A Framework for Content Sequencing from Junior to Senior Mathematics Curriculum

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Abstract
This paper argues for effective sequencing of mathematics content to aid transition from junior (Year 7 to Year 10) to senior mathematics (Year 11 to Year 12) curriculum in Queensland, Australia and provides a tool for sequencing the mathematics content. Planning templates and samples are available to schools; however, it is imperative for teachers to understand the processes that underpin planning. This paper provides a step-by-step systematic sequencing of mathematics concepts. The premise is that depending on the level of assumed prior knowledge and skills students recall and apply, teachers can start teaching from any level. The study draws from constructivism to develop a planning tool that can be adapted to all mathematics subjects and levels, help identify conceptual relationships and skills from lower to upper levels and provide students with the opportunity to build their mathematical knowledge.

Keywords: collaborative planning, mathematics content sequencing, functions and graphs, secondary school mathematics, content break down, concepts

INTRODUCTION

According to Roche et al. (2014, p. 854),

"Given the complexity of mathematics teaching, including addressing curriculum goals, engaging students, catering for the diversity of readiness, connecting mathematics teaching to students' experience, and assessing student learning, to name just a few issues, it is difficult to imagine that teachers of mathematics can perform their role without substantial planning."

Effective planning provides direction and resources for quality curriculum delivery, particularly in the context of mathematics teaching. Furthermore, planning links curriculum requirements in official curriculum documents and commercial and non-commercial resources to how knowledge is developed in class (Li et al., 2009). This paper argues for and provides a tool for understanding and engaging in collaborative planning for effective sequencing of mathematics content for the transition from the *Australian Mathematics Curriculum* (Prep-Year 10) to the *Senior Queensland Mathematical Curriculum* (Year 11-Year 12) (Queensland Curriculum and Assessment Authority [QCAA], 2018)). The *mathematical methods unit 1 on functions* that is taught in Year 11 is used as an example to illustrate the tool.

Planning plays a critical role in enacting the curriculum as it involves "activities related to knowing what to teach and how" (Fernandez & Cannon, 2005, p. 485). What and how teachers teach is critical to students' participation and achievement. Roche et al. (2014, p. 854) noted, as follows:

"Planning for mathematics teaching is important at all levels from sequencing of content and the structuring of lessons to the selection and preparation of manipulatives and worksheets but despite its centrality to curriculum delivery research-based descriptions of the practices of effective mathematics teachers do not emphasize planning."

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Contribution to the literature

- The study argues for collaborative planning and content sequencing in the planning and teaching of mathematics to aid transition from junior to senior mathematics.
- The study develops an original mathematics planning framework on content sequencing that impresses the importance of prior knowledge and the hierarchical nature of mathematics.
- The planning framework is adaptable to different mathematics subjects and levels; and provides opportunity to build mathematical knowledge from familiar to unfamiliar contexts.

Teacher planning directly influences the quality of learning that students receive (González et al., 2020; Grundén, 2020; Li et al., 2009; Roche et al., 2014). For teachers, "planning is seen as an essential part of their work that has consequences for students' learning as well as work situation–planning can cause stress as well as be a way to reduce stress" (Grundén, 2020, p. 80). In fact, planning should focus on improving students' relationship with mathematics through providing a platform that promotes active engagement (Grundén, 2020). Planning is the foundation that sustains the whole curriculum implementation, as it makes a difference in every aspect of curriculum delivery, and consequently contributes to determining teacher quality.

An effective mathematics teacher must be an exceptional planner. "Excellent teachers of mathematics plan for coherently organized learning experiences that have the flexibility to allow for spontaneous, self-directed learning" [Australian Association of Mathematics Teachers (AAMT, 2006, p. 4)].

Australian teachers are expected to plan and teach "mathematical sequences and experiences that encourage students to think flexibly and creatively about concepts to develop 'big picture' thinking" (Davidson, 2019, p. 8). Similarly, the Australian Institute of Teaching and School Leadership (AITSL) expects teachers to design a teaching and learning sequence using curriculum knowledge, content, students' learning strategies, and teaching pedagogies to increase student participation and achievement (AITSL, 2014). This is because, during planning, teachers predict and plan the structure and conditions of the learning space (Munthe & Conway, 2017). Consequently, to ensure that no child is left behind in learning mathematics, planning must be the first port of call.

Supporting the current teachers' planning practices can be a starting point (Sullivan et al., 2013). However, ways of improving the current planning in schools must be explored if teaching and learning is to be enhanced (Attard, 2012). "The curriculum that students experience in classrooms is the product of a complex web of decision-making which is shaped, but not determined, by the formal curriculum documentation" (Sullivan et al., 2013, p. 459). Therefore, curriculum planners such as teachers need to be supported on how to select and organize the crux of the curriculum (O'Neill et al., 2014). Mathematics teachers' understanding of the structure of the subject and how best content can be presented for maximum student participation can be key to effective planning and consequently teaching and learning.

A critical aspect of effective planning is identifying and sequencing content and delivery strategies to optimize acquisition of knowledge, understanding and skills among students (QCAA, 2019). Content sequencing influences student engagement and helps them to develop mathematical knowledge (Kilpatrick et al., 2001). The 'what' of planning informs the 'how', thus teacher effectiveness and learner participation and understanding is not only limited to classroom practice, but how the content is planned, sequenced, and taught.

Collaborative Planning

This study draws from intentional collaboration of teachers as defined by the Queensland Department of Education (DoE). "Providing time and resources for staff to develop and plan units together was suggested as a way of deepening understanding of the Australian Curriculum" (DoE, 2021a, p. 7). Nevertheless, how teachers interrelate during collaboration and how they interpret the curriculum has a strong influence on the planning process (Grundén, 2020). Since teachers enact the curriculum, there is a strong correlation between curriculum planning and delivery material (Superfine, 2008).

