to practice when and how to use the method (Bay-Williams et al., 2022). When students can identify a context where the procedure can be suitably applied, they also have the opportunity for procedure modification (NCTM, 2014), resulting in deeper knowledge. Similarly, "procedural fluency is a comprehensive way of navigating mathematical procedures; it includes mastery of algorithms and strategies, but it also includes knowing when to use them" (Bay-Williams & San Giovanni, 2021, p. 25). However, procedural knowledge is perfected through "practice, and thus is tied to particular problem types" (Rittle-Johnson et al., 2015, p. 119), as mastery of procedures is key to developing this knowledge. As a result, repeated practice and guidance is one critical part of building procedural knowledge (Rittle-Johnson, 2017). Hence, procedural knowledge development is characterised first by being guided, then by mimicking and then through experience, adapting procedures to other complex familiar problems as part of procedural fluency.

Students who operate at high levels of procedural fluency are more likely to integrate and modify familiar procedures to solve complex unfamiliar problems (Blöte et al., 2001). However, in Queensland, simple familiar problems constitute 60% of examination questions and require use of procedures identified in the questions (QCAA, 2018).

In such cases, students have to identify the most appropriate procedure and then apply it correctly and efficiently to pass the examination. Therefore, procedural fluency plays a key role in success in mathematics. This study focused on how procedural flowcharts can support students' procedural fluency in the Mathematical Methods subject.

9.3 Procedural Flowcharts

A flowchart is the most efficient and concrete method with which to illustrate a procedure or multiple procedures to solve a problem (Toyib et al., 2017). The importance of flowcharts in developing procedural knowledge is supported by the definition of procedure established by Rittle-Johnson et al. that it is "a series of steps, or actions, done to accomplish a goal" (p. 588). In addition, a flowchart is effective in a class where students are operating at different levels of prior knowledge, being more advantageous to those at the very low level as it helps in decision-making and provides problem-solving skills (Hooshyar et al., 2015). Importantly, flowcharts play a significant role in promoting independent learning as students can refer to them after encountering a familiar mathematics problem (Marzano, 2017). Apart from showing contradictions and contrasting procedures, flowcharts promote representations of steps and procedures from different perspectives (Andrej, 2018). In procedural knowledge, relationships are sequential, that is, steps follow each other (Hiebert & Leferve, 1986). Consequently, flowcharts are an important tool for a mathematics teacher to teach procedural knowledge because they guide students through a process, allowing learning to be student-centred and to accommodate different levels of understanding among students.

It is a common experience for mathematics teachers to witness students applying procedures inappropriately just because they have memorised them (DeCaro, 2016) and minimising this problem will improve students' participation and achievement. When procedures are taught using flowcharts, decisions are taken at every step. This is because "flowcharts represent a sequence of decision making and information processing" (Marzano et al., 2017, p. 57). They are an "aid to thought" that help in analysing a problem and planning the solution (Ensmenger, 2016, p. 328). Consistent use of flowcharts helps students to develop skills in identifying suitable methods for solving mathematical problems and to become more sophisticated in approaching complex problems (Newton et al., 2020). Superficial procedural knowledge might be limited to accurate and efficient use of one procedure, but deep procedural knowledge involves several approaches and knowing when to apply a particular strategy (Bay-Williams, 2020). As students apply a flowchart, decisions are made depending on how the processes, steps and solution being followed align with the flowchart. Due to the checks and balances provided by the flowchart, students can then determine the most relevant procedures needed to solve a particular problem.

Applying tools that promote multi-solution strategies enhances students' capacity to solve a variety of problems (Siegler, 2003). Teachers can use flowcharts to represent multiple ways or choices to a solution (Marzano, 2017), thus promoting procedural flexibility. Flowcharts guide students through processes, steps and decision-making, all of which are critical for procedural fluency (Marzano, 2017). “When students achieve procedural fluency, they carry out procedures flexibly, accurately and efficiently” (QCAA, 2018 p. 1). Thus, procedural flowcharts are a visual representation of available procedures and corresponding steps, showing all stages of evaluation and alternative paths to a desired result or solution. This study explored teachers' perceptions of the use of procedural flowcharts, based on the research question: “What are teachers' perceptions on how flowcharts can support teaching and learning of procedural fluency in the Mathematical Methods subject?”

9.4 Method

The mixed-methods study informed by constructivism focused on teachers' perceptions on how procedural flowcharts can support the teaching and learning of Mathematical Methods. Quantitative and qualitative data were collected and analysed to gain further insights into participating teachers' perspectives (Creswell, 2015). Ethical approvals were obtained from the Department of Education, Queensland: Reference number: 550/27/2383 and James Cook University Human Research Ethics Committee: Approval number: H8201. The same 16 senior secondary Mathematical Methods teachers who participated in the study described in Chapter 7 participated in this study as well, and watched a 10-minute video presentation based in this instance on procedural flowchart tools developed from a section on Functions in Unit 1 of the Mathematical Methods syllabus. Figure 9.1 is an example of one of the procedural flowcharts used in the presentation.

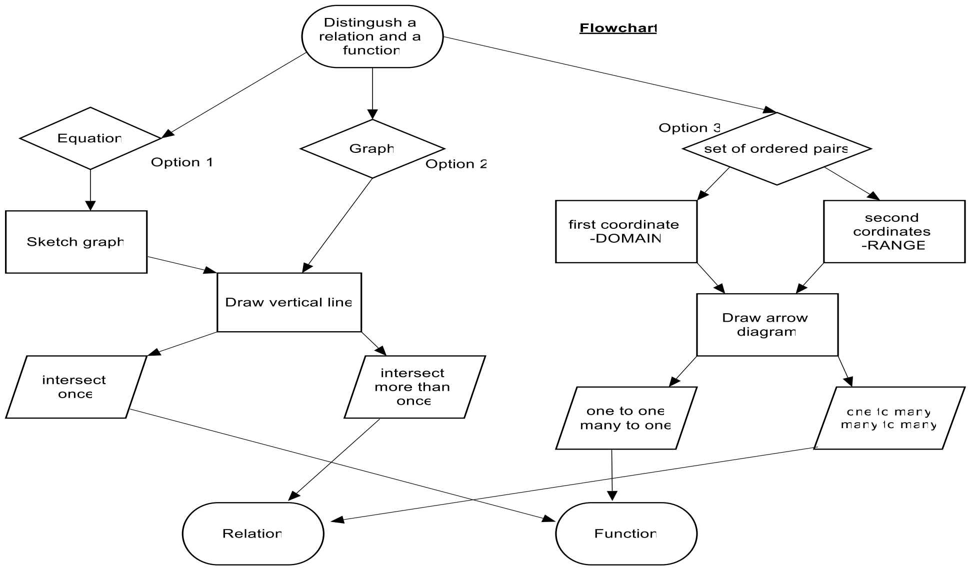

Figure 9.1: Procedural Flowchart on Distinguishing Functions and Relations

Using Figure 9.1, teachers asked the students to determine if a polynomial graph or set of ordered pairs was a function or a relation. The decision was reached after applying the mathematical procedures. As the mathematical procedures were being implemented, they allowed the choices given in the flowchart to be justified. This allowed students to work through independently and be reminded of the steps and procedures that were critical to solving the problem. Students were also expected to learn about features of quadratic functions.

A procedural flowchart on features of quadratic functions in Figure 9.2 shows the procedures needed to determine different features that students are expected to learn in Year 11 was collaboratively developed with participants. Effective teaching of quadratic functions helps students understand how different forms of algebraic representations relate to how features of the functions are determined (Wilkie, 2016), so this was included in the flowchart of quadratic functions. The relationships between these features, as shown in Figure 9.2, help students to build and broaden their understanding of the concept. They also remind students of the key mathematical steps and procedures to solve problems related to the concept. Thus, procedural flowcharts are key in highlighting the vocabulary expected in a concept, such as “intercepts”, “turning points” and “discriminant’, which are important in developing mathematical fluency.

9.5 Data collection and analysis

The participants in this study were given a school term to embed the procedural flowcharts in their teaching and learning of mathematics programs at their schools. They were also asked to respond to the five-point Likert scale questions and the open-ended questions to give this researcher a deeper understanding of their insights from using the procedural flowcharts in the teaching and learning of mathematical methods. The 20-minute semi-structured interviews were conducted with eight participants (the same eight teachers who participated in the study described in Chapter 7) who completed the surveys. The responses to the open-ended questions and semi-structured interviews were then analysed thematically and coded (Creswell, 2015).

9.6 Results

The themes agreed upon after the independent thematic analysis, collaborative reviewing and revision were as follows: (1) procedural flowcharts can foster a classroom environment that stimulates procedural fluency when learning mathematics, and (2) procedural flowcharts can support student-centred teaching and learning of mathematics procedures.

The survey data collected using the five-point Likert scale was analysed as shown in Table 9.1.

Table 9.1: Likert Scale Responses showing Participants Perceptions of how Procedural Flowcharts can Support Teaching and Learning of Mathematical Methods.

Questions	Strongly Agree	Agree	Not Sure	Disagree	Strongly disagree
1. Visual representation of mathematical knowledge enhances teaching and learning of mathematics.	15 94%	1 6%	0 0%	0 0%	0 0%
2. Procedural flowcharts (showing steps and procedures) plays an important role in developing students' mathematical skills.	9 56%	5 31%	1 6%	1 6%	0 0%
3. Procedural flowcharts promote fluency and recall.	11 69%	2 13%	3 19%	0 0%	0 0%
4. Procedural flowcharts can be used to highlight critical vocabulary	11 69%	3 19%	2 13%	0 0%	0 0%
5. Procedural flowcharts are a reference resource that can also be used for revision.	13 81%	3 19%	0 0%	0 0%	0 0%
6. Procedural flowcharts focus on students' learning.	11 69%	3 19%	3 13%	0 0%	0 0%
7. Procedural flowcharts promote independent or collaborative learning.	11 69%	2 13%	3 19%	0 0%	0 0%
8. Procedural flowcharts can help evaluate or give feedback to students on their understanding and correct use of a procedure.	10 63%	5 31%	1 6%	0 0%	0 0%

The results show 15 participants strongly agreed that visual representations of mathematical knowledge enhance teaching and learning of mathematics. At least 13 participants in the survey agreed or strongly agreed that procedural flowcharts support learning of procedural fluency in mathematics. Results show "strongly agree" and "5" were both the mode and median for questions 2-8. Importantly, 11 participants strongly agreed that procedural flowcharts support fluency and recall, highlight critical vocabulary, support student-centred learning and promote independent and collaborative learning. Moreover, 13 of participants strongly agreed that procedural flowcharts are a tool that can play a critical role in revision. Above all, the data shows that 15 of participants agreed or strongly agreed that they can help evaluate or give feedback to students on their understanding and correct use of procedures. Therefore, this study strongly supported procedural flowcharts as a resource that can support teaching and learning of mathematics procedural knowledge.

9.6.1 Theme 1: Procedural Flowcharts can Foster a Classroom Environment that Stimulates Procedural Fluency when Learning Mathematics.

The participants agreed that procedural flowcharts stimulated procedural fluency and the open-ended survey questions showed that participants supported the use of procedural flowcharts in enhancing procedural fluency. These included: (i) teacher-created procedural flowcharts for students to use during explicit teaching phases or targeting students who have not achieved fluency or for students with identified learning needs, (ii) class-generated procedural flowcharts during collaborative teaching phases to show the processes that were applied, and (iii) student-generated procedural flowcharts to show common mistakes or misconceptions. These results demonstrate the flexibility of procedural flowcharts in enhancing fluency.

Feedback from the semi-structured interviews gave greater detail on how procedural flowcharts created a wide range of opportunities for developing procedural fluency in mathematics. Participants' perceptions after applying them as a teaching and learning resource provided some insight into how this resource can help develop students' procedural knowledge and skills.

Participants noted that students were more comfortable with visual representations than just worded steps. In fact, they appreciated that most students were visual learners who responded better to diagrammatical representations than to written steps.
For example, Participant 8 said, "*Because it's a diagrammatic representation, students look at it favourably because it's easier to process and, like I said, most students are visual learners.*" Importantly, during participants' check-in sessions the researcher collaboratively developed procedural flowcharts with participants. Participants 2, 7 and 8 collaboratively developed a procedural flowchart in Figure 9.2 with the researcher which they then used during teaching and learning. Participant 7 went to give an advantage of a procedural flowchart by saying, "*It is steps in diagrammatic form which is easy to process and easy to understand*." Thus, students can follow easily and use the steps to answer problems with minimum help. Participant 2 noted, "*If you had steps just written down in the book, it's hard to flip back through and find the information you're looking for, whereas if it's a diagram, it's easy to find.*" Participant 2 then said, "*They enhance students' memory*". Therefore, flowcharts that are easy to navigate and use

provide a better opportunity to recall and accurately apply information, which assists in the development of procedural fluency. They can help solve most problems in mathematical methods examinations, as indicated by Participant 8 when he said, *"It is very handy for simple familiar questions which are mostly recall and fluency questions, but which are the majority in mathematics examinations."*

As students follow the steps on the procedural flowchart, they work towards developing their procedural knowledge and fluency. Participant 2 made this point when she said, *"Really good how it organises the steps and explains where you need to go if you're at a certain part in a procedure."* In addition, Participant 7 said, "*The cycle approach, the feeding back in, feeding back out, that type of stuff, that's when we are starting to teach students how to think.*" Likewise, Participant 8 observed, "*Complex procedural flowcharts guide students in making key decisions as they work through solutions which is key to critical thinking and judgement and these two are very important in maths.*"
Thus, procedural flowcharts support students' efficiency and flexibility in solving problems, deepening their understanding through reasoning and justification, which are part of mathematics proficiency strands.

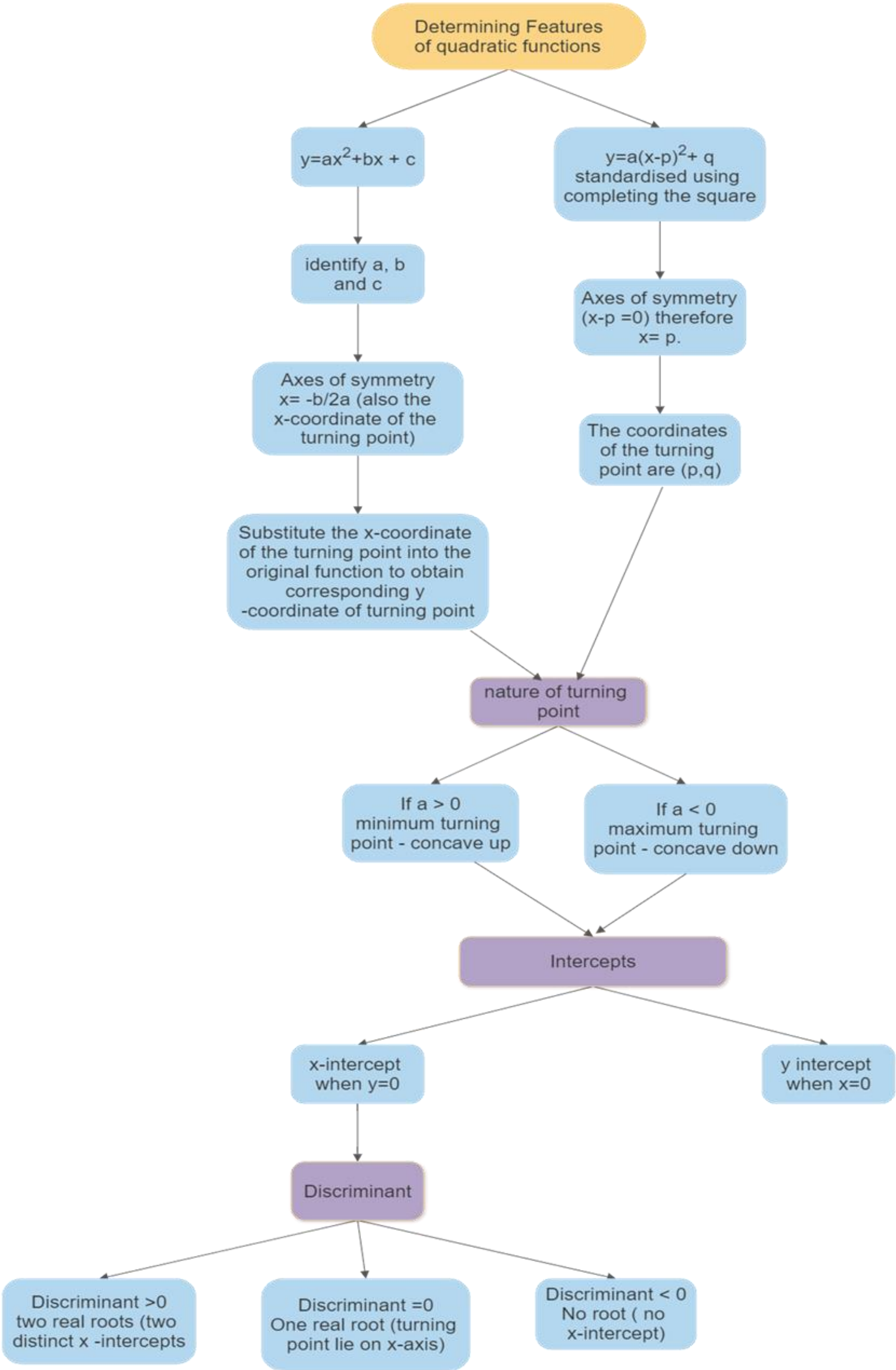

Figure 9.2: Procedural flowchart on features of Quadratic functions.

The flowchart in Figure 9.2 provides an overview of features of quadratic functions in Mathematical Methods. Firstly, students have to match the given equation which will guide the student on steps to follow to determine the coordinates of turning point. Secondly students determine the nature of the turning point by matching the coefficient of the square of the variable with the procedure that has been provided. Finally, students have to determine the value of the discriminant to determine properties of the roots of the function. The value will inform students if the function has two solutions, one solution, or no solutions. Similarly, the procedural flowchart on Transformations in Figure 9.3 was developed by participant 8 and provided different types of transformations needed in Mathematical methods. The flowchart identifies how the transformations can be represented in function notation. It can be noted that the procedural flowchart guides students through procedures, acting as a scaffolding resource. As a diagrammatic representation they can summarise key steps and procedures that are essential to solving a problem. They can be used in assisting students in identifying the most suitable solution as they navigate through the flowchart and match their thinking and proposed solutions with steps in the procedural flowchart. The steps can also help students recall procedures that are needed, for example in Figure 9.2 students have to recall how to determine the discriminant as the formula is not provided.

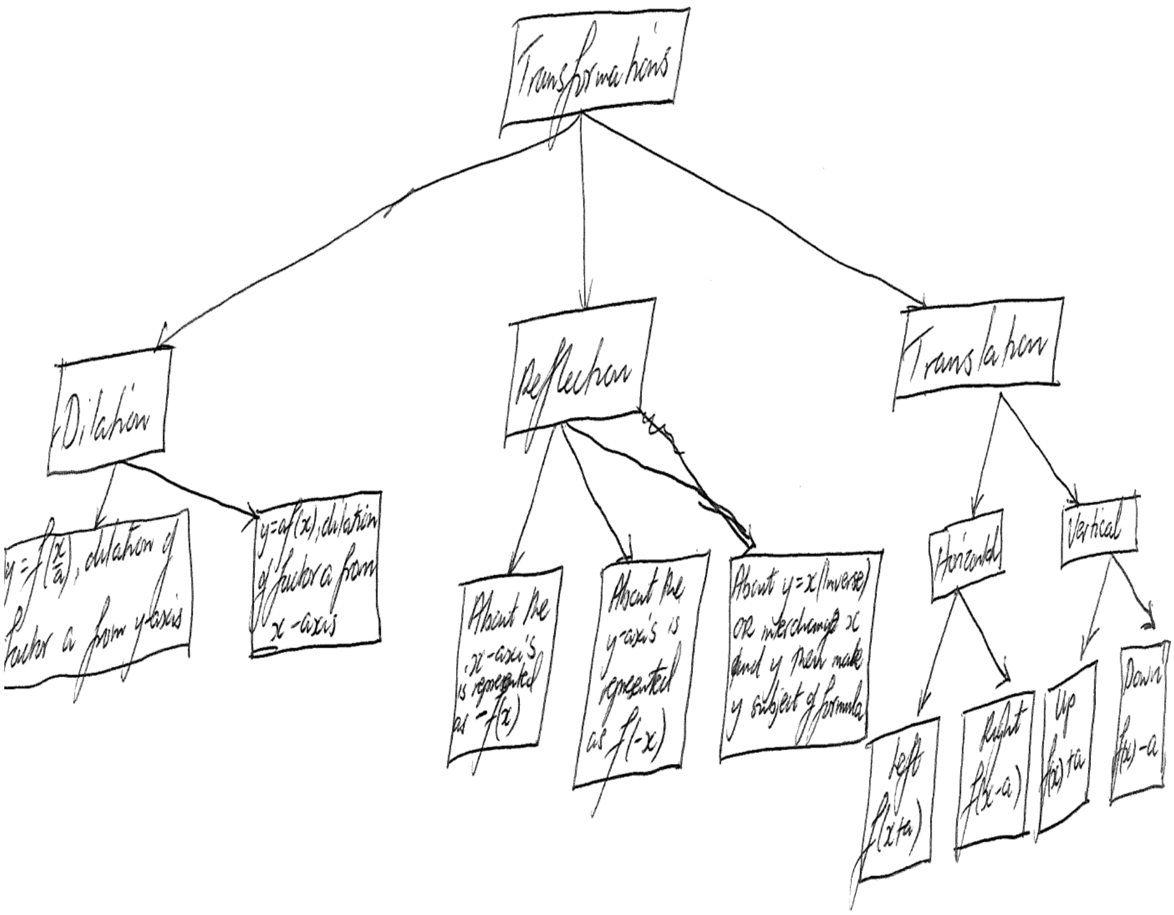

Figure 9.3 Procedural flowchart on Transformation developed by a participant.

Procedural flowcharts provide teachers with the opportunity to determine students' procedural competencies and misconceptions. Participant 8 said, "*I went further to ask my students to create their own procedural flowcharts ... so that I can evaluate if they understand and represent their fluency on the chart.*" Participant 1 included procedural misconceptions: "*I use it to identify the potential students' misconceptions and I'll use it to identify student's competencies*" therefore, enhancing procedural fluency.

9.6.2 Theme 2: Procedural Flowcharts can support Student-centred Teaching and Learning of Mathematics Procedures.

The participants agreed that the use of procedural flowcharts encourages and facilitates independent and student-centred learning. The open-ended survey responses highlighted the use of: (i) student-generated procedural flowcharts after explicit teaching, and (ii) student-generated procedural flowcharts at the end of the lesson as part of lesson consolidation. Participants shared procedural flowcharts in Figures 9.4 and 9.5 that were developed by their students as an alternative way of assessing students' understanding of procedures. The procedural flowchart in Figure 9.4 shows a students' understanding of determining an arithmetic and geometric sequence. It can be the key to solving a question where the sequence is not identified by type, hence the testing so as to identify the sequence given. Such a procedural flowchart shows that a student understands the fundamentals of the sequences section in Mathematical Methods. Likewise Figure 9.5 shows a students' understanding of solving quadratic equations using factorisation. The student's interesting and accurate procedural flowchart shows deeper understanding of the concept. Importantly, the procedural flowcharts can support students' deepening understanding.

