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Chapter 8

Electron swarms in ac electric and magnetic fields

8.1 Introduction

In this chapter, the hydrodynamic time-dependent kinetic theory and Monte Carlo simulation code discussed in the previous chapters are applied to calculate the transport coefficients and other transport properties for inhomogeneous electron swarms under the influence of ac electric and magnetic fields in neutral gases. One of the primary aims of this chapter is to present the first systematic study of non-conservative processes in gases in ac electric and magnetic fields under conditions of an arbitrary phase difference between the fields and varying field configurations. We highlight the explicit modification of the electron transport coefficients brought about by non-conservative collisions of electron attachment and ionization, and emphasize the physical implications which arise from their explicit inclusion.

The hydrodynamic transport theory developed in the previous chapters may be used to consider the electron transport in ac electric and magnetic fields at all frequencies. At low frequencies (less than 1 MHz at the gas pressure of 1 Torr for majority of gases), the so-called quasi-static dc behavior is achieved over the entire cycle of the field. This means that all transport properties are able to relax before the field is changed. However, as the field frequency increases, some interesting kinetic phenomena may occur since electrons do not have enough time to relax their momentum and energy on the time-scale of the changes of the field. The possibility that a swarm averaged property is not able to fully relax before the field changes is often labeled as “temporal non-locality” (White (1996); Bzenič et al. (1999); Petrović et al. (2002); Dujko (2004; 2005); Makabe and Petrović (2006)) in analogy to spatial non-locality. At very high frequencies (generally greater than 1 GHz at the gas pressure of 1 Torr for majority of gases), the electron time-trapping regime is achieved over the entire cycle of the field. In this regime, electrons are not able to follow the electric field on the time-scale of the period of oscillations and essentially the rf-electron swarm behaves as a dc-like swarm for rms properties with an effective electric field.
The temporal non-locality of electron transport in rf fields has been rarely a subject of special studies in both swarm and plasma modeling community. In most cases, the so-called quasi-static approximation for the low field frequencies and/or the effective field approximation for higher field frequencies have been employed. However, both of these approximations usually fail for the range of frequencies critical for any practical applications in plasma processing. Therefore, we have been motivated to investigate the temporal profiles of various electron transport properties in the range of rf frequencies using both a multi-term theory for solving the Boltzmann equation and Monte Carlo simulation technique. In fact, the majority of the results that are presented in this chapter are obtained by a multi-term theory while a Monte Carlo simulation technique is used as a tool to check the sometimes paradoxical manifestation of some transport properties in rf fields. In particular, we have observed a wide range of kinetic phenomena that are generally inexplicable through the use of steady-state dc transport theory. Some illustrative examples include anomalous behavior of the diffusion coefficients in both purely electric and combined electric and magnetic fields, the complex waveforms of transport coefficients due to phases and field orientations between electric and magnetic fields, the time-resolved negative diffusivity and many others. In particular, in this chapter we propose a new additional mechanism for collisional heating in rf electric and magnetic fields caused by the synergism of temporal non-locality and cyclotron resonance effect.

We begin this chapter by investigating the electron transport and associated kinetic phenomena in ac electric fields in CF\textsubscript{4}. CF\textsubscript{4} provides an example of a gas which has applications in a wide range of devices where the electron kinetics plays an important role in device behavior. These applications include various types of radiation detectors (Christophorou \textit{et al.} (1991); Razin (1995)), gas discharge opening switches (Hunter \textit{et al.}, 1988) and gaseous dielectrics (Pradazrol \textit{et al.}, 1996). CF\textsubscript{4} has application in rf plasmas mostly realized in capacitively coupled plasma (CCP) (Kitajima \textit{et al.}, 2000) and inductively coupled plasma ICP (Hioki \textit{et al.}, 2000) for silicon etching. Most of the processing is usually performed in mixtures of CF\textsubscript{4} and argon. The knowledge of electron transport coefficients and in particular the values of electric and magnetic field strengths for which non-conservative collisions (attachment/ionization) may or may not have a significant effect on the drift and diffusion properties in pure CF\textsubscript{4} or its mixture with argon may be important for the operation of these devices. Another motivating factor lies in the fact that the most common two-term techniques employed for calculating the electron transport parameters in plasma modeling may fail for CF\textsubscript{4} due to a strong anisotropy of the velocity distribution function as a consequence of the rapidly raising cross section for vibrational excitation in the region of Ramsauer-Townsend minimum. Thus the only recourse for CF\textsubscript{4} is the exact techniques such as the multi-term theory for solving the Boltzmann equation or Monte Carlo simulation. In this chapter we display the inadequacies of the two-term approximation for solving the Boltzmann equation for CF\textsubscript{4} in ac electric field. The calculations performed in this chapter and those presented in recently published papers that concern the electron transport in dc electric and magnetic fields (Dujko \textit{et al.} (2005; 2006; 2008c)) should be the basis for a comprehensive modeling of a wide range of CF\textsubscript{4} plasma discharges.

The progress and further improvements of modern technology associated with the non-
equilibrium ac magnetized plasma discharges require the most accurate modeling of charged particle transport under the influence of ac electric and magnetic fields in neutral gases. This is the program of the second part of this chapter where the non-equilibrium transport of electrons in gases under the influence of ac electric and magnetic fields is studied via a unified time-dependent multi-term solution of the Boltzmann equation and Monte Carlo simulation technique. In particular, we focus on the time-dependent behavior of electron transport properties in ICP discharges where electric and magnetic fields are radio-frequency. One of the most difficult problems in modeling of ICP is an accurate description of non-local electron kinetics and non-local plasma electrodynamics. The spatial non-locality of electron transport induces a spatial dispersion of the plasma conductivity and associated plasma phenomena: (i) anomalous skin effect (see Kolobov and Economou (1997) and reference therein); (ii) non-monotonic distributions of rf fields and rf current density (Piejak et al. (1995; 2001)); and (iii) negative power absorption (Godyak et al. (1998; 1999); Cunge et al. (2001)).

Analogous to spatial non-locality, the issue of temporal non-locality plays a key role in understanding of electron kinetics. However, the temporal non-locality has been systematically ignored and in the majority of previous works the effective dc or quasi-dc approximations were assumed. These approximations do not accurately account for the relaxation effects of all transport coefficients. In this chapter we study the temporal non-locality of electron transport and associated kinetic phenomena in rf fields. Motivating factors behind this study include: (i) better understanding of power absorption in the weakly collisional and collisional regimes of ICP; (ii) provision of accurate data for fluid modeling of ICP; (iii) overcoming currently used approximations in treatment of rf electron kinetics.

The starting point in our investigation of electron kinetics in ac electric and magnetic fields is examination of the electron transport properties for certain model gases including the Reid ramp model (Reid 1979), the ionization model of Lucas and Saelee (Lucas and Saelee, 1975) and modified attachment model of Ness and Robson (Ness and Robson (1986); Nolan et al. (1997)). The aim of using model gases is to isolate explicit magnetic field effects from effects introduced by various collision processes in real gases, thus enabling us to investigate the fundamental effects of the magnetic field on the temporal profiles of the electron transport properties. The emphasis of this section is the observation and physical interpretation of the properties of the instantaneous and cycle-averaged values of transport coefficients in ac electric and magnetic fields crossed at arbitrary angle and for an arbitrary phase difference between the fields. We begin this section by investigating the numerical accuracy of the present theory and code using the benchmark Reid ramp model for electron swarms in ac electric and magnetic fields. Preliminary benchmark results for the Reid ramp model have been recently presented (Raspopović et al. (2000); White et al. (2002); Dujko (2004)). In this thesis we extend these previous publications by a comprehensive description of electron transport for the most general case of arbitrary field directions and phase differences between the fields. Using both the Reid ramp model and the Lucas-Saelee ionization model for a selected set of conditions, electron transport coefficients obtained by a multi-term theory for solving the Boltzmann equation are compared with those obtained by a Monte Carlo simulation technique. By doing so, we provide benchmarks for future
investigations of charged particle swarms in ac electric and magnetic fields. In addition to model gases, in this chapter we investigate the electron transport properties associated with electron swarms in ac electric and magnetic fields in CF$_4$ and CF$_4$-argon mixtures. The inadequacies of the two-term approximation for ac electric and magnetic fields in both model and real gases are presented.