Indeed, the National Council of Teachers of Mathematics (NCTM) state that: "Effective mathematics teaching begins with a shared understanding among teachers of the mathematics that students are learning and how this mathematics develops along learning progressions" (NCTM, 2014, p. 12). As a result, the level of engagement among teachers during planning influences the quality of the output (Bieda et al., 2020). This study will advocate for a collaborative approach to planning guided by a proposed tool.

Collaborative planning is not only limited to teachers teaching a year level but all mathematics teachers within or across schools. Many teachers look to each other for support to improve and enhance their planning. Thus, school leaders must ensure that collaborative meetings are scheduled for teachers to review and share their experiences and expertise (Clarke et al., 2012). Collaborative planning can present opportunities for teachers to learn from each other, which results in the benefit of students (Gilbert & Gilbert, 2013). Especially,

"when whole grade levels are involved, they create a critical mass for changed instruction at all levels; above all teachers serve as support groups for one another in improving practice" (Darling-Hammond & Richardson, 2009, p. 46). Collaborative professional learning brings teachers to work together resulting in improvements to the whole school system rather than just classroom or grade level improvement (Darling-Hammond et al., 2009). Research also indicates that effective professional learning is a contributing factor in differences in school performance (Darling-Hammond et al., 2009). Moreover, professional collaboration improves planning practice and teacher quality as teachers get an opportunity to discuss, share and document important aspects of teaching and learning (Tricoglus, 2000).

Mathematics planning must support effective teaching and learning at every year level to ensure students' success. Scholars (Kafyulilo, 2013; Konuk, 2018; Lynch, 2017; Schuhl, 2020; Usha, 2010; Voogt et al., 2016) clearly noted that when mathematics planning is done collaboratively:

1. It reminds teachers that all levels/grades play a critical role in developing mathematical knowledge.
2. Teachers are reminded that skills taught at every level/grade are applicable to subsequent levels.
3. It reinforces the notion that mathematical concepts are interlinked.
4. Teachers develop a sense of ownership of the product.
5. It enhances teachers' pedagogical and content knowledge.
6. It brings consistence across year levels.
7. It develops individual and team collective teacher efficacy.
8. It ensures consistent curricular priorities among colleagues.
9. It ensures students learn identified essential mathematics standards.
10. It enhances students learning.
11. Teachers realize teaching is a shared responsibility.
12. It enhances the sense of community and revitalizes enthusiasm towards teaching.
13. Teachers might consider issues that might not have been considered independently.

Linking concepts across year levels demonstrates the hierarchical nature of mathematics and shows that every mathematics teacher at different year levels contribute to building students' mathematical knowledge. It also justifies the importance of collaborative planning within the cohort. Furthermore, students grasp that active participation in lower grades contributes towards success in mathematics at higher levels.

MATHEMATICS PLANNING IN QUEENSLAND

Queensland mathematics teachers have a range of resources at their disposal during planning. Apart from official curriculum documents provided by QCAA, non-official resources that are commercial or non-commercial in nature like textbooks, resources developed by colleagues or mathematics educators' associations and school documents play an important role in planning, delivery and assessment (Roche et al., 2014; Sullivan et al., 2013). Also, web-based resources have grown in influence and use, especially multimedia video resources like YouTube and Khan Academy as they are readily available. The diversity of available resources provides dynamic options to teachers as they can be useful in improving the quality of planning, be it individually or collaboratively.

Queensland schools and teachers are the drivers of the planning process. Undoubtedly, this is important because "curriculum planning is essential for contextualizing curriculum content" (QCAA, 2019, p. 1). Thus, different schools can contextualize content according to students' experiences which might not be shared across schools (Demski & Racherbäumer, 2017). Roche's (2014) findings indicate that planning documents produced by teachers within or across schools vary, with some teachers valuing aspects of planning that others do not. Planning templates and samples from the federal DoE and QCAA have been developed and distributed to schools. However, it is important for teachers to understand the processes that underpin the planning decisions that have led to the creation of such documents (Roche, 2014). Therefore, a guiding tool is necessary to bring consistency and uniformity to the process of planning. Ultimately, we propose a more relational and contextual planning tool underpinned by constructivism that provides a step-by-step systematic sequencing of curriculum content to promote interlinking, coherence and spiraling of mathematics concepts between lower- level and upper-level topics. Constructivism positions learning as a process of building new knowledge from the learner's prior knowledge, beliefs and skills (Garbett, 2011).

Queensland mathematics teachers, as part of planning, are required to create a school specific sequence of content, as the official syllabus document is not regarded as a teaching sequence (Roche et al., 2014; QCAA, 2014, p. 8) which suggests that schools must take responsibility for developing "a spiraling and integrated sequence." Clearly, spiral sequencing deepens knowledge through revisiting concepts, building on previous knowledge, creating new knowledge using prior knowledge and dealing with increased conceptual complexity as learning progresses (Harden, 1999). Above all, the manner in which content is structured in the curriculum facilitates how students learn and

understand complex phenomenon (Bruner, 1977). For example, students are taught fundamental concepts at a lower level of schooling then the concepts are revisited at a higher level to deepen understanding through application, comprehension and interconnections with other concepts.

Queensland schools classify long term planning into three levels: firstly, whole school curriculum and assessment plan, secondly, year level curriculum and assessment plan, and lastly unit overviews (QCAA, 2019). A unit is "a sequence of lessons with a coherent focus, sometimes referred to as a topic sequence" (Roche et al., 2014, p. 854). Whole school curriculum plan "shows learning sequence within and across the year levels" while year level plan "outlines the sequence of learning and reflects the development of knowledge, understanding and skills within a level" and unit overview "links prior and future learning" (QCAA, 2019, p. 3-4). Each level of planning informs the other. Thus, effective planning at all levels has the potential to improve curriculum delivery in Queensland schools.

