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Bayesian Analysis of Masked Competing Risks Data Based on Proportional Subdistribution Hazards Model

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Abstract: Masked issues can emerge when dealing with competing risk data. Such issues are exemplified by the cause of a particular failure not being directly exhibited for all units to observe but only proven to be a subset of possible causes of failure. For assessing the impact of explanatory variables (covariates) on the cumulative incidence function (CIF), a process of Bayesian analysis is discussed in this paper. The symmetry assumption is not imposed on the masking probabilities and independent Dirichlet priors assigned to them. The Markov Chain Monte Carlo (MCMC) technique is utilized to implement the Bayesian analysis. The effectiveness of the developed model is tested via numerical studies, including simulated and real data sets.

Keywords: competing risks; masked causes of failure; subdistribution hazards; MCMC; Bayesian analysis

MSC: 62N99

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1. Introduction

In the presence of competing risks data, inferences based on cause-specific hazard function (CHF) and cumulative incidence function (CIF) are common. Several researchers pointed out that the impact of explanatory variables on CHF of certain failure types may be quite dissimilar from the impact of the corresponding explanatory variables of the CIF [1]. The approaches to cause-specific hazard under proportional hazard formulation do not allow a direct assessment of the impact of explanatory variables on CIF. However, Fine and Gray [1] show that the transformation of cause-specific regression formulation into a regression model for CIF can be achieved by using a complementary log–log transformation.

Previous studies on CIF explored cases where the cause of failure was always observed. For example, Jeong and Fine [2] parameterized CIF directly based on the Gompertz distribution. In addition to that, it can be more natural to model CIF directly rather than indirectly via the cause-specific hazard function when CIF is of primary interest. Shayan et al. [3] extended a two-parameter log-logistic model based on a new four-parameter distribution. They found that their models based on CIF were efficient. Furthermore, Hudgens et al. [4] extended the Jeong and Fine [2] models by using parametric competing risk estimation of the CIF for interval censoring.

On the other hand, some researchers focused on developing regression models that involved CIF, including that of Fine and Gray [1]. They also studied the Cox model and developed some estimating equation-based inference procedures. Fine [5] used a semiparametric regression model based on the CIF and adopted a class of models, including the Cox and proportional odds models. Klein and Andersen [6] developed a method to model CIF directly in a generalized linear model that allows for various link functions. Moreover, Jeong and Fine [7] proposed a parametric regression analysis of the CIF involving the maximum likelihood inferences that were derived to simultaneously fit the parametric models of the CIFs for all causes. Scheike and Zhang [8] introduced a simple and flexible class of regression models containing special case elements of the Fine and Gray model. Their model allows for non-proportional hazards and time-varying covariates.

In engineering experiments, it is often difficult to identify the cause of the failure of a unit; instead, the expert can only identify a set of possible causes. Earlier, Miyakawa [9] discussed this type of data by considering parametric and non-parametric approaches to reliability estimation. Previously, we developed a Bayesian approach to estimate the effect of explanatory variables motivated by incomplete data with masked causes of failure [10,11]. We discussed the effect of covariates on CIF in the presence of a moderate masking level, and preliminary results were introduced [11]. However, this paper considered different levels of masking for the simulated data in addition to an application on a real data set. This paper is organized as follows: Sections 2 and 3 introduce the model construction and the Bayesian computation techniques. We present in Section 4 the results of our evaluation of the model performance where we used simulated data, and Section 5 illustrates our approach, which utilizes an actual data set. Section 6 concludes this paper.

2. Model Structure

In this study, since the covariates' impact is of interest, the proportional hazard model of Fine and Gray [1] is the most common choice for estimating the regression parameters and is given by the following.

$$\lambda_j(T,X) = \lambda_{0j}(T)e^{\beta_j' X} \tag{1}$$

In this equation, j is the cause of interest, and λ_{0j} and β_j are, respectively, the baseline hazard and the vector of the regression coefficients specific to the jth cause of failure. The vector of the covariates is represented by X. The corresponding (CIF) is as follows:

$$F_j(t,X) = P(T \le t, C = j|X) = 1 - e^{-\Lambda_{0j}(t)e^{\beta_j' X}}, \tag{2}$$

where $\Lambda_{0j} = \int_0^t \lambda_{0j}(s)ds$ is the cumulative baseline hazard.

