A Pricing Method for Distribution System Aggregators Considering Differentiated Load Types and Price Uncertainty

Bomiao Liang, Jiajia Yang, Beiping Hou, and Zhiyuan He

Abstract **The utilization of demand response flexibility has become a significant method to cope with the intermittence of renewable energy sources in distributed systems. This paper proposed a new pricing method for demand response resources managed by a distribution system aggregator, which is deduced from analyzing the operating revenue within the timescale from hours to years. In the proposed model, the hourly decision-making of an aggregator is formulated as a newsvendor model and uncertainties in the long-term decisions are modelled by a backward valuation process. It maximizes the benefit of an aggregator by considering the price and quantity uncertainties of distributed load/generation in day-ahead and real-time wholesale electricity markets. Meanwhile, the coexistence of controllable and uncontrollable loads is also considered, where the former refers to electricity consumption from end-users who are equipped with smart devices for energy management, and the latter load demand of passive end-users who have no willingness or capability to participate in the demand response schemes. Finally, numerical studies are carried out to demonstrate the feasibility and effectiveness of the developed model and methods, and the impacts of active end-user percentage on the aggregator operation under the proposed pricing method are also compared and illustrated.**

Index Terms**--Distribution power system, aggregator, load control, pricing, newsvendor model, valuation**

NOMENCLATURE

z, Z	Identical signal for direct load control and its set
X_i	Continuous uniform factor for the active load i
L_i^0	Nominal quantity of an active load i (i=1,2,…,M)
$L_i(z)$	Actual load quantity of an active load i
D_A^0	The quantity of aggregated nominal active load for the concerned aggregator
p_A	Contract energy price determined by aggregator.
S_A	Market size of active load in the area
ρ_A	Market share of the concerned aggregator in active load market
a_1, b_1	Parameters of the active load market
$L_j(t)$	Load of the passive load j (j=1,2,…,N)
$D_P(t)$	Aggregated quantity of uncontrollable load at hour t
S_P	Market size of passive load in the area
ρ_P	Market share of the concerned aggregator in passive load market
p_t	Real-time energy price determined by aggregator
a_2, b_2	Parameters of the uncontrollable load market
π_L, $\mathrm{E}\pi_L$	Long-term aggregated aggregator profit and its expected value
π_D, $\mathrm{E}\pi_D$	Daily aggregator revenue
π_t, $\mathrm{E}\pi_t$	Hourly aggregator revenue
Q_C	Long term fixed contracts of purchase quantity determined by the aggregator
p_C	Long term fixed contract price for the aggregator to purchase electricity
Y	Number of days in the discussed long-term period
$\mathbf{p_D}$	Day-ahead market price for the aggregator to purchase electricity (T-dimension)
T	T=24, number of hours in a day
$\mathbf{Q_D}$	Decision variable of day-ahead purchase quantity (T-dimension)
p_t	Real-time energy price for passive end-users for hour t
z_t	Load control factor for **P.CU** participants at hour t
r_t	Real-time wholesale market price for the aggregator at hour t
s	Salvage price for the surplus power
R	The indifference for every interval
κ_g	Mean reversion rate of market g
θ_g	Long-term equilibrium value of market g
σ_g	Volatility of the market price of market g
g	Market type (g=p_D/r indicating day-ahead /real-time market)
v, V	Price path number and number of price paths
m_1,m_2,m_3	Division of feasible area of p_A, Q_C, $\mathbf{Q_D}[t]$

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η	Upper contract proportion of electricity market
w	Number of the strategy combos
h	Day-ahead hourly sample number
C_A, C_r, C_D	Price caps of contract, real-time, day-ahead transactions
p_{rt0}	Real-time price at the beginning of the day
$a_1,a_2,\ldots,a_8$	Coefficients in the regression expression of day revenue
$A_1,A_2,\ldots,A_7$	Coefficients in the regression expression of long-term revenue

I. INTRODUCTION

The increasing penetration of renewable generations gives rise to higher operation requirements of power systems, both physically and economically [1,2]. Under this circumstance, responses from flexible demands, distributed generation (DG), and storage are supposed to be managed on a large scale to balance the fluctuating and intermittent power generation in future power systems. Among many other attempts, transactive energy (TE) has been regarded as a promising way to stimulate demand response and its integration [3], which is an additive of the traditional electricity market with the new environmental attributes [4]. The application of TE relies on the decentralization and marketization of power industry, which also provides potential opportunities for distribution system aggregators. Aggregators, if properly organized, can bear the main tasks in the integration of flexible demands and distributed generations. Therefore, the future aggregators will serve as active energy consumers or prosumers [5,6], and the market performance of aggregators will be of great significance.

In a distribution power system, the structure of one distribution system operator (DSO) with several aggregators is widely adopted. When enough qualified aggregators are brought in, DSO works like an independent system operator. Many works casted light on the operation strategies of DSO. Ref. [7-9] studied the planning of distribution power systems with new uncertain participants such as DG or electric vehicles. In particular, a chance constrained programming model is developed in [7] to minimize the investment cost, operating cost, maintenance cost, network loss cost, as well as the capacity adequacy cost of DG. Ref. [8] presented a scenario-based comprehensive expansion planning approach for distribution systems which embedded uncertainties due to the wide utilization of plug-in electric vehicles. Ref. [9] proposed a smart substation allocation model (SSAM) to determine the optimal number and allocation of smart substations considering the substation upgrade costs and the customer interruption costs in a given distribution system.

Pricing and bidding strategies of aggregator has been studied for a long while, but no systematic solution is proposed yet to cover the situation where different contracts co-exist in the administration area. Pricing methods are employed to alleviate possible distribution system congestions from the perspective of DSO by methodology such as distribution congestion price-based market mechanism, dynamic tariff, and distribution locational marginal pricing [10-12]. The operation strategy of aggregators is also explored. Ref. [13] discussed the most profitable mid-term capacity limit offering curves. Ref. [14] proposed a DER aggregator's data-driven bidding strategy using the information gap decision theory. Ref. [15] proposed a risk-averse optimal bidding method for electric vehicles and energy storage aggregator in a day-ahead frequency regulation market. A joint market clearing model of energy and reserve capacity for microgrid aggregators aiming at the cost minimization and voltage stability maximization is proposed in [16].

Load control management is usually studied separately from market analysis, neglecting their interaction with different electricity markets such as reserve, real-time, and day-ahead markets. Direct load control is one of the most common methods to conduct load management with a general attempt to minimize disruption [17,18]. Other load management methods mainly fall into the area of demand response, although the specific proposals may differ. Many methods have been employed to calculate the most profitable plan under the demand response scheme. In [19], the profit-maximizing demand response of a load is described with a finite-horizon Markov decision process problem which solved by a dual approximate approach and a row-generation-based solution algorithm. In [20], peak minimization is solved by a hierarchical demand response scheme based on Dantzig-Wolfe decomposition. Demand response proposals can also come in other forms, such as incentive contracts [21] or customer coupons [22].