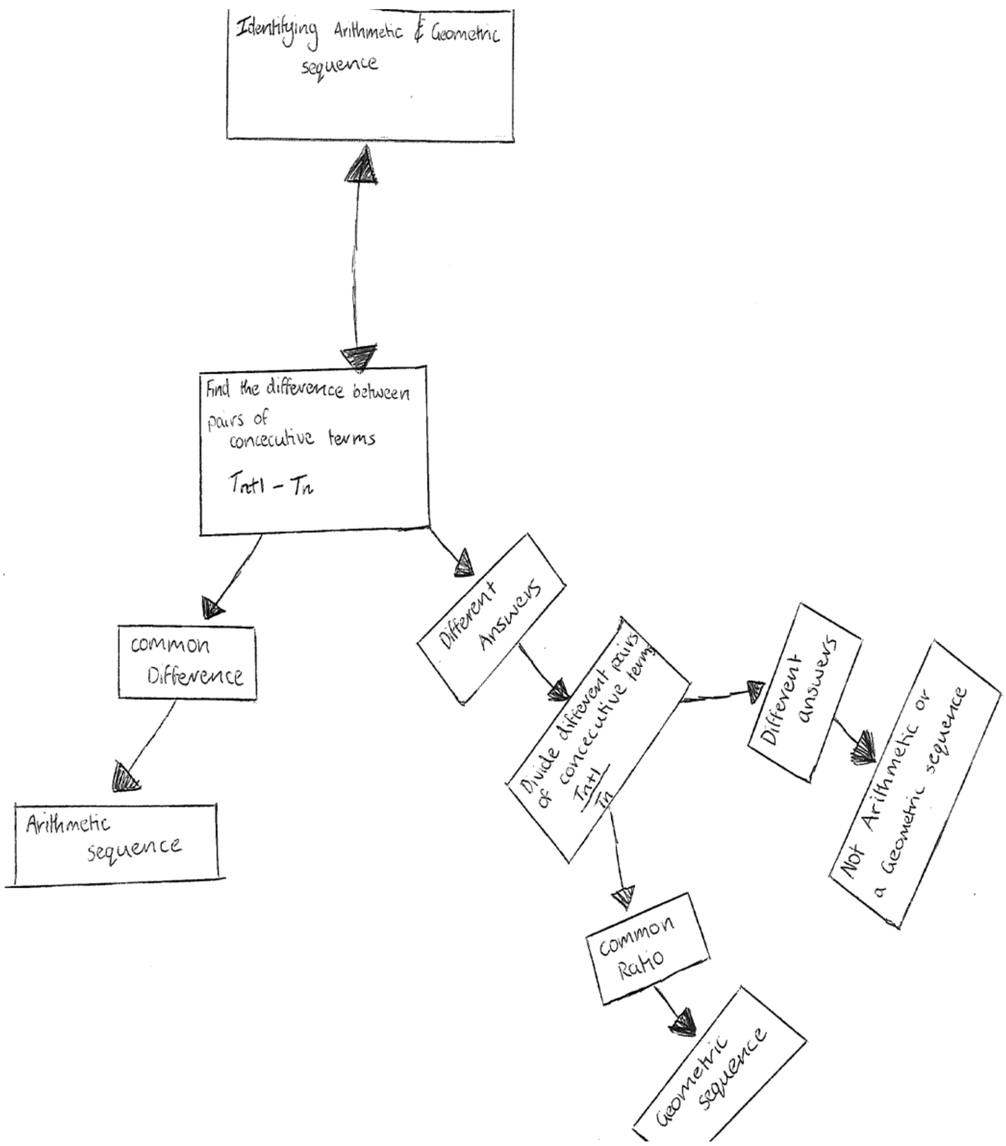

Figure 9.4: Student developed procedural flowchart on Sequences

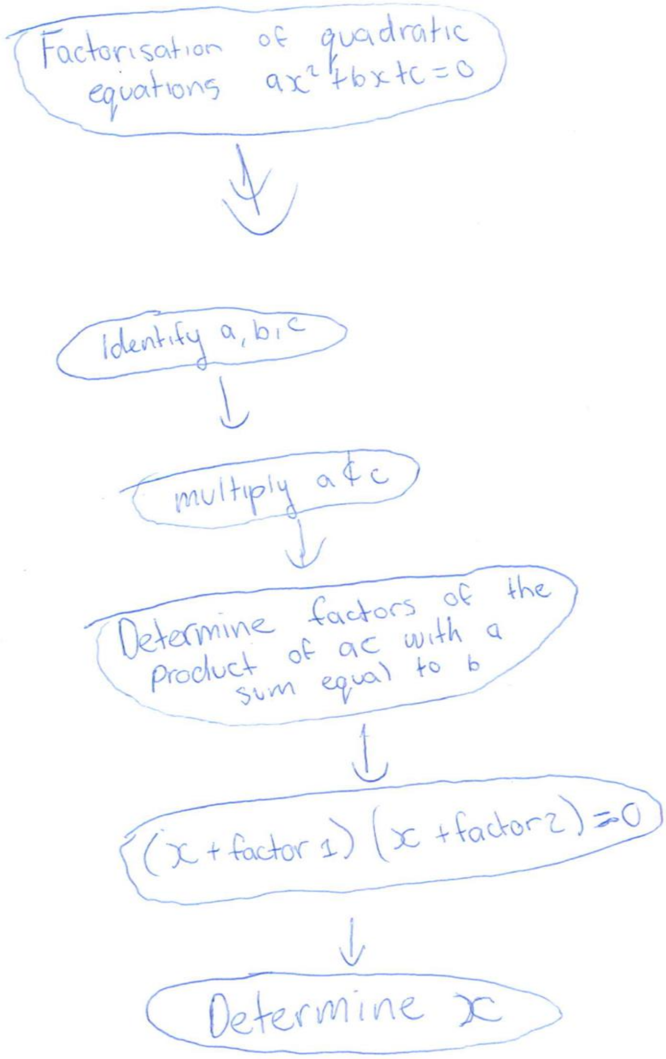

Figure 9.5: Student developed procedural flowchart on factorising quadratic expressions.

The use of procedural flowcharts by participants in the teaching and learning of mathematical methods made the participants realise that the flowcharts promoted independent and student-centred learning. The response from Participant 8 was, "*They promote individual learning and learning which is student-centred*." Participant 6 said that capable students "*can teach themselves without even a teacher.*" The teacher developed procedural flowcharts in Figure 9.2 and 9.3 can be used for students' independent learning. Importantly, independent learning by students can provide a teacher with the opportunity to help struggling students. This view was supported by Participant 5 when he said, "*It gave me the opportunity to work with slower kids as they* [the flowcharts] *promote individual learning*."

Participant responses also indicated that procedural flowcharts encouraged student engagement. When asked about the role of the flowcharts on the development of students' procedural knowledge, Participant 8 said, "*I have witnessed more students engaging more in the YOU DO (student-centred) phase*." The participant went on, "*I was so impressed because students engaged more with the task.*" A similar but more detailed observation was also made by Participant 7, who said,

> "*Mathematics goes from being very dry and dusty to being something which is actually creative and interesting and evolving, starting to get kids actually engaging and having to back themselves, and having to be less passive and more active as learners*."

Moreover, participants noted that the flowcharts helped students understand the importance of procedures if they were to engage effectively with mathematics. Participant 3 shared her observation that procedural flow charts, "*allow the students to move in both directions and it makes them see that the actual responses that they have to give are minimised, rather than seeing every question as separate.*" This was very important, especially for questions that require procedural steps rather than in their most usual form or representation.

The participants agreed that procedural flowcharts play an important role in supporting procedural fluency and engagement in mathematics.

9.7 Discussion

One interpretation of these findings is that participants noted procedural flowcharts can support the development of procedural knowledge and fluency. As highlighted previously, procedural knowledge is knowledge of steps and procedures to reach a solution (Braithwaite & Sprague, 2021). Thus, procedural flowcharts represent a series of steps and procedures that may include several approaches to reach a desired solution to a particular type of mathematics problem. Results of this study show that at least 13 participants agreed or strongly agreed that procedural flowcharts support the development of mathematics skills and promote fluency and recall. Fluency includes an understanding of vocabulary and 14 participants acknowledged that procedural flowcharts highlight the criticality of vocabulary in mathematics procedures. Participants concurred that the flowcharts not only provided steps to be followed but facilitated decision-making through reasoning as students evaluated the correct procedures to follow. Kilpatrick (2001) posited that procedural skills are central to students' learning of mathematics. Thus, practice in solving problems using sequenced steps and procedures promotes accuracy. Additionally, information processing and decision-making help in evaluating how the path to a solution aligns with the available procedures, thus enhancing "efficiency". This study highlighted that multi-solution procedural flowcharts provided an option for students to discover more than one solution, thus enhancing their "flexibility". Using mathematics steps accurately, effectively and efficiently develops fluency (Bay-Williams et al., 2022). Therefore, efficient use of procedural flowcharts helps students develop procedural knowledge and assists in the development of procedural fluency. As a resource, it can also support explicit teaching, which is one of the main pedagogies in Queensland.

The results of this study also show that developing a procedural flowchart during any stage of explicit teaching is beneficial. First, teachers can develop the charts during the "I DO" (teacher-centred) stage by teaching students how to organise the steps, processes and loops for decision-making. The artefacts show the step-by-step presentation of procedures and key stages that require students to decide on which direction to take depending on how the solution is shaping up. Second, the charts can be developed as a class during the "WE DO" (collaborative) stage and, lastly, students can develop them during the "YOU DO" (student-centred) stage. The participants' responses show that having students develop their procedural flowcharts can be an efficient way of checking students' procedural understanding and misconceptions and evaluating their learning. These results are consistent with Raiyn (2016), whose work concluded that visual representations require less time and are easier to process than text. Furthermore, presenting information in different, for example, verbal, numerical and diagrammatical forms, helps students comprehend the phenomenon (Murphy, 2011). When students are given the opportunity to create their own procedural flowcharts, they represent their procedural knowledge and fluency diagrammatically. The artefacts from students demonstrated that procedural flowcharts developed by students can be used by teachers to gain an insight into students' understanding of a procedure. Procedural flowcharts are also a tool that can be used to promote engagement and student-centred learning.

The quantitative data analysis in this study indicated that at least 13 of participants agreed or strongly agreed that procedural flowcharts support independent and student-centred learning, while the qualitative data highlighted the importance of procedural flowcharts during the “YOU DO” stage when using the explicit teaching approach. This is the stage when students are expected to interact and solve familiar problems to what they were taught and practiced as a class in the “I DO” and “WE DO” stages. This is because “routine practice is an extremely powerful instructional tool that not only helps students learn and retain basic skills and facts in a fluent fashion, but has positive outcomes when students attempt higher-order strategies” (Archer & Hughes, 2010, p. 21). The artefacts demonstrated that procedural flowcharts that include more than one simple procedure (system of procedures) may emphasise the ‘bigger picture’ which plays an important role in the development of conceptual understanding. Importantly consistent use of flowcharts helps develop mastery as they are an aid to thinking (Ensmenger, 2016). The participants’ perceptions in this study were consistent with Marzano’s (2017) conclusion that when students come across familiar problems, they can refer to procedural flowcharts as they independently solve them. Likewise, in student-centred learning, students develop knowledge and experiences they have acquired by further exploring using tools and resources as scaffolds (Lee & Hannafin, 2016). When answering the open-ended and interview questions in this study, the participants emphasised that students could use procedural flowcharts during the “YOU DO” stage, providing them with an opportunity to engage with learning using the procedural flowchart as a scaffolding resource and with minimum teacher assistance.

9.8 Chapter Conclusion

This study highlighted that teachers view the use of procedural flowcharts as a resource that can help develop students’ procedural fluency and participation in mathematics and suggests that this approach can be extended to other mathematics subjects at different levels. The present research, therefore, contributes to a growing body of evidence suggesting that representation of mathematical knowledge and processes in non-linguistic forms such as diagrams support participation and achievement. However, the main limitation of this study was the small number of mathematical methods teachers who participated. In terms of future research, it is hoped that this study has provided a basis for further research in use of procedural flowcharts in mathematics teaching and learning. The next chapter outlines the utility of procedural flowcharts in developing problem-solving skills.

Chapter 10: How can Procedural Flowcharts support Mathematics Problem-solving Skills?

A version of this chapter is under review for publication in the Mathematics Education Research Journal.

10.1 Chapter Introduction

This chapter starts with a review of the literature on problem solving as well as the use of visual representations such as procedural flowcharts in mathematics education. It then goes on to discuss the importance of visual representations in learning mathematics, and a Problem Solving and Modelling Task (PSMT) approach from QCAA which provides the context of the study. This is followed by an exploration of a teacher's perceptions of the use of procedural flowcharts in supporting mathematics problem solving skills. An in-depth interview with the senior mathematics teacher and four artefacts produced by her students informed the discussion of the use of procedural flowcharts during a PSMT. The analysis is informed by the stages of problem solving.

Problem solving plays an important role in the teaching and learning of mathematics (see Cai, 2010; Lester, 2013; Schoenfeld et al., 2014). However, research is still needed on tools that teachers can use to support students during problem solving (Lester & Cai, 2016). Although research in mathematics problem solving has been progressing, it has remained largely theoretical (Lester, 2013). Schoenfeld (2013) suggests that researchers should now focus on exploring how ideas grow and are shared during problem solving. Similarly, English and Gainsburg (2016) and Maaß (2010) identified that the development of problem-solving competency in students is an area that researchers should focus on.

A key area that would benefit from further research is the identification of strategies that support students' ability to construct and present their mathematical knowledge effectively during problem-solving, particularly if complex processes such as integration and modification of several procedures are involved (Vale & Barbosa, 2018). Similarly, students face challenges in connecting or bringing all the ideas together and showing how they relate as they work towards the solution (Reinholz, 2020). Research indicates that problem-solving in mathematics is challenging for students (Ahmad et al., 2010) and therefore supporting students' problem-solving skills needs urgent attention (Schoenfeld, 2016). Furthermore, Mason (2016) posits that the crucial yet not significantly understood issue for adopting a problem-solving approach to teaching is the issue of "when to introduce explanatory tasks, when to intervene and in what way" (p. 263). Therefore, teachers also need resources to support the teaching of problem-solving skills, often because they were not taught these skills and approaches when they were school students (Sakshaug & Wohlhuter, 2010).

The purpose of this chapter was to explore, through a teacher's perceptions on the utility of procedural flowcharts in supporting the development of students' problem-solving skills in mathematics. The aim was to investigate if use of procedural flowcharts could support students in planning, logically connecting and integrating mathematical strategies and knowledge and to communicate the solution effectively during problem solving. "Mathematics is the science of patterns, it is natural to try to find the most effective ways to visualise these patterns and to learn to use visualisation creatively as a tool for understanding" (Zimmermann & Cunningham, 1991, p. 3). The use of flowcharts in this study was underpinned by the understanding that visual aids that support cognitive processes and interlinking of ideas and procedures influence decision-making, which is vital in problem-based learning (McGowan & Boscia, 2016). Moreover, flowcharts are effective tools for communicating the processes that need to be followed in problem-solving (Krohn, 1983).

10.2 Problem-Solving Learning in Mathematics Education

The drive to embrace a student-centred problem-solving approach has been a priority in mathematics education (Koellner et al., 2011; Sztajn et al., 2017). In the problem-solving approach, the teacher provides the problem to be investigated by students who then design the strategies to solve it (Colburn, 2000). To engage in problem-solving, students are expected to use concepts and procedures that they have learnt (prior knowledge) and apply them in unfamiliar situations (Matty, 2016). Teachers are encouraged to promote problem-solving activities as they involve students engaging with a mathematics task where the procedure or method to the solution is not known in advance (National Council of Teachers of Mathematics [NCTM], 2000), thus providing opportunities for deep understanding as well as providing students with the opportunity to develop a unique solution (QCAA, 2018). Using this approach, students are given a more active role through applying and adapting strategies to solve a non-routine problem and then communicate the method (Karp & Wasserman, 2015). During problem-solving, they engage with an unfamiliar real-world problem, develop strategies in response, justify mathematically through representation, then evaluate and communicate the solution (Artigue & Blomhøj, 2013).

The process of problem-solving in mathematics requires knowledge to be organised as the solution is developed and then communicated. Polya introduced the use of heuristic strategies as the basic tool to use when developing a problem solution (Klang et al., 2021). Students need to understand the problem, plan the solution, execute the plan and reflect on the solution and process (Polya, 1971). It is therefore guided by four phases: discover, devise, develop and defend (Makar, 2012). Makar expanded on each phase as follows: "discover - connecting context to mathematics, devise – mathematisation of problem, develop - modelling and representational fluency and defend - communicating the process linking purpose to question then evidence and conclusion" (p. 74). When using a problem-solving approach, students can pose questions, develop way(s) to answer problems (which might include drawing diagrams, carrying calculations, defining relationships and making conclusions), interpret, evaluate and communicate the solution (Artigue et al., 2020; Dorier & Maass, 2020). Importantly, the Australian Curriculum, Assessment and Reporting Authority notes that during problem-solving:

> Students solve problems when they use mathematics to represent unfamiliar situations, when they design investigations and plan their approaches, when they apply their existing strategies to seek solutions, and when they verify that their answers are reasonable. Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively. (ACARA, 2014, p. 5)

Therefore, during problem solving students have to plan the solution to the problem and be able to communicate all the key processes involved.

Similarly, mathematical modelling involves problem identification from a contextualised real-world problem, linking the solution to mathematics concepts, carrying out mathematic manipulations, justifying and evaluating the solution in relation to the problem and communicating findings (Geiger et al., 2021). Likewise, in modelling Galbraith and Stillman (2006) suggested that further research is needed in fostering students' ability to transition effectively from one phase to the next. "Mathematical modelling is a problem-solving process that requires students to interpret information from a variety of narrative, expository and graphic texts that reflect authentic real-life situations" (Doyle, 2005 p. 39). Thus, mathematical modelling is part of problem-solving but has additional aspects. Figure 10.1 identifies the main stages that inform mathematics problem-solving from the literature.

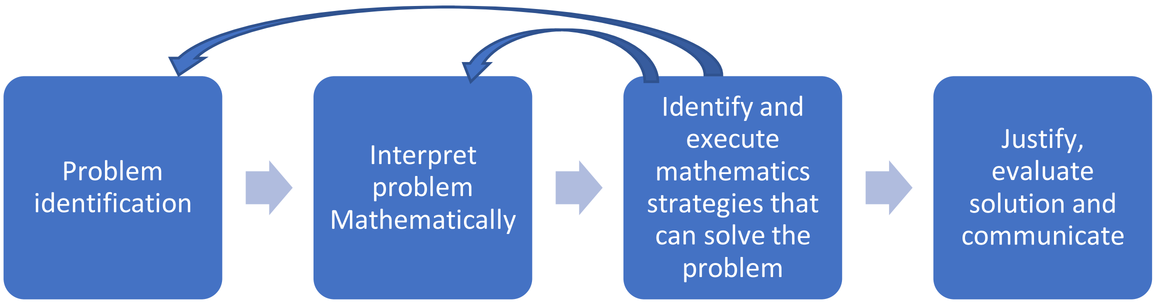

Figure 10.1: Stages of Mathematics Problem-Solving

However, although problem-solving is highly recommended in mathematics education, it presents several challenges for teachers in terms of how they can best support students to connect the processes and mathematics concepts into something coherent that can lead to a meaningful solution (Hacker, 1998). Therefore, relevant tools that support problem-solving and decision-making can make a difference for both mathematics teachers and students (McGowan & Boscia, 2016).

Students can solve problems better if they can think critically (Kules, 2016). Problem-solving requires their active engagement in analysing, conceptualising, applying concepts, evaluating, comparing, sequencing, synthesising, reasoning, reflecting and communicating, which are skills that are said to promote critical thinking (Kim et al., 2012; King, 1995; Moon, 2008); QCAA, 2018). Similarly, the ability to undertake problem solving is supported when students are provided with the opportunity to sequence ideas logically and evaluate the optimal strategy to solve the problem (Parvaneh & Duncan, 2021). However, finding tools that can support problem-solving has been a focus for researchers for a long time but with very limited breakthroughs (McCormick et al., 2015). This study explored how procedural flowcharts as visual representations can support teaching students to organise ideas, executing strategies, justifying solutions and communicating their solution.

10.3 Importance of Visual Representations in Mathematics Learning

As indicated above, visual representations show thoughts in non-linguistic format, which is effective for communication and reflection. "Visual representations serve as tools for thinking about and solving problems. They also help students communicate their thinking to others" (NCTM, 2000, p. 206). In mathematics, visual representation plays a significant role in structuring a problem-solving approach and showing the cognitive constructs of the solution (Owens & Clements, 1998), a view echoed by Arcavi (2003), who said that visual representations can be appreciated as a central part of reasoning and as a resource to use in problem-solving. More importantly, they can be used to represent the logical progression of ideas and reasoning as the solution develops (Roam, 2009). Therefore, use of visual representations such as flowcharts can support problem analysis, problem understanding and solution generation, while communicating the whole process effectively.

Flowcharts have been used to solve problems in different fields for a long time. Significant research (Carlisle et al., 2005; Hooshyar et al., 2018) has noted their use in solving information technology problems in such fields as robotics and programming. They have been used to support independent problem-solving in familiar and unfamiliar situations in vocational training for people with developmental disabilities (Villante et al., 2021), while in health sciences, flowcharts have been used to help appropriate decision-making within given options, which minimises errors and plays a significant role in problem-solving in the field (McGowan & Boscia, 2016). Importantly, in schools, Norton and colleagues (2007) noted that "planning facilitated through the use of flow charts should be actively encouraged and scaffolded so that students can appreciate the potential of flow charts to facilitate problem-solving capabilities" (p. 15). This was because the use of flowcharts in problem-solving provided a mental representation of a proposed approach to solve a task (Jonassen, 2012). The success of flowcharts in problem-solving in different fields can be attributed to their ability to facilitate deep engagement in planning the solution to the problem.

Flowcharts support the process of problem-solving. Creating a flowchart during problem-solving facilitates understanding, thinking, making sense of the problem, investigating and communicating the solution (Norton et al., 2007). Flowcharts can also be used when a logical and sequenced approach is needed to address a problem (Cantatore & Stevens, 2016). Identifying the most appropriate strategy and making the correct decision at the right stage is key to problem-solving. "One of the greatest advantages of a flowchart is its ability to provide for the visualization of complex processes, aiding in the understanding of the flow of work, identifying nonvalue-adding activities and areas of concern, and leading to improved problem-solving and decision-making" (McGowan & Boscia, 2016, p. 213). Teaching students to use visual aids like flowcharts as part of problem-solving supports the ability to easily identify new relationships among different procedures and assess the solution being communicated faster as visuals are more understandable (Vale et al., 2018). Norton and colleagues (2007) posited that using a well-planned and well-constructed flowchart in problem-solving results in a good-quality solution. Flowcharts can also be a two-way communication resource between a teacher and students or among students (Grosskinsky et al., 2019). These authors further noted that flowcharts can help in checking students' progress, tracking their progress and guide them. They can also be used to highlight important strategies that students can follow during the process of problem-solving.

Another aspect of flowcharts is that they can be used to provide a bigger picture of the solution to a problem (Davidowitz & Rollnick, 2001), as teachers can provide ready-made flowcharts to guide students in the problem-solving process. Flowcharts help students gain an overall and coherent understanding of the strategies involved in solving the problem as they promote conceptual chunking (Norton et al., 2007). Importantly, “they may function to amplify the zone of proximal development for students by simplifying tasks in the zone” (Davidowitz & Rollnick, 2001, p. 22). Use of flowcharts by students reduces the cognitive load which then may help them focus on more complex tasks (Berger, 1998; Sweller et al., 2019). Indeed, development of problem-solving skills can be supported when teachers introduce learning tools such as flowcharts, because they can influence the process of problem solving (Santoso & Syarifuddin, 2020). Therefore, the use of procedural flowcharts in mathematics problem-solving has the potential to transform the process.

As stated at the beginning of this chapter, procedural flowcharts are a visual representation of procedures or strategies, corresponding steps, and stages of evaluation of a solution to a problem (Chinofunga et al., 2022c). These authors noted that procedural flowcharts developed by the teacher can guide students during the inquiry process and highlight key strategies and stages for decision-making during the process of problem-solving. This is because “a procedural flowchart graphically displays the information decision action sequences in the proposed order” (Krohn, 1983, p. 573). Similarly, Chinofunga and colleagues (2022c) emphasised that procedural flowcharts can be used to visually represent procedural flexibility as more than one procedure can be accommodated, making it easier to compare the effectiveness of different procedures as they are being applied. They further posited that student-developed procedural flowcharts provide students with the opportunity to comprehensively engage with the problem and brainstorm different ways of solving it, thus deepening their mathematics knowledge. Moreover, a procedural flowchart can be a visual presentation of an individual and group solution during problem-solving.

Research has identified extended benefits of problem-solving in small groups (Laughlin et al., 2006). Vale and colleagues encouraged visual representation of solutions with multi solutions as a tool to teach students problem solving (2018). Giving groups an opportunity to present a solution visually can be a quicker way to evaluate a group solution because visuals can represent large amounts of information (even from different sources) in a simple way (Raiyn, 2016). For example, students can be asked to develop procedural flowcharts individually then come together to synthesise different procedural flowcharts.

The research questions in this study were informed by the understanding that limited resources are available to teachers to support students' problem-solving abilities. In addition, the literature indicates that visual representation can support students' potential in problem-solving. Therefore, the research described in this chapter addressed the following research question: What are teachers' perceptions of how procedural flowcharts support students' problem-solving skills in the Mathematical Methods subject?

10.4 Method

The active involvement of a mathematics teacher in the research described in this chapter brought a wealth of knowledge through their feedback that the researcher could tap into. The interaction between researchers and teachers through workshops and semi-structured interviews promoted an exchange of ideas, while the interaction between teachers and students and the use of procedural flowcharts in teaching and learning provided new insights and opportunities for this research. The method for this study is provided in more detail in Chapter 3 of this thesis.

10.5 Research Context of Phase Four of the Study

In the state of Queensland senior mathematics students engage with three formal assessments (set by schools but endorsed by QCAA) in Year 12 before the end of year external examination. The formal internal assessments consist of two written examinations and a problem-solving and modelling task (PSMT). The PSMT is expected to cover content from Unit 3 (Further Calculus). The summative external examination contributes 50% and the PSMT 20% of the overall final mark, demonstrating that the PSMT carries the highest weight among the three formal internal assessments.

The PSMT is the first assessment in the first term of Year 12 and is set to be completed in four weeks. Students are given three hours of class time to work on the task within the four weeks and write a report of up to 10 pages or 2000 words. The four weeks are divided into four check points, one per week with the fourth being the submission date. On the other three checkpoints, students are expected to email their progress to the teacher. Of particular importance is checkpoint two where students are required to email their draft reports for a general class feedback from the teacher. However, teachers are expected to have provided students with opportunities to develop skills in undertaking problem-solving and modelling task before they engage with this formal internal assessment. These play an important role in developing students' problem-solving skills as they prepare for the formal internal PSMT. The QCAA has provided a flowchart on how a PSMT should be presented (Appendix A).

10.6 Phase Four of the Study

In Phase Four, a teacher's shared experience and observations prompted an in-depth interview with Ms Simon (pseudonym). Ms Simon had explored the use of procedural flowcharts in a problem-solving and modelling task (PSMT) in her Year 11 Mathematical Methods class. This included an introduction to procedural flowcharts, followed by setting the students a task whereby they were asked to use the flowchart to plan how they would approach a problem-solving task. Importantly, procedural flowcharts were used by the students to provide an overview and structure of their proposed solution to the problem. The students were expected to first develop the procedural flowcharts independently then to work collaboratively to develop an alternative solution to the same task. The students developed procedural flowcharts (artefacts) and the in-depth interview with Ms Simon, all of which were analysed. As this was an additional study, an ethics amendment was applied for and granted by the James Cook University Ethics committee, approval Number H8201, as the collection of students artefacts was not covered by the main study ethics approval for teachers.

10.6.1 Participants in Phase Four of the study

Ms Simon and a group of four students were the participants in this study. Ms Simon had studied mathematics as part of her undergraduate education degree, which set her as a highly qualified mathematics teacher. At the time of this study, she was the Head of Science and Mathematics and a senior mathematics teacher at one of the state high schools in Queensland. She had 35 years' experience in teaching mathematics across Australia in both private and state schools, 15 of which were as a curriculum leader. She was also part of the science, technology, engineering and mathematics (STEM) state-wide professional working group. Since the inception of the external examination in Queensland in 2020, she had been an external examination marker and an assessment endorser for Mathematical Methods with QCAA. The students who were part of this study were aged between 17 and 18 years and were from Ms Simon's Mathematical Methods senior class. Two artefacts were from individual students and the third was a collaborative work from the two students.