The Queensland State Schools Improvement Strategy (2021-2025) mentions intentional collaboration as an improvement focus on curriculum delivery. It is defined as "the deliberate actions we take to work together, learn together and improve together" (DoE, 2020, p. 1). Schools have the responsibility to implement the strategy document thus requiring them to put in place mechanisms of collaboration among teachers. It is common practice in education departments the world over to allocate planning time for teachers as a means of enhancing curriculum delivery and student learning (Li et al., 2009). Queensland teachers are allocated five professional collaboration days which are not only limited to planning in subject areas but other activities that the profession demands. Professional collaboration days at the beginning of the year provide an opportunity for long term planning. However, for secondary full-time teachers, an additional 210 minutes a week is also allocated for planning, such as short-term individual planning, preparation, correction and administrative work (Queensland Teachers' Union, 2020). In addition, schools are encouraged to set aside staff curriculum meetings, which might involve all teachers or a sector.

Enhancing Student Participation and Understanding Through Planning

Focusing planning on how students develop mathematical knowledge, skills and understanding enhances participation, as teaching becomes student centered (Grundén, 2020). Therefore, planning should be informed by hypothesizing students' current level of understanding and how to develop it further (Simon, 1995). It is important during planning for teachers to be mindful of students' abilities and learning needs, the goal being for all students to participate and optimally engage (Attard, 2012). As a result, planning that focuses on student learning indirectly develops teachers' pedagogy, content knowledge and practice (Darling-Hammond & Richardson, 2009; Garet et al., 2001; Smith 2007). Student focused planning also enhances student understanding as it anticipates the learning process.

In enacting the curriculum, teachers have the responsibility to identify key topics and provide students with the opportunity to deepen their understanding of such topics (ACARA, 2009). Similarly, QCAA (2013, p. 1) emphasized, as follows:

> "To support the development of complexity and independence of student learning, when planning units of work for a course of study, teachers should consider a range of designing opportunities together with the sequencing, content and interrelatedness of teaching strategies and learning experiences."

Content that is coherently planned provides students with an opportunity to deepen their mathematical knowledge, understanding and skills if they understand the fundamental concepts.

Planning for student understanding focuses on how students develop mathematical knowledge. Procedural knowledge, conceptual knowledge and procedural flexibility is critical for students' development of mathematical knowledge and competency (Rittle-Johnson, 2017). Firstly, procedural knowledge is defined as knowledge of sequences of steps or operations, mathematical rules and facts that can be used to solve problems (Crooks & Alibali, 2014; Rittle-Johnson et al., 2015). Secondly, conceptual knowledge is the "comprehension of mathematical concepts, operations, and relations" (Kilpatrick et al., 2001, p. 5). Thirdly, procedural flexibility involves knowledge and use of varied procedures and the robust application of these to a variety of conditions (Rittle-Johnson & Star, 2007). Conceptual knowledge also plays an important role in flexible problem solving because understanding the conceptual foundations of a procedure will lead to generalizations when confronted with new related problems. The relationship between conceptual and procedural knowledge is bidirectional as they both support the development of the other. However, both rely on students' prior knowledge as a foundation to build from.

Planning that builds on prerequisites helps a teacher to identify gaps in student understanding that are likely to be encountered in class (Reys et al., 2020). A significant number of teachers administer diagnostic tests and studies support the practice as they may stimulate interest in learning and decode forthcoming lessons (John et al., 2013). At the same time diagnostic tests help the teacher to gain understanding of students' prior knowledge, understanding and skills since in most cases

students may be at different levels. However, simply checking prior knowledge is insufficient as teachers must also ensure that planning provides every student with the opportunity to acquire prior knowledge that is critical to engage with new knowledge meaningfully. A comprehensive sequence of learning provides flexibility in a class because students can start from varying levels of competence. For this reason, during sequencing of content an ideal tool must develop a system of linking concepts and determine procedures that are involved in solving problems within a concept.

Content Sequencing in Unit 1 on Functions in the Mathematical Methods Subject

After Year 10 or 16-year-old students in Australia have the option to remain in school or seek vocational traineeships. Students who choose to proceed to senior secondary are expected to engage with a mathematics option of their choice. In the state of Queensland, students who plan on pursuing advanced mathematics are encouraged to engage with 10A curriculum at year 10. However, students who choose to pursue the general Year 10 curriculum can still enroll in advanced mathematics in senior school. The mathematics curriculum from primary school to Year 10 is governed by the Australian Curriculum while the Queensland curriculum, which is developed by QCAA, is followed at senior secondary level. In this paper, the Australian mathematics curriculum (P-10) and the QCAA mathematical methods curriculum documents were used to develop examples on how to apply the proposed tool.

For the purpose of this study, prior knowledge will be defined as prerequisite concepts from lower levels that interlink with concepts at upper levels. Accordingly, assumed prior knowledge is identified from the Australian Curriculum (P-10) that students have engaged with before entering senior secondary school. New knowledge is outlined in the mathematical methods syllabus. "To make decisions about the mathematical content in the planning process, teachers reflect and have considerations in relation to students' abilities and their prior knowledge" (Grundén, 2020, p. 78). Correspondingly, prior knowledge is important in developing quality programs and sequencing as it demonstrates continuity and reinforces the importance of fundamental concepts and structure of mathematics (Reys et al., 2020). The hierarchical nature of mathematics must be the basis of effective planning and classroom practice.

Learning in mathematics is sequential which means basic concepts presented in lower levels must be mastered to enhance the chances of understanding new knowledge (Brosvic & Epstein, 2007). Similarly, Hailikari and Nevgi (2010, p. 2082-2083) emphasize that "Concepts presented in the introductory courses are usually needed throughout the academic career and

should provide building blocks for more advanced courses in the same subject." During planning, teachers have the responsibility of identifying relationships between lower-level and upper-level topics, concepts, and skills, link the two levels and provide students with the opportunity to build from familiar to unfamiliar.