A comparison table as shown in Table I is provided to indicate the current, which compared their object, method, and circumstance. Their assumption and treatment of load control management can also be found in the table.

TABLE I
Comparison Regarding Works in Pricing and Bidding Strategy

Main Methodology	Research focus	Load Control Management
Marginal pricing	Electricity market clearing problem with uniform purchase price and zonal selling prices [23], day-ahead congestion management in distribution systems [10], optimal electric vehicle charging management [12], energy and reserve market clearing with microgrid aggregators [16], uncertainty contained locational marginal price (U-LMP) [24], optimal energy management and marginal-cost electricity pricing in microgrid network [25]	Household demand response [10], electric vehicles deployment [12], prosumers [25]
Mixed integer linear program (MILP)	Energy pricing and dispatch problem faced by a	Simulated as Stackelberg game [26], large

	smart grid retailer [26], optimal dynamic retail electricity pricing of large industrial customers [27], hour-ahead price-based energy management scheme for industrial facilities [28], retailer pricing framework based on the bilevel programming framework and the optimal clustering [29], optimal joint bidding and pricing of profit-seeking load serving entity [30], optimal bidding of electric vehicles and energy storage aggregator in day-ahead frequency regulation market [15]	industrial customers focused [27], price-based demand response [28], classifications of end-users according to their consumption behaviors [29], electric vehicles and energy storage deployment [15]
Conditional value at risk	retailer pricing framework based on the bilevel programming framework and the optimal clustering [29], optimal bidding of electric vehicles and energy storage aggregator in day-ahead frequency regulation market [15]	classifications of end-users according to their consumption behaviors [28], electric vehicles and energy storage deployment [15]
Pricing algorithm and mechanism design	Distribution networks uncertainty management [11], mid-term DSO market scheduling [13], axiomatic approach to efficient prices and cost allocation for a revenue neutral and non-confiscatory day-ahead market [31], nonlinear and randomized pricing for distributed management of flexible loads [32], unified model for pricing under nonconvexity [33], scheduling and pricing for expected ramp capability in real-time power markets [34]	Dynamic tariff for congestion management [11], electric vehicle (EV) commercial charging stations [13], nonlinear and randomized pricing for distributed management of flexible loads [32], flexibility cultivation via pricing strategy [34]
Data mining	Electricity retail price customizing based on load profile clustering analysis [35]	End-users' inherent electricity consumption pattern exploration [35]
Game theory (non-cooperative, Stackelberg)	DER aggregator's data-driven bidding strategy [14], comparison and analysis of fixed-dollar markup and percentage markup price strategies 36], negotiation strategy of discharging price between power grid and electric vehicles [37], Balancing management of strategic aggregators [38], Incentive-based demand response considering hierarchical electricity market [39]	Charging and discharging optimization scheduling model considering EV travel characteristics [37], incentive-based demand response [39]
Multi-agent	Negotiation strategy of discharging price between power grid and electric vehicles [37]	Charging and discharging optimization scheduling model considering EV travel characteristics [37]

Financial valuation of multi-energy conversion, storage, and demand-side management systems under uncertainty is designed in [40], but it focused on the individual operation of each entity such as a CHP, a storage device or a demand-side proposal. In actual, the valuation method is more powerful when dealing with pricing strategies and the design of long-term contracts for aggregators who manage a large number of demand side resources. Historical data and data wrangling algorithms can be exploited to extract more profitable business plans in an uncertain market. In this paper, the business operation of an aggregator is analyzed within three different time scopes: long-term, day-ahead, and real-time. Uncertainties in the long term are modelled by a backward process, so the valuation process can be more reasonable and convincible [41]. The hourly decision-making of an aggregator is formulated by a newsvendor model [42,43] and the situation when controllable and uncontrollable loads co-exist is also considered. Besides, the valuation method is employed to maximize the aggregator benefit while developing pricing and bidding strategies. A satisfied expression is obtained using the least square regression [44], and the corresponding pricing and bidding methods are also determined.

The contributions of this paper are twofold.

(1) The paper proposed a newsvendor model covering several electricity markets and adopted direct load control in a distributed pricing framework for the first time. Load shifting among different markets are represented by a generic model that is analogous to a newsvendor problem. Under this setting, the operation of a distribution system is transferred into a tradable model.

(2) This paper accounted for the situation where differentiated load types coexisted, which is more in line with practical transaction conditions. It monetized the total benefit of congestion management and took into account both direct load control and various kinds of distributed pricing schemes.

The rest of this paper is organized as follows. In Section II, the formulations and market share functions of load in distribution systems are analyzed. Then, the proposed pricing and bidding method for aggregators is presented in Section III. The feasibility of the developed model and method are demonstrated in an example distribution system in Section IV. Finally, Section V concludes the paper.

II. Direct Load Control and Uncontrollable Loads

Separate research on direct load control and uncontrollable loads is quite sufficient, but the coexistence of them are seldom inquired into. However, due to extremely diverse energy scene and numerous energy customers, their coexistence is actually inevitable. The fact that they can substitute each other to certain extent makes the separate research lack of fidelity. In this paper, their coexistence is taken into account, and their influence on each other are also considered in the design of pricing method.

A. Direct Load Control

In each hour, the aggregator provides its contracted active end-users with an identical signal $z \in Z$. In this paper, the direct load control adopts a probabilistic continuous uniform scaling (**P.CU**) scheme, which is able to represent most load characteristics [45]. The continuous uniform factor X_i for the active load i obeys U (0, 1) and is independent of z. L_i^0, the nominal quantity of an active load i (i=1,2,…,M), is segmented by the shared signal z in the actual operation, and the actual load quantity $L_i(z)$ is shown in (1).

$$L_i(z) = L_i^0(1 - z + zX_i) \tag{1}$$

The quantity of aggregated nominal active load for the concerned aggregator D_A^0 is influenced by its contract energy price, p_A. It is assumed that the market size of active load in the area S_A is known, and the market share of the concerned aggregator ρ_A is a function of p_A. The aggregated nominal and actual active load quantity are shown in (2) and (3). The market share function can be defined as (4), which is the logit demand function transformed for market share indication [46,47]. Its first order derivative is expressed in (5). The market model is based on the generalized newsvendor problem with yield risks. These parameters are designated according to the experience from normal commodity markets in this study. In practice, the parameters can be obtained from experience, estimation and consultant. As the relationship is similar to a price elasticity curve, similar approaches can be employed. For the provided parameters, the aggregator can quickly examine its suitability as the actual response quantity and the price is both known after every trial.

$$D_A^0 = \sum_{i=1}^{M} L_i^0 = \rho_A(p_A)S_A \tag{2}$$

$$D_A(z) = \sum_{i=1}^{M} L_i(z) \tag{3}$$

$$\rho_A(p_A) = \frac{e^{-(a_1 + b_1 p_A)}}{1 + e^{-(a_1 + b_1 p_A)}} \tag{4}$$

$$\rho_A'(p_A) = \frac{-b_1 e^{-(a_1 + b_1 p_A)}}{(1 + e^{-(a_1 + b_1 p_A)})^2} = -b_1 \rho_A (1 - \rho_A) < 0 \tag{5}$$

where a_1, b_1 are the parameters of the market, b_1>0.

