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CASE STUDY

A comparative study of marginal loss pricing algorithms in electricity markets

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Abstract

Due to the development of new technologies, change of generation mix and appearance of newly formed energy supply hubs, there is a large year-on-year change in the marginal loss factors in power systems. Since any change of marginal loss factors could have significant impacts on payment of loads and profitability of generators, it is necessary to carry out a comparative study on the loss factor-based locational marginal pricing methods. Considering that a systematic comparison of various locational marginal pricing methods has not been reported in existing publications, this work presents a comparative study of the loss factor-based locational marginal pricing methods that are widely adopted in electricity markets. Advantages and disadvantages of each locational marginal pricing method are explored in detail, and could serve as references in selecting appropriate locational marginal pricing methods in practice. The selected five locational marginal pricing models are tested in two standard power systems, that is, the IEEE 5-bus and 39-bus systems. Then, through numerical experiments and detailed analysis, key findings about the reference point dependency of loss factors, accuracy of loss estimation, load payment, generation income, and market settlement surplus are summarised and elaborated. It is found that marginal loss factors-based locational marginal pricing methods tend to produce a higher market settlement surplus and can lead to a lower generation income than other locational marginal pricing methods.

1 | INTRODUCTION

The locational marginal pricing (LMP) method has been widely employed after being first proposed in [1], since it can provide appropriate price signal for reflecting the short-term operation cost of a given power system. LMP has an advantage in identifying the impacts of the injected power of each node on various line congestions and system power losses. By definition, LMP equals the dual multiplier of the corresponding nodal active power balance constraint in the optimal power flow (OPF) model [2]. Therefore, LMP can be implemented by both alternating current (AC) OPF and direct current (DC) OPF models. In particular, the DC OPF model is currently adopted by the majority of actual electricity markets around the globe, due to its simplicity, robustness and quick convergence.

Since both network losses and reactive power are ignored in the DC OPF model, the attained price signals may be inaccurate. To address this issue, some methods on incorporating network losses into the DC OPF model are proposed. In [2], a linear model for the loss-embedded LMP is presented. Based on the Karush–Kuhn–Tucker (KKT) condition of a general OPF formulation, the relationship among the LMP, the reactive power marginal cost and the congestion cost is first derived; then an iterative algorithm for solving the loss-embedded LMP is developed. In [3], the distributed market slack reference scheme is introduced and a new loss modelling method in the DC OPF-based LMP calculation is proposed. In [4], an iterative DC OPF-based algorithm is presented with a fictitious nodal demand (FND) model employed for calculating LMP. The FND model is proposed to eliminate the power mismatch at the reference bus and the amount of mismatch measures the average system losses. Since it is not accurate to have all the losses absorbed by the reference bus, FND is thus applied to distribute system losses among individual lines. Besides, a matrix loss distribution methodology is developed in [5] for the DC OPF-based marginal loss pricing, where the matrix is used to allocate network losses to system buses. It is verified that such a loss distribution matrix can help make a more accurate representation of the power flow. In [6], a new algorithm for calculating marginal loss factors (MLFs) is proposed, where the transmission loss is expressed in terms of source currents and is independent of the chosen reference bus.

Since existing electricity markets are originally designed based on the premise that most generation resources are fully dispatchable, challenges arise with the rapid growth of variable and less dispatchable renewable energy (RE) generation in power systems. The variations of generation output can result in changes of LMP in power systems, the concept of probabilistic LMP is proposed in [7] to quantitatively model the impact of load forecasting uncertainty on LMP. Furthermore, a bilevel optimisation model is developed in [8] to calculate the LMP intervals under wind uncertainty and this enables a much faster LMP forecasting compared with Monte Carlo-based methods. In [9], a new concept of uncertainty marginal price (UMP) is introduced to define the marginal cost of immunising the next increment of uncertainty at a specific node of power systems. Under a robust optimisation framework, both UMPs and LMPs are derived. The UMP helps allocate the cost of generation reserves to corresponding entities that bring uncertainty. Furthermore, the distribution locational marginal price (DLMP) is employed for settlement of electricity distribution markets in [10–12], and is similar to the concept of LMP in the wholesale electricity market. Due to relatively high power losses, voltage volatilities, and phase imbalances in the distribution network, the determination of DLMP is challenging. Therefore, a three-phase AC OPF-based approach is developed to define and calculate DLMP in [13].

To summarise, in existing research and practice, the integration of network losses into LMP models is usually carried out through modelling the network losses as a linear function of nodal power injections and the loss sensitivity of a nodal injection is referred to as the loss factor. There are several options for calculating MLFs [14, 15], including the direct method, the *B* matrix method, the sensitivity analysis method and the iterative OPF method. Given the system operating point, the sensitivity analysis method would be the most accurate. Although there are already a variety of publications on MLF-embedded LMP models, to the best of our knowledge, systematic comparative studies on exiting LMP methods are not available in existing publication, and are the focus of this work.

The major contributions of this work include two aspects: (1) a systematic comparison of existing MLF-based LMP methods are presented; (2) this comparative study provides decision-making supports for electricity market operators when selecting a specific LMP method in practice.

First, this work presents quantitative comparisons among existing MLF-based LMP methods. Although various LMP methods are available and implemented in practical electricity markets, a systematic comparison among existing LMP methods has not been reported in existing publications. The selected five LMP models that cover all existing loss factor-based LMP methods are tested employing two IEEE standard power systems. Key findings about the dependency of loss factors on reference bus, accuracy of loss estimation, market settlement surplus, payment of loads, income of generators, impacts of transmission congestion on MLFs, and the differences of loss factors between load and generator nodes are elaborated and summarised.

Second, advantages and disadvantages of studied LMP methods are explored in detail, and could serve for references in selecting appropriate LMP methods in practice. In particular, except MLF-based LMP models, the average loss factor (ALF)-based models and reference-independent LMP methods are also tested by numerical experiments. It is found that there is a larger variation in loss factors attained by PF-based methods, and MLF-based LMP methods tend to produce a larger imbalance between consumer payment and generator income. By referring to findings in this work, the electricity market operator can further improve operation efficiency and overcome shortcoming of existing LMP methods in an efficient way.

The rest of the work is structured as follows. Section 2 introduces the decision-making framework of marginal loss pricing models. Then, the numerical experiments for comparative studies, calculation results and corresponding discussions are presented in Section 3. Finally, Section 4 concludes the work by summarising all the key findings.

2 | DECISION-MAKING FRAMEWORK OF MARGINAL LOSS PRICING

The LMP methodology is a dominant approach in power markets. Because of the relatively low speed and convergence problem in a fairly large system while employing the ACOPF-based LMP algorithm, the DC OPF-based LMP methodology is usually used in the power industry to calculate LMPs. A typical feature of the DC OPF model is that the network losses and reactive power are generally ignored. Consequently, the DC OPF-based LMP algorithm can be modelled by linear programming (LP), which guarantees the robustness and speed of LMP calculation [4]. In the power pool market, the market operator (MO) receives energy offers from producers and energy bids from consumers for specified trading interval, and determines the power production of every producer, the consumption level of every consumer, and the price at which every producer/consumer is paid/charged for its energy production/consumption [16]. The objective of market is to pass the generation cost to consumers in a fair and efficient way [17]. Thus, the lossless market clearing model for LMP calculation with the energy price and the congestion price can be formulated as follows.