10.7 Phase Four Data Collection

First, data were collected through a semi-structured interview between the researcher and Ms Simon. The researcher used pre-prepared questions and incidental questions arising from the interview. The questions focused on how she had used procedural flowcharts in a problem-solving and modelling task with her students. The interview also focused on her experiences, observations, opinions, perceptions and results, comparing the new experience with how she had previously engaged her students in such tasks. The interview lasted 40 minutes, was transcribed and coded so as to provide evidence of the processes involved in the problem-solving. Some of the pre-prepared questions were as follows:

1. What made you consider procedural flowcharts as a resource that can be used in a PSMT?
2. How have you used procedural flowcharts in PSMT?
3. How has the use of procedural flowcharts transformed students' problem-solving skills?
4. How have you integrated procedural flowcharts to complement the QCAA flowchart on PSMT in mathematics?
5. What was your experience of using procedural flowcharts in a collaborative setting?
6. How can procedural flowcharts aid scaffolding of problem-solving tasks?

Second, Ms Simon shared her formative practice PSMT task (described in detail below), and four of her students' artefacts. The artefacts that she shared (with the students' permission) were a critical source of data as they were a demonstration of how procedural flowcharts can support problem solving and provided an insight into the use of procedural flowcharts in a PSMT.

10.8 Problem-solving and Assessment Task

The formative practice PSMT that Ms Simon shared is summarised below under the subheadings: Scenario, Task, Checkpoints and Scaffolding (see Appendix A).

Scenario

You are part of a team that is working on opening a new upmarket Coffee Café. Your team has decided to cater for mainly three different types of customers. Those who:

1. consume their coffee fast.
2. have a fairly good amount of time to finish their coffee.
3. want to drink their coffee very slowly as they may be reading a book or chatting.

The team has tasked you to come up with a mode or models that can be used to understand the cooling of coffee in relation to the material the cup is made from and the temperature of the surroundings.

Task

Write a mathematical report of at most 2000 words or up to 10 pages that explains how you developed the cooling model/s and took into consideration the open cup, the material the cup was made from, the cooling time, the initial temperature of the coffee and the temperature of the surroundings.

- Design an experiment that investigates the differences in the time of cooling of a liquid in open cups made from different materials. Record your data in a table.
- Develop a procedural flowchart that shows the steps that you used to arrive at a solution for the problem.
- Justify your procedures and decisions by explaining mathematical reasoning.

- Provide a mathematical analysis of formulating and evaluating models using both mathematical manipulation and technology.
- Provide a mathematical analysis that involves differentiation (rate of change) and/or anti-differentiation (area under a curve) to satisfy the needs of each category of customers.
- Evaluate the reasonableness of solutions.

You may consider Newton's Law of Cooling which states that the rate of change of the temperature of an object is proportional to the difference between its own temperature and the temperature of its surroundings. For a body that has a higher temperature than its surroundings, Newton's Law of Cooling can model the rate at which the object is cooling in its surroundings through an exponential equation. This equation can be used to model any object cooling in its surroundings:

$$y = A_0 e^{kt}$$

Where:

- y is the difference between the temperature of the body and its surroundings after t minutes,
- A_0 is the difference between the initial temperature of the body and its surroundings,
- k is the cooling constant

Checkpoints

Week 1 - Students provide individual data from the experiment and create a procedural flowchart showing the proposed solution to the problem. Teacher provides individual feedback.

Week 2 - Students provide a consolidated group procedural flowchart. Teacher provides group feedback.

Week 3 - Students email a copy of their individually developed draft report for feedback.

Week 4 - Students submit individual final response in digital (PDF format) by emailing a copy to their teacher, providing a printed copy to their teacher and saving a copy in their Maths folder.

Additional requirements/Instructions

- The response must be presented using an appropriate mathematical genre (i.e., a mathematical report).
- The approach to problem-solving and mathematical modelling must be used.
- All sources must be referenced.

10.9 Data Analysis

The Phase Four interview with Ms Simon was transcribed and coded using the four phases of problem-solving identified from the literature review (Figure 10.1). An additional theme on the overarching benefits of procedural flowcharts in supporting problem solving was also used to include data that fell outside the stages of problem-solving. The details of the thematic analysis are provided in Chapter 3 and results are found under Appendix A, B and C.

The students' artefacts in Figures 10.2, 10.3 and 10.4 were analysed based on how they responded to the different stages of problem-solving synthesised from the literature (Figure 10.1) and the QCAA flowchart that guides problem-solving and mathematical modelling tasks (Appendix D). The artefacts were shared between the researcher and his supervisors, the analysis was done independently then reviewed by the researcher and his supervisors. Very little discrepancies were observed except that some stages on the students' procedural flowcharts overlapped between skills.

10.10 Results

This section presents results from the analysis of the interview data and student artefacts. The analysis of data also includes some observations that were made in Phase Three of the study.

10.10.1 Semi-structured Interviews

The thematic analysis of interviews resulted in two themes:

- The utility of procedural flowcharts in supporting mathematics problem-solving.
- The utility of procedural flowcharts in supporting the integration of the four stages of mathematics problem-solving.

In Phase Three, which prompted the targeted Phase Four study described in this chapter, teachers were asked the question, “How have you used procedural flowcharts to enhance teaching and learning of mathematics?” The question was not specific to problem-solving but the teachers’ observations and perceptions strongly related to problem-solving and student-centred learning.

10.10.1.1 Theme 1−The Utility of Procedural Flowcharts Generally Supports Mathematics Problem- Solving.

The visual nature of procedural flowcharts was seen as an advantage to both teachers and students. For students, drawing a flowchart was easier than writing paragraphs to explain how they had arrived at the intended solution. For teachers, the flowchart was easier to process for timely feedback to students.

> *They present steps in diagrammatic form which is easy to process and easy to understand and process… students prefer them more as its in diagrammatic form and I have witnessed more students engaging.* (Participant 8, Phase Three study)
>
> *I find it (visual) a really efficient way for me to look at the proposed individual students processes and provide relevant feedback to the student or for the student to consider. And, you know, once the students are comfortable with using these procedural flowcharts you know, I find it much easier for me to give them relevant feedback, and I actually find that feedback more worthwhile than feedback we used to give them, you know, that was just based on what they wrote in paragraphs,…students get to practice in creating their own visual display, which communicates their intended strategies to solve the problem, then they have opportunities to use it, and fine tune it as they work out the problem …*

> *student developed procedural flow charts, they represent a student's maths knowledge in a visual way.* (Ms Simon).

Identifying students' competencies early was seen as central to successful problem-solving as it provided opportunities for early intervention. Results showed that teachers viewed procedural flowcharts as a resource that could be used to identify gaps in skills, level of understanding and misconceptions that could affect successful and meaningful execution of a problem-solving task. Going through a student developed flowchart during problem solving provided the teachers with insight into the student's level of understanding of the problem and the effectiveness of the procedures proposed to address the problem.

> *I found it quite useful because I can identify what kids or which kids are competent in what, which sort of problem-solving skills. And I can identify misconceptions that students have or gaps in students understanding.* (Participant 1, Phase Three study)

> *It also to me highlights gaps in students' knowledge in unique ways that students intend to reach a solution because the use of the procedural flow chart encourages students to explain the steps or procedures behind any mathematical manipulation that you know they're intending to use. And it's something that was much more difficult to determine prior to using procedural flow charts… I've also used you know, student developed procedural flow charts to ascertain how narrow or wide the students' knowledge is and that's also something that wasn't obvious to make a judgement about prior to using procedural flow charts.* (Ms Simon)

Problem-solving was seen as student-centred. If procedural flowcharts could be used to support problem-solving, then they could facilitate an environment where students were the ones to do most of the work. The students could develop procedural flowcharts showing how they will solve a PSMT task using concepts and procedures they have learnt. The open-ended nature of the problem in a PSMT provide opportunities for diverse solutions that are validated through mathematical justifications. The visual nature of procedural flowcharts makes them more efficient to navigate compared to text.

> *Mathematics goes from being very dry and dusty to being something which is actually creative and interesting and evolving, starting to get kids actually engaging and having to back themselves.* (Participant 7, Phase Three study)

> *As a teacher, I find that procedural flowcharts are a really efficient way to ascertain the ways that students have considered and how they are going to solve a problem … It engages the students from start to finish, you know in different ways this method demands students to compare, interpret, analyse, reason, evaluate, and to an extent justify as they develop this solution.* (Ms Simon)

Similarly, results showed that procedural flowcharts could be used as a resource to promote collaborative learning and scaffolding. Students could be asked to collaboratively develop a procedural flowchart or could be provided with one to follow as they worked towards solving the problem.

> *Sometimes, you know, I get students to work on it in groups as they share ideas and get that mathematisation happening. So, it's really helpful there … I looked at the PSMT and its Marking Guide, and develop a more detailed procedural flowchart for students to use as a scaffold to guide them through the process. So, procedural flowcharts provide a structure in a more visual way for students to know what to do next.* (Ms Simon)

Ms Simon shared her detailed procedural flowchart in Figure 10.2 that she used to guide students in PSMTs.

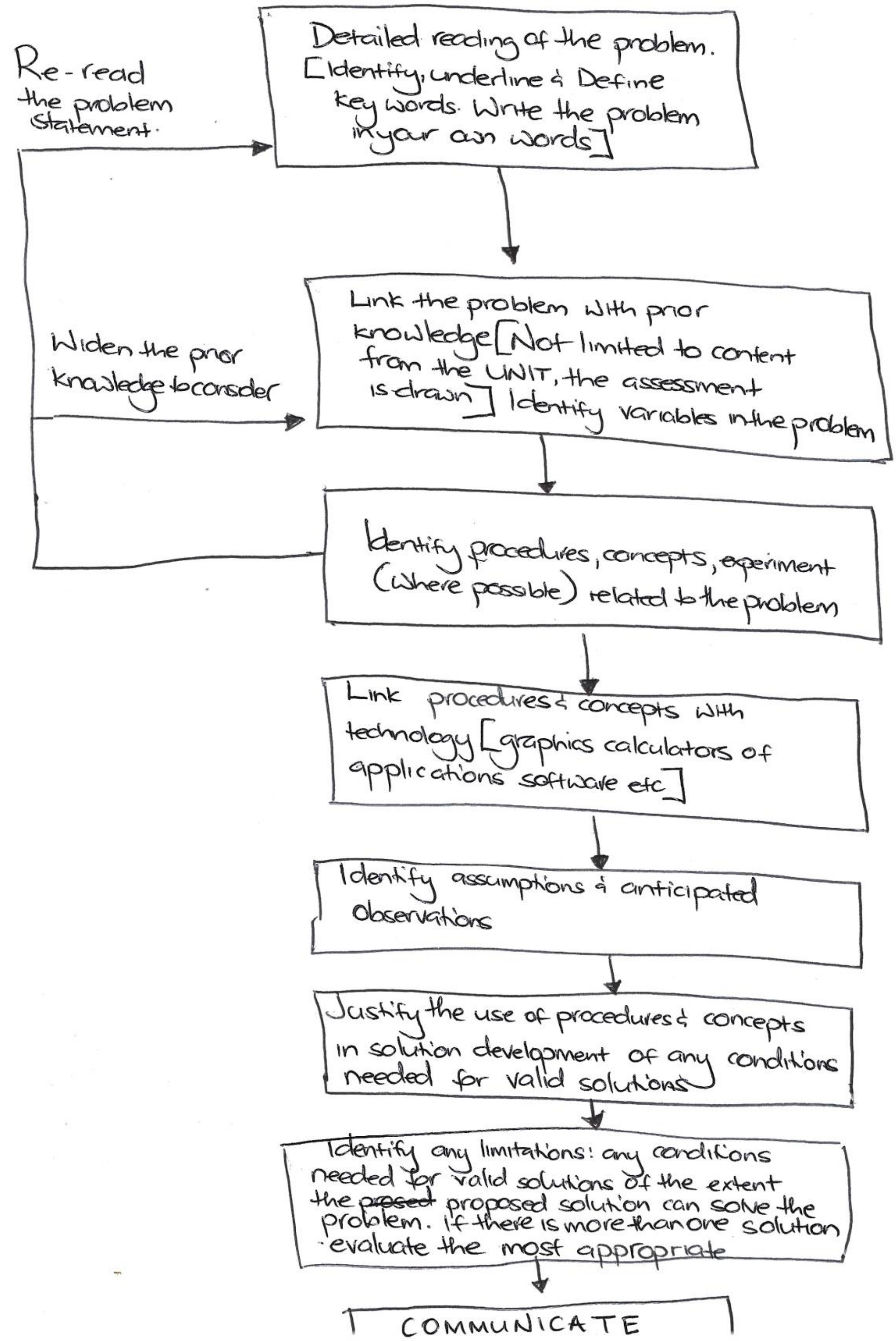

Figure 10.2: Ms Simon’s procedural flowchart on Problem-solving

The participants also observed that procedural flowcharts could be used to promote opportunities for solution evaluation which played an important role in problem-solving. The loops that can be introduced in procedural flowcharts can promote reflection and reasoning as alternative paths are developed or considered as the solution to the problem is being developed. Below are participants' comments referring to Figure 9.1 in Chapter 9 which was shared with participants in the video presentation detailed in Chapter 3.

> *The cycle approach, the feeding back in the feeding back out that type of stuff, y ou know, that is when we starting to teach students how to think*. (Participant 7, Phase Three study)

> *Complex procedural flowcharts like the one you provided guide students in making key decisions as they work through solutions which is key to critical thinking and judgement and these two are very important in maths.* (Participant 8, Phase Three study)

> *I also sincerely believe that procedural flowcharts are a way to get students to develop and demonstrate the critical thinking skills, which PSMTs are designed to assess. Students inadvertently have to use their critical thinking skills to analyse and reason as they search for different ways to obtain a solution to the problem presented in the PSMT … the use of procedural flowcharts naturally permits students to develop their critical thinking skills as it gets their brain into a problem-solving mode as they go through higher order thinking skills such as analysis, reasoning and synthesis and the like … this visual way of presenting solution provides students with opportunities to think differently, which they're not used to do, and it leads them to reflect and compare.* (Ms Simon)

Problem-solving of non-routine problems uses a structure that should be followed. Resources that are intended to support problem-solving in students can be used to support the integration of the stages involved in problem-solving.

10.10.1.2 Theme 2−The Utility of Procedural Flowcharts in Supporting the Integration of the Four Stages of Mathematics Problem-Solving.

Procedural flowcharts can support the flow of ideas and processes in the four stages during problem-solving and modelling task in Mathematical Methods subject. Literature synthesis in this chapter identified the four stages as:

- Identify problem.
- Interpret problem mathematically.
- Identify and execute mathematics strategies that can solve the problem.
- Justify, evaluate solution and communicate.

Similarly, QCAA flowchart on PSMT identifies the four stages as: formulate, solve, evaluate and verify, communicate.

The logical sequencing of the stages of mathematics problem-solving is crucial to solving and communicating the solution to the problem. Procedural flowcharts play an important role in problem-solving through fostering the logical sequencing of processes to reach a solution. Procedural flowcharts show the flow of ideas and processes which provide an overview of how different stages connect into a bigger framework of the solution.

> *Procedural flowcharts help students sum up and connect the pieces together… connect the bits of knowledge together.* (Participant 4, Phase Three study)
>
> *Really good how it organises the steps and explains where you need to go if you're at a certain part in a procedure.* (Participant 2, Phase Three study)
>
> *Potentially, it's also an excellent visual presentation, which shows a student's draft of their logical sequence of processes that they're intending to develop to solve the problem … So, the steps students need to follow actually flows logically. So really given a real-life scenario they need to solve in a PSMT students need to mathematise it and turn it into a math plan, where they execute their process, evaluate and verify it and then conclude … so we use procedural flowcharts to reinforce the structure of how to approach problem-solving … kids, you know, they really struggling, you know, presenting things in a logical way, because they presume that we know what they're thinking*. (Ms Simon)

Procedural flowcharts provided students with opportunities to plan the solution informed by the stages of problem-solving. Teachers could reinforce the structure of problem-solving by telling students what they could expect to be included on the procedural flowchart. Procedural flowcharts can be used as a visual tool to all the critical stages that are included during the planning of the solution.

> *I tell the students, "I need to see how you have interpreted the problem that you need to solve. I need to see how you formulated your model that involves the process of mathematisation, where you move from the real world into the maths world, and I need to see all the different skills you're intending to use to arrive at your solution."* (Ms Simon)

Similarly, procedural flowcharts could visually represent more than one strategy in the "identify and execute mathematics strategies that can solve the problem" stage, thereby providing a critical resource to demonstrate flexibility. When there are multiple ways of addressing a problem, a procedural flowchart can show all possible paths or the relationship between different paths to the solution, thus promoting flexibility.

> *Students are expected to show evidence that they have the knowledge of solving the problem using several ways to get to the same solution. So, it goes beyond the students' preferred way of answering a question and actually highlights the importance of flexibility when it comes to processes and strategies of solving a problem … By using procedural flowcharts, I'm saying to the students, "Apart from your preferred way of solving the problem, give me a map of other routes, you can also use to get to your destination."* (Ms Simon)

The results also indicated that procedural flowcharts could be used to identify strengths and limitations of strategies in the "evaluate solution" stage and thus demonstrate the reasonableness of the answer. Having more than one way of solving a problem on a procedural flowchart helps in comparing and evaluating the most ideal way to address the problem.

> *And I'm finding that, you know, as students go through, and they compare the different processes, you know, the strengths and limitations, literally stare them in the face. So, they don't have to. They're not ... they don't struggle as much as they used to in coming up with those sorts of answers … it's also a really easy way that once the students reach the next phase, which is the evaluating verified stage, they can go back to their procedural flow chart and identify and explain strengths and limitations of their model ... It's a convenient way for students to show their reasonableness of their solution by comparing strengths and weaknesses of all the strategies presented on the procedural flowchart, something that they've struggled with in the past.* (Ms Simon)

The results from the interview show that the procedural flowcharts supported efficient communication of the steps to be followed in developing the solution to the problem. Student developed procedural flowcharts allowed the teacher to have an insight and overview of the solution to the problem earlier in the assessment task. In addition, they provided an alternative way of presenting their solution to the teacher.

> *I expect students to use the procedural flowchart as a way to communicate to me how they're planning to solve the scenario in the PSMT…It's also one of the parts that students are expected to hand in to me on one of the check points, and I find it a really efficient way for me to look at, you know, a proposed individual students processes, and provide relevant feedback to the student to consider in a really efficient way…I just found that it helps students communicate their solution to a problem in lots of different ways that challenges students to logically present a solution.* (Ms Simon)

She went on to say,

> *Students also found it challenging to communicate their ideas in one or two paragraphs, when more than one process or step was required to solve the problem. So, I found that, you know, procedural flowcharts, have filled this gap really nicely, as that provides students with a simple tool that they can use to present a visual overview of the processes they've chosen to use to solve the problem. And so, for me, as a teacher, procedural flowcharts are an efficient way for me to scan the intended processes that an individual student is*

proposing to use to solve the problem in their authentic way and provide them with valuable feedback.

In summary, the teacher's experiences, views and perceptions showed that procedural flowcharts can be a valuable resource in supporting students in all four stages of problem-solving.

Students' Artefacts

The student-generated flowcharts in this part of the research gave an insight into students' thinking as they planned how to solve the problem presented to them. Students were expected to use their metacognitive skills to successfully develop solutions to problems. Their de-identified procedural flowcharts are shown in Figures 10.2 and 10.3.

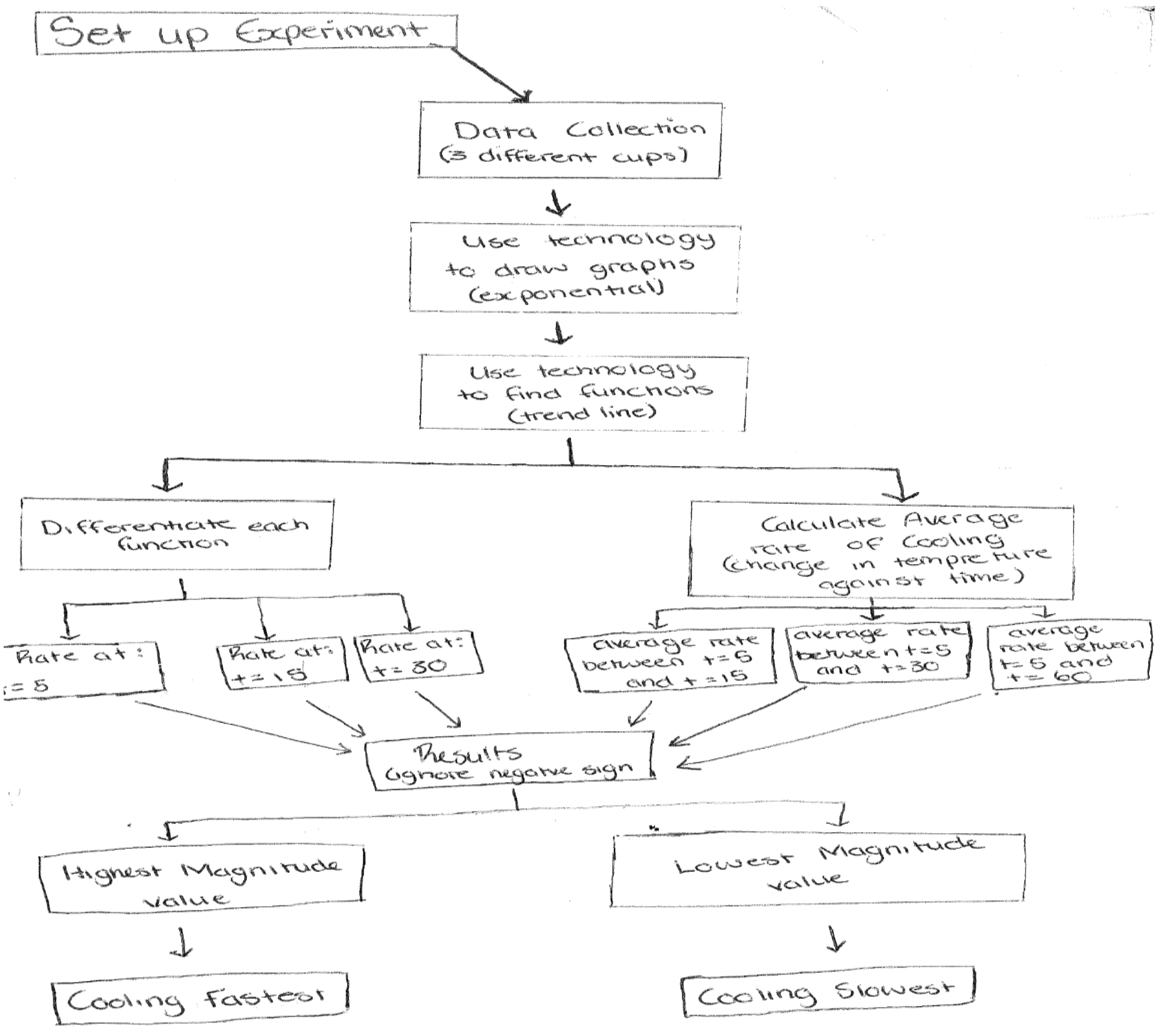

Figure 10.3: Procedural Flow Chart Developed by Student 1

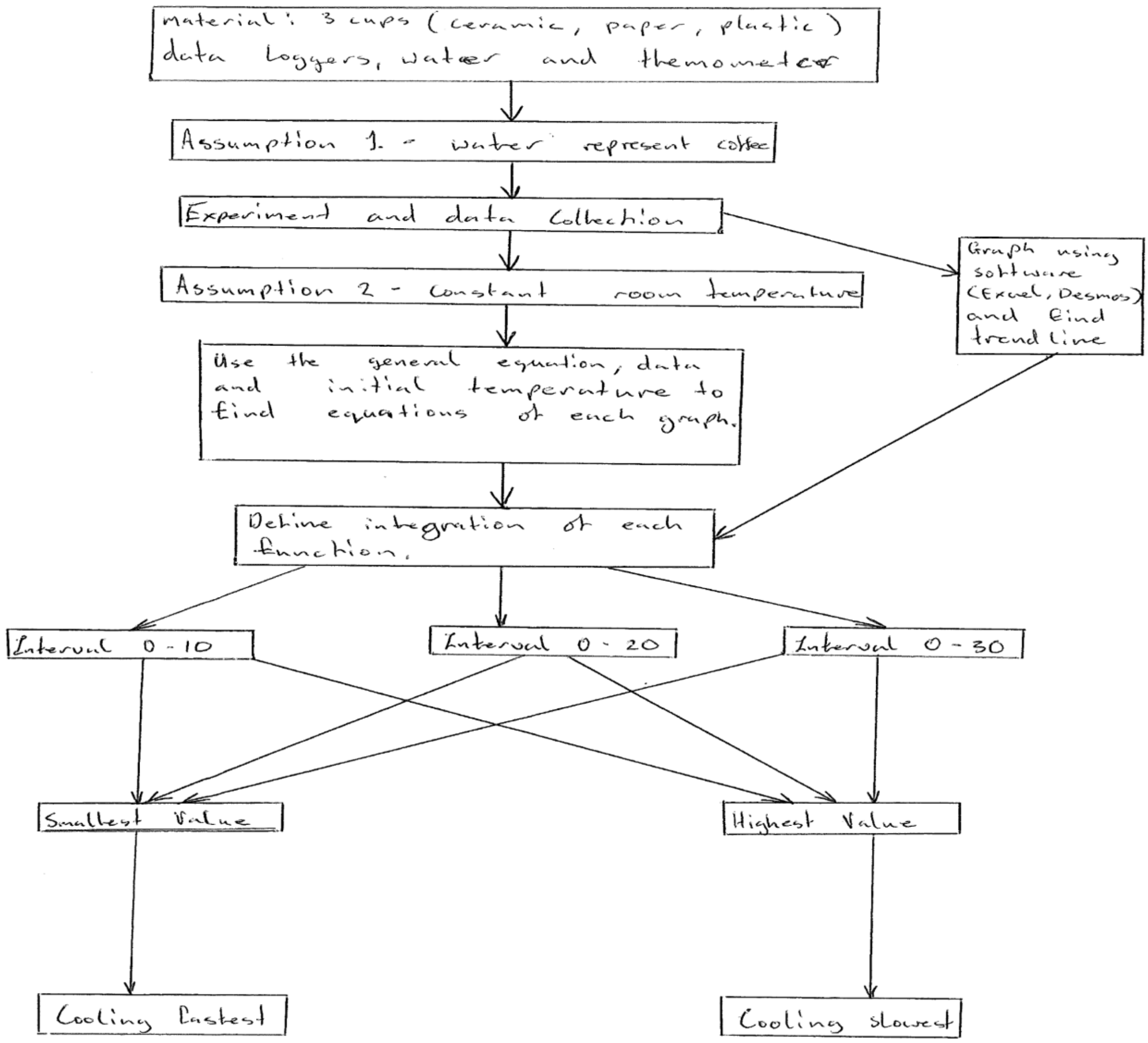

Figure 10.4: Procedural Flowchart Developed by Student 2

Students 1 and 2 also collaboratively developed a procedural flowchart, shown as Figure 10.4.