8.2 Traditional description of the temporal profiles of the transport properties

The traditional approach in the treatment of electron transport in time dependent fields is based on comparison between the frequencies for momentum relaxation ($\nu_m^e(\epsilon)$) (7.1) and energy ($\nu_e(\epsilon)$) (7.2) with the field frequency ($\omega$) (Wilhem and Winkler (1979); Makabe and Goto (1988); Loureiro (1993); White (1996); Dujko (2004; 2005); Makabe and Petrović (2006)). According to this, there are four frequency domains:

(i) low frequency regime ($\omega << \nu_e(\epsilon) << \nu_m^e(\epsilon)$): In this regime, all transport parameters follow the field in a quasi-stationary manner with full modulation and no phase delay with respect to the applied electric field. Therefore, in this low frequency regime, the distribution function and transport properties can be calculated by solving the Boltzmann equation in a dc field for the different time-varying values of the ac field strength.

(ii) intermediate frequency regime ($\nu_e(\epsilon) \approx \omega << \nu_m^e(\epsilon)$): In this regime, all quantities which relax with time constant $\tau_e(\epsilon) = \nu_e^{-1}$ undergo a reduction in modulation and a phase lag in their temporal profile with respect to the applied electric field while all quantities which relax on the time-scale $\tau_m(\epsilon) = \nu_m^e(\epsilon)^{-1}$ remain fully modulated;

(iii) high frequency regime ($\nu_e << \omega \approx \nu_m^e(\epsilon)$): In this regime, all quantities which relax on the time-scale $\tau_m(\epsilon)$ can no longer relax sufficiently before the field changes and undergo a reduction in the modulation and an increase in the phase lag with respect to the applied electric field while all quantities which relax according to the time constant $\tau_e(\epsilon)$ are generally time-independent in this regime. It should be emphasized that in this regime, the so-called “effective field approximation” has played an important role in the traditional description of electron kinetics in ac fields. This approximation may be derived using the momentum balance equation of electrons and is represented by:

$$E_{\text{eff}} = \frac{E_0}{\sqrt{2}} \frac{1}{\sqrt{1 + (\omega/\nu_m^e)^2}}.$$  \hspace{1cm} (8.1)

Within the two-term theory for solving the Boltzmann equation, this approximation is exact for the isotropic component of the distribution function if the total momentum transfer collision frequency is independent of the energy (McDonald and Brown (1949a;1949b)). The effective field approximation was used in investigations of helium and hydrogen where collision frequencies are effectively constant over the range of energies critical for the development of the breakdown in these gases.
(iv) very high frequency regime ($\omega >> \nu_e^m(\epsilon)$): In this regime, the electrons undergo many oscillations between two collisions i.e. the drift velocity oscillate $\pi/2$ out of phase with the field and consequently, on average the field can no longer pump energy into the system and transport approaches to thermal limit.

It is clear that there is no sharp distinction between these frequency regimes due to the complex energy dependencies of the collision frequencies for momentum and energy relaxation. However, there have been many attempts to investigate the electron kinetics in both atomic and molecular gases in different frequency regimes. As an illustrative example, on the basis of quasi-static approximation, the electron transport in domain of low frequencies $\omega << \nu_e$ has been studied by Rogoff et al. (1986) and Seebock and Kohler (1988) while the effective field approximation was employed by Ferreira and Loureiro (1984; 1989) and recently by Trunec et al. (2006) at the high frequency limit $\omega >> \nu_e$. At the intermediate frequency range $\omega \approx \nu_e$, the significant contributions have been made by Winkler and Capitelli and their co-workers (see for example Capitelli et al. (1988) and references therein). Finally, from the intermediate to high frequencies, the electron transport has been studied for molecular gases such as CH$_4$ (Makabe and Goto (1988); Goto and Makabe (1990)).

In summary, two important conclusions emerged from these previous publications involving effective fields. First, as emphasized by White (1996), at any phase of the field a transport property cannot exceed the maximum value of the steady-state dc value of that transport property over the range of applied dc fields $0 \leq E \leq E_0$, where $E_0$ is the amplitude of the ac electric field. This means that any transport property cannot undergo an increase in its modulation for an increasing frequency. One of the most important aspects of this thesis is to emphasize that this is not generally true. This traditional approach is generally applicable only for Maxwell model of interactions and for the case where transport parameters relax exclusively monotonically (White (1996); Dujko (2004; 2005)). In the following, we examine the limitations of such approach and in general highlight some critical aspects of behavior of electron transport coefficients in rf fields which should be considered for a potential application in plasma modeling of CF$_4$ rf discharges. Second, the effective field approximation is not generally valid if one deals with the molecular gases. In contrast to atomic gases where inelastic collision processes usually occur with lower intensity in the region of higher energies, in molecular gases a remarkable vibrational excitation processes at lower energies and significant electronic excitation at medium electron energies take place. As a consequence, the collision frequencies for both the momentum and energy dissipation usually have complex energy dependencies and in general, the effective field approximation fails even at high frequencies. Therefore, we have been motivated to investigate the basic characteristic of the electron transport in rf fields in CF$_4$ having in mind the fact that $\nu_e^{en}$ and $\nu_e$ are complicated functions of electron energy.
8.3 Electron swarms in CF$_4$ and argon-CF$_4$ mixtures in ac electric fields

8.3.1 Preliminaries

Several efforts have been made in order to provide sets of electron-CF$_4$ cross sections based on experimental measurements (Christophorou et al. (1996); Christophorou and Olthoff (1999); Bonham (1994); Morgan (1992)) and based on Boltzmann equation analysis (Hayashi (1987); Nakamura (1991); Stefanov and Pirgov (1993); Bordage et al. (1999)). Despite of these efforts there are two sets of electron-CF$_4$ cross sections which are widely used in plasma modeling community. The first set was determined by Christophorou and co-workers (Christophorou et al. (1996); Christophorou and Olthoff (1999)). Surprisingly, this set of electron-CF$_4$ cross sections gives relatively close agreement with recommended experimental values for electron transport coefficients except for ionization rate coefficients although they did not apply swarm technique for renormalization of cross section to fit the swarm parameters (Bordage et al., 1999). In order to fit the ionization rates these authors have selected to split the cross section for vibrational excitation into two parts. The resonant part at higher energy is assigned a higher energy loss of 5 eV which is somewhat arbitrary. They deduced the higher energy resonance part of the vibrational cross section by subtracting the recommended cross section for non-resonant vibrational excitation, elastic scattering and attachment from the total scattering cross section. However assigning this energy loss to the vibrational excitation cannot be justified easily.

The second set cross sections for electrons in CF$_4$ was published by Kurihara and co-workers (Kurihara et al., 2000). They have employed the so-called direct numerical procedure (DNP) for fitting the initial set of electron-CF$_4$ cross sections published by Nakamura (1991). According to the experimental measurements of Nakano and Sugai (1992) and Stephan et al. (1985) they split the dissociative and total ionization cross section into different channels and thus provide the possibility to construct an exact plasma chemical model. In contrast to Bordage et al. (1996) who have modified cross section for vibrational excitation in order to obtain the best fit for ionization rate coefficient, Kurihara and co-workers decided to choose another approach. Their approach was to modify the cross section for dissociation into ground state fragments, since these data are not known with high accuracy and since it has the strongest effect on the ionization rate coefficient.