Then, the expectation of $D_A(z)$ can be derived from (1) and (3).

$$E[D_A(z)] = E[\sum_{i=1}^{M} L_i^0(1 - z + zX_i)] = (1 - \frac{z}{2})D_{A0} \tag{6}$$

B. Uncontrollable Loads

Uncontrollable loads are demand of passive end-users who have no willingness or capability to participate in the demand response schemes in a distribution system. It is assumed that the concerned aggregator serves N passive loads, and that $L_j(t)$ is the load of the passive load j. The aggregated quantity $D_P(t)$ of uncontrollable load at hour t is the sum of the N loads, as shown in (7). The expected value of $D_P(t)$ is determined by the market size S_P and the market share function ρ_P, as shown in (8). The prediction can be made in advance based on the price information released by the aggregator. ρ_P is a function of the real-time energy price p_t, and its definition and first order derivative are expressed in (9) and (10) [29,30].

$$D_P(t) = \sum_{j=1}^{N} L_j(t) \tag{7}$$

$$E[D_P(t)] = \rho_P(p_P(t))S_P \tag{8}$$

$$\rho_P(p_t) = \frac{e^{-(a_2 + b_2 p_t)}}{1 + e^{-(a_2 + b_2 p_t)}} \tag{9}$$

$$\rho_P'(p_t) = \frac{-b_2 e^{-(a_2 + b_2 p_t)}}{(1 + e^{-(a_2 + b_2 pt)})^2} = -b_2 \rho_P (1 - \rho_P) < 0 \tag{10}$$

where a_2, b_2 are the parameters of the market, b_2>0.

Transformed logit demand function is adopted for both direct load control and uncontrollable loads, as it is "suitable for modelling the global response of customer behavior" [47]. Excavating the relationship between the price and market share can seamlessly serves the need from the newsvendor model to be mentioned in the following. The derivation also applies to for-profit DSO or single aggregator operation. In that case, market share function will be replaced by price elasticity function. When distributed generators are considered, the corresponding load quantities are negative, and the scheme design and model are still applicable without needs of modification.

III. Proposed Pricing and Bidding Method for an Aggregator

On time scopes from one hour to several years, the aggregator has to develop both bidding and pricing strategies. The bidding strategies are made by gaming with its upstream and peer market participants, which can be other aggregators in the distribution system, DSO, or other trading entities. The aggregator may decide to purchase or sell different energy commodities based on its load characteristics. Differently, the pricing strategies are for its clients. The pricing method is of great significance to maximize the benefit of an aggregator, especially when there are a wide variety of clients including demand response participants.

A. Problem Description

The bidding and pricing strategies distribute in the different time scopes. So, the analysis and derivation in this paper are also carried out on three time scopes: long term, day-ahead and real-time. As uncertainty exists in the long term operation, the deterministic value of the aggregator profit π_L cannot be calculated directly, while the objective of the pricing model is to maximize the expected aggregate benefit, Eπ_L. Apparently, π_L is the sum of daily revenues denoted as π_D which is the sum of hourly revenue π_t. The codetermination on the total revenue of different strategies made at different moments is resolved via a backward valuation process as depicted in Fig. 1.

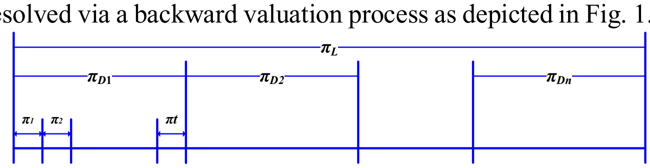


Fig. 1. Abridged general view of the backward valuation process.

1) Long term Strategies

It can be assumed that the aggregator signed long term fixed contracts of quantity Q_C (determined by the aggregator) with upstream or peer participants, at price p_C (determined via negotiation). The aggregator also needs to develop long term pricing strategies for its clients. It is also assumed that the aggregator concerned provides only one contract type for direct load control participants and the contract energy price is p_A. As analyzed above, the value of p_A will affect the aggregate nominal active load quantity D_A^0. Suppose the discussed period consists of Y days, which can be a month, a quarter, a year or several. Therefore, the problem can be described as (11).

$$\max_{p_A, Q_C} \mathrm{E}\pi_L = \sum_{y=1}^{Y} \mathrm{E}\pi_D \tag{11}$$

where the daily revenue π_D for each day is not fixed and cannot be precisely calculated as it depends on the situation and corresponding strategies in each day and hour. The daily situation will be further demonstrated along with the day-ahead strategies, and the mutual effects between strategies at different time scopes will be tackled with a backward process in Section III-C.

2) Day-ahead Strategies

In this model, the aggregator is supposed only to bid in the day-ahead electricity market at the bidding price p_D, which is a known T-dimensional vector, T=24. Accordingly, the decision variable Q_D at day-ahead time scope is also a T-dimensional vector. Then the expected daily revenue π_D of the aggregator can be calculated as (12). Similarly, the hourly revenue for each aggregator is also influenced by the uncertainty of demand and the mutual effects between strategies at different time scopes.

$$\mathrm{E}\pi_D(\mathbf{Q_D}) = \sum_{t=1}^{T} \mathrm{E}\pi_t \tag{12}$$

3) Real-time Strategies

The hourly aggregator business and operation can be modeled with a for-profit newsvendor model, and the aggregate revenue is determined by the combined influence from real-time situation, pricing strategies, and load control strategy. Five minutes prior to [10] each hour in real-time, the aggregator should announce its real-time energy price p_t for passive end-users and load control factor z_t for **P.CU** participants based on the updated generation/demand information. The aggregator should bid at price r_t in real-time wholesale market to cover the reserve gap of the hour, and the surplus is salvaged at a much lower price s. Suppose $r_t, p_t, p_C, p_A > s$, $p_t \le r_t$, Then the hourly aggregator revenue can be calculated as (13).

$$\begin{aligned} \pi_t(p_t, z_t) = {} & p_A D_A(z_t) + p_t D_P(t) - \mathbf{p_D}[t]\mathbf{Q_D}[t] - p_C Q_C \\ & - r_t (D_A(z_t) + D_P(t) - \mathbf{Q_D}[t] - Q_C)^+ \\ & + s(\mathbf{Q_D}[t] + Q_C - D_A(z_t) - D_P(t))^+ \end{aligned} \tag{13}$$

The interaction between different loads, aggregators and DSO across time scopes is depicted in Fig. 2. The narrow arrows indicate the information exchange, while the bold arrows stand for the flow of power. The communication in the left side occurs only once for a certain period, and that in the right happens for every interval (maybe 1h, 30min, 15min, even 5min). The reserve R in Fig.2 stands for the indifference for every interval, which equals to $D_A(z_t) + D_P(t) - \mathbf{Q_D}[t] - Q_C$. The physical meaning is already illustrated along with (13).