Creating a tool to support and improve existing planning practices is of critical importance (Superfine, 2008; Sullivan, 2012, 2013). Not only does a tool provide transparency, accountability and evaluation of the process by stakeholders (O'Neill et al., 2014) but also tools that are flexible can accommodate adjustments during implementation (Grundén, 2020). The proposed tool in Figure 1 will provide a step-by-step systematic sequencing of curriculum content to promote interlinking, coherence and spiraling of concepts. This will cater for mathematical methods students at every level of their mathematics journey in unit 1 at Year 11. Depending on the level of assumed prior knowledge and skills students can recall and apply, teachers can start teaching from any level of sequenced content. The tool can be adapted to all mathematics options and levels although for the purpose of this study, Queensland mathematical methods unit 1 will be considered.

The foundation of the tool is coherence of content so that students can construct new knowledge from assumed prior knowledge. Schuhl (2020) and Usha (2010) argued that for coherence of content to be mastered, mathematics teachers are to be guided by the following questions during collaborative planning:

1. What exactly do students need to know and be able to do in this unit?

2. What prerequisite conceptual understanding and skills fluency are required for all students to effectively learn new knowledge?

3. How do the concepts identify as prior knowledge link with new knowledge?

4. What do we expect students to retain?

Collaborative tackling of these questions provides equity and consistency to students' learning experiences from one teacher/class/level to the next (Schuhl, 2020). As a result, "student learning improves because your entire team is working to ensure each student learns the organized mathematics content from one concept to the next" (Schuhl, 2020, p. 13). The four questions will guide the collaborative tool on concept sequencing being applied on unit 1 of mathematical methods option discussed below.

MATHEMATICAL METHODS UNIT 1 FUNCTIONS AND GRAPHS

Unit 1

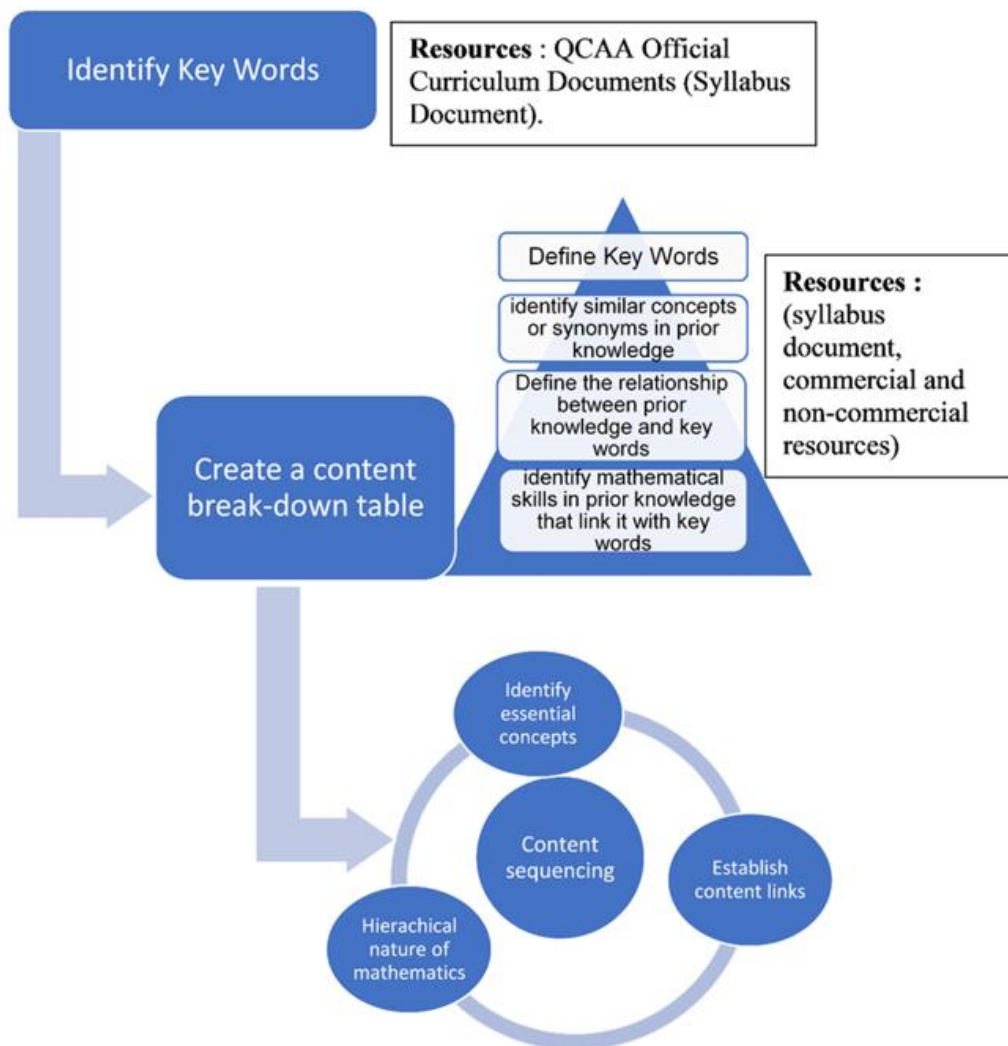Firstly, identify key words from the syllabus document.

Figure 1. Diagrammatic representation of the tool on content sequencing

Functions

In this sub-topic, students will

1. understand the concept of a relation as a mapping between sets, a graph and as a rule or a formula that defines one variable quantity in terms of another.

2. recognize the distinction between functions and relations and use the vertical line test to determine whether a relation is a function.

3. use function notation, domain and range, and independent and dependent variables.

4. examine transformations of the graphs of $f(x)$, including dilations and reflections, and the graphs of $y = af(x)$ and $y = f(bx)$, translations, and the graphs of $y = f(x + c)$ and $y = f(x) + d$; $a, b, c, d \in R$.