In this investigation we employ the cross sections of Kurihara et al. (2000) shown in figure 8.1 (a). In figure 8.1 (b) the energy-resolved frequencies for momentum $\nu_{\text{me}}^{\text{m}}(\epsilon)$ and energy $\nu_{\text{e}}(\epsilon)$ relaxation of electrons at the pressure of 1 Torr are displayed. Using a Monte Carlo simulation technique, this set of cross sections was systematically tested (Dujko et al., 2003). It was found that this set of electron-CF$_4$ cross sections gives an excellent agreement for the ionization rate coefficients with recommended experimental values. In addition, the same set of cross sections was employed for calculation of the electron transport coefficients in dc electric and magnetic fields (Dujko et al. (2005; 2006)). The cross sections for electrons in argon are displayed and detailed in Chapter 7 (see Figure 7.32 (b)). For illustrative purposes we chose an applied electric
field of 100 Td to ensure the average energy of the swarm is in the range where interesting kinetic phenomena may be induced. The gas number density is set to $3.54 \times 10^{22}$ m$^{-3}$ which corresponds to the gas pressure of 1 Torr at 273 K. All calculations were performed for zero gas temperature. The effects of both the electron attachment and ionization on the electron transport are considered.

Figure 8.1: (a) Electron impact cross-section for CF$_4$ (Kurihara et al., 2000) includes elastic momentum transfer (1), three vibrational (2-4) and one electronic excitation cross section (5), attachment cross section (6), seven dissociative ionization cross sections (7-13) and three cross sections for neutral dissociation (14-16); (b) Energy-resolved frequencies for momentum $\nu_e^m(\epsilon)$ and energy $\nu_e(\epsilon)$ relaxation of electrons at the pressure of 1 Torr.

### 8.3.2 Electron transport properties in CF$_4$

In figures 8.2-8.5 we display the variation of the periodic steady-state profiles of the transport properties for electrons in CF$_4$ for various field frequencies. Having in mind the $E/n_0$ dependence of these transport quantities in dc fields (see for example Dujko et al. (2005;2006;2009)) and the time dependence of the electric field $E(t)/n_0 = 100 \cos(2\pi ft)$ Td, one may assume that the time-resolved transport quantity corresponds to the instantaneous electric field in the same way as that transport property in dc field. This is the well-known quasi-static approximation (QSA) or the so-called instantaneous field approximation. In figures 8.2-8.5 we show the temporal profiles of various transport properties obtained by a QSA approximation. QSA has played (and still plays) an important role in modeling a parallel plate ac plasma reactors. In essence, one can solve the time-independent Boltzmann equation for different values of $E/n_0$ and generate look-up tables or analytical expressions of the electron transport parameters as a function of $E/n_0$ and gas composition. Balance equations determining self-consistent electric field and charged particle densities can then be integrated directly.
Figure 8.2: Temporal profiles of the mean energy for electrons in CF$_4$ under the influence of an electric field of the form $E(t)/n_0 = 100 \cos(2\pi ft)$ Td for various applied frequencies.

In figure 8.2 we show the temporal profiles of the mean energy $\varepsilon$, obtained by a QSA and as a function of the field frequency for electrons in CF$_4$. It is important to note that under conditions of QSA, when $E/n_0$ takes zero value we assume thermal values of all transport properties including the thermal values for the mean energy. The results at 1 MHz are practically identical to those obtained by a QSA. This suggests the full energy relaxation at all times for a frequency less than 1 MHz. However, as the field frequency increases, the modulation amplitude decreases while the phase-lag of the temporal profiles of the mean energy with respect to the applied electric field increases. The first obvious failure of the QSA is observed at 5 MHz for the part of the period when the field passes through zero. As the field frequency is further increased to 100 MHz the relaxation of energy is not accomplished, the modulation amplitude is significantly reduced while the cycle-averaged value is increased. In general, the cycle-averaged value of the mean energy for electrons in CF$_4$ displays a maximal property with frequency: at first the cycle-averaged mean energy increases with frequency, attains a maximum and then decreases monotonically with a further increase of frequency. This property has been recently investigated in several time-dependent studies (Makabe and Goto (1988); Goto and Makabe (1990); Loureiro (1993); Sa et al. (1994); Robson et al. (1995)). The maximum occurs at around 50 MHz (not shown). In high-frequency regime, however, the cycle-averaged mean energy begins to decrease due to the temporal trapping. As the field frequency increases, a transition from non-sinusoidal profiles at low frequencies to sinusoidal at higher frequencies is clearly evident. Similar temporal profiles of the mean energy have been recently observed for both the real (CH$_4$: Makabe and Goto (1988); Goto and Makabe (1990); White (1996)); H$_2$: Goto and Makabe (1990) ; N$_2$: Loureiro (1993); SiH$_4$ and Si$_2$H$_6$: Shimada et al. (2003); argon (Raspopović (1999); Raspopović et al. (2005); Trunec et al. (2006)) and model (Reid ramp model: White (1996); Raspopović et al. (2005); Trunec et al. (2006)) and model (Reid ramp model: White (1996); Raspopović et al. (2005); Trunec et al. (2006)).
(1999); White et al. (2002); Trunec et al. (2006); non-conservative model gases (White et al. (1998; 1999b))) gases. Generally speaking, the temporal profiles of the mean energy behave according to the traditional approach for a chosen set of conditions.

![Temporal profiles of the gradient energy parameter for electrons in CF$_4$](image)

Figure 8.3: Temporal profiles of the gradient energy parameter for electrons in CF$_4$ under the influence of an electric field of the form $E(t)/n_0 = 100 \cos(2\pi ft) \ Td$ for various applied frequencies.

In figure 8.3 we show the temporal profiles of the gradient energy parameter $\gamma$ obtained by a QSA and as a function of the field frequency for electrons in CF$_4$. The first signs of the temporal non-locality and failure of the QSA are noticeable at 1 MHz. At frequency of 5 MHz, the height of the sub-maximum associated with a rising field decreases while the height of the sub-maximum associated with a falling field is essentially unaffected. As a consequence, the temporal profile becomes asymmetric. This is a clear sign that energy is not able to fully relax before the field changes. As the field frequency increases, we observe a transition from non-sinusoidal profiles at low frequencies to sinusoidal at higher frequencies. In the high frequency regime, the amplitude of oscillations in the temporal profiles decreases monotonically to zero while the phase-lag with respect to the electric field increases to $\pi$. Therefore, as frequency increases, transition to spatial uniformity in the average energy through the swarm becomes clearly evident. Similar temporal profiles of the gradient energy parameter $\gamma$ were observed for electrons in CH$_4$ (White et al., 2002) and for electrons in some conservative (White, 1996) and non-conservative (White et al. (1998; 1999b)) model gases. Generally speaking, the temporal variation of the gradient energy parameter $\gamma$ has important implications for understanding the effects of non-conservative collisions on the temporal profiles of both the drift velocity and diffusion tensor components. The profiles of this transport property for electrons in non-conservative model gases will be discussed later in a more general context when both the electric and magnetic fields are present.
Figure 8.4: Temporal profiles of the drift velocity for electrons in CF$_4$ under the influence of an electric field of the form $E(t)/n_0 = 100 \cos(2\pi ft)$ Td for various applied frequencies.