Suppose we have N units under observation, which are subject to K competing risks. Let T_i represents the time of failure of the ith unit, which failed due to one of the K causes, and X_i the corresponding vector of covariates (i = 1, 2, . . . , N). Since, in the presence of masking, the unit's failure can only be specified up to a Minimum Random Subset (MRS)$S \subseteq \{1,\ldots,K\}$, the observed data includes the quantities (T, S, and X). Consequently, the likelihood of the ith unit from the data (T_i, S_i, X_i) is represented as $p(T_i, S_i|X_i)$ [12], and can be expressed as follows:

$$p(T_i, S_i|X_i) = \sum\nolimits_{j=1}^{K} p(T_i, C_i = j|X_i)p(S_i|T_i, C_i = j, X_i), j = 1,\ldots,K; i = 1,\ldots,N,$$

where C_i represents the actual cause of the failure of the ith unit. Obviously, term $p(T_i, C_i = j|X_i)$ is equivalent to $f_j(T_i|X_i)$.

$$p(T_i, S_i|X_i) = \sum_{j=1}^{K} f_j(T_i|X_i)p(S_i|T_i, C_i = j, X_i).$$

Following this, the full likelihood for the observed right-censored data can then be written as follows.

$$L = \prod_{i=1}^{n_1} \sum_{j\in S_i} p(S_i|T_i, C_i = j, X_i)f_j(T_i|X_i) \prod_{i=n_1+1}^{n_2} S(T_i|X_i).$$

Here, $n_1, n_2 (n_1 + n_2 = N)$ denote the numbers of failed and right-censored units, respectively. The former likelihood function can be rewritten in terms of subdistribution hazard and (CIF) as follows.

$$L = \prod_{i=1}^{n_1} \sum\nolimits_{j \in S_i} p(S_i | T_i, C_i = j, X_i) \lambda_j(T_i | X_i)(1 - F_j(T_i | X_i)) \times \prod_{i=n_1+1}^{n_2} [1 - \sum\nolimits_{j=1}^{K} F_j(T_i | X_i)].$$

To obtain the final likelihood function, the hazard and cumulative incidence functions can then be substituted by Equations (1) and (2), respectively.

To estimate the actual cause of failure for the *ith* unit, the diagnostic probability equation can be used, which is as follows.

$$p(C_i = j | S_i, T_i, X_i) = \frac{p(S_i | T_i, C_i = j, X_i) \lambda_j(T_i | X_i)(1 - F_j(T_i | X_i))}{\sum_{l \in S_i} p(S_i | T_i, C_i = l, X_i) \lambda_l(T_i | X_i)(1 - F_l(T_i | X_i))} \; ; \; j \in S_i. \quad (3)$$

3. Bayesian Analysis

In this study, WinBUGS software version 1.4.3 (which is a robust and flexible tool for Bayesian survival analysis) is utilized to derive the desired inferences. To apply the Bayesian approach, prior distributions for unknown parameters in addition to the likelihood function of the observed data need to be specified. Hence, the parametric forms for the likelihood function and prior distributions are required. In this study, the likelihood function was constructed based on the proportional hazard model for the subdistribution (i.e., a semiparametric model where the baseline hazard function is unknown). Thus, the proportional hazard model for the subdistribution is rewritten in terms of the counting process, which is the following:

$$I_i(t) = Y_i(t) \lambda_0(t) e^{\beta' X_i}$$

where $Y_i(t)$ represents the at-risk indicator. Here, the risk set at the time of failure for unit i is defined as $\{r : (T_r \geq T_i) \cup (T_r \leq T_i \cap C_r \neq j)\}$, with j representing the cause of interest.