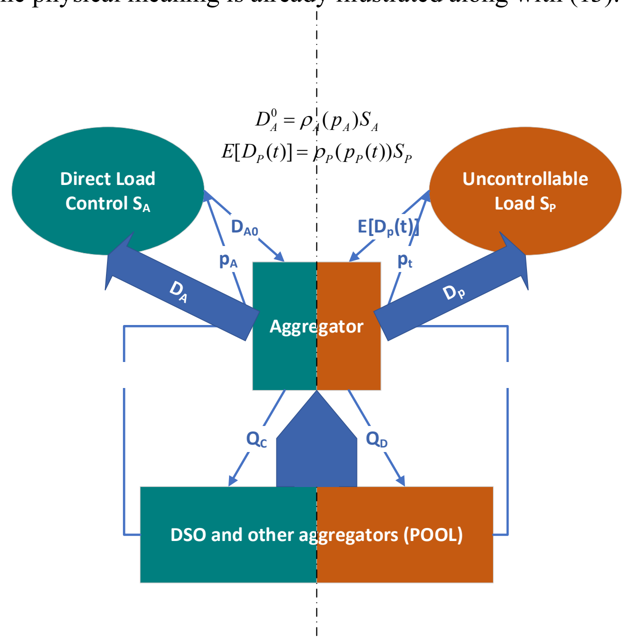


Fig. 2. The interaction between different loads, aggregators and DSO across time scopes.

B. Analysis and Solution

To maximize the revenue over a long horizontal scope at an acceptable computational cost, specific dependencies, state transitions of memorable components, as well as energy substitution across time scopes are neglected. For long-term pricing decision, the approximation still provide sufficient precision, and takes the computational efficiency into account as well. As the revenue of each hour are independent with each other in the model, the aggregated long-term revenue is maximized when the revenue of each hour is maximized. When the situation of each hour is deterministic, the optimization problem can be solved in one single forward process. However, the long term and day-ahead strategies in the paper cannot be optimized directly as the information is incomplete. Fortunately, for any given portfolio of longer-term strategies, the following theorem holds, and the optimal solution and the corresponding objective value can be obtained through finite steps of comparison and search.

Theorem* *For any settled long-term strategy (p_A, Q_C and $Q_D[t]$), there exist only a fixed number of candidate solutions optimizing the hourly strategies (maximum) as depicted in (13) regardless the market size.*

C. Valuation Method

As aforementioned, the solution cannot be obtained straightforward due to the existing mutual effects. To get the optimal solution which accommodates the strategies at different time scopes and the impact of price uncertainty, the optimization model is solved with a valuation method implemented in three steps. These three steps include the forward scenario developing step, the backward model solving step, and the forward result generating step.

1) Development of the Forward Scenario

Price fluctuations are modeled first. The values of a_1, a_2, b_1, b_2, S_A, S_P, p_C, s are already known at the beginning. Then the day-ahead price p_D for each day (p_D is used to represent the element in $\mathbf{p_D}$ during the scenario developing) and the real-time price r_t for each hour in the observed period are generated employing the log-of-price mean reversion process, which represents the main characteristics of the energy price processes [48]. Denote the mean reversion rate by κ_g , the long-term equilibrium value by θ_g , and the volatility of the market price by σ_g (g=p_D/r indicating day-ahead /real-time market) Then the energy price can be modeled as (14), where $du \sim N(1,dt)$. In this step, a sufficient number V price paths should be generated by (15), in which $\varepsilon \sim N(0,1)$.

$$d\ln g = \kappa_g(\theta_g - \ln g)dt + \sigma_g du, g = p_D, r \tag{14}$$

$$\ln g_{t+1} = \ln g_t + \kappa_g(\theta_g - \ln g_t)\Delta t + \sigma_g \varepsilon\sqrt{\Delta t}, g = p_D, r \tag{15}$$

Strategies of longer scopes are also a part of the scenarios. Strategy scenarios are constructed from the long-term scope. Suppose the aggregator is an energy consumer in general when treated as a whole. Then the feasible region of p_A and Q_C are set as [s, C_r] and [0, $\eta(S_A+S_P)$] to start with. Divide [s, C_r] and [0, $\eta\overline{S_A+S_P}$] evenly into m_1–1 and m_2–1 sections, in which $\overline{S_A+S_P}$ is the average of S_A+S_P. Then there are $m_1\times m_2$ long-term strategy combos (p_A^w , Q_C^w) made up by the boundary points. From the analysis above, there exists a unique D_A^{0w} for each p_A^w . Then construct the specific scenarios at the day-ahead scope under each long-term scenario w, (p_A^w , Q_C^w , D_A^{0w}).Similarly, divide the feasible region of the element for each hour in Q_D, [0, $D_A^{0w}+S_P-Q_C^w$], into m_3–1 sections. For each hour under scenario w, there are m_3 day-ahead hourly strategy samples $Q_D[t]^h$. Eventually at the real-time scope, under one specific day-ahead hourly sample h which is developed under long-term scenario w, an hourly model optimizing (13) is obtained.

2) Solution of the Backward Model

Solution of the Backward Model is the core of the real-option-based approach to deal with the uncertainty. By adopting the backward model, the value-to-go can be reckoned with unsettled value. In other words, though appeared as results, the related parameters and variables are actually expressions without fixed values.

First solve every hourly optimization model as (13) as analyzed in Section III-B. The optimal solution under each day-ahead hourly sample h which is developed under long-term scenario (v,w) is denoted as ($p_t^{v,w,h*}, z_t^{v,w,h*}$) and the optimized expected hourly revenue is $\mathrm{E}\pi_t^{v,w,h*}$ .Regress on these $m_1\times m_2\times m_3\times V$ differentiated real-time operation scenarios with the least-square linear regression process, and the relationship between the optimal solutions and scenario settings can be derived as (16)-(18).

$$\mathrm{E}\pi_t^* = f_1(p_A, Q_C, D_A^0, \mathbf{Q_D}[\mathrm{t}]) \tag{16}$$

$$p_t^* = f_2(p_A, Q_C, D_A^0, \mathbf{Q_D}[\mathrm{t}]) \tag{17}$$

$$z_t^* = f_3(p_A, Q_C, D_A^0, \mathbf{Q_D}[\mathrm{t}]) \tag{18}$$

The results in (16)-(18) will be used in the day-ahead strategy decision process. The optimal solution under each long-term scenario w is denoted as $\mathbf{Q_D^{w*}}$ and the optimized expected daily revenue is $\mathrm{E}\pi_D^{w*}$. Similarly, regress on these $m_1\times m_2$ differentiated long-term operation scenarios and reveal the relationship between optimal solutions and scenario settings can be derived as (19) and (20).

$$\mathrm{E}\pi_D^* = f_4(p_A, Q_C, D_A^0) \tag{19}$$

$$\mathbf{Q_D^*} = f_5(p_A, Q_C, D_A^0) \tag{20}$$

Apply (19) and (20) into the (11), the long-term bidding and pricing strategy decision process, then the optimized revenue and the corresponding optimal long-term strategy (Q_C^*, p_A^*) can be settled.