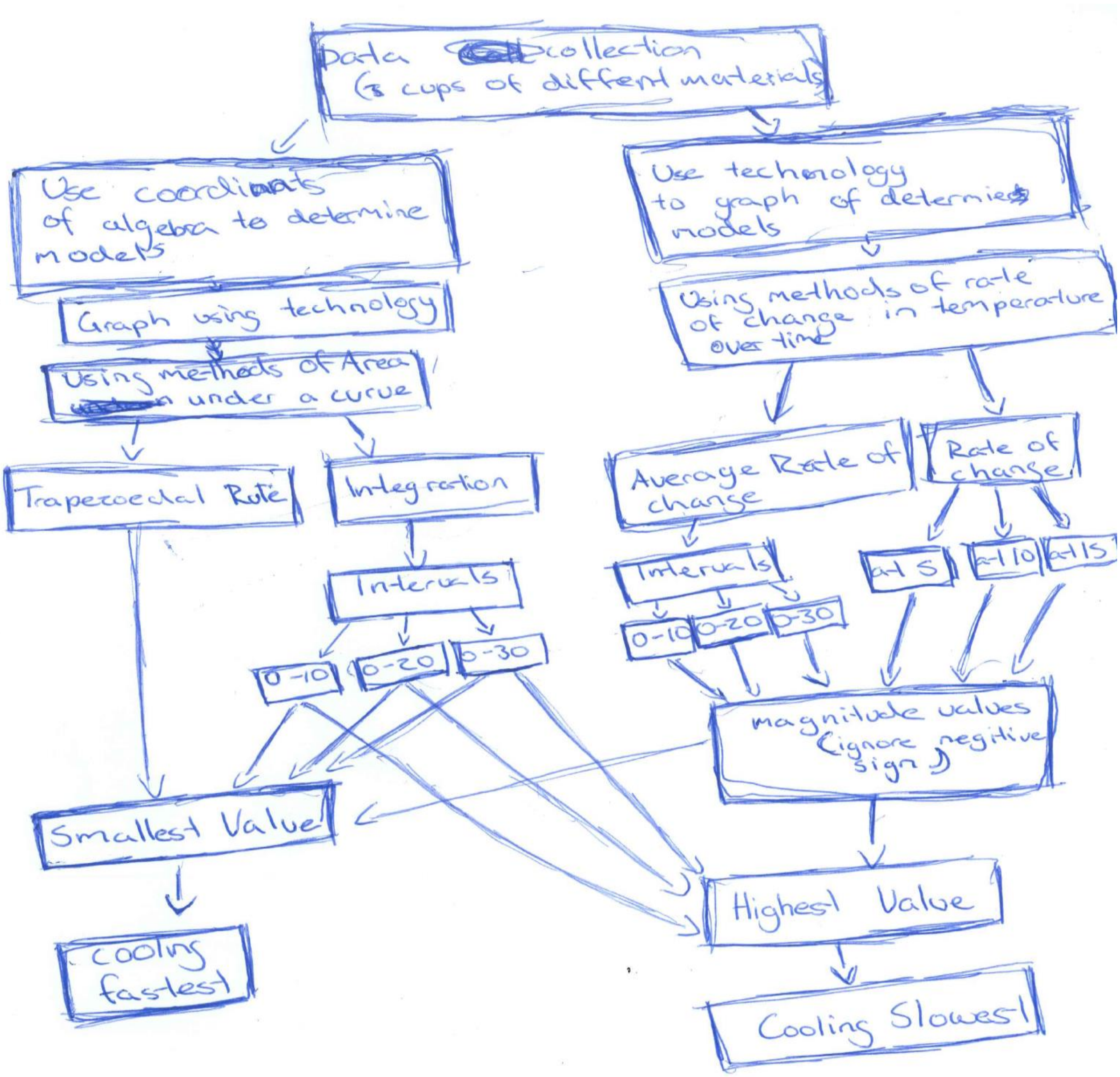

Figure 10.5: Collaboratively-Developed Procedural Flowchart

Table 10.1: Analysis of Students' Procedural Flowcharts on Problem-Solving Stages

Problem-solving stages	Formulate	Solve	Evaluate and verify	Communicate
What is involved?	Involves problem identification and interpreting the problem mathematically. At this stage, task analysis and planning is key.	Involves application of mathematics strategies in solving the problem.	Evaluate if findings can address the problem. This is done by referring the solution (s) to the problem that need to be solved. Sometimes adjustments should be made to better respond to the problem.	Sharing the problem solution.
Student 1	Differentiation -rate of change of temperature against time. **How to collect data**- experiment. **How to formulate models**- technology **How to apply concept of rate of change**-Rate of change at a point. Average rate of change (intervals).	Rate at 5, 15 and 30 minutes for each cup. Average rate at 5-15, 5-30, 5-60 minutes for each cup. Taking note, it's a decreasing graph (cooling).	Checking if rate at a point can solve the problem. Checking if average rate of change of an interval can solve the problem. Checking if the strategies converge to the same conclusion. If not, then which one is the best (help determining strengths and limitations)?	Flowchart shows a reasonable development of steps to solving the problem that includes how mathematics concepts will be used and how different solutions will inform key results and justify the solution. Final report.
Student 2	Integration- area under a curve. **How to collect data**- experiment. **How to formulate models**-algebra and technology **How to apply concept of integration**- definite integration.	Definite integration intervals 0-10, 0-20,0-30 minutes for each cup. Area size determines cooling rate.	Check if definite integration addresses the problem.	Flowchart shows a reasonable development of steps to solving the problem that includes how mathematics concepts will be used and how different

				solutions will inform key results and justify the solution. Final report.
Students 1 and 2 (collaboration)	Differentiation (rate of change of temperature against time) and Integration (area under a curve). **How to collect data**- experiment. **How to formulate models**- technology and algebra **How to apply concept of rate of change**-Rate of change at a point. Average rate of change (intervals). **How to apply concept of integration (area under a curve)**- trapezoidal rule and definite integration.	-Rate at 5, 10 and 15 minutes for each cup. Average rate at 0-10, 0-20, 0-30 minutes for each cup. Taking note, it's a decreasing graph (cooling). -Trapezoidal rule applied between 0-30 minutes. -Definite integration intervals 0-10, 0-20, 0-30 minutes for each cup. Area size determine cooling rate.	Checking if rate at a point can solve the problem. Checking if average rate of change of an interval can solve the problem. Checking if trapezoidal rule can solve the problem. Checking if definite integration can resolve the problem. Checking if the strategies converge to the same conclusion. If not then which one is the best (help determining strengths and limitations).	Flowchart show a reasonable development of steps to solving the problem that include how mathematics concepts will be used and how different solutions will inform key results and justify the solution. Final report.

The procedural flowchart in Figure 10.6 was extracted from a PSMT report (see Appendix E) provided by one of Ms Simon's students. The two artefacts provide a complete example of how procedural flowcharts have been used to support problem solving at senior secondary by Ms Simon.

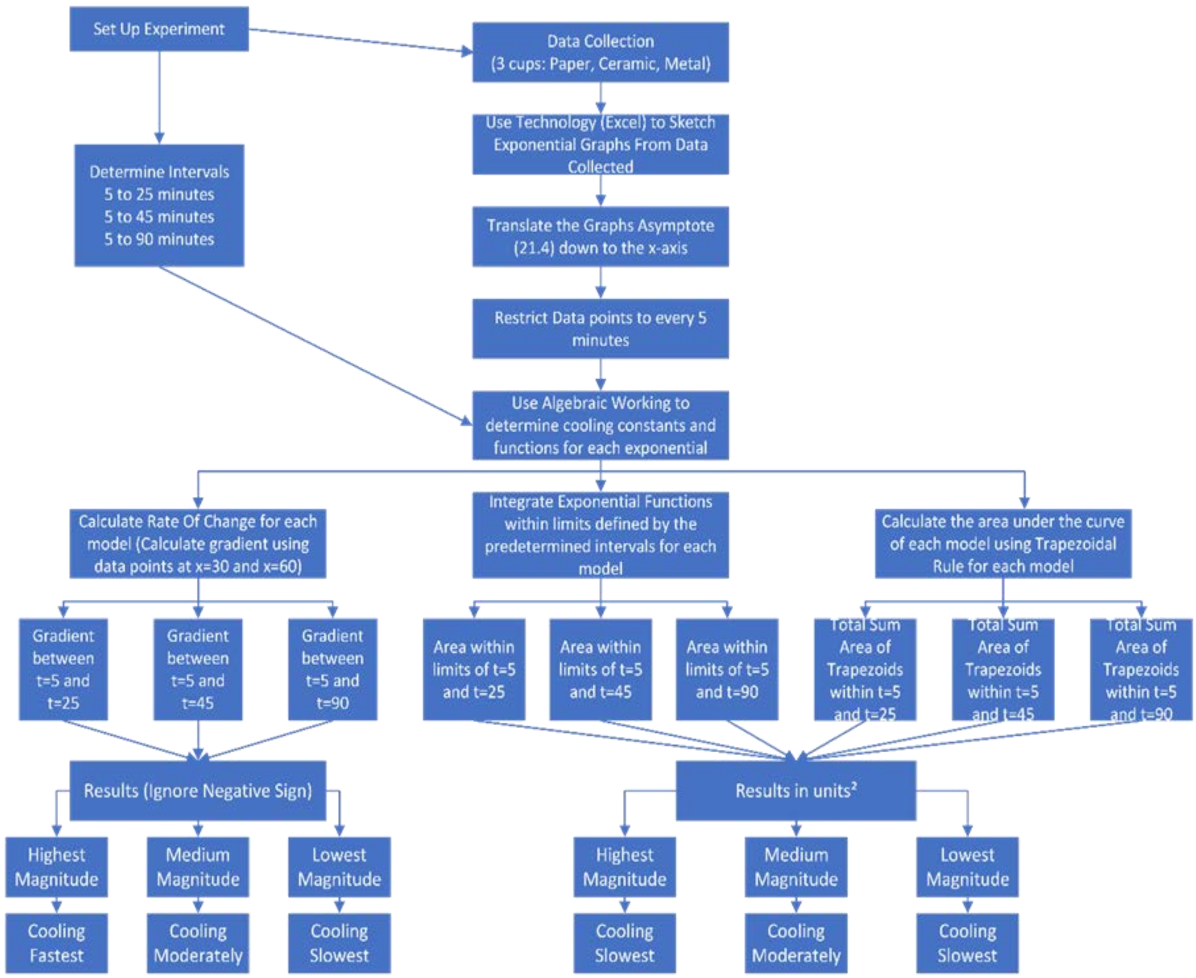

Figure 10.6: Procedural flowchart extracted from a student's PSMT.

The procedural flowchart does not show that the student's calculation of cooling constant was part of evaluating the solution, although the cooling constant was included in the final report as one of the procedures used in reaching the optimal solution. The procedural flowchart may have provided the teacher with the opportunity to remind the student of the significance of the cooling constant. Development of the procedural flowchart provided the student with an opportunity to hypothesise the proposed solution and define how mathematics procedures would contribute to developing and evaluating the solutions. It provided an overview of the solution. The report was developed guided by the procedural flowchart as it includes all the procedures shown in Figure 10.6. However, it went further to provide the different mathematical justification and application of technology in the solution development. Importantly, the procedural flowchart signposted how each procedure will help evaluate and develop the conclusion by providing what the student considered was key to identifying the best solution. In this task, it was the ranking of different cup materials according to customer type.

10.11 Discussion

This discussion is presented as three sections: (1) How procedural flowcharts can support mathematics problem-solving and (2) How procedural flowcharts support the integration of the different stages of mathematics problem-solving. This study highlighted how procedural flowcharts can support mathematics problem-solving, can reinforce the structure of the solution to a problem and can help develop metacognitive skills among students. The different stages involved in problem-solving inform the process of developing the solution to the problem. The focus on problem-based learning has signified the need to introduce resources that can support students and teachers in developing and structuring solutions to problems. Results from this study have also provided discussion points on how procedural flowcharts can have a positive impact in mathematics problem-solving.

10.11.1 Procedural Flowcharts can Support Mathematics Problem-Solving

Procedural flowcharts help in visualising the process of problem-solving. The results described in this chapter show that student-generated flowcharts can provide an overview of the proposed solution to the problem. The study noted that students preferred developing procedural flowcharts rather than writing how they planned to find a solution to the problem. The teachers also preferred visual aids because they were easier and quicker to process and facilitated understanding of the steps taken to reach the solution. These results are consistent with the findings of other researchers (Raiyn, 2016; McGowan & Boscia, 2016). The results are also consistent with Grosskinsky and colleagues' (2019) findings that flowcharts break complex information into different tasks and show how they are connected, thereby enhancing understanding of the process. Consequently, they allow teachers to provide timely feedback at a checkpoint compared to the time a teacher would take to go through a written draft. Procedural flowcharts connect procedures and processes in a solution to the problem (Chinofunga et al., 2022c). Thus, the feedback provided by the teacher can be more targeted to a particular stage identified on the procedural flowchart, making the feedback more effective and worthwhile. The development of a procedural flowchart during problem-solving can be viewed as a visual representation of students' plan and understanding of how they plan to solve the problem as demonstrated in Figures 10.3, 10.4, 10.5 and 10.6.

In this study, Ms Simon noted that procedural flowcharts represented students' knowledge or thinking in a visual form, which is consistent with Owens and Clements' (1998) findings that visuals are cognitive constructs. Consequently, they can facilitate evaluation of such knowledge. This study noted that procedural flowcharts can provide opportunities to identify gaps in students' understanding and problem-solving skills. It also noted that the use of procedural flowcharts may expose students' misconceptions, the depth and breadth of their understanding of the problem and how they plan to solve the problem. This is supported by significant research (Grosskinsky et al., 2019; Norton et al., 2007; Vale & Barbosa, 2018), which identified flowcharts as a resource in helping visualise and recognise students' understanding of a problem and communication of the solution. Thus, providing teachers with opportunities to have an insight into students' thinking can facilitate intervention early in the process. The results in this study showed that when students develop their own plan on how to respond to a problem, they are at the centre of their learning. However, scaffolding and collaborative learning can also support problem-solving.

Vygotsky (1978) posited that in the Zone of Proximal Development, collaborative learning and scaffolding can facilitate understanding. In this study, the results indicated that a teacher-developed procedural flowchart can be used to guide students in developing a solution to a problem. These results are consistent with Davidowitz and Rollnick's study that concluded that flowcharts provide a bigger picture of how to solve the problem. In Queensland, the QCAA has developed a flowchart (see Appendix A) to guide schools on problem-solving and modelling tasks. It highlights the significant stages to be considered during the process and how they relate to each other. Teachers are encouraged to contextualise official documents to suit their school and classes. In such cases, a procedural flowchart acts as a scaffolding resource in directing students on how to develop the solution to the problem. The findings are consistent with previous literature that flowcharts can give an overall direction of the process, help explain what is involved, may help reduce cognitive load and allow students to focus on complex tasks (Davidowitz & Rollnick, 2001; Norton et al., 2007; Sweller et al., 2019).

In addition to being a scaffolding resource, results showed that procedural flowcharts can be developed collaboratively providing students with an opportunity to share their solution to the problem. Being a scaffolding resource or a resource to use in a community of learning highlights the importance of procedural flowcharts in promoting learning within a zone of proximal development, as posited by Davidowitz & Rollnick (2001). Scaffolding students to problem solve and develop procedural flowcharts collaboratively provides students with the opportunity to be at the centre of problem-solving.

Research has identified problem-solving as student-centred learning (Ahmad et al., 2010; Karp & Wasserman, 2015; Reinholz, 2020; Vale & Barbosa, 2018). The process of developing the procedural flowcharts as students plan for the solution provide students with opportunities to engage more with the problem. Results showed that when students developed procedural flowcharts themselves, mathematics learning transformed from students just being told what to do or follow procedures into something creative and interesting. As students develop procedural flowcharts, they use concepts they have learnt to develop a solution to an unfamiliar problem (Matty, 2016), thus engaging with learning from the beginning of the process until they finalise the solution. The results indicated that procedural flowcharts promoted students' ability to not only identify strategies to solve the problem but also determine how and when the conditions were ideal to address the problem, providing opportunities to justify and evaluate the strategies that were used.

Deeper understanding of mathematics and relationships between concepts plays an important role in problem-solving and the results from this study showed that different procedures can be integrated to develop a solution to a problem. The participants observed that procedural flowcharts could support the brainstorming ideas as they developed the flowchart, as ideas may interlink in a non-linear way. Moreover, students are expected at different stages to make key decisions about the direction they will need to take to reach the solution to the problem, as more than one strategy may be available. For example, Student 1 planned on using only technology to develop the models while Student 2 considered both technology and algebra. This showed that Student 2 applied flexibility in using alternative methods, thus demonstrating a deeper understanding of the problem. Equally important, Ms. Simon observed that as students developed their procedural flowcharts while planning the steps to reach a solution, they were required to analyse, conceptualise, reason, analyse, synthesise and evaluate, which are important attributes of deeper understanding. Fostering deeper understanding of mathematics is the key goal of using problem solving (Kim et al., 2012;

King, 1995; Moon, 2008; QCAA, 2018). The results are additionally consistent with findings from Owens and Clements (1998) and Roam (2019), who posited that visual aids foster reasoning and show cognitive constructs. Similarly, logical sequencing of procedures and ways to execute a strategy expected in a procedural flowchart can support deeper understanding, as posited by Parvaneh and Duncan (2021). In a procedural flowchart, students are required to link ideas that are related or feed into another, creating a web of knowledge. Students are also required to identify the ways in which a concept is applied as they develop a solution, and this requires deeper understanding of mathematics. Working collaboratively can also support deeper and broader understanding of mathematics.

The procedural flowchart that was developed collaboratively by the two students demonstrated some of the skills that they did not demonstrate in their individual procedural flowcharts. Like Student 2, the collaboratively-developed flowchart included use of technology and algebra to determine the models for the three different cups. The students considered both rate of change and area under a curve in the task analysis. Apart from planning to use rate at a point, average rate and definite integration, they added the trapezoidal rule. Both average rate and definite integration were to be applied within the same intervals, building the scope for comparison. The trapezoidal rule would also compare with integration. The complexity of the collaboratively-developed procedural flowchart concurred with Rogoff and others (1984) and Stone (1998), who suggested that a community of learning can expand current skills to higher levels than individuals could achieve on their own. It seems the students used the feedback provided by the teacher on their individually-developed procedural flowcharts as scaffolding to develop a much more complex procedural flowchart with competing strategies. Their individually-developed flowcharts might have acted as reference points, as their initial plans were still included in the collaboratively-developed plan but with better clarity. This observation is consistent with Guk and Kellogg (2017), Kirova and Jamison (2018) and Ouyang and colleagues (2022), who noted that scaffolding involving peers, teacher and other resources enhances complex problem-solving tasks and transfer of skills.

10.11.2 Supporting the Integration of the Different Stages of Mathematics Problem-Solving.

Procedural flowcharts support the logical sequencing of ideas from different stages into a process that ends with a solution. Problem-solving follows a proposed order and procedural flowcharts visually display decision and/or action sequences in a logical order (Krohn, 1983). They are used when a sequenced order of ideas is emphasised, such as in problem-solving (Cantatore & Stevens, 2016). This study concurs with Krohn, Cantatore and Stevens, as the results showed that procedural flowcharts could be used to organise steps and ideas logically as students worked towards developing a solution. Students' procedural flowcharts are expected to be developed through the following stages: problem identification, problem mathematisation, planning and execution and finally evaluation. Such a structure can be reinforced by teachers by sharing a generic problem-solving flowchart outlining the stages so that students can then develop a problem-specific version. Importantly, students' artefacts in Figures 10.3 to 10.6 provided evidence of how procedural flowcharts support the different stages of problem-solving stages to create a logical and sequential flow of the solution (see Appendix A). Similarly, Ms Simon noted that while her students had previously had problems in presenting the steps to their solution in a logical way, she witnessed a significant improvement after she introduced procedural flowcharts. Further, the results are consistent with Chinofunga et al.'s (2022c) work that procedural flowcharts can support procedural flexibility, as they can accommodate more than one strategy in the "identify and execute mathematics strategies that can solve the problem" stage. Thus, stages that require one procedure or more than one procedure can all be accommodated in a single procedural flowchart. Evaluating the different strategies is also a key stage in problem-solving.

As students develop the solution to the problem and identify strategies that can address the problem, they also have to evaluate the strategies, reflecting on the limitations and strengths of the solutions they offer. Ms Simon observed that her students had previously struggled with identifying strengths and weaknesses of different strategies. However, she noted that the introduction of procedural flowcharts gave students the opportunity to reflect and compare as they planned the solution. For example, students could have the opportunity to reflect and compare rate at a point, average rate and integration so they can evaluate which strategy can best address the problem. The artefacts identified the different strategies the students used in planning the solution, enabling them to evaluate the effectiveness of each strategy. Thus, enhancing students' capacity to make decisions and identify the optimal strategy to solve a problem aligns with the work of McGowan and Boscia (2016). Similarly, Chinofunga and colleagues' findings noted that procedural flowcharts can be effective in evaluating different procedures as they can accommodate several procedures. The different stages that need to be followed during problem-solving and the way the solution to the problem is logically presented are central to how the final product is communicated.

In this study, procedural flowcharts were used to communicate the plan to reach the solution to a problem. The length of time given to students to work on their problem-solving tasks in Queensland is fairly long (four weeks) and students may struggle to remember some keys processes along the way. Developing procedural flowcharts to gain an overview of the solution to the problem and share it with the teacher at one of the early checkpoints is of significant importance. In this study, Ms Simon expected her students to share their procedural flowcharts early in the process for her to give feedback, thus making the flowcharts a communication tool. The procedural flowcharts developed by the students in Figures 10.3 to10.5 show how students proposed to solve the problem. This result lends further support to the NCTM (2000) findings that visual aids can help students communicate their thinking before applying those thoughts to solving a problem. Ms Simon also noted that before the introduction of procedural flowcharts, students did not have an overall coherent structure to follow, which presented challenges when they wanted to communicate a plan that involved more than one strategy. However, the students' artefacts were meaningful, clearly articulating how the solution to the problem was being developed, thus demonstrating that flowcharts can provide the structure that supports the coherent and logical communication of the solution to the problem by both teachers and students (Norton et al., 2007). The visual nature of the students' responses in the form of procedural flowcharts is key to communicating the proposed solution to the problem.

Visual representations are a favourable alternative to narrative communication. Procedural flowcharts can help teachers to check students' work faster and provide critical feedback in a timely manner. Ms Simon noted that the use of procedural flowcharts provided her with the opportunity to provide feedback faster and more effectively earlier in the task because the charts provided her with an overview of the whole proposed solution. Considering that students are expected to write a report of 2000 words or 10 pages on the task, the procedural flowchart provides the opportunity to present large amounts of information in just one visual representation. Ryan (2016) noted that visual representations can be a quicker way to evaluate a solution and represent large amounts of information.

10.12 Chapter Conclusion

Procedural flowcharts have demonstrated the ability to support problem-solving. Including procedural flowcharts in problem-solving may support teachers and students in communicating efficiently about how to solve the problem. For students, it is a plan that provides the solution overview, while teachers will consider it as a mental representation of students' thinking as they plan the steps to reach a solution. Procedural flowcharts may represent how a student visualises a solution to a problem after brainstorming different pathways and different decision-making stages.

Moreover, the visual nature of procedural flowcharts may make it easy to process and provide timely feedback that in turn might help students engage with the problem meaningfully. Procedural flowcharts may also provide a structure of the problem-solving process and guide students through the problem-solving process. Navigating through stages of problem-solving might be supported by having students design procedural flowcharts first and then execute the plan. The ability of procedural flowcharts to represent multiple procedures, evaluation stages or loops and alternative paths helps students reflect and think about how to present a logically-cohesive solution. Procedural flowcharts have also been identified as a resource that can help students communicate the solution to the problem. They have been noted to support deeper understanding as they may facilitate analysis, logical sequencing, reflection, reasoning, evaluation and communication. Although the in-depth study involved one teacher and three artefacts from her students, it identified the numerous advantages that procedural flowcharts bring to mathematics learning and teaching, particularly in terms of supporting the development of problem-solving skills. The study calls for further investigation on how procedural flowcharts can support students' problem-solving. The next chapter provides an integrated discussion that weaves together the findings from the previous chapters.

Chapter 11: The state of calculus-based mathematics in Queensland and the teaching of mathematics.