Utilizing the Clayton [13] formula, suppose the observed data is $D = \{N_{ij}(t), Y_{ij}(t), X\}$ where $N_{ij}(t)$ represents the counting process of failures due to cause j, occurring up to time t and $Y_{ij}(t)$ being the at risk indicator for cause j. Note that the same formulation was used in Yousif et al. [10]; however, the definition of $Y_{ij}(t)$ is quite different. Let $dN_{ij}(t)$ be a small increment of $N_{ij}(t)$ over interval [t, $t + dt$). Then, $N_{ij}(t)$ and $dN_{ij}(t)$ will be equal to one if the event occurs in [0, t) and [t, $t + dt$), respectively, and zero otherwise. Under non-informative censoring, the likelihood (specific to the *jth* cause of failure) of the data is proportional to the following.

$$\prod_{i=1}^{N} \left[\prod_{t \geq 0} I_{ij}(t)^{dN_{ij}(t)} \right] e^{-I_{ij}(t)dt}$$

This is basically as if the counting process increments $dN_{ij}(t)$ over the time interval [t, $t + dt$) are independent Poisson random variables with means $I_{ij}(t)dt$:

$$dN_{ij}(t) \sim Poisson(I_{ij}(t)dt).$$

and

$$I_{ij}(t)dt = Y_i(t) e^{\beta_j' X_i} d\Lambda_{0j}(t),$$

where $d\Lambda_{0j}(t) = \lambda_{0j}(t)dt$ is the instantaneous probability that the unit at risk at the time t has the event j in the next time interval [t, $t + dt$).

The most popular priors in the literature have been assigned to unknown parameters for prior distributions, namely, regression coefficients, baseline hazards, and masking probabilities. The conjugate prior for Poisson mean is a gamma distribution; thus, it would be convenient if Λ_{0j} were a process in which increments $d\Lambda_{0j}(t)$ were distributed according to gamma distributions, as proposed by Kalbfleisch [14]. They comprise the following form:

$$d\Lambda_{0j}(t) \backsim Gamma\left(c\Lambda^{*}{}_{0j}(t), c\right), j = 1, \ldots, K,$$

where $\Lambda^{*}{}_{0j}(t)$ can be thought of as a prior guess at the unknown baseline hazard function, with c representing the degree of confidence in this guess. Small values of c correspond to high levels of uncertainty concerning the prior beliefs.

The regression coefficients are assumed to be, as common, independently normally distributed.

$$\beta_j \backsim N(\theta_j, \sigma_j^2), j = 1, \ldots, K.$$

For masking probabilities, we assign independent Dirichlet priors. Let $J = 2^{j-1}$ be the number of subsets that contains cause j, and let $S_j = \{S_{j1}, \ldots, S_{j2}\}$ be the collection of potential minimum random sets that contain cause j. Then, the random Dirichlet variables can be defined as follows:

$$(\mu_{ij}(S_{j1}), \ldots, \mu_{ij}(S_{jJ})) \backsim Dir_J(\alpha_j),$$

$$i = 1, \ldots, N; j = 1, \ldots, K; J = 2^{j-1},$$

where $\mu_{ij} = P(S_i|T_i, C_i = j, X_i)$ and $\alpha_j = (\alpha_{j1}, \ldots, \alpha_{jJ})$ are the Dirichlet parameters.

All unknown parameters mentioned above are assumed to be stochastically independent. Thus, the joint posterior distribution of the model's parameters can be defined as follows:

$$P(\beta, \Lambda_0, \mu|D) \propto L(D|\beta, \Lambda_0, \mu) \prod(\beta) \prod(\Lambda_0) \prod(\mu),$$

where D represents observed data.

WinBUGS software version 1.4.3, internally and automatically, identifies and then constructs an efficient simulation approach for each of the related full conditional posterior distributions (i.e., $P(\beta|D, \Lambda_0, \mu)$, $P(\Lambda_0|D, \beta, \mu)$, and $P(\mu|D, \beta, \Lambda_0)$).

4. Simulations

Since we work under a CIF framework, the competing risks data should be simulated in such a way that the subdistribution hazard of the CIF of interest follows the model (1). Then, the simulation introduced by Fine and Gray [1], will be applied to generate the failure times. Let us suppose that there are two events, 1 and 2; Fine and Gray assumed that CIF followed the following model.

$$P(T \le t, C = 1|X) = 1 - \left(1 - p\left(1 - e^{-t}\right)\right)^{e^{\beta_1' X}}$$

The competing cumulative incidence function was computed from the following:

$$P(T \le t, C = 2|X) = P(C = 2|X)P(T \le t|C = 2, X),$$

where $P(C = 2|X) = 1 - P(C = 1|X)$ and $P(T \le t|C = 2, X)$ are assumed to be an exponential distribution with hazard function $e^{\beta_2' X}$.