$$\max \sum_{i\in N} p_i^{\mathrm{cb}} r_i^{\mathrm{b}} - \sum_{j\in M} p_j^{\mathrm{cs}} r_j^{\mathrm{s}} \text{ or } \min \sum_{j\in M} p_j^{\mathrm{cs}} r_j^{\mathrm{s}}, \tag{1}$$

$$\text{s. t.} \sum_{i\in N} p_i^{\mathrm{cb}} = \sum_{j\in M} p_j^{\mathrm{cs}}, \tag{2}$$

$$\sum_{n\in N^{\text{bus}}} f_{l,n}^{\text{PTDF}}\left(\sum_{i\in N_n^{\text{bus}}} p_i^{\text{cb}} + \sum_{j\in M_n^{\text{bus}}} p_j^{\text{cs}}\right) \le P_l^{\max}, \forall l \in L, \quad (3)$$

$$p_i^{\text{b,min}} \le p_i^{\text{cb}} \le p_i^{\text{b,max}}, \forall i \in N, \quad (4)$$

$$p_j^{\text{s,min}} \le p_j^{\text{cs}} \le p_j^{\text{s,max}}, \forall j \in M, \quad (5)$$

where (1) denotes the objective function and is to maximise the net social welfare when both consumers and producers participate in the market and it becomes the minimisation of electricity purchase cost when only producers participate; N/M is the set of consumers and producers; $p_i^{\text{cb}}/p_j^{\text{cs}}$ indicates the dispatched demand/output of the ith consumer/jth producer; $r_i^{\text{b}}/r_j^{\text{s}}$ is the bid/offer price of the ith consumer /jth producer; Equation (2) is the constraint of balance between generation and demand; Equation (3) gives the network constraint and $P_l^{\max}$ is the power limit of branch l; L is the set of branches; $f_{l,j}^{\text{PTDF}}$ denotes the power transfer distribution factor (PTDF) which is used to indicate the relative change of the active power that occurs on a particular branch l due to actual power change at bus j; N_n^{bus} and M_n^{bus} are the set of consumers and producers connected to bus n; N^{bus} is the set of network buses; Equations (4) and (5) represent the limit on generation output and load consumption; $p_i^{\text{b,min}}/p_i^{\text{b,max}}$ is the lower/upper demand limit for the ith consumer; $p_j^{\text{s,min}}/p_j^{\text{s,max}}$ is the lower/upper generation limit for the jth producer.

2.1 | Typical locational marginal pricing model

The nodal pricing method has been widely implemented in a number of US RTOS or ISOs such as PJM, New York ISO, ISO-New England, California ISO, ERCOT and Midwest ISO [18] after proposed, as it can provide appropriate price signal for reflecting the short-term operation cost of a given power system. Nodal pricing has an advantage if identifying the impacts of the injected power of each bus on various line congestions and system power losses. However, the above-mentioned DC OPF-based LMP model is established without losses, and only the marginal energy price and congestion price are derived using dual multipliers in the model. In order to determine the marginal loss component in the LMP, analysis of AC power flow is usually carried out after determining the dispatch plan by (1–5). Therefore, the LMP at a given bus n can be expressed as follows, which is composed of three components, including system energy price, transmission congestion price and the marginal loss price [19].

$$r_n^{\text{np}} = r^{\text{smp}} + r_n^{\text{cp}} + r_n^{\text{mlc}}, \quad (6)$$

where r_n^{np} is the nodal price at the nth bus; r^{smp} is the system marginal energy price; r_n^{cp} represents the congestion price and is calculated using the shadow price of binding constraints and the PTDF; r_n^{mlc} is the marginal loss price and is calculated using the system energy price and the penalty factor.

$$r_n^{\text{cp}} = \sum_{k\in K^{\text{trc}}} r_k^{\text{sdp}} f_{nk}^{\text{PTDF}}, \quad (7)$$

$$r_n^{\text{mlc}} = r^{\text{smp}}\left(1\Big/f_n^{\text{PF}} - 1\right), \quad (8)$$

$$f_n^{\text{PF}} = 1\Big/\left[1 - \left(\Delta P^{\text{loss}}\Big/\Delta P_n^{\text{inj}}\right)\right], \quad (9)$$

where r_k^{sdp} denotes the shadow price of the kth binding constraint; K^{trc} denotes the set of binding constraints; f_{nk}^{PTDF} indicates the PTDF of transmission line k corresponding to bus n; f_n^{PF} is the penalty factor for bus n and is the sensitivity factor of injection P_n^{inj} at bus n to system power losses P^{loss}. ΔP_n^{inj} refers to the change of power injection at bus n; ΔP^{loss} is the change of the total power losses in the network.

$$\begin{aligned} r_n^{\text{np}} &= r^{\text{smp}} + \sum_{k\in K^{\text{trc}}} r_k^{\text{sdp}} f_{nk}^{\text{PTDF}} + r^{\text{smp}}\left(1\Big/f_n^{\text{PF}} - 1\right), \\ &= r^{\text{smp}}\left[1 - \left(\Delta P^{\text{loss}}\Big/\Delta P_n^{\text{inj}}\right)\right] + \sum_{k\in K^{\text{trc}}} r_k^{\text{sdp}} f_{nk}^{\text{PTDF}}. \end{aligned} \quad (10)$$

Besides, in the Canadian electricity market, the congestion component of LMP is slightly modified by introducing the penalty factor to the calculation of congestion price in the LMP formulation as follows [20, 21].

$$r_n^{\text{np}} = r^{\text{smp}}\left[1 - \left(\Delta P^{\text{loss}}\Big/\Delta P_n^{\text{inj}}\right)\right] + \sum_{k\in K^{\text{trc}}} \left(1\Big/f_n^{\text{PF}}\right) r_k^{\text{sdp}} f_{nk}^{\text{PTDF}}. \quad (11)$$

2.2 | Australian national electricity market pricing model

In the Australian national electricity market (NEM), the connection point of a transmission network for a load/generator is defined as the physical point at which the assets owned by a transmission network service provider meet the assets owned by a distribution network service provider/generation supplier. Connection points are also called transmission node identities [22]. The regional reference node (RRN) is defined as the location in each region at which spot prices are determined by the NEM dispatch engine and by reference to which marginal loss factors (MLFs) are calculated. RRNs are typically located near the major load centre in each region, such as the capital city [23]. In the Australian NEM, MLFs are used to represent the change in network losses that occur due to a small increase in load at connection points across the NEM, compared to the change that would occur if the loads were located at the regional reference node (RRN). Conceptually, this can be achieved by modelling a small increase in load at each generation and load connection point in each region in turn, and determining the resultant increase in generation required to meet that load increase assuming it is supplied from a generating unit located at the RRN. Notably, the MLF at the RRN is 1 and the MLF at each bus can be calculated as follows [24].

$$f_n^{\mathrm{MLF}} = \Delta P^{\mathrm{RRN,g}} \Big/ \Delta P_n^{\mathrm{load,incr}} = 1 + \Delta P^{\mathrm{loss}} \Big/ \Delta P_n^{\mathrm{load,incr}}, \tag{12}$$

where f_n^{MLF} and $\Delta P_n^{P\mathrm{load,incr}}$ denote the MLF and load increment at bus n; $\Delta P^{\mathrm{RRN,g}}$ is the change of generation at the RRN; ΔP^{loss} is the change of the total power losses in the network.

The small increase in load at each generation and load connection point means a 1 kW increase of demand at a load connection point, and a 1 kW decrease of output at a generator connection point which is equivalent to a virtual 1 kW increase of load at the generator connection point. The region is defined by regional boundaries here. The regional boundaries in the NEM are selected such that transmission constraints are rarely binding within a region but frequently binding on region boundaries. The National Electricity Rules (NER) allow for boundaries to be reset as required whenever a constraint occurs for greater than 50 h per year [25]. The current NEM is operated based on five interconnected regions that largely follow state boundaries.