11.1 Chapter Introduction

The general discussion chapter in a thesis by publication provides an opportunity to further analyse, reflect and integrate the thesis findings and highlight their implications (Smith, 2015). Following Lewis and colleagues' work on how to write an integrated discussion in an article-based thesis, this chapter weaves together the findings across individual articles, interpreting them, presenting arguments and then giving explanations in relation to existing knowledge (Lewis et al., 2021). In this chapter, I demonstrate the cohesion of the results to supports the teaching of calculus-based mathematics and the discussion points across the thesis, then showcase the new knowledge developed and its contribution to the field (Grant, 2011; Merga, 2015).

This chapter responds to the overarching research questions that informed this research project:

1. What are the trends in Queensland senior students' enrolment in calculus-based mathematics subjects?
2. What pedagogical resources support the planning and teaching of Mathematical Methods for Queensland senior students?

Senior secondary students in Queensland who want to study mathematics can choose either calculus-based or non-calculus-based subjects in mathematics. Research question 1 addressed the trends in students' enrolment in calculus-based subjects from 2010 to 2020. To gain a deeper understanding of the trends and the distribution across the state, the study investigated trends at education district level, incorporating socio-economic status and teacher turnover. As indicated in previous chapters, the trends gave an insight into the declining student enrolment in calculus-based mathematics, the issue that research Question 2 went on to address.

The study developed a planning framework for senior mathematics teachers to use in content and lesson sequencing. Planning is a critical component of teaching; the more effective the planning, the greater the chances of improved teaching. The planning framework in this research took into consideration the interconnection of mathematics content and the hierarchical nature of the subject. The study also focuses on supporting student participation in mathematics through the use of visual representations (concept maps and procedural flowcharts). Similarly, supporting the development of important 21st century skills such as problem solving is vital in teaching and the study provided evidence that procedural flowcharts can support problem-solving in mathematics. The study progressed from understanding trends in student participation in calculus-based mathematics to introducing resources that could support that participation.

The overall purpose of the study was to provide insight into the trends in students' enrolment in calculus-based mathematics in Queensland and then to develop resources that teachers can use to support the teaching and learning of this subject. The quantitative trends analysis in Chapter 4 was conducted to investigate students' enrolment in senior mathematics subjects from 2010 to 2019 under the Queensland Senior Certificate, which was phased out in 2019. Trends showed a high dropout rate in calculus-based options, especially in Mathematical Methods, and a decline then stagnation in students' enrolment in these subjects. Trends also showed a steady increase in students choosing to study non-calculus mathematics subjects. Chapter 5 was also a quantitative study which went further by analysing trends in calculus-based subjects in the new curriculum, which was introduced in 2019. To gain a broader understanding, trends were analysed per education district, taking into consideration socio-economic factors and teacher turnover. The findings showed that districts that were socially advantaged had the highest number of students who selected calculus-based options and the lowest dropout rate. This was in contrast to economically disadvantaged districts where student dropout increased by 45% higher than before the introduction of the new syllabus. The high dropout rate in calculus-based subjects, especially Mathematical Methods, confirmed the problem that this study was addressing through developing pedagogical resources that can help teachers in the teaching of Mathematical Methods.

In Chapter 6, a framework on mathematics content sequencing was developed to guide teachers on how to link junior to senior mathematics. The framework could be used by teachers to help students draw from prior knowledge in their planning at senior secondary level, fostering the developing of new knowledge from prior knowledge. Feedback from the teachers on the framework on content sequencing was included in Chapter 7. The findings from this mixed-methods study indicated that senior secondary mathematics teachers found the framework useful for the logical sequencing of content during their planning.

Chapter 8 was another mixed-methods study that introduced concept maps as a resource for linking prior concepts to new concepts. In this study, teachers viewed concept maps as a resource that can support teaching and learning at senior secondary through developing students' conceptual understanding and as a visual representation of the hierarchical nature of mathematics concepts. Similarly, Chapter 9 was a mixed-methods study that developed procedural flowcharts as a resource to support teaching and learning of mathematics. The resulting perceptions of senior secondary mathematics teachers were that procedural flowcharts can support mathematics procedural fluency. Chapter 10 expanded the use of procedural flowcharts to problem-solving. This qualitative study involved an in-depth interview with a senior mathematics teacher and an analysis of students' artefacts. The findings showed that the use of procedural flowcharts in open-ended problem-solving exercises can support problem solving.

11.2 Trends in Student Enrolment in Senior Mathematics in Queensland

Where there are choices, trends develop based on how individuals choose available options. Trends in senior secondary calculus-based subjects have been a focus of researchers across the western world for a long time (Hodgen et al., 2010b; 2013; Kennedy et al., 2014; Noyes & Adkins, 2016). The importance of calculus-based subjects as preferred prerequisites for STEM courses at tertiary level and drivers of jobs of the future that include data analysis and innovation (Black et al., 2021; Carnevale et al., 2011; Lemaire, 2003; Lyakhova & Neate, 2019; PwC, 2013) has been a focus of most governments (Peters et al., 2017) for some time. In this study, trends were analysed to get a better understanding of student enrolment in calculus-based mathematics.

In the Queensland curriculum from 2010 that was phased out in 2019, students needed to choose from Mathematics A, B, C and Prevocational Mathematics as they entered senior secondary. Mathematics A and Prevocational were non-calculus-based subjects, while Mathematics B and C were calculus-based. However nationally, Mathematics B was classified as intermediate and Mathematics C as advanced and an average of 30% of all mathematics students chose Mathematics B while only 8% chose Mathematics C. As a result, Queensland, the Australian state chosen as the focus for this research, was lagging behind countries like Japan, Singapore and South Korea, with only 30% of senior students studying Advanced Mathematics which is equivalent to Mathematics C in Queensland (Hodgen, 2010). However, under the phased-out curriculum, Queensland was in a better position than some states in Australia, which averaged only 27% enrolment in intermediate level mathematics. The general trend across Australia was that senior high school student participation in intermediate and Advanced Mathematics had been in decline for more than a decade (Kennedy et al., 2014). The sharpest decline in calculus-based subjects in Queensland was witnessed when the new curriculum was introduced, and the national average plummeted by 10% (AMSI, 2022). Australia, and Queensland in particular, have to respond to the decline in calculus-based subjects so that they remain competitive and prepare for the future as both calculus-based options are preferred pre-requisites for tertiary STEM courses. This study expanded the trends analysis to include dropout rates, schools' socio-economic and educational status, enrolment per educational district and schools offering or not offering calculus-based subjects. This provided a better insight into the trends in student enrolment in calculus-based mathematics in Queensland.

11.3 Student Dropout from Calculus-based Mathematics Subjects

Trends analysis of students' participation in the phased-out curriculum in Queensland showed that more students chose calculus-based subjects in Year 11 than in Year 12. This showed a substantial drop in student participation in calculus-based mathematics as students transitioned from Year 11 to 12. The dropout rate was higher in Mathematics B than Mathematics C. From 2010 to 2019, the average dropout rate was 3.76% in Mathematics B and 2.25 % in Mathematics C. In terms of raw numbers every year, 688 students dropped out of Mathematics B and 108 in Mathematics C across Queensland from 2010 to 2019.

When the new Queensland Certificate of Education curriculum was introduced in 2019, Mathematical Methods replaced Mathematics B and Specialist Mathematics replaced Mathematics C. The new curriculum also involved an external examination at the end of the year that constituted 50 % of the overall mark. At Year 11 in 2019, 7207 students in Queensland state schools chose to study Mathematical Methods but only 4495 were still enrolled for Unit 4 in 2020, a 37.63% dropout rate. Similarly, participation statistics in Specialist Mathematics in state schools dropped from 1961 in 2019 to 1465 in 2020, representing a dropout rate of 25.29%.

Students choose subjects as they transition from junior level at Year 10 to senior level at Year 11. Dropping out means a student then decides to leave the subject of first choice. Results show that a large number of students opted out of calculus-based options. Thus, support for mathematics teachers through pedagogical resources is needed if Queensland is to reverse these declining trends. Moreover, teachers will have more resources to draw from when need arises. Resources that can support linking of junior concepts to senior concepts can go a long way towards building students' confidence in mathematics. When students opt for calculus-based mathematics at Year 11 they might be basing their decision on their junior level mathematics understanding. However, if the links between junior and senior concepts are not well established, it might make students doubt their capacity and be less confident of passing calculus-based options. McPhan et al. (2008) emphasised that teachers have to introduce more engaging resources and target mathematical knowledge development if students are to build confidence in their capacity to study mathematics. This supports the development of a planning framework that links students' prior knowledge with new knowledge in their learning of mathematics.

Another trend analysis, per education district, provided an insight into how school location and socio-economic status impacted student enrolment in calculus-based mathematics and dropout rate. Schools located in areas with high socio-economic status had a high number of students in calculus-based mathematics and a lower dropout rate than the state average.

Even though economically and educationally disadvantaged districts might have an equal or a greater number of schools offering the calculus-based subjects compared to socio-economically advantaged districts, their enrolments were far lower than their wealthier counterparts. This was exemplified by Brisbane Central, which had the highest number of students enrolled in calculus-based options but the lowest dropout rate in Queensland. However, schools in areas that were economically and educationally disadvantaged, such as in Mackay, had few students in calculus-based mathematics and a high dropout rate compared to the state average.

These findings are supported by similar findings from ACARA (2014), Bornstein and Bradley (2014) and Broer et al. (2019), who noted that socio-economic status is related to better resources in education and high achievement. Schools in areas regarded as socially advantaged are more likely to be well resourced and more likely to retain experienced mathematics teachers compared to schools in socially disadvantaged areas. In socially disadvantaged areas, there might be fewer opportunities for subject collaboration as the schools are normally small. Moreover, students’ choices of mathematics subjects are positively correlated with socio-economic status (Hascoët et al., 2021; Valero et al., 2015), thus schools with low socio-economic status might struggle to engage students in calculus-based options. To get a better understanding of how school choices and location affected enrolment in calculus-based subjects, the study also looked into the distribution of schools that offered the subjects.

11.4 Distribution of Schools Offering Calculus-based Mathematics Subjects

Not all schools in Queensland offer calculus-based mathematics subjects. In the phased-out curriculum, the number of schools that did not offer Mathematics B had been steadily increasing until the subject was terminated in 2019. The number of schools not offering Mathematics C was fluctuating around 80 during the period of interest, although they were still offering the two non-calculus-based mathematics subjects. The trends in the new curriculum provided more detail, as school enrolment in calculus-based mathematics was found to be correlated to district socio-economic and educational advantage. All schools in socio-economically and educationally advantaged districts offered Mathematical Methods. These districts also had the highest number of schools that offered both Mathematical

Methods and Specialist Mathematics. Conversely, districts that were socio-economically and educationally disadvantaged showed the biggest difference between the number of schools offering Mathematical Methods and those offering Specialist Mathematics. Moreover, these districts also had schools that did not offer both options. These results support the work of Perry (2018), whose findings pointed out that limited educational opportunities and experiences for students from low socio-economic areas resulted in social inequality and less confidence in taking on demanding subjects. Thus, schools not offering calculus-based subjects may indicate that there were no students interested in pursuing the subjects or that resources, both human and/or material, were not available to facilitate teaching and learning of these subjects.

Districts with socio-economic and educational disadvantages covered mostly regional, rural and remote areas. Such areas had a significant population of indigenous communities, which might explain why the majority of indigenous students opt for non-calculus subjects at senior secondary. Moreover, schools in low socio-economic areas may find it difficult to retain teachers as they have high transfer ratings that only attract teachers to serve a minimum of three years and request a transfer to go to schools in urban areas. Schools in socio-economically and educationally advantaged areas had qualified mathematics teachers and did not face the recruitment challenges experienced by regional, rural and remote schools. These results were consistent with an AMSI (2014) report that noted that schools in socially and economically disadvantaged areas may struggle to employ qualified teachers. The same report pointed out that in Queensland, 40% of mathematics teachers in rural, regional and remote were out-of-field teachers. Therefore, most of the students in such schools ended up being taught by non-specialist teachers who probably needed support through resources that could support the teaching and learning. This highlights the need to develop planning frameworks that teachers can use to support the teaching of mathematics.

The high dropout rate, especially in Mathematical Methods, which reached alarming levels when the new curriculum was introduced, and the uneven enrolments and distribution of calculus-based subjects across Queensland is what this thesis has addressed through developing pedagogical resources. With research noting that student engagement is key to participation and achievement, this study developed pedagogical resources that could support the teaching and learning of mathematics (Kilpatrick et al., 2001; Lee & Hannafin, 2016; Varsavsky, 2010). The resources include pedagogical tools that support the development of conceptual and procedural knowledge. These resources were also expanded to support problem-solving at senior level.

11.5 Pedagogical Resources to Support Teaching of Calculus-based Mathematics

Supporting the teaching and learning of calculus-based mathematics must take an all-encompassing approach by addressing planning, developing resources to facilitate teaching and students' participation in the learning and addressing how mathematical knowledge can be developed and represented. Planning informs delivery and delivery can be supported by resources that make learning student-focused and, in this research, help develop mathematical knowledge. This study developed pedagogical resources that started with the creation of a framework that could be used during planning to sequence content logically. Additional resources were then created to support development of mathematical knowledge, with associated examples from Mathematical Methods subject.

11.5.1 Planning: Content Sequencing

Content sequencing is one of the key stages of planning that prompt teachers to think about how they can sequence topics in a way that promotes student understanding. In Queensland, it is the responsibility of a teacher to sequence content during planning as "educators must select the best sequence for students to learn the skills" (Willingham, 2020, p. 44). Effective content sequencing supports student participation and understanding in the learning process. This view is supported by Kilpatrick et al. (2001) and the QCAA (2018), who noted that content sequencing promotes student engagement with mathematics content and gradual development of mathematics knowledge.

Content sequencing is central to planning as it informs how teachers can develop students' knowledge as they engage with the subject matter. At senior secondary level, mathematics teachers are expected to develop teaching and learning plans, unit plans and term planners or lesson sequences. In all these documents, teachers are required to identify the order of topics and concepts to be taught within a unit. This places content sequencing at the centre of any mathematics teacher planning and the order in which content is sequenced has an impact on the effectiveness of the teaching and learning process. When teachers are sequencing content, they are *ipso facto* hypothesising how students will develop new knowledge during the learning process. These views are supported by the findings of Simon (1995) and Lowrie et al. (2018) that planning hypothesises students' current understanding and provides opportunities to gradually expand and deepen mathematics knowledge. This study developed a framework on content sequencing to support mathematics teachers in linking junior to senior level mathematics concepts. The framework can also be used to collaboratively sequence mathematics content in any mathematics subject.

The framework of content sequencing from junior (Years 7 to 10) to senior high school mathematics (Years 11 to 12) developed in this study supports logical and sequential mathematics knowledge development. The framework links the Australian Curriculum: Mathematics (P-10) and Senior Mathematical Methods subjects, demonstrating that students need to have a good understanding of mathematics concepts (as designated in the Australian curriculum) if participation is to be supported at senior secondary level. Teachers' experience of the framework when tested during this research made them to note that prior knowledge was at the centre of effective planning, teaching and learning of mathematics. These views are supported by an ACARA (2014) report that emphasised that effective planning and teaching should provide students with the opportunity to reflect and link their experiences to new knowledge because this promotes engagement, participation and achievement. The framework emphasises the inclusion of prior knowledge during planning which then informs how concepts will be developed during teaching. Thus, effective content sequencing is key to enhancing teaching and learning of mathematics as students are not fast-tracked into new knowledge that they may not understand.

In recent years, there has been significant calls to move away from traditional teacher centred approaches to constructivism. Constructivists believe that new knowledge is constructed from students' prior knowledge (Garbett, 2011; Bruning et al., 2004; Taber, 2019). The pedagogical framework on content sequencing from junior to senior concepts builds from this understanding and emphasises the need to include relevant prior knowledge in knowledge development during planning. This can support teaching and learning, as relevant concepts and skills that are foundational to the development and understanding of a particular senior level mathematics concept can be mapped and included in planning.

Results from this study showed that teachers who participated in the study strongly believed that the four pillars of the framework support content sequencing from junior to senior concepts. The four pillars were: identify key words, backward mapping using a concept breakdown table, identify essential concepts and hierarchical mapping of concepts. Teachers believed that the pillars facilitated identification of prior concepts and skills that were foundational to learning of new concepts (Hailikari & Nevgi, 2010). Identification of essential concepts can help teachers to emphasise and focus on the key concepts that students need to retain from the content being taught to support the learning of future content. Schuhl (2020) identified essential concepts as the key ideas in a unit that help students build conceptual understanding as well as ideas that link across concepts.

The framework was informed by the hierarchical nature of mathematics, which informs how new knowledge builds on prior knowledge. This understanding sets the grounds for collaborative planning across levels, since student participation at senior secondary level depends on understanding of junior level concepts. The effectiveness of teachers at senior schools in developing new knowledge in their students depends on the effectiveness of junior level teachers in teaching the essential concepts needed at senior level. The framework seeks to provide a basis for a systematic and structured way of sequencing mathematics content that include prior knowledge during the planning stage. In this study, the framework linked junior level concepts to senior level Mathematical Methods concepts. The teachers' views in this study confirmed the findings of Reys et al. (2020) that prior knowledge should be included in developing sequenced programs as it demonstrates continuity and reinforces the importance of fundamental concepts in developing new concepts, thus demonstrating the hierarchical structure of mathematics. Importantly, it also helps students understand that every level contributes to future mathematics learning. At the same time, junior level mathematics teachers will be reminded of the importance of mathematics at that level to senior level mathematics. This reinforces how, since the Queensland mathematics curriculum is spiral in nature, new concepts build on concepts taught earlier. Mathematics teachers in Queensland are expected to develop spiral and sequenced content (QCAA, 2014) that gives students opportunities to revisit concepts even as they develop their knowledge (Harden, 1999). The framework is developed to support teachers during this stage in their planning. The framework for content sequencing of mathematical content from junior to senior level further highlights that mathematical knowledge is developmental as knowledge develops from the familiar to the unfamiliar. In this way, the framework on content sequencing can be informed by the structure of the subject and the curriculum needs. This can support mathematics teaching and learning in Queensland.

Effective teaching in mathematics is informed by planning that focuses on students' conceptual and procedural knowledge development. This study focuses on teachers so that they can help students to view mathematics as a web of connected concepts. The understanding that junior mathematics concepts are the foundation of and link to senior calculus concepts will help students appreciate the interconnectedness of mathematics concepts. To this end the framework on content sequencing provides a foundation of this understanding and how teachers can include this understanding during planning. Moreover, the framework on content sequencing showcases the connectedness of concepts at any level. Thus, procedures and skills that students engaged with at junior secondary level remain relevant at senior level and beyond. It reinforces the view that mathematics is a connected system rather than independent concepts that are grasped by rote learning only to be forgotten after a short period. Planning that fosters mathematical knowledge can inform the development of resources that support the building of knowledge during teaching and learning.

11.5.2 Mathematical Knowledge Development

Mathematics knowledge can be divided into two main categories: conceptual and procedural, and these two can be orchestrated when it comes to problem solving tasks. Effective teaching and learning must develop both and integrate them as this will give students deeper mathematical knowledge overall. Rittle-Johnson (2017) reported that the development of conceptual knowledge, procedural knowledge and fluency are central to the development of students' mathematical knowledge and competency. This study emphasised the use of visual representations to develop mathematical knowledge; this is because such representations are easy to understand and retain, simple to show connections, can represent large amounts of information and require less time to process than text (Birbili, 2006; Raiyn, 2016; Shabiralyani, 2015). The literature on the advantages of visual representations in teaching and learning helped this study explore ways of incorporating visuals to support teaching and development of students' mathematical knowledge at senior secondary.

Providing students with the opportunity to present mathematical knowledge in non-linguistic but unfamiliar ways such as visual representations, is key to students' engagement and understanding. Instead of just stating the connection, steps and procedures, visual presentations provide an alternative way of presenting mathematics ideas. This view supports Bay-Williams and San Giovanni's (2021) findings that representing mathematics relationships in a visual form supports the teaching of mathematics. Presenting information in alternative forms, for example non-verbal ways such as maps, helps students comprehend the phenomenon (Murphy, 2011). It prompts students to think about what they are doing and where thinking is involved, learning is taking place.

Student-developed visual representation of mathematical processes are a visual representation of their mathematical knowledge. Results from this study showed that teachers found visual representation in the form of concept maps and procedural flowcharts provided insight into students' mathematical knowledge (see representations in Chapter 8, 9 and 10). Concept maps and procedural flowcharts can therefore be used to represent students' mathematical understanding and can thus be an assessment resource, as posited by Ho et al. (2017) and Bell (2017). Students can develop concept maps to show how concepts interlink and in so doing teachers can identify common errors and misconceptions in their understanding (see students' artefacts in Chapter 8). Likewise, as an alternative to the actual use of a procedure to solve a given mathematics problem, students can create a procedural flowchart to provide a generalised way of solving a particular problem (see students' artefacts in Chapter 9). A procedural flowchart can thus also expose students' errors and misconceptions involving a particular mathematics problem. Teachers can also provide concept maps and procedural flowcharts with gaps for students to complete as an assessment. Results of this study showed that concept maps and procedural flowcharts can be easy and faster way for teachers to provide feedback compared to the written exercises that most teachers use. The degree to which student-generated concept maps and procedural flowcharts can vary in sophistication and depth of mathematical knowledge can thus also demonstrate the range of students' knowledge and thinking skills.

11.5.2.1 Concept Maps: Conceptual Knowledge and Beyond

The understanding that mathematics is a web of concepts that are related and that senior level concepts develop from junior concepts is key in teaching students conceptual understanding. The development of concept maps at senior secondary level supports students' understanding of how mathematics concepts are interlinked, supporting the view of Novak (2010), Hartsell (2021) and Watson et al. (2016) that concept maps provide a visual representation of how concepts are interlinked. Concept maps deepen students' understanding as they reflect and determine how concepts are connected. In this study, senior secondary mathematics teachers identified concepts maps as a resource that can be used for unit consolidation, that is, of how concepts related to and within a unit interconnect, which can support understanding, simply because as visual representations are easy to process and retain.

Effective teaching should enable students to integrate concepts during problem solving. Participants in this study identified that concept maps can be used to demonstrate how solving a complex problem may involve bringing together different concepts. This finding aligns with Beat (2015), Fonteyn (2007) and Kinchin et al. (2019), whose work found that concept maps facilitated the integration of concepts and new ideas to form complex ideas and relationships. Understanding the interlinking of concepts can support students' ability to integrate them, thus deepening their mathematical understanding. Likewise, artefacts provided by teachers and students in Chapter 8 show concept maps can be used to break down a topic into a connected web of concepts that might be easier to understand and manipulate.

In this study, teachers found that the visual nature of concept maps could support the linking of prior knowledge to senior concepts, thus supporting mathematical knowledge development. In this study, concept maps were also used to support the framework on content sequencing, as they linked junior to senior level concepts, thus reinforcing the hierarchical nature of mathematics (see concept maps in Chapter 8). The teachers involved in this study also noted that concept maps could be used to identify essential concepts in a unit, and identifying key concepts is one of the key pillars of the framework on content sequencing. The findings in this study are supported by Llinas et al. (2018) and Groffman and

Wolfe (2019), who posited that concept maps provide opportunities to connect fundamental and other related concepts. In summary, the concept maps in this study complemented the framework on content sequencing from junior to senior concepts. Importantly, the conceptual and procedural knowledge complemented each other as they developed and this plays an important role in supporting student participation.

11.5.2.2 Procedural Flowcharts: Procedural Knowledge and Beyond

Teachers are expected to help students develop procedural knowledge. When students know the steps in key mathematics procedures and can apply them flexibly, efficiently and accurately, they acquire procedural fluency. Operating at high levels of procedural fluency enhances modification of procedures to solve complex problems (Blöte et al., 2001). This study explored the use of procedural flowcharts in supporting procedural fluency.

Senior secondary mathematics teachers who participated in this study noted that procedural flowcharts can support mathematics procedural knowledge and fluency. They identified procedural flowcharts to represent procedure(s) and corresponding steps to solve a particular mathematics problem. This finding is in line with the definition of procedural knowledge reported by Braithwaite and Sprague (2021). The teachers in this study noted that procedural flowcharts provided the opportunity to visually present different methods or procedures, thus supporting flexibility. The steps to the solution in a procedural flowchart are sytematically ordered and, when followed correctly, would result in a specific solution being reached, thus enhancing accuracy. Procedural flowchats are also meant to eliminate steps and decisions that lead to undesired solutions. The teachers in this study noted that procedural flowcharts offered opportunities to evaluate given procedures to a solution, thereby supporting students' capacity to determine the optimal procedure to solve a given problem and thus supporting efficiency (see procedural flowcharts collaboratively developed by teachers and from individual teachers in Chapter 8). All these views from senior mathematics teachers are supported by the work of Bay-Williams et al. (2022), which identified that procedural fluency involves using procedures flexibily, efficiently and accuractly.

Developing procedural flowcharts facilitates the deepening of mathematical knowledge. Knowledge of the steps and procedures to solve a problem is needed and developing a meaningful and logical procedural flowchart visually involves deeper understanding of how procedures are used to solve problems. Teachers identified that procedural flowcharts can also be used to highlight and deepen mathematical vocabulary as the steps and procedures to a solution are developed. Additionally, procedural flowcharts may be useful in minimising and correcting common errors and misconceptions in mathematics procedures, as only the relevant steps and procedures are available to choose from. Success in developing a procedural flowchart, either collaboratively or individually, can also be a resource to measure understanding.

The development of procedural flowcharts independently or collaboratively makes them a resource that can be used flexibly to engage students in the learning process. Artefacts developed by teachers and students included in Chapter 9 provide an alternative to representing procedural knowledge. Teachers emphasised the use of procedural flowcharts in any or all of the three stages of explicit teaching. When teachers use explicit instruction, they demonstrate the skill first (I DO stage); they then engage students by guiding them into using a skill or procedure (the WE DO stage) and finally release the responsibility to the students through unprompted practice (the YOU DO stage) (Archer & Hughes, 2010). Teachers in North Queensland and most other schools in the state are expected to use explicit instruction during teaching and learning. The teachers in this study identified that procedural flowcharts could support student engagement.