Figure 8.4 displays the temporal profiles of the drift velocity obtained by a QSA and as a function of the field frequency for electrons in CF$_4$. Temporal profiles of the drift velocity shown in figure 8.4 behave similarly to those for CH$_4$ (Jelenak et al. (1995); White (1996); Bzenić et al. (1999)) and those for SiH$_4$ and Si$_2$H$_6$ (Shimada et al., 2003). In the quasi-static regime, we observe the instantaneous negative differential conductivity (NDC) exists over a certain phase of the field. This suggests that the swarm has sufficient time to relax over the entire cycle of the electric field and the drift velocity follows the field in a quasi-stationary manner. As the frequency is increased, however, the height of the sub-maximum associated with a rising field magnitude increases, while the height of the sub-maximum associated with a falling field magnitude decreases. This means that at low frequencies, and in low energy part of the cycle, the swarm cannot fully relax before the field changes while in the high energy part of the cycle the relaxation times are sufficiently short that NDC effect remains in these phases. A further increase of frequency reduces the time for relaxation and limits the occurrence of NDC only in the high energy phase of the field where the field is rising. At sufficiently high frequencies, where the inequality $\omega >> \nu_e$ holds over the entire cycle of the field, the swarm is not able to relax over the entire cycle of the field and hence the NDC effect vanishes. We note there is a transition from triple-peaked non-sinusoidal profile at low frequency (less than 1 MHz) to sinusoidal profiles at high frequency. The phase delay of the temporal profile of the drift velocity with respect to the electric field increases from 0 to $\pi/2$ rad at high frequencies. The modulation amplitude displays a maximal property with increasing frequency. An enhancement of modulation amplitude is clearly evident at 500 MHz. In contrast to H$_2$ (Goto and Makabe, 1990), the maximum does not occur at the same frequency as the maximum in the cycle-averaged energy. Generally speaking, the temporal profiles for CF$_4$ are qualitatively similar to those associated with CH$_4$ and have in
general the same physical origin.

Figure 8.5: Temporal profiles of the diffusion coefficients for electrons in CF$_4$ under the influence of an electric field of the form $E(t)/n_0 = 100 \cos(2\pi f t)$ Td for various applied frequencies.

The importance of the temporal profiles of the drift velocity shown in figure 8.4 lies with the following facts. The convolution between the electric field and drift velocity yields the power deposited into electron swarms by external electric field. Obviously, the power deposition to electrons is strongly dependent on the shape of the temporal profile of the drift velocity which can be quite different from the expected sinusoidal profile according to the traditional approach and/or from the symmetric non-sinusoidal profile according to the QSA. There is another important aspect associated mainly with the early studies of the time-resolved NDC effect. Low order truncations of the combined Legendre-Fourier expansion of the velocity distribution function could not represent the rapidly changing profile of time-resolved NDC (Goto and Makabe (1990)). This illustrates how important is to perform calculations with the highest degree of accuracy in investigations of the time-resolved electron kinetics in rf fields. The presented temporal profiles for the drift velocity obtained by a multi-term theory for solving the Boltzmann equation are compared with those obtained by a Monte Carlo simulation technique and excellent agreement was found (Dujko (2004; 2005)).

In figure 8.5 we show the temporal profiles of the longitudinal and transverse diffusion coefficients obtained by a QSA and as a function of the field frequency for electrons in CF$_4$. In the quasi-static regime, the phenomenon of anisotropic diffusion is clearly evident and the errors associated with assuming isotropic diffusion can be quite large. We observe that the inequality $n_0 D_T > n_0 D_L$ holds over the entire cycle of the field (except for zero phase where diffusion is essentially isotropic). This is a clear sign that the swarm has sufficient time to relax over the entire cycle of the electric field and both the longitudinal and transverse diffusion coefficients follow the field in a quasi-stationary manner. However, as the field frequency increases, a very
specific and unexpected behavior of the longitudinal diffusion coefficient develops. The anomaly is reflected in the following. Firstly, the longitudinal diffusion coefficient peaks during or just after the phases when the electric field changes the sign and secondly, for a brief phase period, the longitudinal diffusion coefficient becomes larger than the transverse diffusion coefficient. At the lowest frequencies of 1 MHz, we may observe a small but pronounced spike of \( n_0D_L \) at the phase where electric field changes sign. As frequency increases, the spike grows in both magnitude and relative duration and becomes the dominant feature in the profile. As a consequence, \( n_0D_L \) becomes larger then \( n_0D_T \) which is opposite to what may be expected from the dc electric field conditions where diffusion of electrons is strongly anisotropic with \( n_0D_T > n_0D_L \) for the similar energy range. It is interesting to note that at 500 MHz, \( n_0D_L \) is greater than \( n_0D_T \) over the entire cycle of the field while in the limit of the highest frequency considered in this work, the re-establishment of the phenomenon of anisotropic diffusion (\( n_0D_T > n_0D_L \)) is clearly evident. One may expect isotropic diffusion at even higher field frequencies for a chosen set of conditions. In addition to the anomalous behavior of \( n_0D_L \) as the field frequency increases, we note the following: (i) there is an apparent increase in the phase-lag of \( n_0D_L \) with respect to the applied electric field; (ii) there is a transition from non-sinusoidal profiles at low frequencies to sinusoidal at higher frequencies; and (iii) there is a maximal property in the cycle-averaged value of \( n_0D_L \).

In contrast to the longitudinal diffusion coefficient, the behavior of the transverse diffusion coefficient may be explained using the traditional description of the temporal profiles of the electron transport properties in ac fields. We observe that as the field frequency increases, the modulation amplitude is decreased and the shape of the temporal profile is changed from non-sinusoidal to sinusoidal. In the limit of high frequencies, both the modulation amplitude and cycle-averaged value are significantly reduced.

So physically, why does the phenomenon of anomalous behavior exist for the longitudinal component of the diffusion tensor? The effect of anomalous anisotropic diffusion has been observed and explained independently by groups at James Cook University (White et al., 1995) and Keio University (Maeda et al., 1997). Thus this section is not intended to be a comprehensive investigation of the anomalous anisotropic diffusion in CF\(_4\) but rather the current aim is to check the standard explanation of this phenomenon which can be found in these references. To this end, let us consider once more the temporal profiles of the diffusion coefficients, drift velocity and energy gradient parameter shown above (for example at the field frequency of 5 MHz). At times prior to the field reversal, electrons at the front of the swarm have higher energies than those at the back, thus \( W\gamma < 0 \) and inequality \( n_0D_L < n_0D_T \) follows. When the sign of the field is changed, the directional reversal of \( W \) and the local instantaneous average velocities along the direction of motion of the swarm occurs on the time-scale for momentum relaxation (\( \tau_m \)) while \( \gamma \) due to an inability to relax on the same time-scale remains virtually constant during this time. In this brief time a very specific, unusual and anomalous situation develops: electrons at the front of the swarm have lower energy than those at the back. Under these conditions, \( W\gamma > 0 \) and \( n_0D_L \) is enhanced to exceed \( n_0D_T \). As time progress, \( \gamma \) passes through zero and the previous situation is re-established: electrons at the front of the swarms have higher energies than those at the back. At the same time, \( \gamma \) reaches a value equal in magnitude but opposite
in sign to that which it had before the field reversal. This process occurs at the time-scale for energy relaxation ($\tau_e$). As a consequence, $n_0D_L$ begins to decrease and drops below $n_0D_T$ since the sign of $\gamma$ is changed.

In order to check the standard explanation of the anomalous behavior of the longitudinal diffusion coefficient (White et al. (1995); Maeda et al. (1997)) the spatially dependent electron swarm parameters such as the number of electrons, average energy and average velocity are calculated using a Monte Carlo simulation technique. Similar results cannot be found in the literature (preliminary results have been recently published in a conference paper (Dujko, 2004) and in this chapter we present the first spatially resolved transport properties under conditions that lead to the development of the anomalous anisotropic diffusion. In figures 8.6 (a) and (b) we show the spatially dependent average energy and average velocity, respectively, during one half of the field period at different phases. As can be seen, at the phases of $\pi/10$ rad and $2\pi/10$ rad, we may observe an increase of the local spatially dependent average energies which is the sign of the phase delay of the mean energy with respect to the electric field. As the electric field gradually decreases, the local average energies decrease too. However, after the field passes through the phase of $5\pi/10$ rad there is still no change of the slope of the local average energy which indicates the failure to achieve energy relaxation. This is a consequence of the predominantly elastic collisions that are not efficient in energy exchange. At $5\pi/10$ rad we can see that field is changing sign and that slow electrons in the tail are easily accelerated as controlled by the field while the fast electrons at the opposite end are still maintaining high velocity in the opposite direction. This leads to an increase of $n_0D_L$. In the phases, starting from $8\pi/10$ rad the slope of the spatially dependent average energy is changed while the spatially dependent drift velocity at the head of the swarm decreases. In the subsequent phases the slope of the spatially dependent mean energy continually increases while the spatially dependent drift velocity continually decreases. Therefore, the longitudinal diffusion decreases.