Besides, since a load increment is equivalent to a negative power injection at a certain bus, therefore, Equation (12) can be further transformed as follows [24].

$$f_n^{\mathrm{MLF}} = \frac{\Delta P^{\mathrm{RRN,g}}}{\Delta P_n^{\mathrm{load,incr}}} = 1 + \frac{\Delta P^{\mathrm{loss}}}{\Delta P_n^{\mathrm{load,incr}}} = 1 - \frac{\Delta P^{\mathrm{loss}}}{\Delta P_n^{\mathrm{inj}}}. \tag{13}$$

The locational signals of MLF are defined as follows:

For MLFs less than 1, it indicates that network losses will increase as more generation is dispatched at that bus and decrease as more loads are taken. The smaller the MLF when it is below 1, the greater the increase (or decrease) in network losses for the same magnitude of change. Connection points in transmission networks where there is an overall net injection into the network will tend to have MLFs less than 1. This would normally be expected to apply to generators. However, this will also apply to loads situated in transmission networks where the local level of generation is greater than the local load.

For MLFs greater than 1, it indicates that losses will increase as more loads are consumed and decrease as more generation is dispatched. The higher the loss factor is above 1 the greater the increase (or decrease) in losses for the same magnitude of change. Connection points in transmission networks where there is an overall net load tend to have MLFs greater than 1. This would normally be expected to apply to load; however, this will also apply to generation situated in transmission networks where the local load is greater than the local level of generation.

In Australia NEM, the spot market for each trading interval is solved independently of all other spot market trading intervals. Even though centralised day-ahead forecasts of future prices are carried out by the Australian Energy Market Operator (AEMO), most responsibility for decision-making rests with participants, such as the unit commitment decisions, are left to market participants. Market clearing process of NEM while considering the MLF is summarised as follows [24]:

1. The generator submits offer prices to AEMO referenced to its connection point;
2. These offer prices are divided by the generating unit loss factor to refer the price to the RRN;
3. The offer prices of the equivalent generating unit at the RRN are entered into an offer stack of offer prices submitted by all generating unit (all referred to the RRN);
4. The offer stack is arranged into a merit order of offers received, ranging from cheapest to most expensive;
5. The equivalent generating units are dispatched according to this merit order until sufficient generation is available to meet demand;
6. The most expensive generating unit dispatched sets the marginal price;
7. Each Generator is paid for its generation through the AEMO settlements system. The price paid is equal to the marginal (spot) price multiplied by the relevant loss factor.

Notably, in the Australian NEM, the final nodal price is also composed of three components, namely the energy, losses and congestion prices [26].

2.3 | Reference-independent marginal loss pricing model

Since electrical energy is delivered to customers via the transmission network, properties of power networks including transport capacity and transport losses have to be included in the marginal-based spot price. Although it is pointed out in [24] that network properties can be integrated into the spot price as separate physical components, they are not totally independent of each other. Then, a novel method is proposed in [18] to obtain a truly reference-independent LMP decomposition such that all three components of LMP at each bus, which corresponds to the marginal energy price, the marginal congestion price and the marginal loss price, will be invariant despite the choice of the system reference bus. The proposed fully reference-independent LMP model can be summarised as follows where only single-sided offers from generators are considered [18].

$$\min \sum_{j\in M} p_j^{\mathrm{cs}} r_j^{\mathrm{s}}, \tag{14}$$

$$\text{s. t.} \sum_{j\in M} p_j^{\mathrm{cs}} - \sum_{i\in N} p_i^{\mathrm{load}} - P^{\mathrm{loss}} = 0, \tag{15}$$

$$P^{\mathrm{loss}} - \sum_{n\in N^{\mathrm{bus}}} f_n^{\mathrm{MLF}} \left(\sum_{j\in M_n^{\mathrm{bus}}} p_j^{\mathrm{cs}} - \sum_{i\in N_n^{\mathrm{bus}}} p_i^{\mathrm{load}} \right) + P^{\mathrm{offset}} = 0, \tag{16}$$

$$\sum_{n\in N^{\mathrm{bus}}} \rho_{l,n}\left(\sum_{i\in N_n^{\mathrm{bus}}} p_i^{\mathrm{cb}} + \sum_{j\in M_n^{\mathrm{bus}}} p_j^{\mathrm{cs}} - f_n^{\mathrm{LDF}} \times P^{\mathrm{loss}}\right) \le P_l^{\mathrm{max}} \ \forall l \in L, \tag{17}$$

$$p_j^{\mathrm{s,min}} \le p_j^{\mathrm{cs}} \le p_j^{\mathrm{s,max}} \ \forall j \in M, \tag{18}$$

$$f_{l,n} = \frac{\left\{\left[(Z_{k_1,n} + Z_{k_2,n})(V_{k_1}^* - V_{k_2}^*) + (Z_{k_1,n}^* - Z_{k_2,n}^*)(V_{k_1} + V_{k_2})\right] \big/ 2z_l^*\right\}^{\mathrm{RE}}}{(V_n + Z_{n,n} I_n^*)^{\mathrm{RE}}}, \tag{19}$$

$$f_n^{\mathrm{MLF}} = \sum_{l\in L} R_l \times 2I_l \times f_{l,n}, \tag{20}$$

$$E_n = \sum_{l\in L} I_l^2 \times R_l / 2; \ f_n^{\mathrm{LDF}} = E_n \Big/ \sum_{n\in N^{\mathrm{bus}}} E_n, \tag{21}$$

where (14)–(18) denote the DC OPF-based market clearing; P^{loss} represents the system loss; p_i^{load} is the load demand of consumer i; f_n^{MLF} is the MLF at bus n; P^{offset} is the variable introduced to offset the doubled system loss caused by the MLF; f_n^{LDF} indicates the loss distribution factor at bus n; Equations (19) and (20) calculate the MLF at bus n; Since the power losses of a certain branch l is assigned to its corresponding terminals, E_n denotes half of the power losses on branch l that is assigned to bus n for calculating the loss distribution factor at bus n; R_l represents the resistance of branch l in the transmission network; $f_{l,n}$ is the reference independent real-power distribution factor for the calculation of MLFs; k_1 and k_2 are the sending and receiving buses, respectively, for branch l; Z is the network impedance matrix; z_l is impedance of branch l; V_n is the complex voltage at bus n; I_n and I_l are the current injection at bus n and current through branch l, respectively; Equation (21) is to obtain the loss distribution factor at bus n using the FND model proposed in [4].

In order to solve the above reference-independent LMP model, the initial values of system status, such as line flows and generation outputs, are needed to obtain E_n and f_n^{LDF} in (21), P^{offset} in (16), and f_n^{MLF} in (20). This can be carried out by running an initial ACOPF. Then, the above reference-independent DC-based model can be applied for LMP calculation with fully reference-independent decomposition.

2.4 | Determination of marginal loss factors

From above analysis on existing marginal loss models, it can be found that the calculation of MLFs is critical in the marginal loss pricing. In particular, there are two different algorithms of calculating MLFs: (1) OPF-based algorithm; (2) power flow (PF)-based algorithm. In the OPF-based algorithm, the loss sensitivity of each node is calculated by re-solving the OPF problem after implementing a unit change of load or generation output at each node. For the PF-based algorithm, the PF formulation is re-solved each time for the loss sensitivity analysis. Figure 1 presents the processes of MLF calculation.