5. recognize and use piece-wise functions as a combination of multiple sub-functions with restricted domains.

6. identify contexts suitable for modelling piece-wise functions and use them to solve practical problems (taxation, taxis, the changing velocity of a parachutist).

Review of Quadratic Relationships

Recognize and determine features of the graphs of $y = x^2, y = ax^2 + bx + c, y = a(x - b)^2 + c$, and $y = a(x - b)(x - c)$, including their parabolic nature, turning points, axes of symmetry and intercepts.

Inverse Proportions

In this sub-topic, students will

1. examine examples of inverse proportion.

2. recognize features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{(x-b)}$, including their hyperbolic shapes, their intercepts, their asymptotes and behaviour as $x \to \infty$ and $x \to -\infty$.

Powers and Polynomials

In this sub-topic, students will

1. identify the coefficients and the degree of a polynomial.

2. expand quadratic and cubic polynomials from factors.

3. recognize and determine features of the graphs of $y = x^3, y = a(x - b)^3 + c$ and $y = k(x - a)(x - b)(x - c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$.

4. use the factor theorem to factorize cubic polynomials in cases where a linear factor is easily obtained.

5. solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained.

6. recognize and determine features of the graphs $y = a(x - b)^4 + c$, including shape and behavior.

7. solve equations involving combinations of the functions above, using technology where appropriate.

Graphs of Relations

In this sub-topic, students will

1. recognize and determine features of the graphs of $x^2 + y^2 = r^2$ and $(x - a)^2 + (y - b)^2 = r^2$, including their circular shapes, centers, and radii.

2. recognize and determine features of the graph of $y^2 = x$, including its parabolic shape and axis of symmetry.

Exponential Functions 1

Indices and the index laws

In this sub-topic, students will

1. recall indices (including negative and fractional indices) and the index laws.

2. convert radicals to and from fractional indices.

3. understand and use scientific notation.

Applying the Tool to Functions and Graphs

Importance of keywords

The Oxford Advanced Learner's Dictionary (Hornby et al., 2000) define a keyword (noun) as a main idea or concept that is very important in a particular context. Keywords "provide significant clues for the main points about the sentence" (Li et al., 2020, p. 8196). Therefore, a keyword is one which is essential to the meaning of a sentence. Definitions of some keywords help in identifying prerequisites of the concept as they provide more detail about the key word. For example,

1. **What exactly do students need to know and be able to do in this unit?**

Key words in the syllabus highlight critical skills and concepts as well as link prerequisites to new concepts. When they are closely analyzed by teachers, different concepts not directly mentioned in the syllabus will emerge as prerequisites. An example of a definition that can directly link to prerequisites is the definition of a relation. A relation is a set of ordered pairs (Evans et al., 2019). Ordered pairs are points on a Cartesian plane that are represented in the form (x, y). The definition helps to realise the importance of a Cartesian plane in understanding relations and any other concepts related to them. In the ordered pairs we derive the domain and range. For students' understanding, it is critical to ensure that every student understands a Cartesian plane and can identify all x and y values that satisfy a graph represented on the Plane. How x values will be manipulated to give corresponding y values is called mapping.

Key words that are repeated or mean the same can be combined or expanded under one unifying name.

Examples:

1. Shapes and intercepts, asymptotes shapes and behavior and features, center and radii can all be brought under features of graphs.

2. Coefficients, variables, formula and algebraically can fall under algebra.

3. Factors, factor theorem, factorize linear and non-linear functions (linear, quadratic and cubic) will be looked at under factorization.

4. Mapping, domain, range, sets, independent, and dependent variable under relations.

5. Index laws, negative, and fractional indices fall under indices.

6. Translation, reflection, and dilation under transformations.

7. Solving linear quadratic and simultaneous equations will fall under solving equations.

Curriculum Mapping of Concepts

Curriculum mapping is a critical tool used to display the comprehensive coherence of the curriculum (Levin & Suhayda, 2018), investigate the degree of how concepts in a curriculum are interlinked (Vashe et al., 2020) and improve communication among teachers on content, skills and teaching and learning (Koppang, 2004). Curriculum mapping promotes long term planning as it reflects topics or content, concepts to be covered and skills both new and old to be mastered in a specific period (Koppang, 2004). The investigation of content connectedness will help educators identify gaps that might be addressed during teaching to help students gain a deeper understanding (Vashe et al., 2020). Curriculum mapping involves the creation of visual representation of linked displays. However, curriculum mapping is not only limited to a diagrammatic linking of curriculum content but also structure and assessments which are beyond the scope of this study.

Mapping provides visual displays, which are quick to understand and easy to compare. "Mapping is a visual representation of information and can be in the form of tables, flow charts or textual information" (Ervin et al., 2013, p. 310). Undoubtedly, diagrams or visual displays enhance explanatory power (Peterson et al., 2021). Tables, scope, and sequence charts provide a visual representation of knowledge. "Graphical displays are more effective than text for communicating complex content because processing displays can be less demanding than processing text" (Ioanna, 2002, p. 262). Concept breakdown tables and flowcharts will be used in this study to present a diagrammatic representation of how content is broken down and sequenced to realize coherent planning.

The tables and flowcharts can also be used to demonstrate how content develops from familiar to complex unfamiliar, that is from prior knowledge to new knowledge. Therefore, "a careful examination of such a chart reveals how the sequence of activities related to a particular unit is organized in a spiral approach, giving students repeated opportunities to develop and broaden concepts" (Reys et al., 2020, p. 55). Spiraling involves building from assumed prior knowledge or from what is known and navigating through to complex phenomenon.