During the I DO stage, the teacher can develop a procedural flowchart on how to solve a particular type of mathematics problem. This procedural flowchart will include all the necessary steps needed to develop the solution. The teacher can also introduce different procedures and their corresponding steps that can be added to the flowchart. During the WE DO stage, the class can develop a procedural flowchart using worked examples the teacher will have used during the I DO stage. The third and last option of using procedural flowcharts during explicit instruction is during the YOU DO stage, when students develop procedural flowcharts themselves on how they can solve or have solved a particular type of mathematics problem. The teachers participating in this study noted at this stage that students could also use procedural flowcharts as a reference as they engaged with a particular type of mathematics problem. This allowed students to work through at their own pace, as procedural flowcharts can also be a scaffolding resource. Importantly, developing procedural flowcharts can deepen students' understanding and provide opportunities for generalisation of mathematics procedures. However, the study noted that the use of procedural flowcharts was not limited to supporting procedural knowledge and fluency but could also support problem solving.

Queensland's problem solving and modelling task (PSMT) at senior secondary level provides students with an opportunity to write a report. Planning for the solution can be complex, as students are expected to analyse a real-world open-ended problem and develop a solution using mathematics concepts and procedures. Students are expected to engage with this task over four weeks and each week has a checkpoint that is used to determine the students' progress and are expected to identify mathematics concepts and procedures to use in developing the solution. Senior secondary school teachers who participated in the study identified procedural flowcharts as a resource that students could use to communicate the key steps and procedures for solving the problem. This finding supports Vale and Barbosa's (2018) work that further research was needed on supporting students' ability to construct and effectively present their mathematical knowledge during problem-solving. An insight into the overview on how students plan to solve a problem can be beneficial to students and their teachers in terms of checking if the student interpreted the question correctly. Procedural flowcharts were used at an early checkpoint in this study for students to provide an overview of how they plan to solve the given problem.

Scaffolding represented as a flowchart on how to approach a PSMT is available to teachers and students (QCAA, 2018). This flowchart shows the key stages involved in the process. In this study a student-developed procedural flowchart can supplement the QCAA flowchart by providing more detail on how the solution will be developed. It can also demonstrate how the different stages link as the solution is being developed. Results from this study showed that procedural flowcharts can support the logical representation of the stages involved in the PSMT and the results help in addressing the finding by Galbraith and Stillman (2006) that students need help in linking the different stages involved in problem-solving. Procedural flowcharts can support how a solution to a problem is interconnected, even though more than one procedure is included. The flexibility of a procedural flowchart to accommodate more than one procedure and provide opportunities to justify and evaluate the different procedures is vital in demonstrating and accommodating the key elements of developing a PSMT solution. In a PSMT, students have to justify why they used their chosen mathematics procedures. This can be done by evaluating solutions developed by the procedures in relation to the problem identified. A procedural flowchart can reinforce the structure of the PSMT, which will help students when writing the report, thus supporting communication. Development of procedural flowcharts provides opportunities for students to analyse a problem and link it to mathematics concepts (mathematisation), identify procedures and their relationships in addressing the problem, evaluate how and what the procedures address and then synthesise the solution. To develop a logical procedural flowchart, students have to reflect on how the procedures they are proposing addresses the problem they have identified. Moreover, flowcharts are a visual representation of a sequence of steps or stages (Marzano, 2017) in a complex system; thus, they support communication where different procedures, steps and stages are involved.

11.6 Implications of the Study

11.6.1 Trends Analysis

Trends in student enrolment, especially in key subjects such as calculus-based mathematics, have implications for different sectors of society. Trends analysis is important for the education sector as, in this case, it provides an insight into the state of calculus-based mathematics in Queensland. Trends can also inform policy as policy makers need to have an insight of what is working and what is not. Importantly, stakeholders such as teachers, parents, industry, universities, education departments, politicians and the wider community can then dissect the implications of these trends to inform their planning.

This study provided insight into the trends in student enrolment in the phased-out and current mathematics curricula in Queensland. Understanding trends between the two curricula help in determining changes, if any. This evaluation can be done by comparing trends in student enrolment in the two curricula and using the findings to inform future policy, considering mathematics is compulsory in Queensland. The trends might inform targeted intervention and resource mobilisation or inform administrators what is working and what is not.

This study was the first in Australia (and Queensland in particular) to track student enrolment in calculus-based mathematics at both levels (Year 11 and 12), including socio-economic indices for districts from the ABS, schools' index of community socio-educational advantage values from ACARA and schools transfer ratings from the DoE. Most available studies have focused on enrolment at the end of Year 12. What this study offered was an insight into dropout rates in calculus-based mathematics from 2010 to 2020 across Queensland. Comparing dropout rates from the phased-out curriculum to the new curriculum provided a new understanding on how student enrolment is changing, namely, that the dropout rate has increased at an alarming rate.

In Australia, and in Queensland in particular, calculus-based mathematics subjects are the preferred prerequisites for STEM courses at tertiary level. Students' enrolment in calculus-based subjects at senior secondary level have a direct impact on tertiary entry. The trends can provide a basis for comparing Queensland with other jurisdictions in Australia and beyond. Furthermore, since mathematics is a key subject in Queensland education, understanding students' enrolment at post-compulsory level can help in evaluating the state's STEM education implementation in which mathematics, especially calculus options, plays a central role.

STEM is a critical area of focus for most education departments as individual and societal prosperity depends on it. The trends can inform educational stakeholders in areas that need attention to align with the vision of Australia of being an economic powerhouse. The trends can be used to project Queensland's future STEM workforce and tertiary enrolment growth in relevant sectors. Moreover, the trends can also inform parents, teachers and the education department about areas where students need more support. One of the key trends observed in this study was high dropout rate in calculus-based mathematics, especially in Mathematical Methods, which demonstrates calculus-based subjects are losing students and require key stakeholders to intervene and work together.

This study proposes resources to help address the problem of high dropout rates. It offers concrete steps that span planning, teaching and learning of calculus-based mathematics and with a focus on supporting student participation. The resources used in this study are key foundational tools that require more research and focus at senior mathematics level in order to support every student who chooses to study this subject. Retaining all students who choose to study calculus-based options is a key first step before pushing for more students to study the subject.

11.6.2 Pedagogical Resources (Framework on Content Sequencing)

In Australia, the junior curriculum is national and the senior curriculum is state developed. The framework on content sequencing developed in this study is a novel approach that provides cohesion between the two systems, thus allowing teachers and students to make a smooth transition from junior to senior mathematics. The framework on content sequencing in mathematics is a pioneer framework that focuses on such a key sector of mathematics planning. The framework, in which the development of complex concepts is linked to familiar concepts, can help teachers in their planning and in their understanding and teaching of senior mathematics concepts. Importantly, the framework emphasises the importance of every year level, and every mathematics teacher's responsibility and understanding that for their students to progress to mathematics at a higher level, they need to understand mathematics at the level they are teaching them.

The framework on content sequencing from junior to senior mathematics also provides a guiding framework that can be used by teachers across different school levels to effectively sequence mathematics content. This framework advocates for collaborative content sequencing across levels, thus providing opportunities to share pedagogical knowledge through identifying prerequisite concepts and how they can be used to develop new knowledge. Including prior knowledge and skills during planning means the plan will be readily available to be delivered whenever necessary. It will also help teachers think about how new knowledge can be effectively taught from prior knowledge before engaging students. Reflection and mapping play an important role in preparing teachers to have deeper content knowledge and think about how they will teach concepts. Ensuring that every student has sufficient and required prior knowledge and skills to engage with new knowledge increases the chances of students engaging and understanding new knowledge.

11.6.3 Pedagogical resources (Concept maps)

Concept maps can help students build conceptual understanding and this study expanded the use of concept maps in senior secondary mathematics to support the linking of prior knowledge to new knowledge. In other words, concept maps can be used to link junior mathematics concepts to concepts at senior level. Use of concept maps at senior secondary, which has been limited (see Schroeder et al., 2018), needs to be encouraged as it provides a visual representation of how concepts are connected.

Nor should the use of concept maps be limited to linking concepts in one unit or at a single school level but rather they should be used to link foundational concepts to new concepts. This study suggests that at senior secondary level in mathematics, concept maps can be a visual representation of how new concepts are developed from prior concepts. This is underpinned by the constructivist learning approach, which has grown in influence in the teaching and learning of mathematics. An additional advantage of student-drawn concept maps at senior secondary level is that they give teachers an insight into their students' mathematical understanding.

Another benefit of concepts maps in mathematics is that teachers can provide quick feedback because the maps are easier to process than calculations and text, they reinforce and provide an overall overview of the unit and they can be used by students during revision.

11.6.4 Pedagogical Resources (Procedural Flowcharts).

Procedural flowcharts are a versatile resource for supporting procedural fluency in mathematics teaching and learning. Developing students' procedural fluency deepens students' skills and use of procedures and steps to solve familiar problems. At senior secondary school level in Queensland, the QCAA requires 60% of the questions in any examination, be they formative or summative, to be familiar questions that mostly involve fluency and application of known procedures. Thus, introducing procedural flowcharts at senior level may support students' chances of solving familiar problems.

This study expanded the use of flowcharts beyond just representing steps in a procedure. Instead, it advocates for procedures and steps to mathematics problems to be presented on a procedural flowchart instead of just writing steps. It suggests that teachers introduce development of procedural flowcharts as a tool for checking understanding instead of answering questions through calculations. Teachers can ask students to develop a procedural flowchart that generalises how a particular type of mathematics problems can be solved, thereby deepening students' understanding through generalisation. This study also suggests that a class-or group-developed procedural flowchart can promote collaborative learning. Instead of marking-student developed procedural flowcharts, teachers can distribute the procedural flowcharts among students and ask them to first apply them to solve related questions then evaluate and discuss solutions. This strategy can also help identify and correct misconceptions and common errors among students.

Engaging students to think independently in mathematics classes can sometimes be a challenge, as students are comfortable just following teachers' examples in familiar problems. Including procedural flowcharts in mathematics teaching and learning provides students with the opportunity to reflect and think about the procedures and steps involved. To deepen students' mathematical thinking, teachers can give students problems that require more than one procedure and ask them to develop a multi-solution procedural flowchart, thus, making students more engaged and active in their learning. When students are developing procedural flowcharts, they are using a lot of critical skills that are useful in problem solving.

This study suggests that students should develop procedural flowcharts as an initial stage in solving open-ended mathematics questions, which will benefit both teachers and students. It will give teachers an insight into what their students are thinking and planning to include in the solution and will help students analyse the task, reflect, organise their thoughts, logically sequence the solution and then evaluate it. The procedural flowchart also gives teachers an opportunity to check if students have understood the problem and if their planned solution is relevant. It is also a tool for timely feedback.

Generally, the role of concept maps and procedural flowcharts in representing students' mathematics knowledge and how these tools support critical skills should prompt a policy consideration on assessment. First, QCAA can emphasise the need for a flowchart in the PSMT report to give a visual overview of the solution. Second, QCAA can introduce assessment methods other than short response and open response questions in senior mathematics. This change would be beneficial to visual learners as they can use visual representation instead of solving problems using calculations.

These pedagogical resources have additional implications for research as use of visual representations in mathematics teaching and learning is gaining momentum. This study is among very few available studies focusing on developing both conceptual and procedural knowledge through the use of visual representations, but it is all the more timely as there have been repeated calls to support students' participation in calculus-based mathematics. However, limited research has been conducted on how teachers and students can be supported by resources.

The use of information technology in teaching and learning also supports the use of visual representation. Free application software programs such as GitMind and XMind are available for the development of concept maps and flowcharts, aligning these resources to the 21st century skills needed in education. The software allows online collaboration, which is important if a teacher wants students to work collaboratively on a task. The use of concept maps and procedural flowcharts does not limit teachers to explore any teaching and learning approach of their choice but supports effectiveness.

11.7 Conclusion

This chapter has discussed the overall study of trends in calculus-based mathematics as a senior secondary school subject choice, pedagogical resources to support teaching and learning and their implications for practice. Enrolment trends in calculus-based mathematics in Queensland and pedagogical resources to support teaching and learning of mathematics were discussed. The key findings presented from the trends analysis include the high dropout rates and declining enrolment in calculus-based mathematics subjects, particularly in socially-disadvantaged districts. The pedagogical resources to address the declining trends include a framework on content sequencing from junior to senior concepts and visual representation of mathematics using concept maps and procedural flowcharts. Data collected from a sample of senior mathematics teachers show that teachers view the framework on content sequencing as central to developing new knowledge from prior knowledge. The pedagogical resources (concept maps and procedural flowcharts) were used to develop conceptual and procedural knowledge, which are the key constituents of mathematical knowledge. The participants also acknowledged that concept maps and procedural flowcharts could contribute to the development of mathematical knowledge, assessment, students' engagement and participation in mathematics. Use of procedural flowcharts was also expanded to support problem-solving. The implications of the current trends and the pedagogical resources for education, policy and research are informed by the findings and literature in this study. Chapter 12 will articulate the study's contribution to knowledge and provide recommendations to support teachers in enhancing the teaching and learning of mathematics. The limitations of the study will also be explored.

Chapter 12: Conclusion

12.1 Chapter Introduction

A conclusion chapter should clearly articulate factual, conceptual and secondary conclusions, the study's original contribution to the discipline, recommendations and limitations of the study (Trafford et al., 2014). As all these attributes of the conclusion chapter are being explored, new questions will arise, providing opportunities for future studies (Lovitts, 2007). It is important to remind the reader that the major thrust of this thesis has been to analyse trends of student enrolment in calculus-based mathematics in Queensland and develop pedagogical resources that will support the teaching and learning of that subject.

12.2 Trends in Student Enrolment in Calculus-based Mathematics in Queensland

Analysis of trends can be used to measure progress, evaluate a program and determine areas of concern that might need intervention. The state of Queensland has been lagging behind other Australian states, like New South Wales, in investigations into trends in student enrolment in mathematics (see, for example, Jaremus et al., 2018), as the last analysis was done in 2008. The switch to a new senior secondary curriculum in Queensland in 2019 meant some changes in mathematics subjects, both in content and title, and the introduction of an external examination. Thus, trends analysis was ideal to evaluate if student enrolment in the phased-out curriculum persisted in the new curriculum. However, available research had previously focused on student enrolment in Year 12 mathematics only. This study investigated the latest trends in student enrolment in calculus-based mathematics subjects in Queensland. The analysis included student dropout data, the distribution of calculus subjects in the state, the links between student enrolment and a school's ICSEA value (see ACARA, 2013), socio-economic indices for area (SEIFA) value (see ABS, 2018a) and a school's transfer ratings (DoE, 2019, 2020). The study also undertook a comparative analysis of student enrolment in calculus-based mathematics at educational district level.

The trends analysis in this study provided an insight into student enrolment in calculus-based mathematics at senior secondary level in Queensland between 2010 and 2020. Over the period, there were more students studying calculus-based subjects in Year 11 than in Year 12. Each year a substantial number of students (an average of 688) who chose Mathematical Methods at Year 11 were opting out by the time they got to Year 12. The results showed that when the new curriculum was introduced in 2019 the number of students who had left Mathematical Methods by 2020 was 2712 out of 7207, representing a dropout rate of 37.6%. Females had a slightly lower dropout rate than males, although males had consistently dominated the subject for the decade under study. The dropout rate was also high among Indigenous students compared to non-Indigenous students. Generally, there was a high dropout rate in calculus-based mathematics subjects and Mathematical Methods (Mathematics B) had higher dropout rate than Specialist Mathematics (Mathematics C) and this was consistent yearly.

Enrolments and dropout rates were also influenced by socio-economic status, educational advantage or disadvantage and school transfer rating. Schools in areas with a low SEIFA index (less than 50) had lower enrolments and higher dropout rates than those in areas with a higher index. This trend was also witnessed in schools with low ICSEA values (less than 1000), as they had lower enrolments and higher dropout rates than those with high values. Likewise, enrolments in schools with low teacher transfer ratings (rating of 1) were high and the dropout rate was low compared to schools with high transfer ratings (rating 2 to 7). Importantly, a higher percentage of schools in high socio-economic and educationally advantaged areas offered both calculus-based subjects compared to those in disadvantaged areas. The findings of this study indicate that enrolments in calculus-based mathematics subjects are influenced by socio-economic status, educational advantage and school transfer rating. Thus, it can be concluded that dropout rates are inversely correlated to the socio-economic and educational advantage of a school. Therefore, availability of resources influences students' choices and retention in calculus-based mathematics subjects.

12.3 Pedagogical Resources

Pedagogical resources that engage students are some of the key resources that teachers need to support teaching and learning of mathematics (Cook, 2008). Such resources can make a big difference in school settings as they help students engage with the learning process, which eventually encourages greater participation. This study was conceptualised within a constructivist epistemology. In constructivism, knowledge is not transmitted but is created through active interaction and new knowledge is developed from prior knowledge (Jenkins, 2000; Lew, 2010; Narayan et al., 2013). Purposively sampled senior secondary mathematics teachers were presented with pedagogical resources for use during teaching for a full school term. The teachers were actively involved in teaching Mathematical Methods as they used the resources to support their classroom practise. Moreover, participants' teaching experiences of mathematics provided the opportunity for innovation as they used the resources to support delivery. Feedback and opinions from the participants were used in optimising the use, as well as improving and evaluating the pedagogical resources. This study developed pedagogical resources that included a planning framework on content sequencing from junior to senior content. The study also developed concept maps and procedural flowcharts to support students' development of mathematical knowledge. Procedural flowcharts were also used to support problem solving in Mathematical Methods subject.

12.3.1 Planning Framework on Content Sequencing

The planning framework on content sequencing was developed through literature synthesis and was informed by the understanding that mathematics planning should support the development of new knowledge from prior knowledge. The framework was a critical tool and addressed a foundational section of mathematics planning that had previously received very little attention in mathematics education research. The framework addressed this gap and at the same time started the conversation on building a foundation for further research in content sequencing. The framework links the Australian Curriculum: Mathematics developed by ACARA (national) and the Queensland Senior Mathematics Curriculum developed by QCAA (state). The framework on content sequencing links concepts from junior levels, which are regarded as prior knowledge, to concepts at senior level, which would be new knowledge for students.

The perceptions of the senior secondary school teachers on the framework on content sequencing was used in this study. The participants viewed the framework very positively as a resource that supported the process of identifying prior knowledge and mapping it to new knowledge during content sequencing in mathematics. Furthermore, the framework promoted the hierarchical nature of mathematics, that is, foundational concepts that link to higher level concepts were identified first then mapped to senior concepts, which in turn provided the basis for collaborative planning by teachers across school levels.

12.3.2 Concept Maps

Development of conceptual knowledge is key to success in mathematics as it is a discipline that explores and proves new relationships (Bingölbali & Coşkun, 2016). This study investigated the utility of concept maps in linking prior knowledge to new knowledge and the identification of essential concepts. Concept maps can be used to visually link prior concepts to new concepts. In this study concept maps were used to link prior mathematics concepts at junior secondary level to concepts at senior level. Therefore, concept maps can be used as a resource to deliver content after applying the framework on content sequencing, as concept maps complement the framework.

Concept maps can enrich the learning environment. They can be used to deepen students' mathematical understanding through integrating concepts, especially from simple familiar to complex unfamiliar or by breaking up a complex concept into several familiar concepts. This study's findings also supported research (for example, Watson et al., 2016) that student-developed concept maps can be used to assess students' conceptual knowledge. Participating teachers indicated that consolidation of units or topics can be supported by using concept maps as they can provide an overview of how concepts are linked, and the mental representation of students conceptual understanding. From all this, it can be concluded that concept maps can support teaching and learning of mathematics.

12.3.3 Procedural Flowcharts

Procedural flowcharts are a visual representation of steps and procedures to solve a specific type of mathematics problem. The concept of procedural flowcharts was developed from the use of flowcharts in different areas of teaching and learning (see, for example, Marzano, 2017). This study introduced procedural flowcharts in a diverse and flexible way to provide guidance and support communication of processes involved when engaging with mathematics at senior secondary level. The adaptation of procedural flowcharts by the participants in the mathematics teaching and learning positioned them as a key resource that could support student development of mathematical knowledge and skills.

Developing students' mathematics procedural knowledge and fluency supports participation and engagement. Procedural flowcharts go beyond supporting the development procedural knowledge but can represent more than one procedure, thereby supporting flexibility. The opportunity of presenting more than one procedure supports efficiency, as students have to choose the optimal solution. The steps provided on procedural flowcharts can be used by students to solve problems, which requires identifying and using a procedure and thus leads to accuracy. Participants in the study noted that procedural flowcharts can promote flexibility, efficiency and accuracy, thus developing procedural fluency, confirming that this study produced a visual representation resource that can support both procedural knowledge and procedural fluency.

Procedural flowcharts can be used flexibly during the teaching and learning of mathematics. Participants noted that teacher-developed procedural flowcharts could guide students to engage independently with mathematics questions, while class-developed procedural flowcharts can support collaborative learning and deeper understanding of steps and procedures involved in solving mathematics problems. When students develop procedural flowcharts, they engage in an unfamiliar way of representing their mathematical understanding; this requires reflection, which in turn develops a deeper understanding of mathematics (Murphy, 2011). Participants also noted that student-developed procedural flowcharts can be a visual representation of students' procedural knowledge, which can be used by teachers to assess their students' understanding. Teachers who participated in this study observed that procedural flowcharts could support procedural fluency and student participation and engagement in mathematics.

Problem-solving is increasingly an approach of choice in the teaching and learning of mathematics (Chan & Clarke, 2017; Russo & Minas, 2020). In Queensland, all students in Year 11 and 12 undertake tasks where they use concepts, procedures, and skills they have learnt to engage with problem-solving and modelling. They engage with an open-ended question to develop a unique solution by writing a report over a period of four weeks. Scheduled check points to assess students' progress are a requirement. In relation to the question used in this study, procedural flowcharts were used to support the problem-solving structure and sequence the steps and procedures in a logical way.

Student artefacts and teacher feedback showed that students used procedural flowcharts to develop a visual representation of how they planned to solve the problem. Moreover, the flexible nature of procedural flowcharts allowed students to include several procedures they expected could solve the problem. Importantly, procedural flowcharts also provided opportunities to evaluate the procedures that were identified. Analysis of the students developed procedural flowcharts showed evidence that students analysed, planned, reflected, evaluated and communicated the solution through the visual representation. Given the above, procedural flowcharts can support mathematics problem-solving in mathematics.

12.4 Limitations of the Study

They were some limitations in this study. The number of participants involved was small which, while providing quality and valuable data, was limiting in terms of the generalisability of the results and the application of quantitative data analysis (Albers, 2017). Although there were diverse views from different participants in this study, involving more participants was going to be more representative of Queensland senior mathematics teachers and increase the chances of getting much more diverse perceptions. Including a bigger sample of mathematics teachers with diverse professional experience would have provided richer feedback, as this study focused on providing resources for teachers. The opinion of junior level teachers, particularly on the content sequencing framework, could also have helped, especially as junior mathematics level concepts form prior knowledge.

Except for the artefacts produced by students, data was primarily collected from 16 teachers. More artefacts from students, who are key stakeholders in teaching and learning, could have provided more generalisable insight on how the resources impacted the teaching and learning from their perspective. Additionally, more student-developed artefacts could have provided a better insight into how students engaged with the pedagogical resources. Observation of classes where teachers used the pedagogical resources that were developed in this study might also have provided insight into the effectiveness of those resources. Moreover, the study focused only on Mathematical Methods, which is a calculus-based mathematics subject, although the pedagogical resources that were developed could benefit all mathematics as a discipline. However, the limitations do not affect the value of this study in supporting teachers in their teaching of mathematics.

12.5 Opportunities for Future Study and Recommendations

Future research opportunities might arise mainly from expanding the methodological perspective. Validating the quantitative findings in the student enrolment trends analysis with qualitative data could have provided more detail and a broader view of the trends because it would have provided the opportunity of triangulating the data (Creswell, 2014). Collecting qualitative data from stakeholders such as teachers, students, parents and education officials could have provided a holistic picture of trends. Involving different voices might also have helped in understanding how targeted interventions and research might bring about change. Expanding the trends analysis that involved all the data components involved in this study's trends analysis to other jurisdictions in Australia and beyond will help in making a comparative analysis.

The framework on content sequencing from junior to senior concepts is novel but more research is needed to validate the framework in other settings beyond the Queensland curriculum. The framework can also be validated in other mathematics subjects and at different levels, as content sequencing is not limited to secondary school settings. While this study validated the framework constructs with the research participants, further research is needed with a larger group of participants. Observing teachers collaboratively using the planning framework on content sequencing might also help improve the framework. How the planning framework will influence unit planning, instruction during delivery and students' achievement can be new areas to explore.

Visual representation of mathematics has always shown potential for engaging students (Murphy, 2011; Raiyn, 2016). There is need for further study in the use of concept maps and procedural flowcharts beyond the scope of this study, which aimed to provide resources for teachers. Research could also be instituted on instructional design where concept maps and procedural flowcharts are used. Studies that involve student feedback and observations on how students use concept maps and procedural flowcharts may present interesting and relevant insights into participation, engagement and achievement. Research might also help to determine the optimal time to use concept maps and procedural flowcharts during teaching and learning.

Further studies are needed on how to include procedural flowcharts as an alternative assessment resource and how to develop the associated marking rubrics. More research is needed into the use of procedural flowcharts to support different instructional approaches. Results from this study provide some insight into how development of procedural flowcharts can support problem solving skills but further research that involves a bigger sample size and more diverse students with different capabilities is needed. This could help to determine if similar results can be obtained in different contexts.

This study recommends the development of pedagogical resources to support teachers so that students who initially chose to study calculus-based mathematics are retained. This study contributes to the development of pedagogical resources, but more research is needed so that diverse students can be catered for. The study recommends collaborative planning during content sequencing as a way of upskilling and bringing uniformity to how new mathematics knowledge is developed. Moreover, planning that includes prior knowledge also caters for diversity in learning among students. The framework on content sequencing can help teachers be more effective in their teaching and in engaging students with learning, as mathematics knowledge will develop gradually.