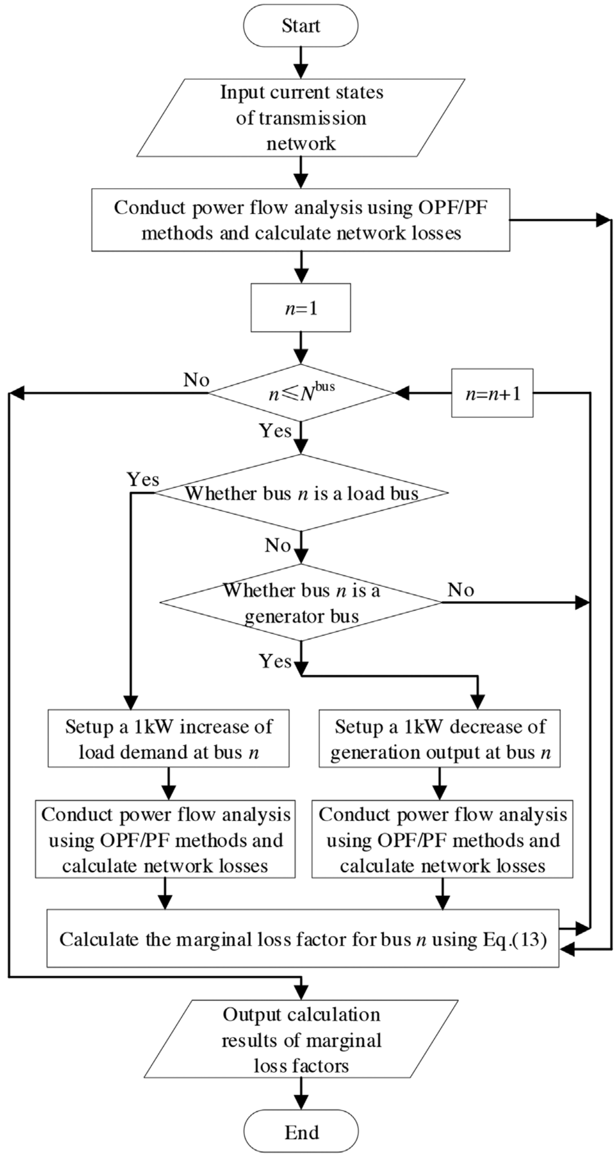

FIGURE 1 Illustration of marginal loss factor calculation processes

Notably, in the PJM electricity market, MLF represents the percentage increase in system losses caused by a small increase in power injection or withdrawal [27]. The OPF-based algorithm is able to calculate the MLFs under this circumstance. However, in the Australian NEM, MLF is defined as the marginal losses to deliver electricity to that node from the RRN [28]. Consequently, it needs a PF-based in order to control the source of power supply.

3 | COMPARATIVE STUDY OF LMP ALGORITHMS

3.1 | Selected LMP algorithms

Since there exist two methods for MLF calculation, in this comparative study, both OPF- and PF-based MLFs are studied. Besides, there is also another category of loss factors,

TABLE 1 Details of selected methods in the comparative study

	Loss calculation		Loss factor	
Method	**OPF**	**PF**	**Marginal**	**Average**
OPF-M	√	×	√	×
OPF-A	√	×	×	√
PF-M	×	√	√	×
PF-A	×	√	×	√
RI-LMP	×	×	×	×

Note: √ and × indicate the algorithm is and is not included, respectively.

namely the ALFs [29]. The philosophy behind ALF-based LMP method is to allocate network losses to generators and consumers equally while ignoring the impact of network topology. When using ALFs, network losses can be equally assigned to generators and consumers, or 100% to consumers. In order to compare the efficiency of MLF- and ALF-based pricing methods, it is assumed that network losses are equally assigned to consumers and generators. ALFs are calculated as follows [29].

$$f^{\mathrm{ALF}} = P^{\mathrm{loss}} \Bigg/ \left(\sum_{i \in N} P_i^{\mathrm{D}} + \sum_{j \in M} P_j^{\mathrm{G}} \right), \tag{22}$$

$$f^{\mathrm{D,ALF}} = 1 + f^{\mathrm{ALF}};\ f^{\mathrm{G,ALF}} = 1 - f^{\mathrm{ALF}} \tag{23}$$

where $P_i^{\mathrm{D}} / P_j^{\mathrm{G}}$ is the load demand/generation output of consumer i/producer j; $f^{\mathrm{D,ALF}}$ and $f^{\mathrm{G,ALF}}$ are ALFs for consumers and producers, respectively.

Then, five different LMP methods are selected for the comparative study, including (1) OPF MLF (OPF-M)-based LMP; (2) OPF ALF (OPF-A)-based LMP; (3) PF MLF (PF-M)-based LMP; (4) PF ALF (PF-A)-based LMP; and (5) the reference independent LMP (RI-LMP) method proposed in [18]. Table 1 presents the details of each method. In this section, the selected five methods are tested in two standard power systems (the IEEE 5-bus and 39-bus systems) under congested and uncongested conditions, respectively. Notably, DC model-based LMP methods can usually derive LMP results without a marginal loss price component. Even when power losses are incorporated in a DC-based LMP model, it still requires a running of AC model-based OPF/PF beforehand in order to calculate the MLFs. In this case study, all the calculation discussed below is based on an AC model. Besides, all computations are carried out on a desktop computer with an Intel Core i7-6700 CPU at 3.4GHz, 16.0 GB RAM. The Matpower package installed in the Matlab software is used for power flow analysis. The linear programming solver embedded in Matlab is used for solving the electricity market clearing problems.

Tables 2 and 9 present specifications of different cases employed in the case study. First, in order to analyse impacts of RRN locations on the final locational marginal price (LMP) results, Cases 1, 3, 7, 8 of the 5-bus power system, Cases 1–6 of the 39-bus power system are selected. In Cases 1, 3, 7, 8 of the 5-bus system, offer parameters of generators and the setup of load demand are consistent, but the location of RRN varies from case to case. This is the same for Cases 1–6 of the 39-bus system.

Furthermore, since offer prices of generators determine, to a great extent, LMP values, Cases 1, 2 and Cases 4, 5 of the 5-bus system are selected to study impacts of offer prices on LMP results, where the offer prices of generators differ among these cases but the setup of the load demand and RRN are consistent. Similarly, in the 39-bus system, two groups of offer prices are also selected and compared between Cases 1, 2, 3 and Cases 4, 5, 6.

Besides, Cases 1 and 4 are selected to examine impacts of load demand changes on LMP results in the 5-bus system, thus both offer prices and RRN are consistent in Cases 1 and 4 but the load demand varies. Finally, the LMP results are studied when the offer prices of generators and load demand change simultaneously in Cases 3, 6 of Table 2.

3.2 | Results and analysis for the 5-bus system

The PJM 5-bus system [27] is adopted for simulation and the original index of buses is modified from A–E to 1–5. Table 2 gives the data configuration for eight different cases. Considering that the reference bus (also known as slack bus or swing bus) should be able to balance the active and reactive power in power systems, all buses with generators are tested as reference bus in turn.

3.2.1 | Calculation results for uncongested 5-bus system

The loss factors obtained by each algorithm are presented in Figures 2 and 3. Tables 3–5 compare the revenue collected from loads, the income of generators, and the market settlement surplus under different LMP algorithms in the uncongested 5-bus system.

In practical electricity markets, when a specific LMP algorithm is employed, generators and consumers will be paid/charged with the corresponding nodal prices. The prices paid by electricity consumers can diverge from the prices paid to generators. The imbalance between total consumer payment and total generator income is referred to as the market settlement surplus [30]. In practical electricity markets, the market settlement surplus can be positive or negative. The allocation, distribution and recovery of the settlement residues are managed by the corresponding electricity market operator.