In this simulation, we use R software to run the simulations (see Appendix A). It is assumed that there is one covariate X, which takes on values 0.5 or −0.5, whereas the true parameter values are assumed to be (p, β_1, β_2) = (0.7, −0.9, 1.9). The censored times are generated from the uniform distribution *U[1,7]*. The simulations achieved the following data sets: 55% of units failed due to cause 1, 45% failed due to cause 2, and 24% were right-censored units. The cause of failure is masked randomly with equal chances for all units. Furthermore, different data sets with different masking percentages were generated.

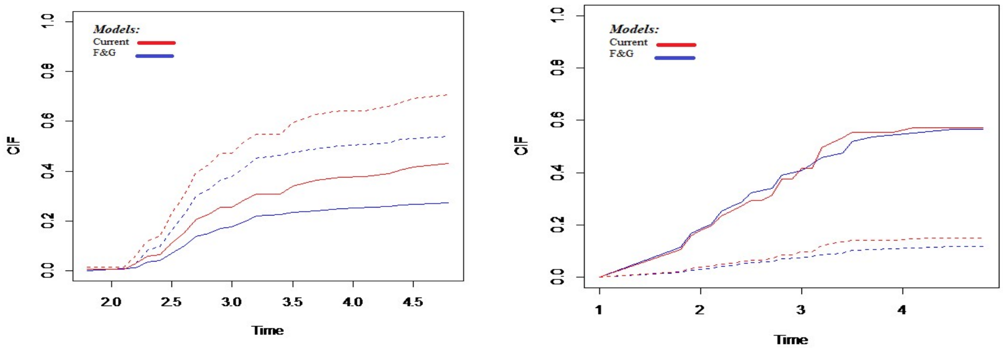
Four MCMC chains, each of 4000 iterations with a burn-in of 1000, were run, and the convergence was achieved by monitoring the time series plots, auto-correlation function plots, and Gelman and Rubin multiple sequences diagnostics. Generally, the results demonstrate that the developed model performs very well compared to the Fine and Gray model, which has no masked causes of failure. However, the high levels of masking can affect its performance. Table 1 shows the posterior estimations of the regression coefficients, such as the estimated coefficient (β), standard deviation (SD), and Monte Carlo error (MCE). The values of MCE suggest that the posterior estimates of all regression parameters are accurate. It can be seen that the estimators of the current model are close to those from the Fine and Gray model; however, the proposed model exhibits some evidence of sensitivity to the level of masking.

Table 1. Estimated Regression Coefficients.

Masked Units (%)	Model	Cause 1			Cause 2		
		β	SD	MCE	β	SD	MCE
	FG *	−0.887	0.344		1.915	0.422	
(26%)	Current	−0.795	0.313	0.003	1.737	0.472	0.006
(49%)	Current	−0.679	0.329	0.003	1.291	0.380	0.004

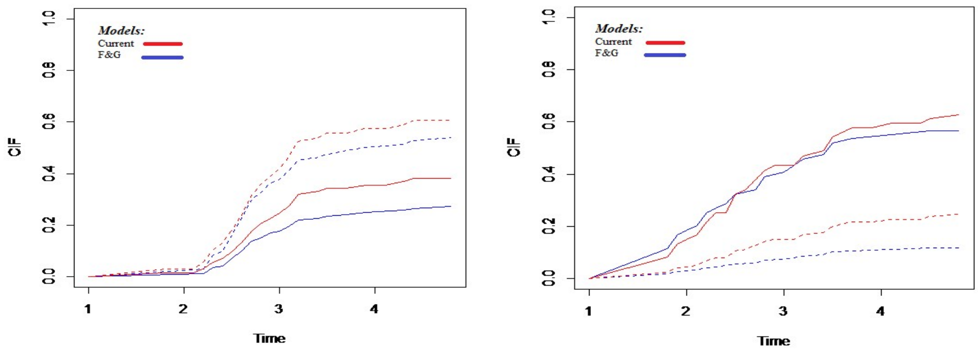
* FG: Fine and Gray. $(p, \beta_1, \beta_2) = (0.7, -0.9, 1.9)$; sample size = 100.