Mapping of a unit plays an important role in providing a visual representation of knowledge. It provides resources to visualize how concepts are developed from foundational principles to new or future developments, hence exposing the complications involved in learning (Wilson et al., 2016). In this instance, a breakdown table formulated from the syllabus document can be a starting point. Collaborative mapping of mathematical concepts brings together teachers' knowledge and understanding of the topic or concepts under consideration. Done collaboratively, the exercise will provide an opportunity for teachers to have better insight on how prior knowledge will link with new knowledge.

Researchers (Gurupur et al., 2015; Novak, 2010; Reina, 2018) identified the following advantages of mapping:

1. Breaking down concepts and linking them to develop high cognitive skills.
2. Lay the foundation of how concepts will be developed.
3. Teachers share content knowledge as the map is being developed.
4. Developing deeper conceptual understanding.
5. Showcase the importance of prior knowledge.
6. Teachers become better prepared to teach.
7. Other planning documents like unit plans and term planners will use it as a foundation.
8. Gives teachers an opportunity to interrogate the syllabus.
9. Expands the knowledge and scope of key concepts which enhance teaching and learning.
10. Pictorial representation of knowledge which is easy to understand and adjust when need arises.
11. Help create connection activities or tasks as a new concept is being introduced.

Concept Breakdown Table

The concept break-down table will be instrumental in addressing the following questions:

2. **What prerequisite conceptual understanding and skills fluency are required for all students to effectively learn new knowledge?**

3. **How do the concepts identify as prior knowledge link with new knowledge?**

Concept breakdown tables explore how the key words link to prior knowledge. They include defining key words, identifying similar assumed prior knowledge concepts and linking assumed prior knowledge to new knowledge. This aspect of the proposed tool is necessary because mathematical language is content specific (Harmon et al., 2005). In addition, it is important to note that mathematics terminology increases in complexity as students progress from lower to higher levels of school. "Students who lack the formal language of mathematics have difficulties reasoning and communicating about mathematics" (Ben-Hur, 2006, p. 67). Similarly, mathematical language has been identified as a hindrance to students as they engage with new concepts (Schuhl, 2020). Including mathematical vocabulary in the proposed tool will demonstrate how language changes as concepts develop and reinforces the importance of terminology in enhancing teaching and learning.

For example, at Year 9 and Year 10 levels, parabolas are referred to as quadratic equations. Likewise ordered pairs on a Cartesian plane in Year 7 is a mapping of x onto y. The concept breakdown tables can be made available to students to dissuade their view of mathematics "as a series of unrelated procedures and techniques that have to be committed to memory" (Swan, 2006, p. 162). The views are influenced by how they are taught and consequently how they learn (Wong et al., 2001). Therefore, the planning process undertaken by teachers has a strong impact on how students are taught. Lack of coherence of content will promote students' memorization of procedures if concepts are taught in isolation. Mathematics has a highly connected web of concepts and skills, therefore these must be firmly consolidated to provide a basis for new learning (Australia Academy of Science, 2015, p. 17). Above all, concept break-down tables provide "a clear line-of-sight

Table 1. Concept break-down table: Linking junior senior mathematical methods concepts for unit 1: Functions

Keywords (QCAA mathematical methods unit 1)	Definition of keys words where applicable	Assumed prior knowledge similar concept (Australian Curriculum)	Link between assumed prior knowledge from Australian Curriculum and key words
Relations	Ordered pairs	Cartesian plane, ordered pairs	On ordered pairs the set all x (first) coordinates represent the domain which is also an independent variable and the set of y (second) coordinates is the range which is also a dependent variable. A vertical line is a line parallel to the y-axis (Year 7 & Year 8). The relationship between the x and y is the rule, formula, equation or mapping, arrow diagrams.
Transformations (reflection, translation & dilation)	Changing a shape using: Turn, flip, slide, or resize.	Flip, slide, & enlargement	Rules of translation-translating horizontally or vertically. Reflection about the x and y axis (Year 7). Enlargement & reduction as a form of dilation (Year 9).
Piece-wise	Combination of multiple sub functions	Combining linear & non-linear equations & graphs	Distinguish linear and non-linear using highest powers of variables (degree). Represent linear and non-linear equations graphically (Year 9 &Year 10).
Inverse proportion	When one value increases and the other decreases	Direct proportion	For direct proportion Increase in one variable result in an increase in another variable (Year 9) which is opposite for inverse proportion.
Features of the graphs (including quartic)	Characteristics of graphs	Linear & non-linear graphs	Calculate intercepts, increasing and decreasing graphs. Distinguish between linear and non-linear graphs comparing shapes. Graph quadratic equations, identify intercepts and turning points (Year 9-Year 10A).
Algebra	Rules to manipulate symbols		Identify coefficients (Year 7), group & simplify like terms (Year 7), general substitution (Year 7-Year 9), making one variable a subject of formula (Year 9-Year 10A).
Expand	Multiply factors	Distributive law	Removing brackets using distributive laws (Year 8-10A).
Factorization	Express as a product of several factors	Factors	Factorize algebraic (Year 9 & Year 10A) and quadratic expressions (Year 10). Factor theorem and remainder theorem to find factors of polynomials (Year 10A).
Solve equations	Find solutions in a balanced system through algebraic manipulation.	-Linear equations -Quadratic equations (factorization, quadratic formulae, completing the square, & graphically) -Simultaneous equation (substitution & elimination)	Solve linear equations (Year 7 & Year 8). Solve quadratic equation using quadratic equations (Year 9), factorization, and completing the square (Year 10 & Year 10A). Completing the square can also be used to standardize a quadratic function and the equation of a circle to determine coordinates of center and radius. Solve simultaneous equation (Year 10A) Equations show the relationship between variables (mapping) (Year 7- Year 10A).
Indices	Power or superscript	Exponents	Write surds in indicial notation, index laws, negative indices, fractional indices, & solve simple indicial equations (Year 8-Year 10A).
Scientific notation	When a number between 1 and 10 is multiplied by a power of 10		Expressing numbers to scientific notation (Year 9).

for the development of students' cognitive skills across year levels" (DoE, 2021b, p. 23). Thus, a concept breakdown table will influence students' views on mathematics as it will demonstrate mathematical concepts are interconnected and hierarchical, therefore procedures and skills are transferable. Table 1 highlights the relationship between assumed prior knowledge and new knowledge for unit 1 of the mathematical methods option.