Results in Figures 2 and 3 show that the OPF-A and PF-A methods produce ALFs with similar values, while there could be a big difference between MLFs calculated by the OPF-M and PF-M methods. In particular, MLFs obtained by the PF-M method tend to be either the largest or the smallest among these five methods, which is obvious in Cases 2, 3, 7, and 8. In Cases 1, 4, 5 and 6, when the PF-M loss factors are not the largest or the

TABLE 2 Offer parameters of generators under each case

	Offer prices ($/MWh)					Load (MW)			
Case	G1	G2	G3	G4	G5	L2	L3	L4	RRN
1	14	15	30	35	10	300	300	300	4
2	14	15	40	35	10	300	300	300	4
3	14	15	30	35	10	300	300	300	5
4	14	15	30	35	10	600	0	300	4
5	30	15	10	20	50	600	0	300	4
6	30	15	10	20	50	600	0	300	5
7	14	15	30	35	10	300	300	300	1
8	14	15	30	35	10	300	300	300	3

Note: G1–G5 represent generators 1–5; L2 to L4 indicate loads 2–4.

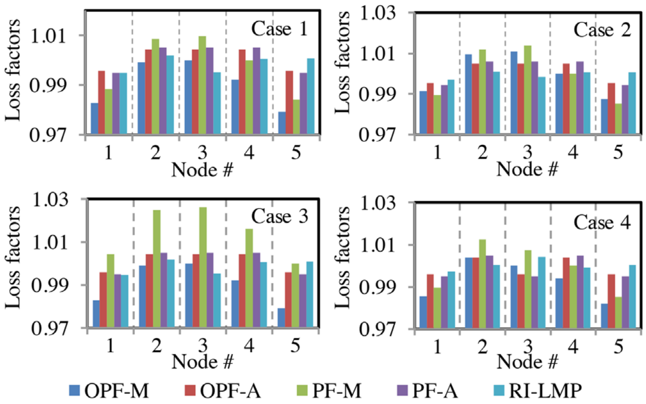

FIGURE 2 Loss factors obtained by different algorithms under Cases 1–4 in the uncongested 5-bus system

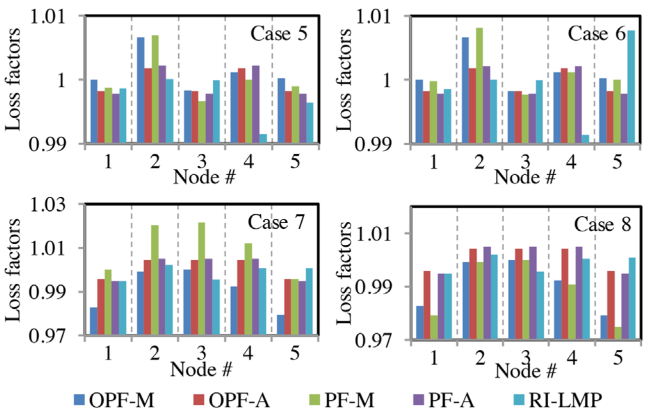

FIGURE 3 Loss factors obtained by different algorithms under Cases 5–8 in the uncongested 5-bus system

TABLE 3 Revenue collected from loads under different LMP methods in the uncongested 5-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	26923.0	27113.2	26905.1	26877.2	27108.1
2	31715.2	31649.9	31768.3	31681.5	31479.7
3	26923.0	27113.2	26905.3	26449.6	27107.1
4	27018.4	27108.4	27022.4	26930.9	26885.3
5	27130.2	27048.4	27159.9	27094.1	26961.7
6	27130.2	27048.4	27161.9	27064.3	26962.2
7	26923.0	27113.2	26905.6	26564.9	27106.4
8	26923.0	27113.2	26901.0	27135.8	27102.1

Note: Maximum value ($) Minimum value ($)

TABLE 4 Income of generators under different LMP methods in the uncongested 5-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	26721.4	27113.4	26658.6	26877.2	27397.6
2	31444.9	31649.8	31434.8	31681.5	31809.8
3	26721.4	27113.4	26656.5	26449.7	27396.0
4	26801.9	27108.5	26755.0	26930.9	27149.5
5	27055.3	27048.3	27067.2	27094.0	27204.4
6	27055.3	27048.3	27068.6	27064.4	27205.5
7	26721.4	27113.4	26658.5	26564.9	27391.6
8	26721.4	27113.4	26661.2	27135.9	27381.4

Note: Maximum value ($) Minimum value ($)

smallest, they are usually the second largest or second smallest. However, loss factors obtained by the other algorithms do not show an obvious trend as those of the PF-M algorithm. Meanwhile, loss factors in Figures 2 and 3 indicate that the load nodes (nodes 2, 3 and 4 in Cases 1,2,3,7 and 8; nodes 2 and 4 in Cases 4, 5 and 6) tend to have a higher loss factor than the generator nodes (nodes 1 and 5 in Cases 1,2,3,7 and 8; nodes 1, 3 and 5 in Cases 4, 5 and 6), which complies with the previous analysis in Section 2.2. MLFs obtained by RI-LMP for load nodes sometimes are smaller than MLFs for generator nodes. This may be because the RI-LMP method is derived based on the DC model and is less accurate in measuring network losses. Notably, loss

TABLE 5 Market settlement surplus under different LMP methods in the uncongested 5-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	201.60	−0.143	246.496	−0.0881	−289.50
2	270.29	0.072	333.478	−0.0267	−330.06
3	201.60	−0.143	248.780	−0.1267	−288.83
4	216.45	−0.144	267.477	0.06756	−264.16
5	74.942	0.117	92.7617	0.0539	−242.70
6	74.942	0.117	93.3472	−0.0516	−243.37
7	201.60	−0.143	247.183	−0.0095	−285.23
8	201.60	−0.143	248.783	−0.0116	−279.32

Note: Maximum value ($) Minimum value ($)

factors obtained by the OPF-based methods are independent of chosen reference bus, since OPF is reference independent.

Nodal prices for the uncongested 5-bus system are also calculated. Different from the loss factors, there is no any obvious trend found in the nodal prices obtained by these five methods. This is because nodal prices are determined by both the offer prices of generators and loss factors at each node. Then, changing trend of loss factors in Figures 2 and 3 is weakened by the marginal energy price in the final nodal prices.

The revenue collected from loads and payment to generators under the five LMP methods are presented in Tables 3 and 4, respectively. On the demand side, the electricity market operator tends to collect the lowest revenue from loads when using the PF-A method. On the contrary, loads are very likely to make a high payment for purchasing electricity under all the other methods. On the supply side, the RI-LMP method can always bring the highest income to generators. Also, the ALF-based LMP methods can produce a higher income for generators than the MLF-based LMP methods, as shown in Table 4. Consequently, the market settlement surplus produced by the OPF-M and PF-M methods is much higher than those by the OPF-A, PF-A and RI-LMP methods, as given in Table 5. Besides, in existing electricity markets, such as the Australian NEM [31] and regional electricity markets in the United State [30], the market settlement surplus is usually refunded to loads. Therefore, when adopting the MLF-based LMP methods, genertaors would have a lower income than under the ALF-based LMP and RI-LMP schemes.

3.2.2 | Calculation results for congested 5-bus system

In the congested 5-bus system, the transmission capacity of branch 2–3 and branch 4–5 is set as 200 and 240 MW, respectively. Then, the five LMP methods in Table 1 are tested again under the eight cases in Table 2. Figures 4 and 5 present the obtained loss factors.