3) Forward Result Generating

Adopt (Q_C^*, p_A^*) as the long-term strategy, and the optimal day-ahead strategy can be derived according to (19) and (20). As for any specific hour, the solving method in Section III-B will develop the optimal strategy according to the settled strategies from longer scopes. The step provides a full circle of the strategy developing, which develops the daily and hour under the optimal long-term strategy and one designated scenario. The results can be employed for advice and may be further differentiated in the short-term operation.

The entire decision process based on valuation is depicted with a flowchart as shown in Fig. 3.

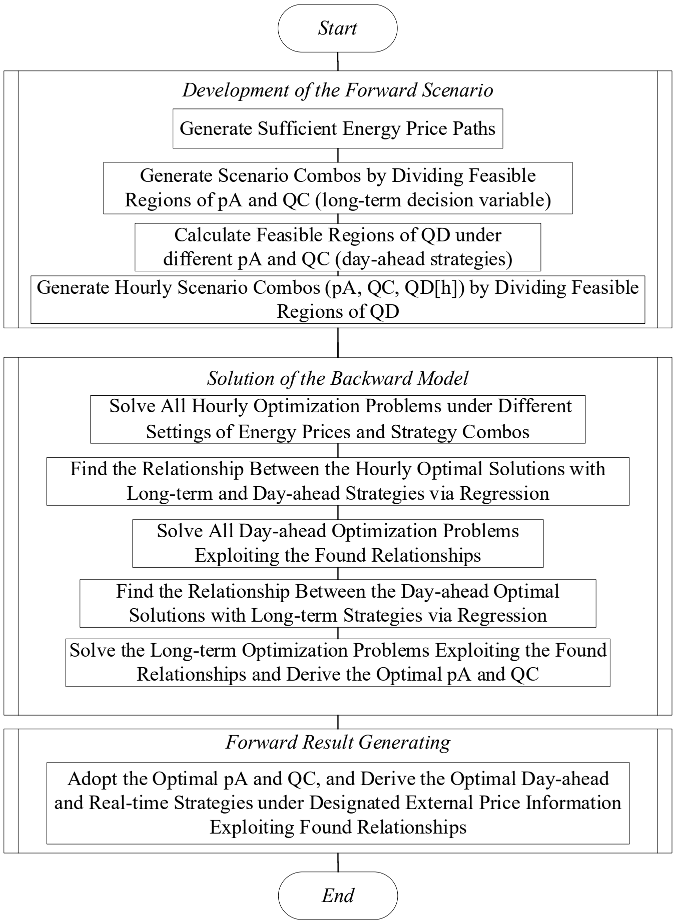


Fig. 3. Flowchart of the solution process.

IV. Case Study and Discussions

A distributed market with differentiated load types and price uncertainty is employed to illustrate the function of the proposed pricing method for aggregators. The performance of different observation intervals, different load compositions (percentage of active end-users), and strategies under different time scope are also derived.

A. Data Specifications in the Cast Study

The parameters employed in the price generation (log-of-price mean reversion process) is derived from the historical statistics of PJM data [49] (the day-ahead and real-time price data of 2015 in specific): $\kappa_{p_D}=\kappa_r=1.69$; $\theta_{p_D}=33.94$, $\theta_r=33.34$; $\sigma_{p_D}=66\%$, $\sigma_r=83\%$. p_C for direct load control is set to be 23.72\$/MWh (the minimal price among the historical day-ahead price), and the salvage price s is set to be half of p_C (11.86 \$/MWh). The price caps C_A=90\$, C_r=2700\$, C_D=2700\$. The parameters of the market $a_1/b_1=a_2/b_2=-100$, b_1=0.025, b_2=0.0125, and the value in the Table II is the whole market size, which is S_A+S_P, and the proportion of S_A:S_P=0.7:0.3.

TABLE II
Whole Demand Market Size of the Typical Day [49]

Hour	S_A+S_P (MW)	Hour	S_A+S_P (MW)	Hour	S_A+S_P (MW)	Hour	S_A+S_P (MW)
1	5181	7	3645	13	11425	19	8892
2	4046	8	6155	14	11676	20	9645
3	3488	9	8882	15	11677	21	9443
4	2940	10	9945	16	10252	22	8712
5	3001	11	10112	17	9369	23	7376
6	2815	12	10000	18	8494	24	6059

The price paths are generated according to the market parameters derived, setting V to be 1000 for regression fineness. The average day-ahead and real-time price of the V paths are as shown in Fig.4 (only 7 days is displayed due to space limitation).

B. Numerical Results and Analysis

Different products of related variables are regressed on and the R2 scores are compared to find the appropriate expression. Based on the R2 comparisons of different expressions, an appropriate expression of day revenue with average R2 score above 0.9 is found.

$$\begin{aligned}f_1(p_A,Q_C,D_A^0,\mathbf{Q_D}[\mathrm{t}])=&a_1p_A+a_2p_{da}+a_3p_{rt0}+a_4Q_c\\&+a_5\mathbf{Q_D}[\mathrm{t}]+a_6D_A^0+a_7p_AQ_C+a_8p_A\mathbf{Q_D}[\mathrm{t}]\end{aligned}\tag{21}$$

where p_{rt0} is the real-time price at the beginning of the day (which is known for sure at the decision moment). Other products of the variables are also compared, but neglected due to their trivial improvement on R2 score.

As can be inferred from (21), only the 5th and 8th item matters in the decision of $\mathbf{Q_D}$[t]. And the recommended value can be settled by a first order derivative $\mathbf{Q_D}$[t]. Setting the derivative to be 0, the function f_5 can be derived for long-term strategy making. Obviously, the optimal $\mathbf{Q_D}$[t] will appear at the boundaries, when no solution exists.