Figure 8.6: The spatially dependent average energies (a) and average velocities (b) during one half of the period of the field at different phases as indicated on the graph.
Another proof of anomalous behavior of the rf diffusion has been obtained by sampling the numbers of electrons. In figures 8.7 (a) and (b) we show the spatial distributions of the forward and backward moving electrons in the periodic steady-state, respectively. For clarity, we present the spatial distributions of the number of electrons in the less number of phases during the one half period of the field. As expected, the number of forward moving electrons in the phase ranges from 0 to $\pi/10$ rad increases. However, when the field changes the sign the number of the forward moving electrons increases. Together with an increase of the average velocity, this leads to an enhancement of the diffusion flux of electrons in the same direction. As can be seen, the number of forward moving electrons increases until the slope of the spatially dependent average energy does not change. Conversely, the number of backward moving electrons decreases until the slope of the spatially dependent average energy does not change, as shown in figure 8.7 (b).

**Figure 8.7:** The spatial distributions of the forward (a) and backward (b) moving electrons during one half of the period of the field at different phases as indicated on the graph.

To illustrate the failure of the two-term approximation for CF$_4$ in ac electric fields, in figure 8.8 we show the temporal profiles of the diffusion coefficients obtained by the two-term approximation (dashed lines) and multi-term theory (solid lines) over a range of the field frequencies. The $l_{\text{max}} = 7$ was required to achieve $0.5\%$ accuracy for all transport coefficients and properties over the entire cycle of the field. The inadequacies of the two-term approximation are clearly evident, particularly for the transverse diffusion coefficient. Surprisingly, at low frequencies the two-term approximation is very accurate for the calculation of $n_0D_L$. However, as the field frequency increases, the two-term approximation decrease in accuracy and at the frequency of 500 MHz, the errors associated with $n_0D_L$ can be as high as 200%. Note that the two-term approximation contains errors generally less than 30% in the calculation of the spatially-homogeneous quantities over the range of frequencies considered. However, the errors associated with the transverse diffusion coefficient are much larger. The two-term approximation is inadequate for the calculation of $n_0D_T$ over the entire cycle of the field regardless of the field frequency. We observe that the two-term approximation predicts the temporal profiles that are wrong in mag-
Figure 8.8: Comparison of the two-term and multi-term profiles of the diffusion coefficients for electrons in CF$_4$ at various field frequencies.

magnitude and out of phase with respect to those obtained by a multi-term theory. As can be observed, the two-term approximation generally tends to overestimate the converged multi-term results. At high frequencies the maximum deviation between the two- and converged multi-term profiles occur for mean energies for which the momentum transfer cross section in elastic collisions is at its minimum and rapidly increasing cross sections for vibrational excitations are significant. Under these conditions, one may expect a significant anisotropy of the distribution function in velocity space and hence the subsequent failure of the two-term approximation follows. Therefore, even at the very high frequencies, one cannot expect an improvement in accuracy of the two-term approximation due to very low energy thresholds for the collision processes that lead to vibrational excitation of the CF$_4$ molecule. These examples clearly show the importance of performing calculations of highest degree of accuracy in studies of kinetic phenomena in rf fields. As outlined by White et al. (2003), one may be never sure about the accuracy of the two-term approximation and the best option would be to use a multi-term theory for solving the Boltzmann equation or Monte Carlo simulation technique.

8.3.3 Electron transport properties in CF$_4$-argon mixtures

In this section we investigate the time-dependent behavior of electron swarms in CF$_4$-argon mixtures. We do not propose this section to be a comprehensive investigation of the temporal profiles of the electron transport properties in ac electric fields, but rather aim to demonstrate the ability of the code to handle mixtures and to discuss some interesting phenomena associated with these mixtures. Figures 8.9-8.14 display the temporal profiles of various transport properties for different CF$_4$-argon mixtures at four applied field frequencies. A seven-term approximation was generally needed in the $l$-index for all mixtures. Convergence in the $\nu$-index was found to be poor at low frequencies and for high fractions of argon and $\nu_{\text{max}}$ up to 90 were required.
Figure 8.9: Temporal variation of the mean energy with the frequency for various CF$_4$-argon mixtures.

Figure 8.9 displays the variation of the mean energy with the field frequency for various CF$_4$-argon mixtures. As expected, the mean energy is significantly affected by addition of argon to CF$_4$. With an increasing fraction of argon, we observe the following: (i) the cycle-averaged mean energy increases; (ii) the phase-lag with respect to the electric field is increased; and (iii) the modulation amplitude is decreased. These phenomena are attributable to the decreasing contribution of inelastic processes that lead to vibrational and/or electronic excitation of CF$_4$ molecule. For an increasing fraction of argon the large energy losses in these inelastic processes are decreased and hence an increase of the mean energy follows. Our calculated temporal profiles for the gradient energy parameter as a function of the field frequency for various CF$_4$-argon mixtures are shown in figure 8.10. In general, for an increasing fraction of argon we observe similar variations in the temporal profiles with respect to those observed previously for the mean energy.

In figure 8.11 we show the variation of the reaction rate with the field frequency for various CF$_4$-argon mixtures. For 100% CF$_4$ the reaction rate is negative over the entire cycle. This indicates that the attachment rate dominates the ionization rate. As expected, for low frequencies the reaction rate peaks when the electric field has its maximal value. When the field frequency is increased, this is not true any more due to the phase delay with respect to electric field. For low frequencies and for an increasing fraction of argon, the peak values are decreased while the minimum values are essentially unaffected. This means that the electron attachment processes are reduced and ionization processes take place and become significant during the phases where electric field reaches the maximal value. For a short period of time at the field frequency of 5 MHz, the reaction rate is positive reflecting the fact that the ionization
rate exceeds the electron attachment rate associated with the CF$_4$ molecule. Temporal profiles of the mean energy presented in figure 8.9 support these arguments. In the limit of the highest fraction of argon considered here (e.g. 50-50 (CF$_4$-argon mixture)), we observe that the reaction rate is modulated with a frequency of $4\omega$. This results from a complex interplay of temporal non-locality and energy dependent collision frequencies for non-conservative collisions.

Perhaps the most striking phenomenon at high frequencies is an increase of the reaction rate for an increasing fraction of argon. If argon concentration increases, one might intuitively expect a decrease in the reaction rate. At frequency of 200 MHz this is not true for mixtures of 10% and 20% of argon while at frequency of 500 MHz the opposite situation holds: the reaction rate monotonically increases with an increasing fraction of argon. This phenomenon follows from the time-dependent behavior of the mean energy. Although the concentration of CF$_4$ decreases, the mean energy is significantly increased for an increasing fraction of argon. As a consequence, the collision frequency for attachment processes is significantly increased and an enhancement of the reaction rate follows. Note that in this energy range the ionization processes are not operative. Generally speaking, when the field frequency is increased, the complex waveforms associated with the reaction rate are not present. At 200 and 500 MHz, we observe a large phase-lag between the temporal profile of the reaction rate and electric field. We may observe that the symmetric profile of the reaction rate for 100% CF$_4$ becomes asymmetric and triangular with a fast increase and slower decrease as the concentration of argon is increased.