In the congested 5-bus system, the OPF-A and PF-A methods still produce ALFs with similar values. Different from the ALFs, there is a big difference in MLFs calculated by OPF-M and PF-M methods. In cases 1, 2, and 8, the OPF-M MLFs are larger than the PF-M MLFs, whereas in cases 3, 4, and 7, OPF-M MLFs become smaller than the PF-M MLFs. In the congested 5-bus system, the PF-M method also tends to produce the largest or the smallest loss factors, especially under cases 3, 7 and 8. Moreover, results of nodal prices indicate that when congestion happens, the difference of nodal prices obtained by different LMP methods is slight. This means the loss factors have less impact on final nodal prices than network transmission congestions. Therefore, the difference of nodal prices would be mainly determined by the congestion component. Tables 6–8 give the revenue collected from loads, income of generators, and the market settlement surplus, respectively.

Comparing with results in the uncongested situation, when congestion happens, the MLF-based LMP methods still tend to collect higher revenue from loads. Differently, load revenue collected by the OPF-A is exceeded by the OPF-M method in Table 6. Thus, loads become more likely to make a higher payment under MLF-based LMP methods than under ALF-based LMP schemes. Meanwhile, in the supply side, income brought by the OPF-M is the largest in Table 7, which is the lowest in

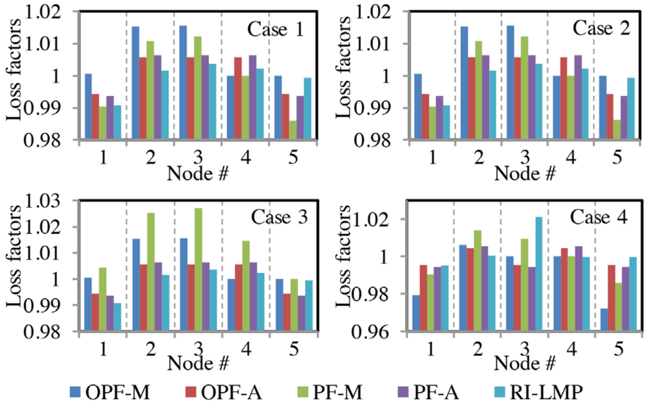

FIGURE 4 Loss factors obtained by different algorithms under Cases 1–4 in the congested 5-bus system

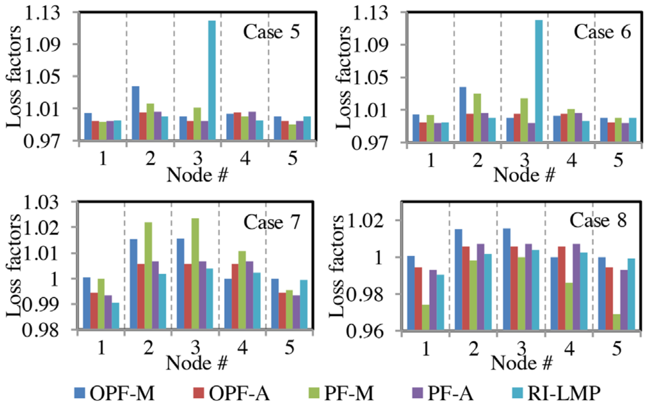

FIGURE 5 Loss factors obtained by different algorithms under Cases 5–8 in the congested 5-bus system

TABLE 6 Revenue collected from loads under different LMP methods in the congested 5-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	25976.6	25831.9	25893.5	25853.6	25629.2
2	25976.6	25831.9	25893.5	25853.6	25629.2
3	25596.8	25555.4	25704.9	25562.2	25626.8
4	28128.9	28141.5	28291.2	28168.7	27015.3
5	60075.1	59357.6	59540.9	59383.2	60509.4
6	59852.5	58903.8	59728.1	58958.4	60441.4
7	25724.5	25658.8	25844.6	25672.1	25630.6
8	26200.5	26088.5	25822.1	26122.2	25630.5

Note: Maximum value ($) Minimum value ($)

TABLE 7 Income of generators under different LMP methods in the congested 5-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	13364.5	13226.2	13044.1	13247.9	13255.0
2	13364.5	13226.2	13044.1	13247.9	13255.0
3	13213.8	13174.3	13251.0	13181.1	13264.7
4	20178.4	20784.2	20613.1	20811.3	18959.0
5	43478.5	43317.0	43232.2	43342.2	42868.8
6	42982.5	42775.4	43160.1	42823.1	42810.7
7	13286.9	13224.1	13286.9	13237.5	13267.9
8	13665.4	13558.3	13062.2	13616.1	13273.2

Note: Maximum value ($) Minimum value ($)

Table 4. Moreover, the PF-M and PF-A methods tend to have the same capability in bringing income to generators. In contrast, generators would likely to have the smallest income when using the OPF-A and RI-LMP methods if network congestion happens.

In terms of the market settlement surplus, the MLF-based LMP methods including OPF-M and PF-M still tend to have a higher market settlement surplus than their corresponding ALF-based LMP methods, as shown in Table 8. Besides, due to network congestion, the settlement surplus in Table 8 gets much higher than those in the uncongested situation.

TABLE 8 Market settlement surplus under different LMP methods in the congested 5-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	12612.1	12605.7	12849.4	12605.7	12374.3
2	12612.1	12605.7	12849.4	12605.7	12374.3
3	12383.0	12381.2	12453.9	12381.2	12362.1
4	7950.5	7357.29	7678.10	7357.43	8056.29
5	16596.6	16040.6	16308.7	16041.0	17640.7
6	16870	16128.4	16568.0	16135.3	17630.6
7	12437.6	12434.7	12557.6	12434.7	12362.8
8	12535.2	12530.3	12759.9	12506.1	12357.4

Note: Maximum value ($) Minimum value ($)

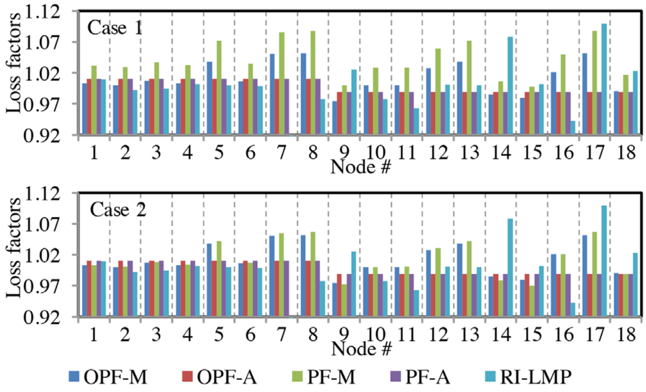

FIGURE 6 Loss factors obtained by different algorithms under Cases 1–2 in the uncongested 39-bus system

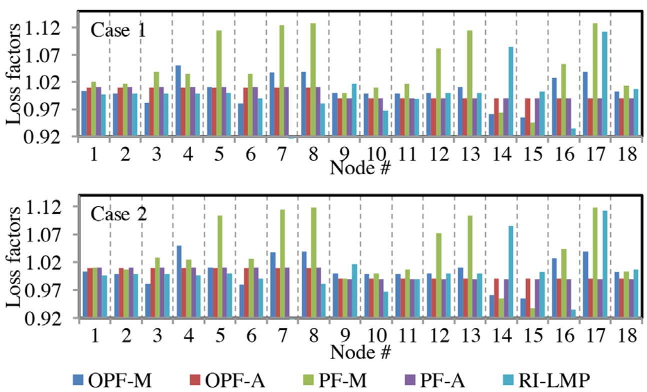

FIGURE 7 Loss factors obtained by different algorithms under Cases 5–6 in the uncongested 39-bus system

3.3 | Results and analysis for the 39-bus system

To further compare the five LMP methods, the IEEE 39-bus system [32] is adopted for simulation. Load buses 8, 12, 16, 18, 20, 24, 28, 29 and generator buses 30–39 in the original 39-bus system are renumbered as buses 1–8 and buses 9–18, respectively, in this subsection. The data configuration for six different cases is presented in Table 9.