The study recommends the use of concept maps and procedural flowcharts to develop mathematical knowledge (conceptual and procedural knowledge). The use of concept maps in linking prior to new knowledge can help students' understanding of mathematics because visual representations are easy to process. Development of procedural knowledge is very important as most familiar questions in formative and summative examinations require recall, fluency and comprehension of procedures. Thus, procedural flowcharts provide a resource to develop procedural fluency. Finally, the study recommends the use of procedural flowcharts to help students plan how to develop a solution during problem-solving and assessment tasks. Although this study focused on developing resources for calculus-based mathematics, in particular Mathematical Methods, these resources are recommended for any mathematics subjects at any level.

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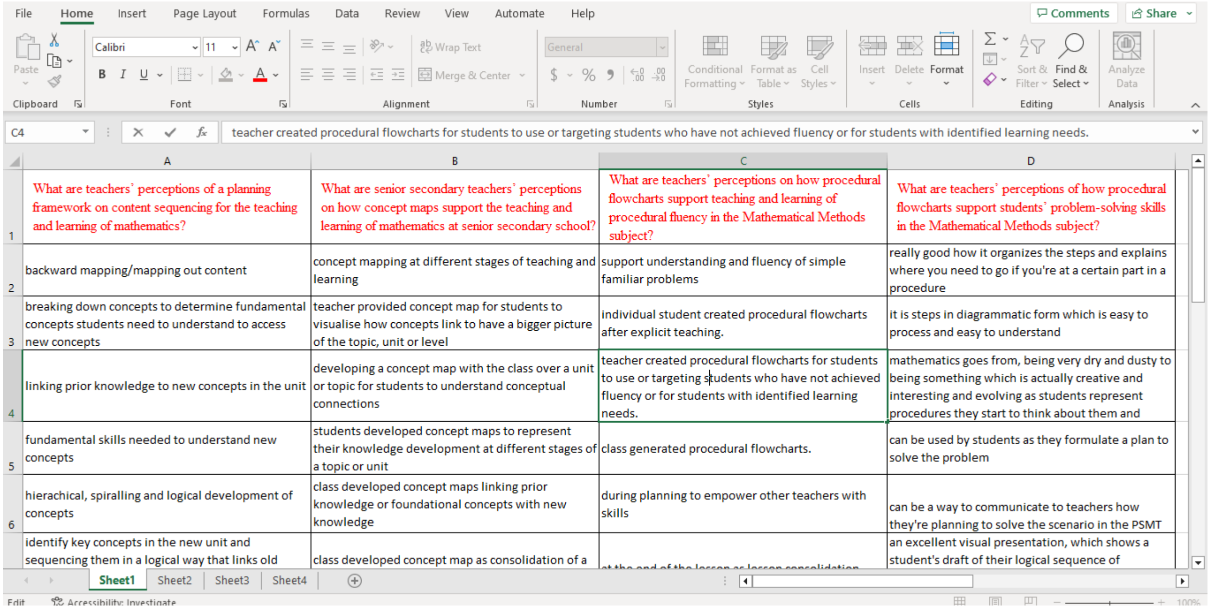
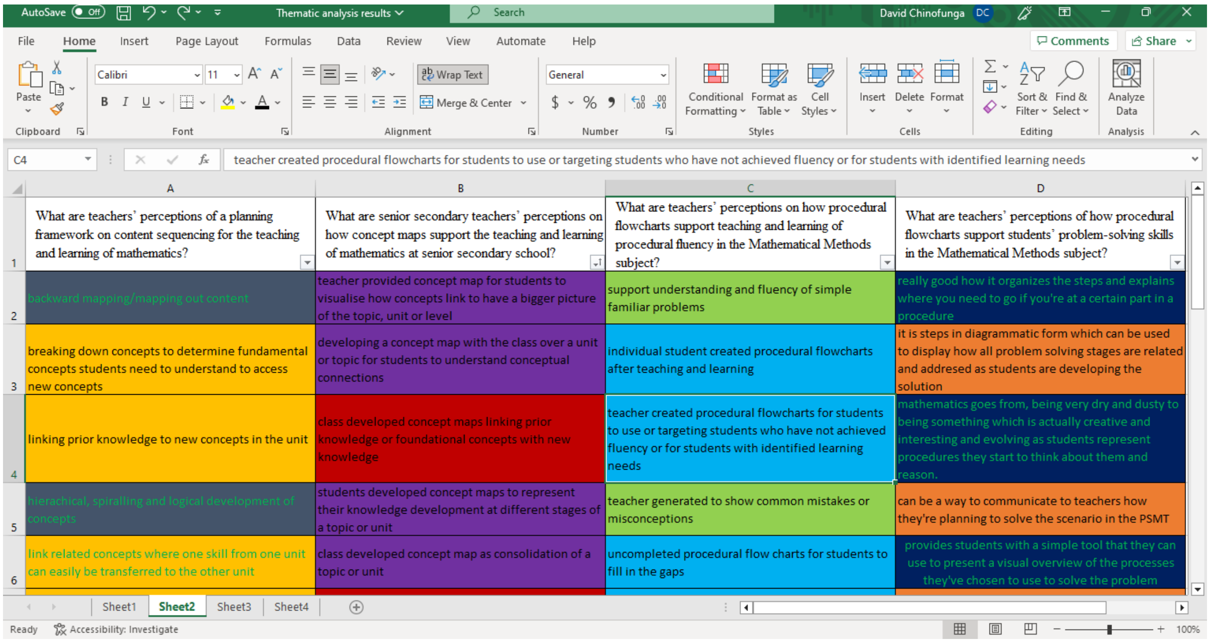
Appendix A: Thematic Analysis Results- initial codes



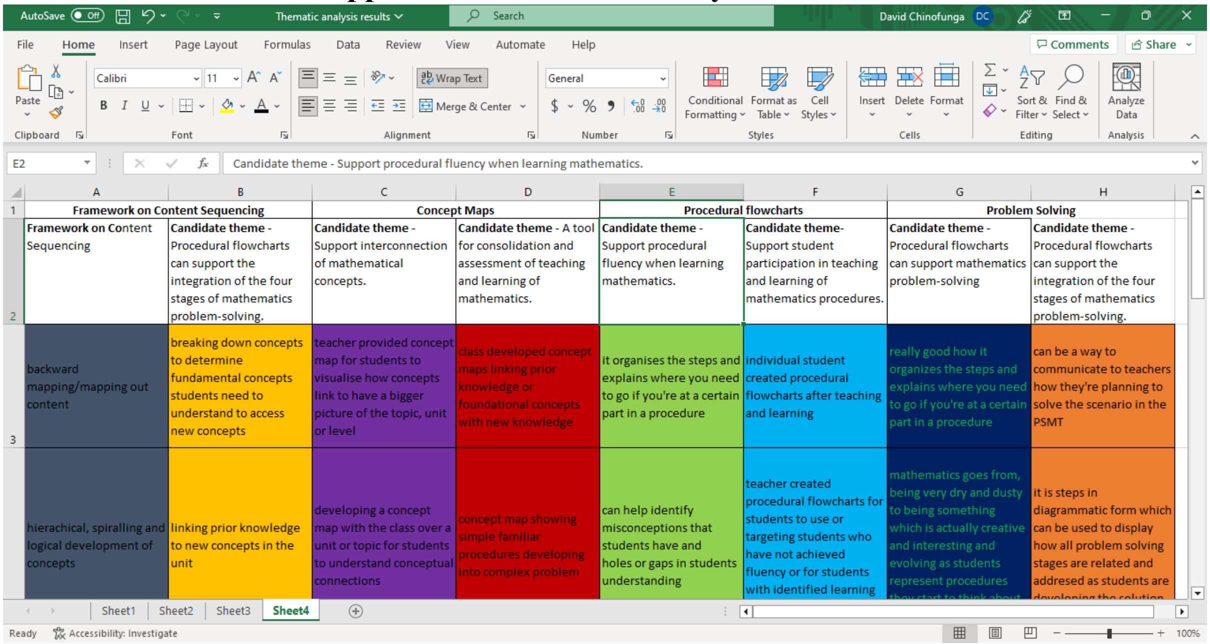
	A	B	C	D
1	What are teachers' perceptions of a planning framework on content sequencing for the teaching and learning of mathematics?	What are senior secondary teachers' perceptions on how concept maps support the teaching and learning of mathematics at senior secondary school?	What are teachers' perceptions on how procedural flowcharts support teaching and learning of procedural fluency in the Mathematical Methods subject?	What are teachers' perceptions of how procedural flowcharts support students' problem-solving skills in the Mathematical Methods subject?
2	backward mapping/mapping out content	concept mapping at different stages of teaching and learning	support understanding and fluency of simple familiar problems	really good how it organizes the steps and explains where you need to go if you're at a certain part in a procedure
3	breaking down concepts to determine fundamental concepts students need to understand to access new concepts	teacher provided concept map for students to visualise how concepts link to have a bigger picture of the topic, unit or level	individual student created procedural flowcharts after explicit teaching.	it is steps in diagrammatic form which is easy to process and easy to understand
4	linking prior knowledge to new concepts in the unit	developing a concept map with the class over a unit or topic for students to understand conceptual connections	teacher created procedural flowcharts for students to use or targeting students who have not achieved fluency or for students with identified learning needs.	mathematics goes from, being very dry and dusty to being something which is actually creative and interesting and evolving as students represent procedures they start to think about them and
5	fundamental skills needed to understand new concepts	students developed concept maps to represent their knowledge development at different stages of a topic or unit	class generated procedural flowcharts.	can be used by students as they formulate a plan to solve the problem
6	hierachical, spiralling and logical development of concepts	class developed concept maps linking prior knowledge or foundational concepts with new knowledge	during planning to empower other teachers with skills	can be a way to communicate to teachers how they're planning to solve the scenario in the PSMT
	identify key concepts in the new unit and sequencing them in a logical way that links old	class developed concept map as consolidation of a	[illegible]	an excellent visual presentation, which shows a student's draft of their logical sequence of

Appendix B: Thematic Analysis- Categorising codes.



	A	B	C	D
1	What are teachers' perceptions of a planning framework on content sequencing for the teaching and learning of mathematics?	What are senior secondary teachers' perceptions on how concept maps support the teaching and learning of mathematics at senior secondary school?	What are teachers' perceptions on how procedural flowcharts support teaching and learning of procedural fluency in the Mathematical Methods subject?	What are teachers' perceptions of how procedural flowcharts support students' problem-solving skills in the Mathematical Methods subject?
2	backward mapping/mapping out content	teacher provided concept map for students to visualise how concepts link to have a bigger picture of the topic, unit or level	support understanding and fluency of simple familiar problems	really good how it organizes the steps and explains where you need to go if you're at a certain part in a procedure
3	breaking down concepts to determine fundamental concepts students need to understand to access new concepts	developing a concept map with the class over a unit or topic for students to understand conceptual connections	individual student created procedural flowcharts after teaching and learning	it is steps in diagrammatic form which can be used to display how all problem solving stages are related and addresed as students are developing the solution
4	linking prior knowledge to new concepts in the unit	class developed concept maps linking prior knowledge or foundational concepts with new knowledge	teacher created procedural flowcharts for students to use or targeting students who have not achieved fluency or for students with identified learning needs	mathematics goes from, being very dry and dusty to being something which is actually creative and interesting and evolving as students represent procedures they start to think about them and reason.
5	hierachical, spiralling and logical development of concepts	students developed concept maps to represent their knowledge development at different stages of a topic or unit	teacher generated to show common mistakes or misconceptions	can be a way to communicate to teachers how they're planning to solve the scenario in the PSMT
6	link related concepts where one skill from one unit can easily be transferred to the other unit	class developed concept map as consolidation of a topic or unit	uncompleted procedural flow charts for students to fill in the gaps	provides students with a simple tool that they can use to present a visual overview of the processes they've chosen to use to solve the problem

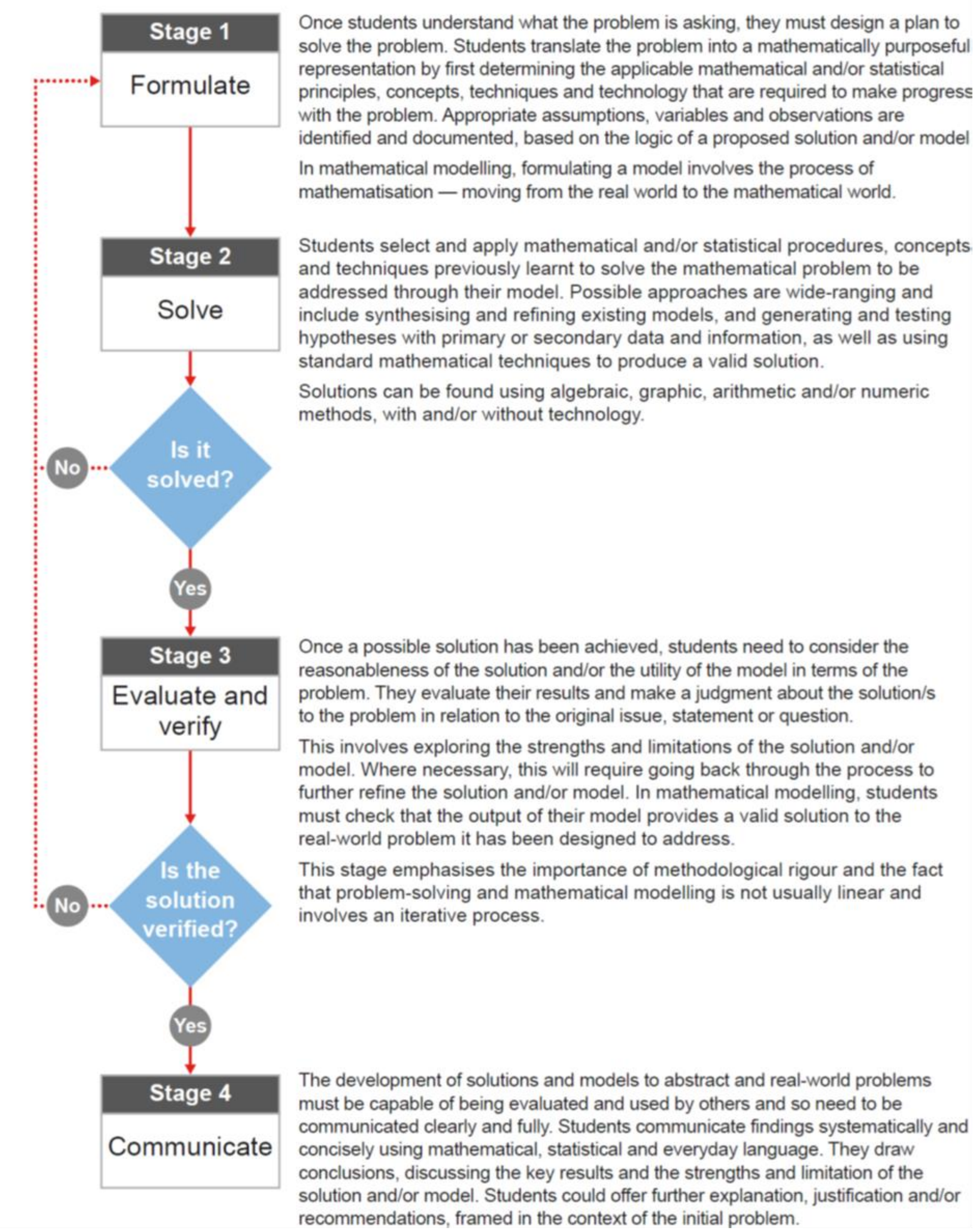
Appendix C: Thematic Analysis- Themes



Framework on Content Sequencing		Concept Maps		Procedural flowcharts		Problem Solving	
Framework on Content Sequencing	**Candidate theme -** Procedural flowcharts can support the integration of the four stages of mathematics problem-solving.	**Candidate theme -** Support interconnection of mathematical concepts.	**Candidate theme -** A tool for consolidation and assessment of teaching and learning of mathematics.	**Candidate theme -** Support procedural fluency when learning mathematics.	**Candidate theme-** Support student participation in teaching and learning of mathematics procedures.	**Candidate theme -** Procedural flowcharts can support mathematics problem-solving	**Candidate theme -** Procedural flowcharts can support the integration of the four stages of mathematics problem-solving.
backward mapping/mapping out content	breaking down concepts to determine fundamental concepts students need to understand to access new concepts	teacher provided concept map for students to visualise how concepts link to have a bigger picture of the topic, unit or level	class developed concept maps linking prior knowledge or foundational concepts with new knowledge	it organises the steps and explains where you need to go if you're at a certain part in a procedure	individual student created procedural flowcharts after teaching and learning	really good how it organizes the steps and explains where you need to go if you're at a certain part in a procedure	can be a way to communicate to teachers how they're planning to solve the scenario in the PSMT
hierachical, spiralling and logical development of concepts	linking prior knowledge to new concepts in the unit	developing a concept map with the class over a unit or topic for students to understand conceptual connections	concept map showing simple familiar procedures developing into complex problem	can help identify misconceptions that students have and holes or gaps in students understanding	teacher created procedural flowcharts for students to use or targeting students who have not achieved fluency or for students with identified learning	mathematics goes from, being very dry and dusty to being something which is actually creative and interesting and evolving as students represent procedures	it is steps in diagrammatic form which can be used to display how all problem solving stages are related and addresed as students are

Appendix D: An approach to problem-solving and mathematical modelling

(Adapted from QCAA, 2018, p.15).



Appendix E: Student 4: PSMT Response.

Mathematical Methods

1.0 Introduction

Mathematics is applied within everyday life constantly.

"*Newton's law of cooling states that the rate at which an object cools is proportional to the difference in temperature between the object and the object's surroundings*" (carolinastaff, 2022). This law can be surmised that in relation to an object with a temperature higher than its surroundings, the exponential function of this objects cooling would be modelled by a decreasing exponential modelled as $T = T_0e^{kt}$

This report is founded upon data collected and mathematically analysed from an experiment tracking the temperature – time relationship of boiled water across a period of 240 minutes (4 hours). The intention of this report is to determine the best fit cup for customers within three categories, the first wanting to consume their coffee quickly, as they have limited time; the second wanting to have a good amount of time to finish their coffee; and the third customer type wanting to drink their coffee very slowly, as they may be reading a book or chatting.

Intervals were generated based upon approximate preparation time for a cup of coffee and average timeframes for customers in cafés. These intervals are as followed (all in minutes), 5 to 25 giving a 20-minute period, the second being 5 to 45 providing a 40-minute period and the last interval being 5 to 90 resulting in an 85-minute period. These intervals were constructed by rounding the time required to brew various types of coffees in different systems (Johnson, 2021), giving a starting point of 5 minutes and lengthy timeframes to cater for variety of customers fitting one of three categories, such as somebody eating breakfast for the third category.

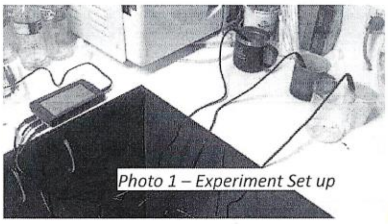
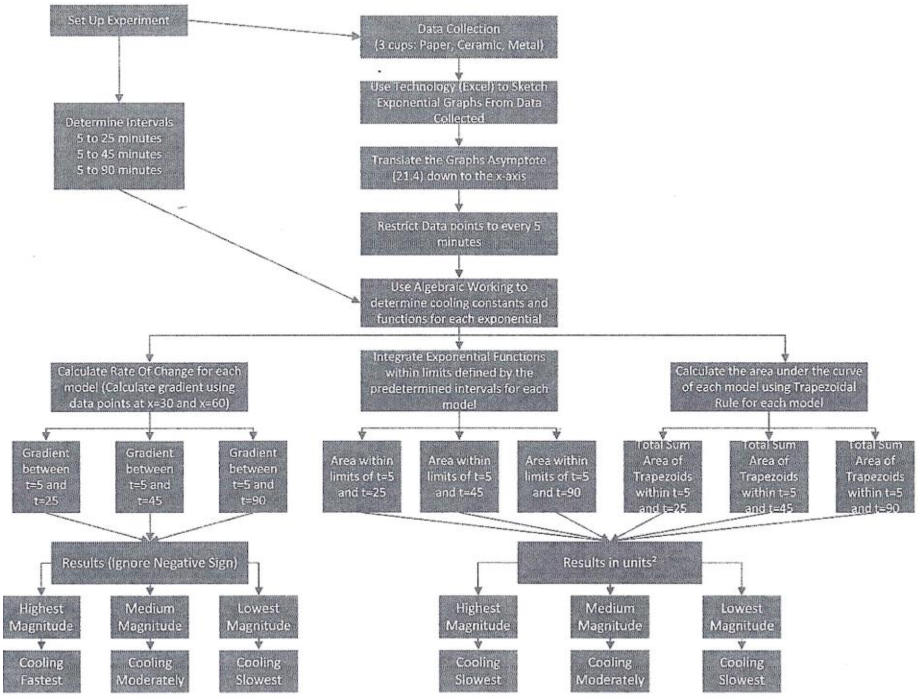


Photo 1 – Experiment Set up

The test was conducted utilising Vernier data loggers and temperature probes, over the course of 4 hours testing the temperature of three containers each containing 250mL of boiled water with data recorded at 30 second intervals.

Throughout this report mathematically determined models are compared to fit intervals for customers to determine cooling speeds of various containers, this will be achieved by using mathematical calculations such as Integration, Rate of Change and Trapezoidal Rules. On the left is a procedural flowchart documenting the process:



Before completing the equations, it is believed that the best fit for the proposed intervals would be in order of Paper, Ceramic then Metal.

1

2.0 Considerations

2.1 Observations

The data chosen to be analysed was the algebraically generated model determined from the cooling constants worked from the data points collected from the experiment. The data used to generate the algebraic data was first translated then restricted to data collected from 0 onwards at 5-minute intervals.

The algebraically generated table is shown on the right, for the table of data used to generate this table refer to Appendix.

The report was sufficiently structured based upon various observations made.

- The models prepared to represent the cooling of each cup were constructed with technology and algebraic working.
- The cooling models of each cup are depicted as decreasing exponentials modelled to the equation $y = T_0 e^{kt}$ where k is the cooling constant of the function and A_0 is the temperature at x=0.
- The algebraically determined data contains the translated data modelled by the function $f(t) = 52.0e^{kt}$ where k is the cooling constant from the respective cup.
-

2.2 Assumptions

Within this report it has been assumed that:

- Although the ingredients for coffee would influence the boiling point, as would impurities within the water (Wikipedia, 2022), for the sake of the experiment, these impurities would not be present, and water used within the coffees is distilled beforehand as well as the ingredients of coffee would not influence the boiling point.
- The boiling point of water is closer to 80°C rather than 100°C due to the increased elevation and subsequent higher altitude (USDA - FSIS, 2012) of Ravenshoe, as it is the highest town in Queensland at 930 meters above sea level (Wikipedia, 2021).
- Room temperature remains at a constant of 21.4°C across the 4-hour period the experiment was conducted for despite realistic fluctuations caused by physical changes. Without this assumption the asymptote of the exponentials approach would fluctuate, and the area found under the curve found by mathematical calculations would be disturbed.

Mathematic Cooling Experiment - Dataset 5 Algebraically Generated Data Points			
x	y1	z1	u1
Time (mir	Paper Cup Temperature (°C)	Ceramic Cup Temperature (°C)	Metal Cup Temperature (°C)
0	52.0	52.0	55.0
5	47.1	46.9	50.5
10	42.7	42.2	46.4
15	38.6	38.1	42.6
20	35.0	34.3	39.1
25	31.7	30.9	35.9
30	28.7	27.9	33.0
35	26.0	25.1	30.3
40	23.6	22.6	27.8
45	21.4	20.4	25.6
50	19.3	18.4	23.5
55	17.5	16.6	21.6
60	15.9	14.9	19.8
65	14.4	13.5	18.2
70	13.0	12.1	16.7
75	11.8	10.9	15.3
80	10.7	9.9	14.1
85	9.7	8.9	12.9
90	8.8	8.0	11.9
95	7.9	7.2	10.9
100	7.2	6.5	10.0
105	6.5	5.9	9.2
110	5.9	5.3	8.5
115	5.3	4.8	7.8
120	4.8	4.3	7.1
125	4.4	3.9	6.5
130	4.0	3.5	6.0
135	3.6	3.1	5.5
140	3.3	2.8	5.1
145	3.0	2.6	4.7
150	2.7	2.3	4.3
155	2.4	2.1	3.9
160	2.2	1.9	3.6
165	2.0	1.7	3.3
170	1.8	1.5	3.0
175	1.6	1.4	2.8
180	1.5	1.2	2.6
185	1.3	1.1	2.4
190	1.2	1.0	2.2
195	1.1	0.9	2.0
200	1.0	0.8	1.8
205	0.9	0.7	1.7
210	0.8	0.7	1.5
215	0.7	0.6	1.4
220	0.7	0.5	1.3
225	0.6	0.5	1.2
230	0.5	0.4	1.1
235	0.5	0.4	1.0
240	0.5	0.4	0.9

3.0 Mathematical Concepts / Procedures

To analyse the cooling models of the chosen containers, models were created using technology such as Excel in which line graphs depicted the decreasing exponential function modelled by the formula previously stated.

3.1 Exponential Formula

The three exponentials depicting the cooling of the Paper, Ceramic and Metal cup are modelled by the following equation: $T_0 e^{kt}$ where T_0 is the starting temperature and k is the cooling constant which when substituted is a negative due to the decreasing nature of the Models.