Figure 8.12 displays the variation of the drift speed with the field frequency for various CF$_4$-argon mixtures. For all frequencies considered here the modulation amplitude monotonically
Figure 8.11: Temporal variation of the reaction rate with the field frequency for various CF₄-argon mixtures.

Figure 8.12: Temporal variation of the drift speed with the frequency for various CF₄-argon mixtures.

increases with increasing argon concentration. At low frequencies the signs of NDC are evident for all fractions of argon in the mixture. In general, the full understanding of the NDC effect in mixtures of NDC gas-atomic gas requires a detailed comparison of relative magnitudes of the appropriately weighted energy transfer collision frequencies for elastic and inelastic collisions in association with the effects of an inability of drift velocity to relax as the field frequency increases. This is beyond the scope of this work and we defer this to a future investigation.
In figures 8.13 and 8.14 we show the variation of the longitudinal and transverse diffusion coefficients with the field frequency for various CF$_4$-argon mixtures. At high frequencies, we observe that the longitudinal diffusion coefficient monotonically increases with an increasing concentration of argon. At low frequencies the height and duration of the maxima in the profile of $n_0D_L$ increase. The variation of the $n_0D_T$ profiles with argon concentration at low frequencies is quite striking, with an apparent inversion of the profile when electric field change the sign.
The cycle-averaged value of $n_0D_T$ is a monotonically increasing function of argon concentration for the field frequencies of 5, 20 and 200 MHz. At 500 MHz, however, the cycle-averaged value of $n_0D_T$ firstly decreases and then start to continually increases with an increasing fraction of argon. The basic trends in the profiles of both $n_0D_L$ and $n_0D_T$ can be understood by considering transition in the predominant mechanism of scattering with mixture ratio.

8.4 The influence of magnetic field on the electron transport properties

8.4.1 Benchmark comparison: Reid ramp model

In this section we consider the Reid ramp model (6.1) for electron swarms in ac electric and magnetic fields. The amplitude of the electric field is set to 12 Td while the magnetic field amplitudes assume the values between 0-2000 Hx. We consider the reduced angular frequency range: $(1 \times 10^{-17}) - (2 \times 10^{-14})$ rad m$^3$s$^{-1}$ and the phase difference between the fields of 0°, 30°, 60° and 90° degrees. Under conditions of dc electric and magnetic fields, the Reid ramp model has become the standard test for Boltzmann equation solutions (Ness (1994); White et al. (1999a)) and Monte Carlo simulations (Raspopović et al. (1999); Petrović et al. (2002); Dujko (2004; 2005)), particularly in the light of the known failure of the two-term approximation for this model. Aside from some conference papers with preliminary results, for an ac electric field only, the Reid ramp model has been investigated in theses of White (1996), Bzenić (1997) and Raspopović (1999). The first systematic comparison between the temporal profiles for the electron transport coefficients obtained by a Monte Carlo simulation technique and direct numerical procedure (DNP) for solving the Boltzmann equation under conditions of an ac electric field was given by Maeda et al. (1997).

Before embarking on a discussion of our results calculated under conditions of arbitrary phases between the fields and varying field configurations, we first present the results of benchmark calculation for electron transport in rf electric and magnetic fields. A multi-term theory for solving the Boltzmann equation was tested using a Monte Carlo simulation technique over a range of conditions for which the traditional approximations and assumptions fail. This was necessary step in order to have the full confidence in the adequacy of the time dependent transport theory and corresponding computer codes. For comparisons between the electron transport coefficients obtained by these two independent techniques, the following conditions are considered. The electric and magnetic fields are $\pi/2$ out of phase and have the following forms: $E(t)/n_0 = 10\sqrt{2}\sin(2\pi ft)$ Td and $B(t)/n_0 = 500\cos(2\pi ft)$ Hx. The gas number density is set to $3.54 \times 10^{22}$ m$^{-3}$ which correspond to the pressure of 1 Torr at 273 K. In our Monte Carlo simulations, the initial electron velocity distribution is a Maxwellian with the mean starting energy of 1 eV. All transport quantities obtained by a multi-term theory for solving the Boltzmann equation represent an $l_{\text{max}} = 4$ truncation. Certain trends in temporal profiles of electron transport coefficients as a function of frequency of the applied fields are addressed. These trends
will be discussed later in a more general context. In this section we are entirely focused on a comparison between the electron transport properties obtained by various techniques.

In figure 8.15 we show the temporal profiles of the mean swarm energy as a function of frequency of the applied fields. We observe that the results from a multi-term theory are consistent with those predicted by a Monte Carlo simulation technique over the entire range of the field frequencies. We observe that modulation amplitude decreases with increasing frequency. As a consequence, the cycled-averaged value of the mean energy displays a maximal property with frequency. In addition, as the field frequency increases, the phase-lag of the temporal mean energy profile with respect to the applied electric field also increases.

![Figure 8.15: Comparison of the Boltzmann equation (black) and Monte Carlo (red) results for ε over a range of applied field frequencies for the Reid ramp model.](image)

In figures 8.16 (a) and (b) we show the temporal profiles of the longitudinal and transverse drift velocity components as a function of frequency of the applied fields. Results from the Boltzmann equation analysis coincide very well with the Monte Carlo results. We observe that the longitudinal drift velocity component peaks at the phase or just after the phase of the peak in the electric field. It becomes equal to zero or very small in phases where the magnetic field is dominant. Both components of the drift velocity show an additional oscillatory-type behavior in the frequency interval from 25 to 100 MHz. Clearly, this is a sign of cyclotron like motion where electrons may complete the large fractions or even whole circular orbits.

In figures 8.17 (a) and (b) we show longitudinal ($n_0D_{zz}$) and transverse component along the $E \times B$ direction ($n_0D_{xx}$) of the diffusion tensor. Large scatter in diffusion data at low frequency obtained in Monte Carlo simulations is the result of poor statistics which requires following a large number of electrons and their complete relaxation. Perhaps the most distinct property of both diffusion coefficients is the presence of ‘negative’ diffusion. This effect appears at those phases of the field where the electric field is low in magnitude. Transverse diffusion coefficient along the $E \times B$ direction becomes negative for frequencies above 25 MHz while for $n_0D_{zz}$, the negative excursions in the profiles are delayed to 50 MHz and higher. These results
Figure 8.16: Comparison of the Boltzmann equation (black) and Monte Carlo (red) results for $W_z$ and $W_x$ over a range of applied field frequencies for the Reid ramp model.

independently confirm the effect of ‘negative’ diffusion first observed by Raspopović et al. (2000).

Figure 8.17: Comparison of the Boltzmann equation (black) and Monte Carlo (red) results for $n_0D_{zz}$ and $n_0D_{xx}$ over a range of applied field frequencies for the Reid ramp model.

8.4.2 The effects of the magnetic field strength on the electron transport properties in ac electric and magnetic fields

In this section we investigate the effects of the magnetic field strength on the temporal profiles of the electron transport properties in ac electric and magnetic fields. The variation of the temporal profiles of the mean energy $\varepsilon$ with $B_0/n_0$ for four applied reduced angular frequencies
\( \omega/n_0 \) is displayed in figure 8.18. For the magnetic field-free case, as \( \omega/n_0 \) increases, the following interesting points can be observed: (i) the modulation amplitude decreases; (ii) the phase-lag with respect to the applied electric field is increased; and (iii) the cycle-averaged value displays a maximal property with frequency. Hence the temporal profiles of the mean energy for the magnetic field-free case behave according to the traditional approach in treatment of electron transport for a chosen set of conditions.

Figure 8.18: The variation of the temporal profiles of the mean energy with \( B_0/n_0 \) in a crossed field configuration for various \( \omega/n_0 \) for the Reid ramp model. The amplitude of electric field is 12 Td.