The next question after the concept breakdown table should emphasize identification of the important concepts that must be learnt to prepare students.

4. What do we expect students to retain?

Essential concepts represent the most critical content from the content domains–the deep understandings that are important for students to remember long after they have forgotten how to carry out specific techniques or apply particular formulas (NCTM, 2018, p. 11). They are the big ideas in a unit (Schuhl, 2020), that play an important role in building students' mathematical conceptual understanding. Mapping concepts helps identify the essential concepts that students must retain.

Table 2. Grouping concepts under main concepts

1. Numbers	2. Relations	3. Algebra	4. Graphs
-Indices	-Relations	-Relations	-Transformation of graphs
-Scientific notations	-Solve equations	-Algebra	-Relations
-Relations		-Combination of multiple functions	-Combination of multiple functions
		-Inverse proportion	-Features of graph
		-Algebra	-Inverse proportion
		-Expand	
		-Factorization solve equations	

Determining essential concepts

Scholars (Ervin et al., 2013; Harden, 2001) emphasized the need to make main conceptual conceptions by synthesizing concepts that are interlinked. The main concepts are identified below:

Relations–number/algebra/graphs

Transformations (reflection, translation & enlargement)–algebra/graphs

Combination of multiple sub functions–graphs/algebra

Inverse proportion–algebra/graphs

Features of graphs–graphs

Algebra–algebra

Expand–algebra

Factorization–algebra

Solve equations–relations/algebra

Indices–number/algebra

Scientific notation–number

Creating a table such as the one shown in Table 2 with the main concepts identified in the conceptual connections and listing all the other concepts students must learn under the corresponding main concept will help teachers to check if there are some concepts left out. Secondly, it provides an opportunity to further link, expand or collapse the main concepts.

Table 2 shows different concepts that are repeated under a range of main conceptions. Hence, it can be condensed to identify only the essential concepts that students must retain. For example, relations are found under all four main concepts hence eliminating the need to have relations as one of the main concepts. Additionally, in the Australian Curriculum: mathematics, numbers and algebra have a linked relationship and thus can be combined into one concept. Also, graphs have different features and characteristics, for example, if the *x variable* in a hyperbolla, $y = \frac{1}{x}$ is increased to a very big value (positive infinity) the value of *y* turns to zero. Subsequently different types of graphs can be renamed as characteristics and features of graphs. Thus, the essential concepts can be distilled down to numbers, algebra, and characteristics and features of graphs.

Content Sequencing

The main conceptual connections identified in this unit on functions are number, relations, algebra, and graphs. Using the main conceptual connections, instead of essential concepts which may be too broad, will ensure all concepts to be taught are included. It is important to include all the assumed prior knowledge from the concept breakdown table in their hierarchical order to show the structure of knowledge development. "Mathematics is a hierarchical subject, where new learning builds on earlier learning in a highly connected way" (Australian Academy of Science, 2015, p. 17). The hierarchical nature of mathematics means concepts increase in complexity as they develop; hence, assumed prior knowledge must generally follow levels of hierarchy to new knowledge as shown in Figure 2.

HOW THE PLANNING TOOL INFLUENCES EFFECTIVE TEACHING OF MATHEMATICS?

Teachers have a responsibility to ensure that mathematics learning is effective. Mathematics teachers are expected to unpack subject matter, sequence content, provide students with an opportunity to connect prior knowledge to new knowledge and gradually release support for students (Stoll et al., 2012). Similarly, effective teaching and learning require students to have suitable, relevant, and applicable prior knowledge, new knowledge that interconnects and can be expanded to other concepts as well as allow students to link concepts (Novak, 2010).

The hierarchical nature of mathematics and spiral sequencing of concepts across levels make senior level mathematics teaching and learning highly dependent on junior level mathematical understanding. The amount and quality of prior mathematics knowledge a student possesses determines how the student builds new mathematical knowledge (Schneider et al., 2011). This is a prerequisite for successful achievement of learning outcomes (Achmetli et al., 2019). Thus, operating at high levels of understanding of prior knowledge helps students identify different methods of solving a mathematical problem and choosing the most efficient one (Newton et al., 2020). The connection of critical and relevant prior knowledge and corresponding new

negative numbers
fractions
surds
indices (numbers only)
scientific notations
Cartesian Plane
sets
independent and dependent variable
domain and range
general substitution into linear and nonlinear relationships(mapping)
identify coefficients
grouping and simplify like terms
subject of the formulae
distributive law
factorisation of linear and quadratics expressions
direct proportion
solve linear and quadratic equations
simultaneous equations
indicial equations
factorise cubic functions (Remainder and Factor Theorem)
transformation of graphs
linear, quadratic functions and their inverse
piecewise-defined functions
hyperbolic functions
cubic functions
quartic functions
circles (graph, recognize characteristics and features)

Figure 2. Sequenced content using the tool

knowledge, as emphasized in concept-break-down tables, is critical for effective teaching and learning.