Moreover, Figure 1 compares the estimated CIF between Fine & Gray model and the developed model with 26% masked units. Figure 2 shows another comparison where there is a 49% masked units [11]. Obviously, the CIF curves are comparable and show a substantial consistency with Fine & Gray CIF curves. This indicates that the developed model performs very well compared to the Fine & Gray model, which has no masked observations.



(**a**) (**b**)

Figure 1. Comparison of the CIF's Current Model (26% Masked Observations) with F&G Model for: (**a**) Cause 1, solid line: X = 0.5. Dashed line: X = −0.5. (**b**) Cause 2, solid line: X = 0.5. Dashed line: X = −0.5.



(**a**) (**b**)

Figure 2. Comparison of the CIF's Current Model (49% Masked Observations) with F&G Model for: (**a**) Cause 1, solid line: X = 0.5. Dashed line: X = −0.5. (**b**) Cause 2, solid line: X = 0.5. Dashed line: X = −0.5.

5. An Application

The developed approach is applied to the data set reported in Klein and Basu [15]. The failure times of insulation systems for electric motors were recorded with their corresponding causes of failure. These systems have three possible types of failures: Turn, Phase, and Ground. The systems were subject to different stress levels $(Z_1 = 190, Z_2 = 220, Z_3 = 240)$ where 20 units are tested at each level.

The data were manipulated as masked and partly interval-censored data to meet the requirements of the developed method. The results were obtained with 50,000 iterations, burn-in of 20,000 and thinning to every 10th iteration for each of the five (5) chains.

The MCMC chains showed that a good mix and convergence was obtained. Table 2 introduces the posterior estimates: the mean, median, standard error, 95% credible interval (CI), and p-value of the regression coefficients. The p-values indicate that the regression coefficients are insignificant for the three causes of failure; the different stress levels do not significantly influence the CIF. The posterior means of the masking probabilities for the full masked units computed using Equation (3) are shown in Table 3. It can be noted that 50% of the masked units are failed due to cause Turn (i.e., units 5, 37, 39, 40, and 58), 25% due to Phase, and 25% due to Ground. Furthermore, Figure 3A–C compare the CIF of the three causes for each stress level. Noticeably, the CIF curves from both models are nearly identical at the first stress level. However, for the two other stress levels, the CIF curves of cause Turn show a gradual increase as the stress level increases, unlike causes Phase and Ground, which are relatively stable. Figure 3D–F compare the cumulative incidence functions by examining the different stress levels for each cause of failure. It is clear that changing the stress levels does not influence causes Phase and Ground, while in the case of cause Turn, there is a slight and steady increase. However, since this increase is meaningless according to the p-value from Table 2 and may be caused by unknown factors, it can be deduced that the stress used in the experiment is not a factor that affects the probability of failure for the three causes.

Table 2. Posterior Summaries of the Regression Coefficients (20% masked observations).

Parameter	Mean	Median	SE	95% CI		p-Value
				2.5%	97.5%	
β_{0T}	−16.77	−16.65	5.292	−27.49	−6.7250	0.0015
β_{0P}	−7.715	−7.615	6.925	−21.74	5.6760	0.2670
β_{0G}	−13.82	−13.60	7.016	−28.19	−0.5025	0.0488
β_{1T}	17.81	17.62	10.68	−2.546	39.470	0.0949
β_{1P}	−2.121	−2.177	14.14	−29.60	26.230	0.8808
β_{1G}	10.50	10.170	14.21	−16.74	39.420	0.4593

Table 3. Masking Probabilities for the Full Masked Units.