2

4.0 Mathematical Models

Cooling Constants for Three Mathematical Models:

Paper Cup Cooling Constant

$T = T_0e^{-kt}$ $at\ (0, 52)\ where\ x = t\ and\ y = T$
$52 = T_0e^{-k(o)} \rightarrow 52 = T_0e^0 \rightarrow 52 = T_0 \times 1 \rightarrow \boldsymbol{T^0 = 52}$
Create Two Simultaneous Equations with points from Data
$T = 52e^{-kt}$
1) $(30, 24.8)$
2) $(60, 13.7)$
$Equation\ 1: 24.8 = 52e^{-k(30)} \rightarrow 24.8 = 52e^{-30k}$
$Equation\ 2: 13.7 = 52e^{-k(60)} \rightarrow 13.7 = 52e^{-60k}$
$$\frac{Equation 1}{Equation\ 2}$$
$$\frac{24.8 = 52e^{-30k}}{13.7 = 52e^{-60k}} \rightarrow 1.810218978 = e^{-30k+60k} \rightarrow 1.810218978 = e^{30k}$$
$\log 1.810218978 = \log e^{30k} = \log 1.810218978 = 30k \log e$
$$k = \frac{\log 1.810218978}{30 \log e}$$
$= 0.01978159401 \rightarrow Cooling\ Constant\ of\ Paper\ Cup$

Ceramic Cup Cooling Constant

$T = T_0e^{-kt}$ $at\ (0, 52)\ where\ x = t\ and\ y = T$
$52 = T_0e^{-k(o)} \rightarrow 52 = T_0e^0 \rightarrow 52 = T_0 \times 1 \rightarrow \boldsymbol{T^0 = 52}$
Create Two Simultaneous Equations with points from Data
$T = 52e^{-kt}$
1) $(30, 22.2)$
2) $(60, 11.9)$
$Equation\ 1: 22.2 = 52e^{-k(30)} \rightarrow 22.2 = 52e^{-30k}$
$Equation\ 2: 11.9 = 52e^{-k(60)} \rightarrow 11.9 = 52e^{-60k}$
$$\frac{Equation 1}{Equation\ 2}$$
$$\frac{22.2 = 52e^{-30k}}{11.9 = 52e^{-60k}} \rightarrow 1.865546218 = e^{-30k+60k} \rightarrow 1.865546218 = e^{30k}$$
$\log 1.865546218 = \log e^{30k} = \log 1.865546218 = 30k \log e$
$$k = \frac{\log 1.865546218}{30 \log e}$$
$= 0.02078512962 \rightarrow Cooling\ Constant\ of\ Ceramic\ Cup$

Metal Cup Cooling Constant

$T = T_0e^{-kt}$ $at\ (0, 55)\ where\ x = t\ and\ y = T$
$55 = T_0e^{-k(o)} \rightarrow 55 = T_0e^0 \rightarrow 55 = T_0 \times 1 \rightarrow \boldsymbol{T^0 = 55}$
Create Two Simultaneous Equations with points from Data
$T = 55e^{-kt}$
1) $(30, 26.5)$
2) $(60, 15.9)$
$Equation\ 1: 26.5 = 55e^{-k(30)} \rightarrow 26.5 = 55e^{-30k}$
$Equation\ 2: 15.9 = 55e^{-k(60)} \rightarrow 15.9 = 55e^{-60k}$
$$\frac{Equation 1}{Equation\ 2}$$

$$\frac{26.5 = 55e^{-30k}}{15.9 = 55e^{-60k}} \rightarrow 1.666666667 = e^{-30k+60k} \rightarrow 1.666666667 = e^{30k}$$

$$\log 1.666666667 = \log e^{30k} = \log 1.666666667 = 30k \log e$$

$$k = \frac{\log 1.666666667}{30 \log e}$$

$$= 0.0170275208 \rightarrow Cooling\ Constant\ of\ Metal\ Cup$$

Cooling Constants:

Paper Cup: 0.01978159401

Ceramic Cup: 0.02078512962

Metal Cup: 0.0170275208

Algebraically Determined Functions for Each Model:

Paper: $52.0e^{-0.0197819401}$

Ceramic: $52.0e^{-0.02078512962}$

Metal: $55.0e^{-0.0170275208}$

Note: Ceramic cools fastest followed by paper then metal cup.

Integration:

Paper Cup:

5-25: $\int_5^{25} 52.0\, e^{-0.01978194010t}\, dt = 778.03\ units^2$

5-45: $\int_5^{45} 52.0\, e^{-0.01978194010t}\, dt = 1301.84\ units^2$

5-90: $\int_5^{90} 52.0\, e^{-0.01978194010t}\, dt = 1938\ units^2$

Total Area: $\int_0^{240} 52.0\, e^{-0.01978194010t}\, dt = 2605.91 units^2$

Ceramic Cup:

5-25: $\int_5^{25} 52.0e^{-0.02078512962t}\, dt = 766.93\ units^2$

5-45: $\int_5^{45} 52.0e^{-0.02078512962t}\, dt = 1273\ units^2$

5-90: $\int_5^{90} 52.0e^{-0.02078512962t}\, dt = 1869.51\ units^2$

Total Area: $\int_0^{240} 52.0e^{-0.02078512962t}\, dt = 2484.74\ units^2$

Metal Cup:

5-25: $\int_5^{25} 55.0e^{-0.0170275208t}\, dt = 856.18\ units^2$

5-45: $\int_5^{45} 55.0e^{-0.0170275208t}\, dt = 1465.25\ units^2$

5-90: $\int_5^{90} 55.0e^{-0.0170275208t}\, dt = 2268.75\ units^2$

Total Area: $\int_0^{240} 55.0e^{-0.0170275208t}\, dt = 3175.81\ units^2$

Note: Ceramic cools fastest followed by paper then metal cup.

Trapezoids:

$$\frac{1}{2}(a+b)\times h = A\ units^2 \quad Where\ a = Side\ 1, b = Side\ 2\ and\ h = 5$$

Paper Cup:

(5-25) a-b where a = y at x= Time ; a=47.1 at x=5

5-10 $a = 47.1\,;\ b = 42.7\,;\ A^2 = \frac{1}{2}(47.1 + 42.7) \times 5 = 224.5$

10-15 $a = 42.7\,;\ b = 38.6\,;\ B^2 = \frac{1}{2}(42.7 + 38.6) \times 5 = 203.25$

15-20 $a = 38.6\,;\ b = 35\,;\ C^2 = \frac{1}{2}(38.6 + 35) \times 5 = 184$

20-25 $a = 35\,;\ b = 31.7\,;\ D^2 = \frac{1}{2}(35 + 31.7) \times 5 = 166.75$

TOTAL = $(A^2 + B^2 + C^2 + D^2) = 224.6 + 203.25 + 184 + 166.75 = 778.5\ units^2$

(5-45) a-b where a=y at x=Time

5-25 $TOTAL = 778.5$

25-30 $a = 31.7\,;b = 28.7\,;\ E^2 = \frac{1}{2}(31.7 + 28.7) \times 5 = 151$

30-35 $a = 28.7\,;\ b = 26\,;\ F^2 = \frac{1}{2}(28.7 + 26) \times 5 = 136.75$

35-40 $a = 26\ ;\ b = 23.6\ ;\ G^2 = \frac{1}{2}(26 + 23.6) \times 5 = 124$

45-40 $a = 23.6\ ;\ b = 21.4\ ;\ H^2 = \frac{1}{2}(23.6 + 21.4) \times 5 = 112.5$

TOTAL = $(5-25)^2 + E^2 + F^2 + G^2 + H^2 = 778.5 + 151 + 136.75 + 124 + 112.5 = 1302.75units^2$

(5-90) a-b where a=y at x=time

5-45 $TOTAL = 1302.75$

45-50 $a = 21.4\ ;\ b = 19.3\ ;\ I^2 = \frac{1}{2}(21.4 + 19.3) \times 5 = 101.7$

50-55 $a = 19.3\ ;\ b = 17.5\ ;\ J^2 = \frac{1}{2}(19.3 + 17.5) \times 5 = 92$

55-60 $a = 17.5\ ;\ b = 15.9\ ;\ K^2 = \frac{1}{2}(17.5 + 15.9) \times 5 = 83.5$

60-65 $a = 15.9\ ;\ b = 14.4\ ;\ L^2 = \frac{1}{2}(15.9 + 14.4) \times 5 = 75.75$

65-70 $a = 14.4\ ;\ b = 13.0\ ;\ M^2 = \frac{1}{2}(14.4 + 13.0) \times 5 = 68.5$

70-75 $a = 13.0\ ;\ b = 11.8\ ;\ N^2 = \frac{1}{2}(13.0 + 11.8) \times 5 = 62$

75-80 $a = 11.8\ ;\ b = 10.7\ ;\ O^2 = \frac{1}{2}(11.8 + 10.7) \times 5 = 56.25$

80-85 $a = 10.7\ ;\ b = 9.7\ ;\ P^2 = \frac{1}{2}(10.7 + 9.7) \times 5 = 51$

85-90 $a = 9.7\ ;\ b = 8.8\ ;\ Q^2 = \frac{1}{2}(9.7 + 8.8) \times 5 = 46.25$

TOTAL (5-90) = $(5-45)^2 + I^2 + J^2 + K^2 + L^2 + M^2 + N^2 + O^2 + P^2 + Q^2$

$= 1302.75 + (101.7 + 92 + 83.5 + 75.5 + 68.5 + 62 + 56.25 + 51 + 46.25 =$

$1939.7\ units^2$

Ceramic Cup Trapezoids:

(5-25) a-b where a = y at x= Time ; a=47.1 at x=5

5

10-15 $a = 46.4\,;\ b = 42.6\,;\ B^2 = \frac{1}{2}(+) \times 5 = 222.5$

15-20 $a = 42.6\,;\ b = 39.1\,;\ C^2 = \frac{1}{2}(+) \times 5 = 204.25$

20-25 $a = 39.1\,;\ b = 35.9\,;\ D^2 = \frac{1}{2}(+) \times 5 = 187.5$

TOTAL = $(A^2 + B^2 + C^2 + D^2) = 242.25 + 222.5 + 204.25 + 187.5 = 856.5\ units^2$

(5-45) a-b where a=y at x=Time

5-25 $TOTAL = 856.5$

25-30 $a = 35.9\,; b = 33.0\,;\ E^2 = \frac{1}{2}(+) \times 5 = 172.25$

30-35 $a = 33.0\,;\ b = 30.3\,;\ F^2 = \frac{1}{2}(+) \times 5 = 158.25$

35-40 $a = 30.3\ ;\ b = 27.8\ ;\ G^2 = \frac{1}{2}(+) \times 5 = 145.25$

45-40 $a = 27.8\ ;\ b = 25.6\ ;\ H^2 = \frac{1}{2}(+) \times 5 = 133.5$

TOTAL = $(5-25)^2 + E^2 + F^2 + G^2 + H^2 = 856.5 + 172.25 + 158.25 + 145.25 + 133.5$

$1465.75\ units^2$

(5-90) a-b where a=y at x=time

5-45 $TOTAL = 1465.75$

45-50 $a = 25.6\,;\ b = 23.5\,;\ I^2 = \frac{1}{2}(25.6 + 23.5) \times 5 = 122.75$

50-55 $a = 23.5\ ;\ b = 21.6\ ;\ J^2 = \frac{1}{2}(23.5 + 21.6) \times 5 = 112.75$

55-60 $a = 21.6\ ;\ b = 19.8\ ;\ K^2 = \frac{1}{2}(21.6 + 19.8) \times 5 = 103.5$

60-65 $a = 19.8\ ;\ b = 18.2\ ;\ L^2 = \frac{1}{2}(19.8 + 18.2) \times 5 = 95$

65-70 $a = 18.2\ ;\ b = 16.7\ ;\ M^2 = \frac{1}{2}(18.2 + 16.7) \times 5 = 87.25$

70-75 $a = 16.7\ ;\ b = 15.3\ ;\ N^2 = \frac{1}{2}(16.7 + 15.3) \times 5 = 80$

75-80 $a = 15.3\ ;\ b = 14.1\ ;\ O^2 = \frac{1}{2}(15.3 + 14.1) \times 5 = 73.5$

80-85 $a = 14.1\ ;\ b = 12.9\ ;\ P^2 = \frac{1}{2}(14.1 + 12.9) \times 5 = 67.5$

85-90 $a = 12.9\ ;\ b = 11.9\ ;\ Q^2 = \frac{1}{2}(12.9 + 11.9) \times 5 = 62$

TOTAL (5-90) = $(5-45)^2 + I^2 + J^2 + K^2 + L^2 + M^2 + N^2 + O^2 + P^2 + Q^2$

$= 1495.75 + (122.75 + 112.75 + 103.5 + 95 + 87.25 + 80 + 73.5 + 67.5 + 62$

$= 2270\ units^2$

Note: Ceramic cools fastest followed by paper then metal cup.

Rate Of Change for the Algebraically determined models:

$$\frac{y_2 - y_1}{x_2 - x_1}$$

Intervals Data $Rate\ Of\ Change = \frac{y_2 - y_1}{x_1 - x_1} = m$

Paper Cup:

7

5-25 (5, 47.1) | (25,31.7) $Rate\ Of\ Change = m_1 = \frac{31.7-47.1}{25-5} = -0.77$

5-45 (5, 47.1) | (45, 21.4) $Rate\ Of\ Change = m_2 = \frac{21.4-47.1}{45-5} = -0.6425$

5-90 (5,47.1) | (90, 8.8) $Rate\ Of\ Change = m_3 = \frac{8.8-47.1}{45-5} = -0.45059$

Ceramic Cup:

5-25 (5, 46.9) | (25, 30.9) $Rate\ OfChange = m_1 = \frac{30.9-46.9}{25-5} = -\frac{4}{5} \approx -0.8$

5-45 (5, 46.9) | (45, 20.4) $Rate\ Of\ Change = m_2 = \frac{20.4-46.9}{45-5} = -0.6625$

5-90 (5, 46.9) | (90, 8.0) $Rate\ Of\ Change = m_3 = \frac{8.0-46.9}{90-5} = -0.45765$

Metal Cup:

5-25 (5, 50.5) | (25, 35.9) $Rate\ Of\ Change = m_1 = \frac{35.9-50.5}{25-5} = -0.73$

5-45 (5, 50.5) | (45, 25.6) $Rate\ Of\ Change = m_2 = \frac{25.6-50.5}{45-5} = -0.6225$

5-90 (5, 50.5) | (90, 11.9) $Rate\ Of\ Change = m_3 = \frac{11.9-50.5}{90-5} = -0.45412$

Note: Metal cup had the lowest rate of change of cooling for all cups in all three intervals, followed by paper cup then lastly ceramic cup.

5.0 Evaluation

5.1 Limitations

This report faced limitations as follows

- The scientific equipment accessed to complete this report was found been uncalibrated and not of newest standards, limiting the accuracy of the results gained. The testing equipment was also limited in numbers which impacted the ability for multiple tests to be ran at once, causing minor delay.
- Due to large collection of data, the results had to be limited to intervals of 5 minutes which does not accurately reflect fluctuations in the temperature over time and as a result gives less insight into the cooling of the cups.
- The boiling temperature of the water was varied for each cup due to environmental factors and physical properties out of human control such as the altitude the test was taken at.
- The water was boiled in an electric kitchen kettle that was found to be incapable of boiling higher than 80°C impacting the accuracy of results.

5.2 Strengths

- Different mathematical justifications that involve calculus which is the mathematics of change were applied in this report to in developing the solution. The diversity in methods help in seeking the most consistent cup from different processes and procedures.
- Use of data derived from an experiment which was not biased or influenced by myself. The data provided the opportunity to present a unique solution to the problem.

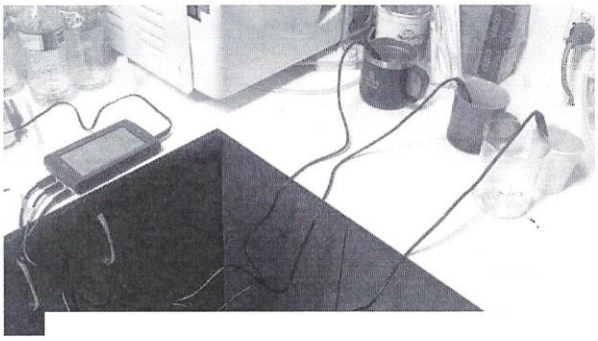
6.0 Conclusion

All mathematics justifications used in developing this solution identified that ceramic cools faster followed by paper then metal cup. This solution seeks to identify and justify the type of cup to be allocated to three categories of customers which are those who want to take their coffee very fast in a very short time then those who want to take coffee in a fair amount of time then lastly those who will take the coffee very slow than every other category. This report can conclude that ceramic will be ideal for customers who want to take coffee fastest followed by paper cup and then metal for those who will take the longest time.

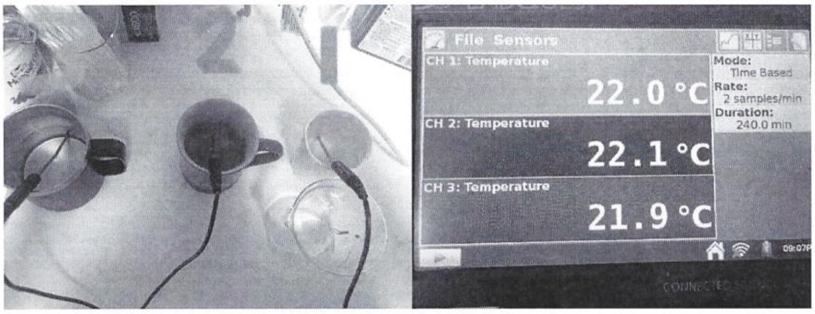
Appendix

1.0 Photographs

1.1 Experiment Layout with Vernier data logger and three chosen cups.



1.2 Cooling Model – Order of Cups



Picture 2 - Order of Cups both physically and on the data logger

Appendix F: Survey instrument.

Feedback Activity 1: Planning Framework (content sequencing)-Mathematics
To help us improve this framework, please give us feedback on the following:

	Strongly Disagree	Disagree	Not Sure	Agree	Strongly Agree
Collaborative content sequencing is a critical component of mathematics planning and teaching as it provides a clear link between relevant and significant assumed prior knowledge and corresponding new knowledge.	1	2	3	4	5
Collaborative content sequencing helps teachers to have a deeper understanding of concepts to be taught.					
Collaborative content sequencing enhances teachers content knowledge.					
Content sequencing places assumed prior knowledge, skills and conceptual connections at the centre of mathematics knowledge development.					
Content sequencing helps teachers in hypothesising effective delivery methods.					
Collaborative content sequencing reinforces teachers' responsibility of effective teaching of mathematics concepts at every level.					
Collaborative content sequencing fosters a common agenda of focusing on how students develop mathematical knowledge.					
Collaborative content sequencing makes mathematics teaching a collective responsibility as students understanding and participation at higher levels depend on lower levels.					

(Write in the space provided)

1. How can we enhance collaborative mathematics planning in schools?

2. How do you identify assumed knowledge critical to new knowledge?

3. How do you identify essential concepts (concepts that students must return in a topic)?

4. How do you sequence content in a Unit?

5. Was the rationale for this framework realised?

Feedback Activity 2: Teaching and learning resources- Mathematics

	Strongly Disagree	**Disagree**	**Not Sure**	**Agree**	**Strongly Agree**
Reteaching prior knowledge to students operating at low levels enhance students' participation and achievement	1	2	3	4	5
Effective mathematics teaching involves linking prior knowledge and vocabulary to new knowledge.					
Visual representation of mathematical knowledge enhances teaching and learning of mathematics.					
Procedural flowcharts (showing steps and procedures) plays an important role in developing students' mathematical skills.					
Procedural flowcharts promote fluency and recall.					
Procedural flowcharts can be used to highlight critical vocabulary					
Procedural maps are a reference resources that can also be used for revision.					
Procedural flowcharts focus on students learning.					
Procedural flowcharts promote independent or collaborative learning.					
Procedural flowcharts can help evaluate or give feedback to students on their understanding and correct use of a procedure.					

Use of procedural flowcharts can help students identify relationships in mathematical concepts.					
Concept maps help students understand how mathematical concepts are related.					
Student or teacher developed concept maps can be used to link prior knowledge to new knowledge.					
Concept maps facilitate consolidation of learning.					
Concept maps facilitate a visual evaluation of students learning.					
Concept maps give an overview of a topic.					
Concept maps helps identify key concepts in a topic.					
Concept maps promote integration of concepts that deepen mathematical understanding.					
The hierarchical nature of mathematics make concept mapping central to teaching and learning of mathematics.					

(Write in the space provided)

1. How do you activate your students' prior knowledge in your class?

2. How have you used procedural flow charts to teach steps/procedures and skills in your teaching?

3. How have you used concept maps to link concepts in your teaching?

4. What can be improved on this framework?

5. Was the rationale for this framework realised?

Appendix G: Semi structured interview questions

Semi structured Interview Questions

Planning Framework (Content sequencing)

1. How would you define/describe collaborative mathematics planning in your school?
2. What informs content sequencing as you go through planning?
3. How would/did the framework that is being proposed enhance content sequencing at your school?

Teaching and learning resources

1. How do you teach your students mathematics procedural knowledge (knowledge of procedures—steps to take to accomplish a goal)?
2. How would/did procedural flowcharts enhance the teaching of mathematics procedural knowledge?
3. How do you teach your students mathematics conceptual knowledge (knowledge of concepts or principles)?
4. How would/did conceptual maps enhance the teaching of mathematics conceptual knowledge?

Appendix H: Information Sheet for Principals

INFORMATION SHEET FOR PRINCIPALS

PROJECT TITLE: An Investigation into supporting the teaching of calculus-based senior mathematics in Queensland.

The study is being conducted by David Chinofunga and will contribute to the research project in Doctor of Philosophy (Research) at James Cook University.

As a Principal of a school, if you agree to be involved in the study, you will be asked to give permission to the researcher to collect data from senior Mathematics teachers timetabled to teach Year 11 and/or 12. The data will be collected through surveys that will comprise of rating scale, open answer questions and interviews for selected teachers.

Your school taking part in this study is voluntary, and it can stop taking part in the study at any time without explanation or prejudice.

Your teachers' responses and contact details will be strictly confidential. The data from the interview will be used in research publications. You will not be identified in any way in these publications.

If you have any questions about the study, please contact – **David Chinofunga, Philemon Chigeza or Subhashni Taylor.**

Principal Investigator:

David Chinofunga

PhD Candidate

College of Arts, Society and Education

James Cook University

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James Cook University

Phone:

Email :subhashni.taylor@jcu.edu.au

If you need counselling because of this research project, please contact:

JCU Counselling Service

Office hours: 9:00am - 4:00pm

Phone:

Level 1, Building B1 (Library)

Headspace

2/42 Grafton St, Cairns City QLD 4870

Phone:

If you have any concerns regarding the ethical conduct of the study, please contact:

Human Ethics, Research Office

James Cook University, Townsville, Qld, 4811

Phone: (07) 4781 5011 (ethics@jcu.edu.au)

Appendix I: Information Sheet for Teachers

INFORMATION SHEET FOR TEACHERS

PROJECT TITLE: An Investigation into supporting the teaching of calculus-based senior mathematics in Queensland.

As a time-tabled senior mathematics teacher, you are invited to take part in a study being conducted by David Chinofunga and will contribute to the research project in Doctor of Philosophy (Research) at James Cook University.

Your school principal has approved your participation, if you agree to be involved in the study, you will be invited to complete an initial survey and interview based on planning, teaching and learning in Mathematics. You will then be invited to take part in a webinar that will introduce and provide training in using a framework and associated pedagogical resources. The framework and associated pedagogical resources are a set of planning documents, teaching and learning resources and associated examples. A second survey and interview will be carried out after you have interacted with the framework and associated pedagogical resources. The surveys will be comprised of rating scales and open answer questions. The two surveys and interviews combined will take approximately 20 minutes and 60minutes respectively, while the webinar will take only 30minutes to complete.

Taking part in this study is voluntary, and you can stop taking part in the study at any time without explanation or prejudice.

Your responses and contact details will be kept strictly confidential as the survey responses will be given pseudonym names and will not be shared by your school. The data from the study will be used in research publications. You will not be identified in any way in these publications.

If you have any questions about the study, please contact – **David Chinofunga, Philemon Chigeza or Subhashni Taylor.**

Principal Investigator:	**Primary Supervisor:**	**Secondary Supervisor:**
David Chinofunga	Dr Philemon Chigeza	Dr Subhashni Taylor
DEd Candidate	College of Arts, Society and	College of Arts, Society and
College of Arts, Society and	Education	Education
Education	James Cook University	James Cook University
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Email: david.chinofunga@my.jcu.edu.au		

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Phone: (07) 4781 5011 (ethics@jcu.edu.au)

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Appendix K: Information sheet for Students

INFORMATION SHEET FOR STUDENTS

PROJECT TITLE: An Investigation into supporting the teaching of calculus-based senior mathematics in Queensland.

The study is being conducted by David Chinofunga and will contribute to a research project in Doctor of Philosophy (Research) at James Cook University. The study focuses on the teaching of calculus-based senior mathematics and your mathematics teacher has agreed to participate.

With your permission, your teacher would like to share artefacts that you have produced while engaging with the pedagogical resources developed in this study. If you agree for your artefact to be included in the study, you will be invited to sign a consent form.

Releasing your artefact for this study is voluntary, and you can withdraw your consent at any time without explanation or prejudice. Whether or not you participate will not affect your relationship with your teacher in any way.

Your responses and contact details will be strictly confidential. The artefact will be used in research publications such as a thesis and journal articles. You will not be identified in any way in these publications.

If you have any questions about the study, please contact – **David Chinofunga, Philemon Chigeza or Subhashni Taylor.**

Principal Investigator:	**Primary Supervisor:**	**Secondary Supervisor:**
David Chinofunga	Dr Philemon Chigeza	Dr Subhashni Taylor
PhD Candidate	College of Arts, Society and Education	College of Arts, Society and Education
College of Arts, Society and Education	James Cook University	James Cook University
James Cook University	Phone:	Phone:
Phone:	Email:philemon.chigeza@jcu.edu.au	Email :subhashni.taylor@jcu.edu.au
Email: david.chinofunga@my.jcu.edu.au		

If you need counselling because of this research project, please contact:

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