3.3.1 | Calculation results for uncongested 39-bus system

Figures 6 and 7 give the values of loss factors in the uncongested 39-bus system. In Figures 6 and 7, MLFs obtained by the PF-M method tend to be the biggest or the smallest among the five methods, which can be verified by results of cases 1–6. For only

TABLE 9 Offer parameters of generators under each case

	Offer prices of generation unit ($/MWh)										
Case	G1	G2	G3	G4	G5	G6	G7	G8	G9	G10	RRN
1	14	15	18	20	30	10	11	25	50	40	30
2	14	15	18	20	30	10	11	25	50	40	31
3	14	15	18	20	30	10	11	25	50	40	32
4	14	18	15	20	30	11	10	25	40	50	33
5	14	18	15	20	30	11	10	25	40	50	34
6	14	18	15	20	30	11	10	25	40	50	35

Note: G1–G10 denote generators at buses 9–18.

TABLE 10 Revenue collected from loads under different LMP methods in the uncongested 39-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	38293.0	37824.5	38390.5	36781.5	36114.8
2	38293.0	37824.5	38391.0	37825.1	36115.3
3	38280.3	37811.9	38390.8	37848.2	36114.3
4	38366.7	37827.0	38650.1	39200.9	37498.2
5	38366.7	37827.0	38722.0	39826.2	37511.5
6	38366.7	37827.0	38732.6	37146.0	37506.4

Note:
Maximum value ($) Minimum value ($)

TABLE 11 Income of generators under different LMP methods in the uncongested 39-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	37503.7	37824.6	37319.3	36781.4	38541.0
2	37503.7	37824.6	37320.6	37825.0	38541.0
3	37491.2	37812.0	37320.4	37848.2	38546.7
4	37570.6	37827.0	37011.8	39201.0	38134.7
5	37570.6	37827.0	36913.3	39826.2	38134.0
6	37570.6	37827.0	37053.3	37171.6	38132.8

Note:
Maximum value ($) Minimum value ($)

TABLE 12 Market settlement surplus under different LMP methods in the uncongested 39-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	789.31	−0.039	1071.21	0.0666	−2426.1
2	789.31	−0.039	1070.41	0.0366	−2425.7
3	789.05	−0.039	1070.36	0.0359	−2432.4
4	796.14	0.0175	1638.29	−0.068	−3234.3
5	796.14	0.0175	1808.68	−0.035	−3199.6
6	796.14	0.0175	1679.39	−25.61	−2908.9

Note:
Maximum value ($) Minimum value ($)

TABLE 13 Revenue collected from loads under different LMP methods in the congested 39-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	43787.8	43779.5	45965.4	43810.6	41098.9
2	44203.8	44194.8	46196.0	44228.3	42685.3
3	44289.2	44280.0	46041.8	44314.0	43159.0
4	46000.3	45908.3	45349.0	45913.8	36448.1
5	46555.8	46462.8	45216.0	46467.4	37910.8
6	44030.7	43942.4	45861.1	43953.7	36247.9

Note:
Maximum value ($) Minimum value ($)

TABLE 14 Income of generators under different LMP methods in the congested 39-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	42338.9	42746.1	42414.0	42721.4	41936.1
2	42667.7	43111.0	42399.0	43084.4	43731.7
3	42725.4	43179.4	42188.4	43152.5	44249.7
4	43754.9	44302.2	42272.5	44307.3	36124.6
5	44303.3	44856.6	42019.3	44860.8	37606.2
6	41810.5	42336.4	42860.0	42347.5	35920.1

Note:
Maximum value ($) Minimum value ($)

quite a few nodes in Figures 6 and 7, MLFs produced by the PF-M is exceeded by those of the OPF-M and RI-LMP methods. These are consistent with the results of the 5-bus system.

For the nodal price, all the OPF-M, OPF-A and RI-LMP are able to obtain reference-independent nodal prices. Nodal prices calculated by the PF-M and PF-A methods change with the location of chosen reference bus. The revenue collected from loads, income of generators, and the market settlement surplus in the 39-bus system is demonstrated in Tables 10 to 12.

The MLF-based LMP methods tend to collect higher revenue from loads than ALF methods in the uncongested system. Under the MLF-based LMP methods, the market operator pays less to generators than ALF-based LMP and RI-LMP methods. Thus, the market settlement surplus is the highest under

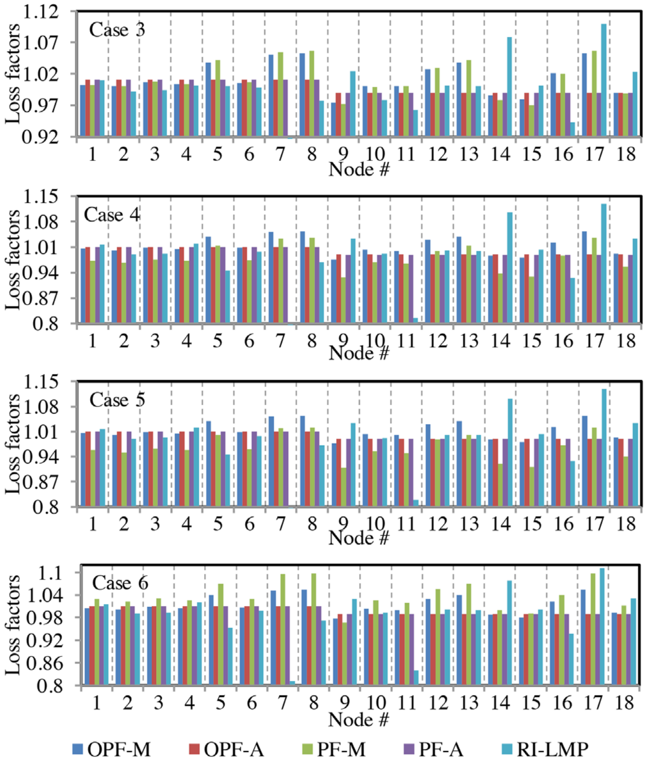

FIGURE 8 Loss factors obtained by different algorithms under Cases 1–2 in the congested 39-bus system

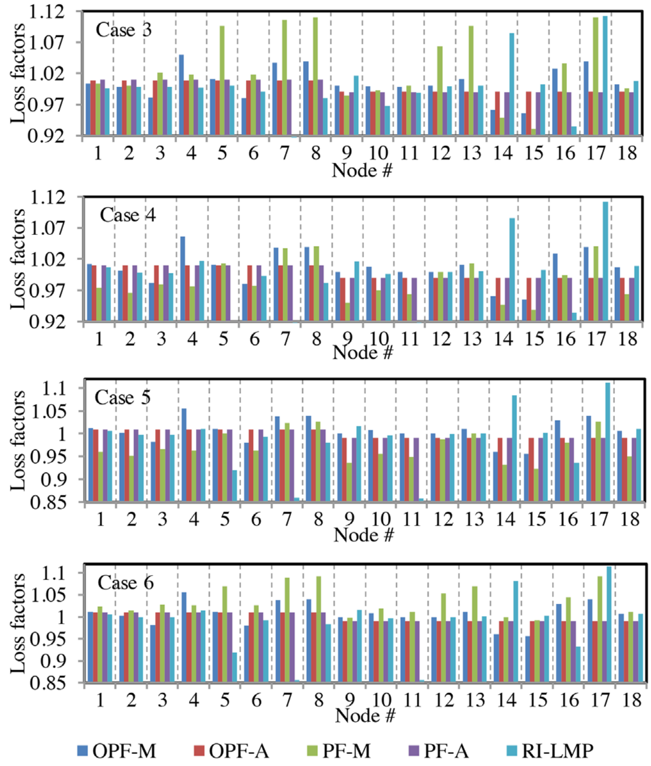

FIGURE 9 Loss factors obtained by different algorithms under Cases 5–6 in the congested 39-bus system

OPF-M and PF-M, which is followed by the OPF-A and PF-A. The RI-LMP has the smallest settlement surplus. As mentioned before, if settlement surplus is returned to loads, the adoption of MLF-based LMP methods will result in a low income of generators.