Relevant prior knowledge provides the foundation from which new knowledge can be developed. Students have a better chance of participating and achieving in mathematics when links are developed between what students already know and new concepts (Australian Curriculum, & Assessment and Reporting Authority [ACARA], 2018a, 2018b; QCAA, 2018). For example, the Cartesian plane, creating a table of values of linear and non-linear relationships may enhance students' understanding of independent and dependent variables, domain and range and mapping of functions and relations. To illustrate this, when students are asked to create a table of values for a linear relationship at Year 8 level, they substitute $x-values$ in the given relationship to obtain corresponding $y-values$. Importantly teachers can emphasise that the $y-value$ obtained is dependent on the $x-value$ substituted, thus defining independent and dependent variables. Knowledge of the Cartesian plane is vital when representing the relationship graphically. Importantly all the $x-values$ in the table of values of the linear relationship satisfy the graph, hence defining the domain of the graph, since domain is a "set of all the first coordinates of the ordered pairs in a relation (Evans et al., 2018, p. 215). Correspondingly, the $y-values$ of the table of contents will define the range of the linear relationship. However, restricting a domain involves considering only a smaller portion of a domain. Inequality solutions when displayed on a number-line can also be used to indicate only the part that satisfies the solution. Similarly restricting a domain is considering only the part that

makes a relation a function, hence inequalities might be prior knowledge in enhancing students' understanding of restricting a domain. In addition, inequalities can also help build foundational knowledge for piece-wise functions as piece-wise functions have "different rules for different subsets of the domain" (Evans et al., 2018, p. 231). Thus, a piece-wise function has the domain divided into different sections which can be defined by inequalities. Knowledge of linear and non-linear relationships at Year 9 level can facilitate students' understanding of different rules for different sections of a piece-wise function.

Tables of values are not limited to linear relationships but can also be extended to non-linear relationships that include parabolas, hyperbolas, exponential and logarithmic graphs to mention just a few. It follows that as students are creating the tables of values, they are mapping an independent variable to a dependent variable. At Year 8 level, the linear relationship is the rule or formula for mapping the variables. Grouping all $x-values$ in one set and all $y-values$ in another set, then using arrows to match all corresponding ordered pairs will demonstrate an arrow diagram. Different relationships shown from arrow diagrams will allow the teacher to introduce conditions for a relationship to be defined as a function or not. Similarly, when linear and non-linear relationships are represented diagrammatically from the tables of values on the Cartesian plane, students can be asked to use the vertical line test to determine if the relationships are for functions or not. Different ways of determining if relationships are functions or not will enhance flexibility and deeper understanding of the concept.

From junior secondary level students are expected to represent relationships graphically. The relationship between the rule of the relationship and the shape of the graph must be emphasized. In fact, "the likelihood of information being maintained in memory increases when students' brains are prepared in advance to 'catch' the new input" (McTighe & Willis, 2019, p. 99). To develop mastery of features and shapes of graphs in Year 11, prior knowledge on features and shapes of graphs from lower levels is significant. For example, linear relationships are represented by straight lines while quadratic relationships are represented by a concave shape. Features and shapes can also include turning points that are expected to be covered in Year 9 when non-linear graphs are introduced. Other points such as intercepts and tables of values can also be important when emphasizing the zeros on intercepts. Most of the graphs in Year 11 are also in Year 10A curriculum; hence, it is important for teachers to start by recapping the assumed prior knowledge. Teaching and learning that facilitate students' understanding of fundamental prior knowledge while developing the basic knowledge to new and deeper meaning at the same time promotes students' understanding and participation. Furthermore, when teaching and learning in mathematics start from prior knowledge, it not only facilitates the retention of ideas but also deepens mathematical knowledge by integrating the ideas and creating effective mathematical meaning (Kilpatrick, 2001). Indeed, "the most significant variable in learning something new is prior knowledge (McTighe & Willis, 2019, p. 99). Thus, students with high cognition of prior knowledge are better positioned to use both procedural and conceptual learning effectively and efficiently (Newton et al., 2020). In fact, mathematical understanding is enhanced when students are presented with the opportunity to adapt or reflect on their prior experience and knowledge and make connections between concepts, resulting in a gradual development of new knowledge (ACARA, 2018; Lowrie et al., 2018). Similarly, effective teaching involves "activating prior knowledge by making explicit connections to new learning" (DoE, 2021b, p. 14). Starting with familiar then progressing to unfamiliar concepts enhances participation, knowledge building and understanding, which should be complemented by mathematics vocabulary advancement.

CONCLUSION

The planning tool can reinvigorate the pedagogical dialogue as classroom teachers collaboratively plan to deliver effective teaching of mathematics. To reiterate, a central premise of this paper is that effective sequencing of mathematics content can aid the transition from junior mathematics (Year 7 to Year 10) curriculum to senior mathematics (Year 11 to Year 12) curriculum in Queensland and provide a tool for sequencing the

mathematics content. The potential implementation of this planning tool can mean that the hierarchical nature of mathematics and spiral sequencing of concepts across levels can be articulated more explicitly. The connection of critical and relevant prior knowledge and corresponding new content knowledge, as emphasized in the concept-break-down tables can be effectively addressed in the teaching and learning. However, there are potential limitations when implementing this tool which focuses mainly on the spiral sequencing of mathematics concepts across levels. The limitations might include a lesser focus on catering for individual student needs, diversity of readiness and connecting mathematics teaching to the students' diverse everyday experiences.

The paper has suggested that there is an urgent need to enhance collaborative planning for effective sequencing of mathematics content between lower- level and upper-level topics and across different level mathematics subjects. The paper has proposed a step-by-step systematic sequencing of mathematics content to promote interlinking, coherence and spiraling of concepts between the *Australian Curriculum* (Prep–Year 10): *Mathematics* and the *Senior Queensland Mathematical Curriculum*: *Mathematical Methods Unit*. The paper identified that depending on the level of assumed prior knowledge and skills students recall and apply, teachers can start teaching from any level of the sequenced content. The paper has suggested that the tool can be adapted to all mathematics subjects and levels; help identify relationships between lower- level and upper-level topics, concepts, and skills; link the two levels and provide students with the opportunity to build their mathematical knowledge from the familiar to unfamiliar contexts. The aim is to encourage further research, dialogue and professional development to (re)conceptualize collaborative planning for effective sequencing of mathematics content.

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