When aggregated to calculate the day revenue for long-term strategy making (p_A and Q_C), only the 1st, 4th,6th and 7th items are relevant, where others can be regarded as constant. It should be noted that D_A^0 is a complicate function of p_A and changeable during the day, which should be tackled in series. The revenue of the day can be expressed as (22), and that of the period is the sum of those days.

$$f_4(p_A,Q_C,D_A^0)=A_0+A_1p_A+A_4Q_c+\mathbf{a_6D_A^0}+A_7p_AQ_C\tag{22}$$

where $A_1=\sum_{t=1}^{24}a_1^t$, $A_4=\sum_{t=1}^{24}a_4^t$, $\mathbf{a}_6=[a_6^t]_{24}$, $\mathbf{a}_6\mathbf{D_A^0}=[a_6^t\mathbf{D_A^0}[t]]_{24}$, $A_7=\sum_{t=1}^{24}a_7^t$. Note that $\mathbf{D_A^0}$ is the function of p_A and market share, where the former stays the same during the study period and the latter varies each hour in the day. The day number is marked as the left superscript in the following when need. Observation length of one week, month, season, and year for decision is considered.

The proposed method is first applied to an observation interval of one week. The corresponding coefficient in long-term strategy making is as listed in Table III. And the R2 scores of the regression expression for each hour are depicted in Fig.5.

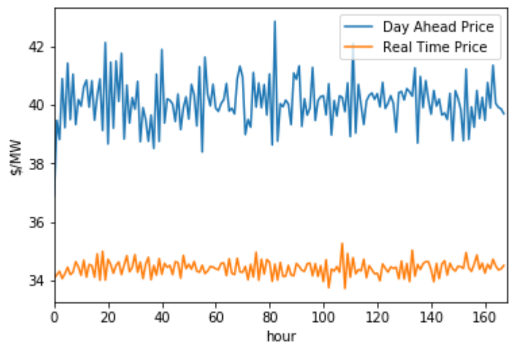


Fig.4. Average day-ahead and real-time price of the V paths. (First 7 days)

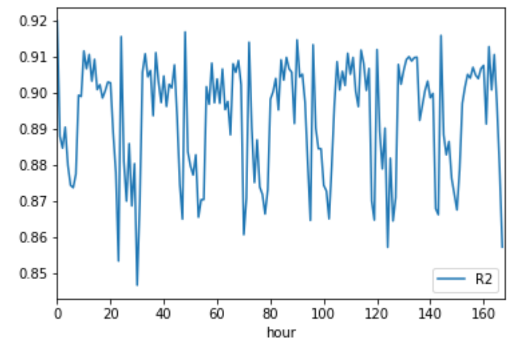


Fig. 5. R2 scores of the regression expression for each hour in the week.

According to (2), (4) and (34), calculate the first order derivatives of p_A and Q_C, and seek the optimal long-term strategy which maximize the expected period revenue. The recommended p_A and Q_C can be calculated from (35-36),

TABLE III
Whole Demand Market Size of the Typical Day

Day	Coefficients			
	A_1	A_4	$\mathbf{a}_6$	A_7
1	-22156.9	-211.567	[0.88, 0.01, 0.54, 0.95, -0.41, 0.82, -1.13, -0.71, 0.11, 0.15, 1.09, 0.52, 0.87, -0.52, 0.89, -0.09, 0.79, 0.28, 0.52, 0.57, 0.47, 0.20, 0.36, -0.67]	6.481
2	-21638	-218.33	[1.17, -0.87, 1.01, 0.65, -0.09, 0.74, -0.77, -1.07, -0.08, 0.58, 0.53, 0.91, 0.13, 0.98, 0.49, 0.65, 0.58, 0.61, 0.30, 0.79, 0.62, -0.32, -0.41, 0.70]	7.821
3	-22287.3	-155.528	[1.02, 0.12, 0.83, 0.32, 1.00, 0.50, -1.25, -0.99, 0.48, -0.52, 1.17, -2.15, 1.09, -3.49, 1.09, -1.33, 0.92, -0.25, 0.53, 0.78, 0.58, 0.76, -1.10, 0.55]	0.671
4	-22626.2	-211.527	[0.80, 0.72, -1.01, 0.99, -0.60, 1.01, -1.67, -0.30, -0.59, 0.60, 0.72, 0.47, 0.48, 0.52, 0.52, 0.10, 0.84, 0.21, 1.11, 0.56, 0.83, -0.03, 0.31, -0.19]	6.430
5	-21936.2	-192.172	[0.98, -0.39, 0.84, 0.48, 0.70, 0.89, -3.28, -0.16, -1.66, 0.93, -0.17, 1.09, 0.05, 0.54, 0.55, 0.83, 0.42, 0.01, 0.86, 0.04, 0.52, 0.37, -0.60, 0.74]	4.557
6	-22173.5	-234.21	[0.39, 0.47, 0.27, 0.69, 0.25, 0.84, -1.21, -0.69, 0.01, 0.63, 0.72, 0.52, 0.63, 0.88, 0.60, 0.48, 0.29, 0.56, 0.81, -0.10, 0.96, 0.53, -0.59, 0.46]	8.405
7	-21912.3	-217.14	[0.88, -0.28, 0.92, -0.01, 0.93, 0.45, -1.11, -1.06, 0.00, 0.76, 0.82, 0.77, 0.59, 0.58, 0.21, 0.88, 0.28, -0.11, 0.89, 0.21, 0.74, 0.33, -0.16, 0.22]	7.733

which are 34.22$/MW, 3675.42MW. And the expected revenue of the week is 4.65M$.

Same procedures are applied to one month (30 days) and one season (90 days). Note that the training data of the selected one week is the same as that of the first 7 days in the month and season. Their average and minimal R2 scores are compared in Fig 6. Similarly, derive the first order derivatives of p_A and Q_C, and seek the optimal long-term strategy which maximize the expected period revenue. The suggested long-term strategies, expected average day revenue is compared in Table IV.

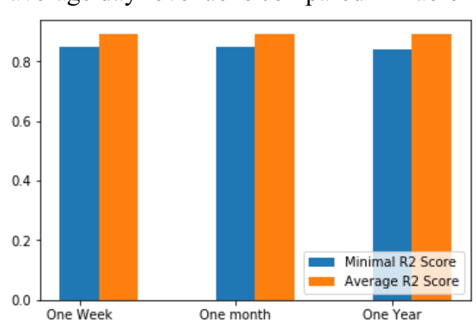


Fig. 6. Average and minimal R2 scores in Different Observation Length.

TABLE IV
Long Term Strategies Suggestion and Expected Average Day Revenue in Different Observation Length

Observation Length	Strategy Suggestion	Expected Average

	p_A($/MW)	Q_C(MW)	Day Revenue ($)
1 week (7 days)	34.22	3675.42	664663.86
1 month (30 days)	31.37	3181.95	236160.10
1 season (90 days)	31.22	3170.11	220107.85

A price path for test under same distribution is generated (as partly shown in Fig.7), and the revenue employing suggested long-term strategies under the testing price path is revealed in Table V.

Taking one week as example, the optimal day-ahead decision Q_D of each hour in the first week of the observed week are mostly zero except for Day2, as demonstrated in Fig 8.

Impacts of the percentage of smart devices, namely the proportion of active load, are also compared and illustrated. The suggested p_A and Q_C, along with the corresponding expected aggregate revenue under different proportions of S_A:S_P is compared in Table VI. The comparison is also based on one week, and the same testing price path is also employed here to convey one possible outcome.