3.3.2 | Calculation results for congested 39-bus system

In the congested 39-bus system, the transmission capacity of branch 4–14 and branch 17–18 is both set as 50 MW and 100 MW, respectively. The six cases as shown in Table 9 are adopted. Figures 8 and 9 show the loss factors for the congested 39-bus system.

Results of loss factors in Figures. 8 and 9 comply with the previous analysis, namely the PF-M method tends to produce loss factors with the highest (in cases 1, 2, 3 and 6) or lowest values (in cases 4 and 5) compared with other methods. Tables 13 to 15 give the revenue collected from loads, income of generators, and the market settlement surplus for the congested 39-bus system. In general, the MLF-based LMP methods collect higher revenue from loads but make lower payment to generators than ALF-based LMP algorithms. Consequently, the PF-M method has the highest market settlement surplus due to its highest load revenue from loads but lowest generation payment, as shown in Table 15.

TABLE 15 Market settlement surplus under different LMP methods in the congested 39-bus system

Case	OPF-M	OPF-A	PF-M	PF-A	RI-LMP
1	1448.9	1033.45	3551.41	1089.15	−837.25
2	1536.1	1083.83	3797.22	1143.84	−1046.4
3	1563.8	1100.53	3853.32	1161.53	−1090.7
4	2245.4	1606.16	3076.54	1606.52	724.243
5	2252.5	1606.21	3196.69	1606.54	684.698
6	2220.3	1606.0	3001.14	1606.16	821.198

Note: Maximum value ($) Minimum value ($)

4 | CONCLUSIONS

MLF-based LMP methods have been widely adopted in existing electricity markets to distinguish the contribution of each bus to system losses. Driven by the evolution of power systems, there is a large year-on-year change in MLFs, such as in the Australian NEM. Since changes in MLFs have significant financial implications for market participants, it is of great importance to conduct a comparative study for the various LMP methods. This work carried out a comparative study for the selected LMP methods, and key findings from numerical experiments are summarised in Table 16.

Besides, it is also found that if compared with transmission congestions, impacts of loss factors on nodal prices will be less significant. Although load buses are normally expected to have

TABLE 16 Summary of key findings and supportive results in the comparative study

Property	Key finding	Support result
Reference dependency	(1) OPF based LMP methods (OPF-M, OPF-A) and the RI-LMP method are able to produce reference-independent loss factors and nodal prices. (2) In contrast, PF based LMP methods (PF-M, PF-A) are reference dependent.	Figures 2–9
Loss estimation accuracy	Loss factors produced by the PF-M method are either larger or smaller than those of other methods.	Figures 2–9
Load payment	MLF based LMP methods tend to collect higher revenue from loads than ALF based LMP methods.	Tables 6, 10, and 13
Generation income	(1) MLF based LMP methods tend to produce a lower income for generators than ALF based LMP methods. (2) The PF-M method tends to produce the lowest income to generators.	Tables 4, 11, and 14
Market settlement surplus	(1) MLF based LMP methods produce a higher market settlement surplus than ALF based LMP methods. (2) The PF-M method produce the largest while the RI-LMP tends to produce the smallest settlement surplus among all the methods. (3) When there is no congestion, ALF based LMP methods can produce a zero settlement surplus and it will increase to non-zero if congestion happens.	Tables 5, 8, 12, and 15

MLFs greater than 1.0 and generator buses have MLFs less than 1.0, MLFs obtained in this study are sometimes less than 1.0 for load buses, and are greater than 1.0 for generator buses. In other words, when using LMF LMP methods, some buses will benefit from the favourable MLFs due to their special location. However, this problem can be avoid while using the ALFs since loss factors are allocated to loads and generators by ignoring the impact of network topology.

In practical electricity markets, when selecting LMP schemes, attentions should be paid to the key aspects that are summarised in Table 16 of this work. First, the OPF-based methods for loss factor calculation are recommended rather than the PF-based methods, because there is a larger variation in loss factors attained by PF-based methods and this is not beneficial to the market stability. Second, although the MLF algorithm can distinguish contributions of each bus to the system total power loss, MLF-based LMP methods tend to produce a larger imbalance between consumer payment and generator income. An outstanding settlement imbalance represents a significant impediment to market efficiency. Meanwhile, when calculating MLFs using PF-based methods, the market settlement imbalance can be further enlarged. Therefore, it is necessary for decision-makers to carry out a comprehensive evaluation about the feasibility and efficiency of the MLF-based LMP methods, especially when the MLF is calculated by the PF method.

NOMENCLATURE

Indices and sets

i	Index of consumers, $i \in N$
j	Index of producers, $j \in M$
l	Index of branch in the transmission network
k	Index of binding constraints
N/M	Set of consumers/producers in the market
$N_n^{\mathrm{bus}}/M_n^{\mathrm{bus}}$	Set of consumers/producers connected to bus n
L	Set of branches in the transmission network
N^{bus}	Set of network buses
K^{trc}	Set of binding constraints in the market clearing model

Parameters

$r_{\mathrm{i}}^{\mathrm{b}}/r_{\mathrm{j}}^{\mathrm{s}}$	Bid/offer price of consumer i/producer j
$P_l^{\max}$	Power limit of transmission branch l
$f_{l,j}^{\mathrm{PTDF}}$	PTDF for branch l corresponding to bus j
$p_i^{\mathrm{b,min}}/\mathrm{p}_i^{\mathrm{b,max}}$	Lower/upper demand limit of consumer i
$p_j^{\mathrm{s,min}}/p_j^{\mathrm{s,max}}$	Lower/upper generation limit of producer j
f_n^{PF}	Penalty factor for bus n
P_n^{inj}	Power injection at bus n
P^{loss}	Power losses of a power system
f_n^{MLF}	MLF at bus n
$\Delta P_n^{\mathrm{load,incr}}$	Load increment at bus n
$\Delta P^{\mathrm{RRN,g}}$	Change of generation at the RRN
ΔP^{loss}	Change of network losses
f_n^{LDF}	Loss distribution factor at bus n
$f^{\mathrm{D,ALF}}$	ALF for consumers
$f^{\mathrm{G,ALF}}$	ALF for producers

Variables

$p_i^{\mathrm{cb}}/p_j^{\mathrm{cs}}$	Dispatched load demand/generation output of consumer i/producer j
r_n^{np}	Nodal price at the nth bus
r_n^{smp}	Marginal energy price component of the nodal price
r_n^{cp}	Congestion price component of the nodal price
r_n^{mlc}	Marginal loss price component of the nodal price
r_k^{sdp}	Shadow price of the kth binding constraint in the market clearing model

Abbreviations

LMP	Locational marginal pricing
OPF	Optimal power flow
FND	Fictitious nodal demand
MLF	Marginal loss factor
ALF	Average loss factor
LP	Linear programming

MO	Market operator
PTDF	Power transfer distribution factor
NEM	National electricity market
RRN	Regional reference node
AEMO	Australian energy market operator
PF	Power flow

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