The application of the magnetic field in the low frequency regime lowers the mean energy. At the lowest \( \omega/n_0 \) of \( 1 \times 10^{-17} \) rad m\(^3\) s\(^{-1}\) and for an increasing \( B_0/n_0 \), the temporal profiles of the mean energy become more narrow while the peaks remain pretty much the same and essentially unaffected by the action of the magnetic field. On the other hand, as \( B_0/n_0 \) increases, the minimal values of the mean energy undergo a reduction and phase delay with respect to the applied electric field. This is the reason why the temporal profiles become asymmetric and why the modulation amplitude is a monotonically increasing function of \( B_0/n_0 \).

The asymmetry in the temporal profiles of the mean energy becomes more evident as \( \omega/n_0 \) is further increased to \( 1 \times 10^{-15} \) and \( 1 \times 10^{-14} \) rad m\(^3\) s\(^{-1}\). As \( B_0/n_0 \) increases, there is a large reduction in the extremes in the mean energy profiles. The peaks are shifted towards lower phases while the minimums are shifted towards higher phases where the electric field reaches its maximal value. As a consequence, the temporal profiles become asymmetric and triangular with a fast increase and a slower decrease. Due to reduction of the peak values, the modulation amplitude is decreased in comparison with the profiles at lower frequencies. Perhaps
the most striking phenomenon observed in the temporal profiles in the high frequency regime is an enhancement of the mean energy as $B_0/n_0$ increases. As can be observed, at frequency $5 \times 10^{-14}$ rad m$^3$s$^{-1}$ the mean energy displays a maximal property with $B_0/n_0$. This is a clear sign that the well-known magnetic cooling phenomenon in dc electric and magnetic fields does not carry over directly to the ac electric and magnetic fields. Before embarking on a discussion associated with the electron heating mechanisms in ac electric and magnetic fields, we first look at the behavior of the temporal profiles of the drift velocity components.

Figure 8.19: The variation of the temporal profiles of the longitudinal drift velocity component with $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

The variation of the temporal profiles of the longitudinal drift velocity component $W_z$ with $B_0/n_0$ for various applied reduced angular frequencies $\omega/n_0$ is displayed in figure 8.19. Among many interesting points when $B_0/n_0 = 0$ Hx and for an increasing $\omega/n_0$, one may observe an enhancement of the modulation amplitude at $1 \times 10^{-14}$ rad m$^3$s$^{-1}$. In other words, the modulation amplitude displays a maximal property with $\omega/n_0$. This property is inexplicable through the traditional approach in treatment of electron transport in ac electric fields.

In the low frequency regime and as $B_0/n_0$ increases, the temporal profiles of $W_z$ become more narrow while the peak values are essentially unaffected. In the asymptotic limit of strong magnetic field and when electric field goes through zero value, the drift along the electric field direction is significantly reduced. The absence of oscillations from the temporal profiles is a clear sign that the collision frequency dominates the cyclotron frequency. As the field frequency is further increased and for an increasing $B_0/n_0$, the temporal profiles lose their symmetry. The
shape of the temporal profiles becomes more and more triangular with a slow increase and a fast increase (during the period when magnetic field increases) leading to strong oscillations. It must be emphasized that these oscillations occur when cyclotron frequency exceeds collisions frequency. Essentially they are imprints of the individual cyclotron motion on the collective averaged property, the longitudinal drift velocity component in this case. In the high frequency regime, the peak values are less than those at lower frequencies but more importantly for an increasing $B_0/n_0$, one may observe a significant reduction of the phase delay between $W_z$ and electric field. Such behavior can strongly affects the efficiency of electron heating mechanism in ac electric and magnetic fields, as explained in later sections.

Figure 8.20: The variation of the temporal profiles of the transverse drift velocity component with $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

Special characteristic of electron transport in $E(t) \times B(t)$ is the transverse drift velocity component along the $E \times B$ direction $W_x(t)$. The variation of the temporal profiles of this transport coefficient with $B_0/n_0$ for various applied reduced angular frequencies $\omega/n_0$ is displayed in figure 8.20. As can be observed, the temporal profiles of $W_x$ are modulated at twice the field frequency as they follow the product $E(t) \times B(t)$. In the low frequency regime and as $B_0/n_0$ increases, this transport quantity increases, reaches the maximum and then start to decrease in the asymptotic limit of high magnetic field. Similar trends are exhibited in $W_x$ in dc $E \times B$ fields regardless of the type of the gas considered and energy-dependence of the cross sections (Ness, 1994). For an increasing $B_0/n_0$ the position of extremes in $W_x$ are shifted towards higher phases in such a manner that the symmetry of the profiles essentially remains unaltered. At the same time there are no changes in the extremes of $W_x$ and consequently, the mean value in the
cycle-averaged sense of this transport quantity is zero.

As the field frequency increases, one may observe dramatic changes in the temporal profiles of $W_x$. The first signs of the asymmetry in the temporal profiles of $W_x$ become evident at the field frequency $1 \times 10^{-15}$ rad m$^3$ s$^{-1}$. In the high-frequency regime the asymmetry associated with the temporal profiles of $W_x$ is more evident. There are two important aspects associated with the asymmetry of the $W_x$-temporal profiles. The first aspect concerns the asymmetry between the maximal and minimal values. For example, at the reduced field frequency $1 \times 10^{-15}$ rad m$^3$ s$^{-1}$ we observe that the minimal values can be much larger in magnitude than the corresponding maximal values. Moreover, in the high frequency regime the temporal profiles for some $B_0/n_0$ do not change the sign. Such behavior can be ultimately linked to the relation between the cyclotron and collision frequency. Therefore, the magnetic field induces an effective force in the $E \times B$ direction producing a macroscopic drift in that direction while along other directions the electrons simply oscillate around the initial positions. However, as $B_0/n_0$ is further increased, the symmetry in the temporal profiles of $W_x$ is re-established and the mean value in a cycle-averaged sense is zero. Under these conditions when the cyclotron frequency exceeds the collision frequency and the magnetic field dominates the collisions (the so-called magnetic field-dominated regime), the reduction of the Larmor radius associated with an increasing $B_0/n_0$ acts to reduce both $W_x$ and $W_z$, because electrons are ‘held’ by the magnetic field lines (Ness (1994); White et al. (1997); Raspopović et al. (2000; 2005); Dujko et al. (2005; 2006)).

The second aspect associated with the asymmetry of the $W_x$-temporal profiles concerns the phase delays of maximal, minimal and zero values. Note that in high-frequency regime and for all $B_0/n_0$ the temporal profiles go through zero at the same phases. As the applied reduced frequency increases, this property is absent from the profiles and $W_x$ changes sign at higher phases. For an increasing $B_0/n_0$, this phase delay becomes more evident. In the limit of high $B_0/n_0$, strong oscillations are induced due to cyclotron motion.

We now turn our attention to the diffusion coefficients in ac electric and magnetic fields in a crossed field configuration. The variation of the temporal profiles of the longitudinal diffusion coefficient $n_0D_{zz}$ with $B_0/n_0$ for various applied reduced angular frequencies $\omega/n_0$ is displayed in figure 8.21. In figures 8.22 and 8.23 we show the variation of the temporal profiles of the transverse diffusion coefficients $n_0D_{yy}$ and $n_0D_{xx}$ with $B_0/n_0$ for various $\omega/n_0$, respectively. For the magnetic field-free case, the two transverse diffusion coefficients are identical and their behavior is well described by traditional approach in treatment of electron transport in ac electric fields. On the other hand, the longitudinal diffusion coefficient $n_0D_{zz}$ shows typical anomalous behavior: first, when electric field changes the sign, a peak, rather than minimum occurs and second, for some field frequencies that peak exceeds the transverse diffusion coefficient which is also unexpected.