From the above numerical results, it can be inferred that iterations on different scale of observation periods can help to find the optimal decision for the moment. Suggestion for hours are also included in the strategy package, but with limited fineness, as the method is designed for long-term pricing. The suggestions will serve the aggregators best with aid of day operation which designed of short-term operation decision, which is also under study in future study.

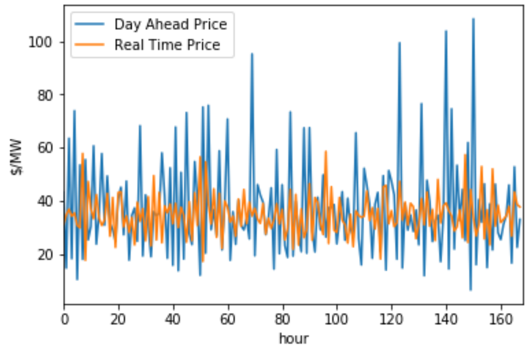


Fig. 7. Test price path of day-ahead and real-time price. (First 7 days)

TABLE V
Aggregate Revenue under Test Case for Different Observation Length

Observation Period	1 week (7 days)	1 month (30 days)	1 season (90 days)
Expected Aggregate Revenue ($)	4652647.02	7084802.94	19809706.30
Aggregate Revenue Under Test Case ($)	5115275.71	7216737.35	19638870.72

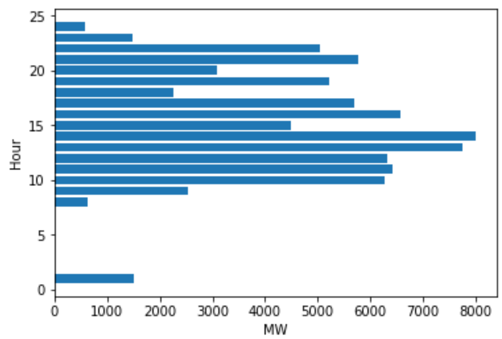


Fig. 8. Optimal day-ahead decision Q_D of each hour in Day2.

TABLE VI
Long Term Strategies Suggestion and Expected Aggregate Revenue under Different Proportions of S_A:S_P

S_A:S_P	Strategy Suggestion		Expected Aggregate Revenue ($)
	p_A($/MW)	Q_C(MW)	
0.8:0.2	43.73	3467.69	6711576.69
0.7:0.3	34.22	3675.42	4652647.02
0.6:0.4	27.78	3583.44	4038596.82
0.5:0.5	22.24	3087.45	3420146.84
0.4:0.6	-13.06	5311.66	5277829.35

V. Conclusion

A distributed aggregator pricing method for long-term differentiated load types, which can maximize the expected aggregate revenue considering price uncertainty, is designed in the paper. It monetized the total benefit of load shifting and congestion management with a tradeable model and took into account various kinds of distributed pricing schemes which is more in line with practical transaction conditions. The proposed aggregator pricing method also applies to non-profit situation, when replacing the objective of the newsvendor model to social welfare maximization. Future research will study the situation where there exist transshipping or game behaviors between several aggregators. Ascending of distributed intermittent energy share, such as solar or wind, will expose the operation to new uncertainty and threat. AI technology is also apt to be embedded in the model-based method, which possesses the potential to further improve the computational accuracy and efficiency. The situation as well as those with multiple uncertainty sources and new power operation requirements will be also studied in further study.

VI. Appendix

Proof of the Theorem*** The optimization problem as (13) can be described as (23), according to (6).

$$\max_{z_t, p_t} \mathrm{E}\pi_t = \begin{cases} \mathrm{E}\pi_t^+ & \mathrm{E}[D_A(z_t) + D_P(t)] \le \mathbf{Q_D}[t] + Q_C \\ \mathrm{E}\pi_t^- & \mathrm{E}[D_A(z_t) + D_P(t)] > \mathbf{Q_D}[t] + Q_C \end{cases} \quad (23)$$

where

$$\mathrm{E}\pi_t^+ = (p_A - s)(1 - \frac{z_t}{2})\rho_A S_A + (p_t - s)\rho_P S_P + (s - \mathbf{p_D}[t])\mathbf{Q_D}[t] + (s - p_C)Q_C$$

$$\mathrm{E}\pi_t^- = (p_A - r_t)(1 - \frac{z_t}{2})\rho_A S_A + (p_t - r_t)\rho_P S_P + (r_t - \mathbf{p_D}[t])\mathbf{Q_D}[t] + (r_t - p_C)Q_C$$

First check the ideal situation when the supply happens to meet the demand, a relationship between the related decision variables as (24) can be derived (expectation). The criterion function is denoted as $\Gamma(t)$.

$$\Gamma(t) = (1 - \frac{z_t}{2})D_A^0 + \mathrm{E}[D_P(t)] - \mathbf{Q_D}[t] - Q_C = 0 \quad (24)$$

For one-dimensional optimization, this point is actually a turning point. In the two-dimensional optimization, the equation stands for a cutting line dividing the area into two parts: the surplus part and the deficit. Substitute (2), (4), (8), (9) into (24), and the relationship as (25) is derived. Bring (5) and (10) into (25), and the derivatives as (26) and (27) show that p_t and z_t have one-to-one correspondence, i.e. $p_t(z_t)$ are bijective. Denote the cutting line as $z_t = f^{\#}(p_t)$.

$$e^{a_2 + b_2 p_t} + 1 = \frac{S_P}{\mathbf{Q_D}[t] + Q_C - (1 - \frac{z_t}{2})\rho_A S_A} \quad (25)$$

$$\frac{dp_t}{dz_t} = \frac{-\rho_A S_A S_P e^{-(a_2 + b_2 p_t)}}{2b_2(\mathbf{Q_D}[t] + Q_C - (1 - \frac{z_t}{2})\rho_A S_A)^2} < 0 \quad (26)$$

$$\frac{dz_t}{dp_t} = \frac{-2b_2 S_P e^{a_2 + b_2 p_t}}{\rho_A S_A (e^{a_2 + b_2 p_t} + 1)^2} < 0 \quad (27)$$

Then the two parts divided by the cutting line are studied separately. For the deficit part, the derivatives of $\mathrm{E}\pi_t^-$ on p_t and z_t are derived as (28) and (29). It can be seen that $\mathrm{E}\pi_t^-$ increases monotonically on p_t and changes monotonically on z_t. For the surplus part, the derivatives of $\mathrm{E}\pi_t^+$ on p_t and z_t are derived as (30) and (31). Similarly, $\mathrm{E}\pi_t^+$ does not have any extreme or non-derivative points on its domain.