Probability	Causes of Failure		
	Turn	Phase	Ground
P^3	0.182	0.661	0.157
P^5	0.78	0.119	0.101
P^{11}	0.553	0.241	0.206
P^{16}	0.19	0.190	0.620
P^{17}	0.164	0.162	0.674
P^{37}	0.964	0.016	0.020
P^{38}	0.037	0.028	0.936
P^{39}	0.808	0.093	0.099
P^{40}	0.712	0.141	0.148
P^{42}	0.173	0.704	0.124
P^{54}	0.118	0.792	0.090
P^{58}	0.887	0.050	0.063

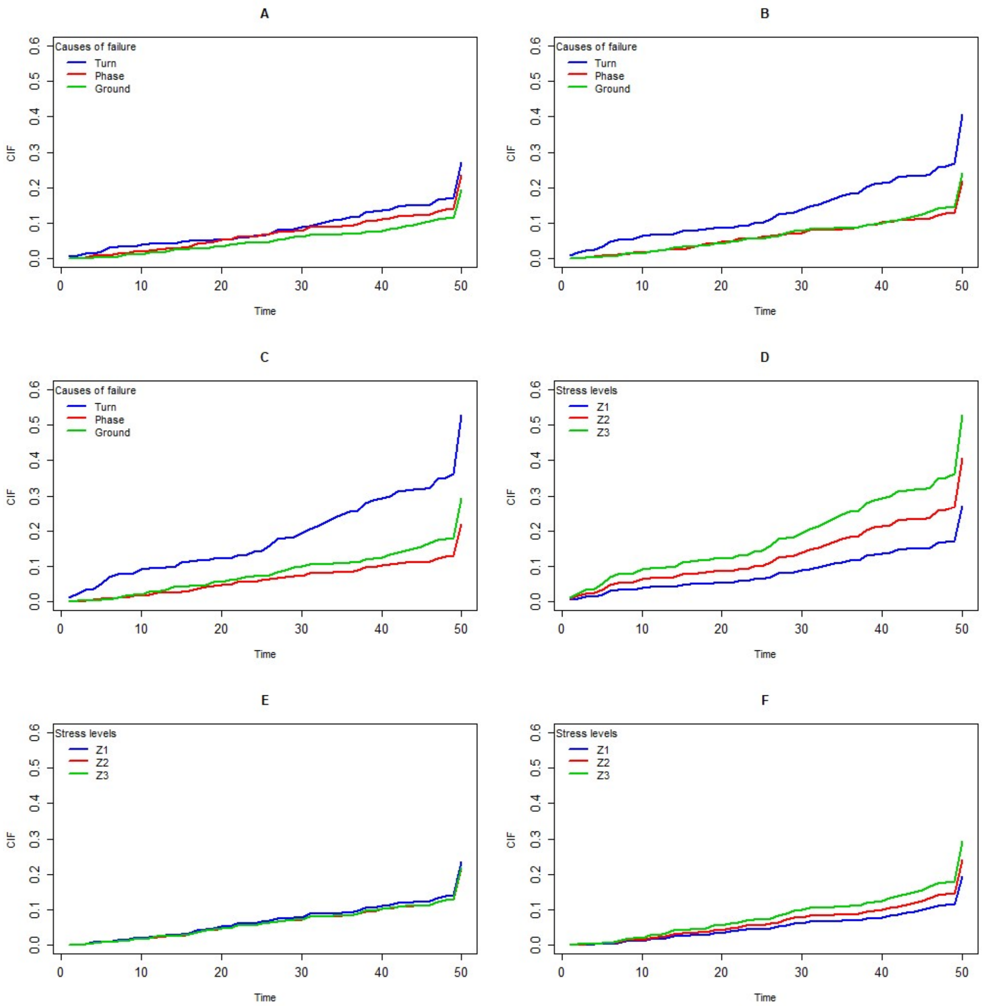


Figure 3. Comparisons of cumulative incidence functions (CIF) from: (**A**) three causes for stress level 1, (**B**) three causes for stress level 2, (**C**) three causes for stress level 3, (**D**) three levels of stress for cause turn, (**E**) three levels of stress for cause phase, and (**F**) three levels of stress for cause ground.

6. Conclusions

In this paper, Bayesian analysis for competing-risk data under a cumulative incidence function framework was derived for cases where the cause of failure was masked for some units. The developed method provides an assessment of covariates' effect on the CIF without imposing the common assumptions utilized in the literature (i.e., symmetry assumption and independence of the competing risks). The introduced method is feasible according to the results obtained from simulated data sets. However, one drawback of this model is its sensitivity toward high levels of masking. Moreover, instead of a semi-parametric analysis, one can also develop a parallel full parametric Bayesian analysis, which might be more suitable in some situations.

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Appendix A

R code for generating the failure times

```
generate.my.times1 <- function(N1,p)
{
temp1 <- function(x,y)
{
#return(-1*log(1-((1-((1-y*(1-(1-p)^exp(x*beta)))^exp(-1*x*beta)))/p)))
a <- y*(1-(1-p)^exp(x*beta))
b <- 1-((1-a)^exp(-1*x*beta))
c <- 1- (b/p)
return(-1*log(c))
}
stime1 <- NULL
i<-1
while(length(stime1) < N1)
{
u <- runif(1,0.9,1)
z<- ifelse(i<=N1/2,0.5,-0.5)
stime1[i] <- temp1(z,u)
i <- i + 1
}
return(stime1)
}
```

References

1. Fine, J.P.; Robert, J.G. A Proportional Hazards Model for the Subdistribution of a Competing Risk. *Am. Stat. Associat.* **1999**, *94*, 496–509. [CrossRef]
2. Jeong, J.H.; Jason, F. Direct Parametric Inference for the Cumulative Incidence Function. *J. R. Stat. Society. Ser. C Appl. Stat.* **2006**, *55*, 187–200. [CrossRef]
3. Shayan, Z.; Ayatollahi, S.; Zare, N. A parametric method for cumulative incidence modeling with a new four-parameter log-logistic distribution. *Theor. Biol. Med. Model.* **2011**, *8*, 43. [CrossRef] [PubMed]
4. Hudgens, M.G.; Li, C.; Jason, P.F. Parametric Likelihood Inference for Interval Censored Competing Risks Data. *Biometrics* **2014**, *70*, 1–9. [CrossRef] [PubMed]
5. Fine, J.P. Regression Modeling of Competing Crude Failure Probabilities. *Biostatistics* **2001**, *2*, 85–97. [CrossRef] [PubMed]
6. Klein, J.P.; Andersen, P.K. Regression Modeling of Competing Risks Data Based on Pseudovalues of the Cumulative Incidence Function. *Biometrics* **2005**, *61*, 223–229. [CrossRef] [PubMed]
7. Jeong, J.-H.; Jason, P.F. Parametric Regression on Cumulative Incidence Function. *Biostatistics* **2007**, *8*, 184–196. [CrossRef] [PubMed]
8. Scheike, T.H.; Zhang, M.-J. Flexible Competing Risks Regression Modeling and Goodness-of-Fit. *Lifetime Data Anal.* **2008**, *14*, 464–483. [CrossRef] [PubMed]
9. Miyakawa, M. Analysis of incomplete data in competing risks model. *IEEE Trans. Reliab.* **1984**, *33*, 293–296. [CrossRef]
10. Yousif, Y.; Elfaki, F.A.M.; Hrairi, M.; Adegboye, O.A. A Bayesian Approach to Competing Risks Model with Masked Causes of Failure and Incomplete Failure Times. *Math. Probl. Eng.* **2020**, *2020*, 8248640. [CrossRef]
11. Yousif, Y.; Elfaki, F.A.M.; Hrairi, M. Regression Analysis of Masked Competing Risks Data under Cumulative Incidence Function Framework. *Austrian J. Stat.* **2020**, *49*, 25–29. [CrossRef]
12. Kuo, L.; Yang, T.Y. Bayesian reliability modeling for masked system lifetime data. *Stat. Probab. Lett.* **2000**, *47*, 229–241. [CrossRef]
13. Clayton, D. Some Approaches to the analysis of Recurrent Event Data. *Stat. Methods Med. Res.* **1994**, *3*, 244–262. [CrossRef] [PubMed]

14. Kalbfleisch, J. Non-Parametric Bayesian Analysis of Survival Time Data. *J. R. Stat. Soc.* **1978**, *40*, 214–221. [CrossRef]
15. Klein, J.P.; Basu, A.P. Weibull Accelerated Life Tests When There Are Competing Causes of Failure. *Commun. Stat.-Theory Methods* **1981**, *10*, 2073–2100. [CrossRef]