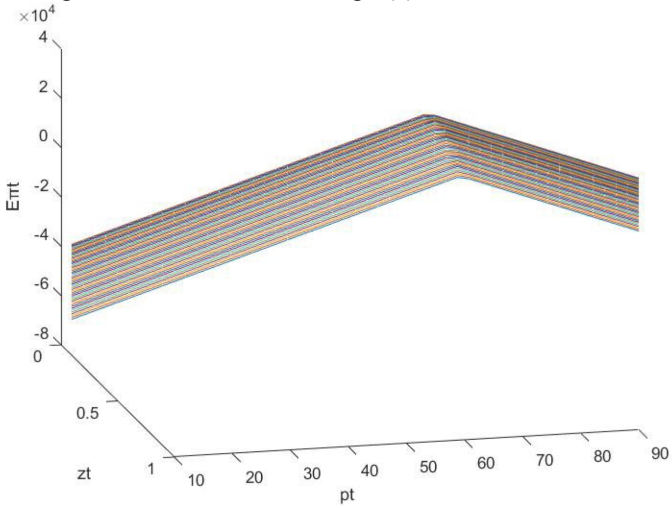
$$\frac{\partial \mathrm{E}\pi_t^-}{dp_t} = (1 - b_2(1 - \rho_P)(p_t - r_t))\rho_P S_P > 0 \quad (28)$$

$$\frac{\partial \mathrm{E}\pi_t^-}{dz_t} = -\frac{1}{2}(p_A - r_t)\rho_A S_A \quad (29)$$

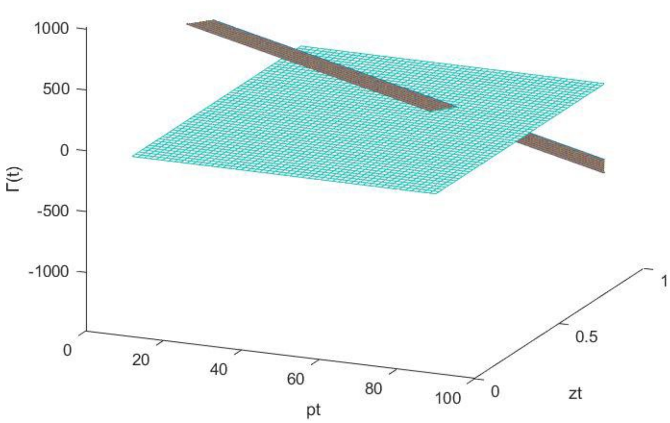
$$\frac{\partial \mathrm{E}\pi_t^+}{dp_t} = (1 - b_2(1-\rho_P)(p_t - s))\rho_P S_P \quad (30)$$

$$\frac{\partial \mathrm{E}\pi_t^+}{dz_t} = -\frac{1}{2}(p_A - s)\rho_A S_A < 0 \quad (31)$$

As $\mathrm{E}\pi_t$ is made up of $\mathrm{E}\pi_t^+$ and $\mathrm{E}\pi_t^-$, it is derivable except on the cutting line, and the optimal value will be spotted by searching the boundaries and the cutting line, which are $z_t = 0, z_t = 1, p_t = s, p_t = r_t(C_p), z_t = f^{\#}(p_t)$. Then the optimal solution can be spotted by searching these lines. The relationship between $\mathrm{E}\pi_t$ and z_t, p_t is depicted in Fig.9(a), and the left side of (24) is also compared with zero plane in Fig.9(b). The solution plane is smooth as analyzed above, which is quite intuitive from the depiction of Fig.9(a). The intersection of the cutting line is demonstrated in Fig.9(b).



(a)



(b)

Fig. 9. Solution diagram: (a) the relationship between $\mathrm{E}\pi_t$ and z_t, p_t, (b) the criterion of the piecewise function $\mathrm{E}\pi_t$.

On $p_t = s$ and $p_t = C_p$, the possible solutions will appear at their intersections with $z_t = 0$, $z_t = 1$, $z_t = f^{\#}(p_t)$ as (29) and (31) still hold.

Similarly, on $z_t = 0$ and $z_t = 1$, (28) still holds and $\mathrm{E}\pi_t^-$ increases monotonically on p_t. As for (30), $\rho_P S_P$ remains positive for all possible p_t and decreases monotonically on p_t according to (10), and the other element $1 - b_2(1-\rho_P)(p_t - s)$ decreases monotonically on p_t as well according to the derivation in (32). Then it can be derived that $\mathrm{E}\pi_t^+$ is a unimodal function on p_t, with the maximum at $1-b_2(1-\rho_P)(p_t-s)=0$.The optimal p^* can be calculated by apply a simple ternary search on $p^* = s + \frac{1}{b_2} + \frac{1}{b_2}e^{-(a_2+b_2p^*)}$ .Then except from the intersections of $z_t = 0$ and $z_t = 1$ with $z_t = f^{\#}(p_t)$, the other candidate optimal points on $z_t = 0$ and $z_t = 1$ are $(p^*,0), (p^*,1)$, $(s,0)$, $(s,1)$, $(r,0)$, $(r,1)$.

$$\frac{\partial(1 - b_2(1-\rho_P)(p_t - s))}{dp_t} = -b_2(1-\rho_P)[b_2\rho_P(p_t - s) + 1] < 0 \quad (32)$$

On the feasible part of the cutting line $z_t = f^{\#}(p_t)$, (24) holds, and $\mathrm{E}\pi_t$ can be expressed as (33). Substituting z_t by $f^{\#}(p_t)$ in (23), $\mathrm{E}\pi_t$ turns to a univariate function which is also a monotonically increasing function according to (34). Then the maximum solution on $z_t = f^{\#}(p_t)$ will appear at the upper limit of the feasible region of p_t in $f^{\#}$ when applies. Thus the related candidate optimal point can be its intersection with $p_t=r_t$, namely $(r_t, 2 - 2\frac{\mathbf{Q_D}[t] + Q_C}{\rho_A S_A} + \frac{2S_P}{\rho_A S_A(1 + e^{a_2 + b_2 r_t})})$, or intersection with $z_t=0$, namely $(\frac{1}{b_2}\ln\frac{S_P - \mathbf{Q_D}[t] - Q_C + \rho_A S_A}{\mathbf{Q_D}[t] + Q_C - \rho_A S_A} - \frac{a_2}{b_2}, 0)$.

$$\mathrm{E}\pi_t = p_A(1 - \frac{z_t}{2})\rho_A S_A + p_t\rho_P S_P - \mathbf{p_D}[t]\mathbf{Q_D}[t] - p_C Q_C \quad (33)$$

$$\frac{\partial \mathrm{E}\pi_t}{dp_t} = \frac{b_2 p_A S_P e^{a_2 + b_2 p_t}}{(e^{a_2 + b_2 p_t} + 1)^2} + \rho_P S_P > 0 \quad (34)$$

From the analysis and derivations above, for any settled – long-term strategies, there exist only a fixed number of candidate optimal points regardless the market size. Thus, ***Theorem**** is proved.

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