As expected, for an increasing $B_0/n_0$ the longitudinal diffusion coefficient $n_0D_{zz}$ undergoes a significant reduction. The variation of the $n_0D_{zz}$-temporal profiles with $B_0/n_0$ at the lowest reduced angular field frequency ($\omega/n_0 = 1 \times 10^{-17}$ rad m$^3$ s$^{-1}$) is quite striking. When the magnetic field is applied, one may observe an apparent inversion of the temporal profile. As
Figure 8.21: The variation of the temporal profiles of the longitudinal diffusion coefficient $n_0D_{zz}$ with $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

$B_0/n_0$ increases, the temporal profiles become more narrow while the peak values do not change. However, the minimum values are a monotonically decreasing function of $B_0/n_0$. As a consequence, the modulation amplitude monotonically increases with $B_0/n_0$. For an increasing $B_0/n_0$ there are no phase delays of the extremes with respect to the electric field. The longitudinal diffusion coefficient $n_0D_{zz}$ peaks when electric field reaches its maximal value while the values of minimums are found to correspond to the maximum of the magnetic field. In the asymptotic limit of high $B_0/n_0$, diffusion along the electric field direction is significantly reduced for an increasing magnetic field and decreasing electric field. In other words, at high values of $B_0/n_0$, the trajectories of the electrons become frozen and diffusion is allowed only at times when the magnetic field goes through zero.

As $\omega/n_0$ is further increased, the phase of diffusion maximums is shifted to the phase of the zero magnetic field thus reducing the anomalous behavior of $n_0D_{zz}$. At the highest values of $B_0/n_0$, strong oscillations are induced due to the cyclotron motion. This may lead to the negative diffusion, a phenomenon previously observed by a Monte Carlo simulation technique (Raspopović et al., 2000) and Boltzmann equation analysis (White et al., 2002). The presence of negative diffusion coefficients is one of the most striking phenomena observed in the electron transport in ac electric and magnetic fields. This phenomenon cannot be extrapolated from the quasi-stationary (projection of the dc values to the instantaneous values of the electric field) behavior. While the full appreciation and understanding of this kinetic phenomenon requires full understanding of the relaxation profiles of the diffusion coefficients in dc electric and magnetic fields.
fields (see Chapter 7), it is clear that the magnetic field induced cyclotron motion is the cause.

Figure 8.22: The variation of the temporal profiles of the transverse diffusion coefficient $n_0 D_{yy}$ with $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

In a crossed field configuration, the transverse diffusion coefficient $n_0 D_{yy}$ describes diffusion of the swarm along the magnetic field direction. At the lowest reduced angular field frequency considered here, as $B_0/n_0$ increases the temporal profiles become more narrow while the minimal values of $n_0 D_{yy}$ are shifted towards higher phases. As a consequence, the modulation amplitude increases and $n_0 D_{yy}$ loses symmetry as $B_0/n_0$ increases. It is interesting to note that two peaks are formed and it is found that their position correspond to the maximum of the electric field. The existence of these sharp peaks is a clear sign of an inability of the swarm to achieve the full relaxation of energy/momentum before the field is changed. The cycle-averaged value of $n_0 D_{yy}$ is a monotonically decreasing function of $B_0/n_0$.

As $\omega/n_0$ is further increased, the modulation amplitude is significantly reduced due to a large phase delay of the extremes in $n_0 D_{yy}$ with respect to the electric field. In the asymptotic limit of $B_0/n_0$, the temporal profiles are asymmetric, almost triangular with a fast increase and a slower decrease. At the highest $\omega/n_0$ considered here, this transport quantity has a very small modulation and it shows the maximal property with $B_0/n_0$ in exactly the same way as the mean energy. This is a clear sign that in a crossed field configuration, the magnetic field only indirectly affects the diffusion in the $y$-direction, through the magnetic field’s action to cool the swarm. Thus we would expect $n_0 D_{yy}$ to reflect the changes in the component of the temperature tensor $T_{yy}$.
Figure 8.23: The variation of the temporal profiles of the transverse diffusion coefficient $n_0D_{xx}$ with $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

On the other hand, the transverse diffusion coefficient perpendicular to the magnetic field $n_0D_{xx}$ undergoes a large reduction with the magnetic field at all field frequencies in comparison with the coefficient $n_0D_{yy}$ (see Fig. 8.23). This follows from the magnetic anisotropy effect: the magnetic force acts in the $x-z$ plane reducing the diffusion perpendicular to the magnetic field. As longitudinal diffusion coefficient $n_0D_{zz}$ in the low-frequency regime, $n_0D_{xx}$ peaks when electric field reaches its maximal value while the position of minimums correspond to those phases where the magnetic field peaks. In the high frequency regime, as $B_0/n_0$ increases $n_0D_{xx}$ begins to behave as $n_0D_{zz}$. If we take a careful look at the figures 8.21 and 8.23, then we may observe that in the limit of high $B_0/n_0$ the temporal profiles of $n_0D_{xx}$ and $n_0D_{zz}$ are almost identical over all frequencies. This suggests that in the limit of strong $B_0/n_0$, the diffusion in the $x-z$ plane is essentially isotropic over all frequencies. The maximum deviations from isotropy in this plane exist at those phases of the field where the electric field is small and the additional ‘oscillations’ are present. Figure 8.24 shows the cycle-averaged values of the diagonal elements of the diffusion tensor as a function of $B_0/n_0$ for various $\omega/n_0$. The above observations are clearly evident. As for $n_0D_{zz}$, the negative diffusion is present in the temporal profiles of $n_0D_{xx}$ at certain frequencies and $B_0/n_0$.

The variation of the temporal profiles of the Hall diffusion coefficient with $B_0/n_0$ for various $\omega/n_0$ is shown in figure 8.25. As already emphasized in Chapter 6, the Hall diffusion coefficient represent the sum of the off-diagonal components of the symmetrical part of the diffusion tensor. These coefficients may be viewed as a measure of the interaction of the ‘primary’ diffusion
Figure 8.24: The cycle-averaged values of the diagonal elements of the diffusion tensor as a function of $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

fluxes in the $x$ and $z$ directions with the magnetic field to produce ‘secondary’ fluxes in $z$ and $x$ directions. The Hall diffusion coefficient exhibits similar additional oscillatory behavior to the other coefficients. We observe that the temporal profiles of this transport quantity can assume both positive and negative values for all $B_0/n_0$ considered and over all frequencies. At the lowest $\omega/n_0$ of $1 \times 10^{-17}$ rad m$^3$s$^{-1}$ and as $B_0/n_0$ increases, the phase of the diffusion maximums is shifted to the phase where the magnetic field is zero. Both maximums in a half-period of the field have the same height for all $B_0/n_0$. However, as $\omega/n_0$ is further increased to $1 \times 10^{-15}$ rad m$^3$s$^{-1}$, the sub-maximum associated with a falling magnetic field and rising electric field significantly decreases. This is indicative of an inability of this transport property to achieve the full relaxation before the field is changed. In the high frequency regime, $n_0 D_{\text{Hall}}$ is significantly reduced and strong oscillations are induced due to the cyclotron motion.

To complete the present investigation of Reid’s ramp model for ac electric and magnetic fields, we present the temporal variation of the spatially homogeneous energy distribution function. In figure 8.26 (a) the contour plots of the spatially homogeneous distribution function $f_0^{(0)}(\epsilon, t)$ are given for the magnetic field-free case (black line) and for $B_0/n_0 = 200$ Hx while in figure 8.26 (b) we display $f_0^{(0)}(\epsilon, t)$ for $B_0/n_0 = 500$ Hx. The applied reduced angular frequency $\omega/n_0$ is set to $1 \times 10^{-14}$ rad m$^3$s$^{-1}$. We observe that when $B_0/n_0$ increases, the electrons tend to be pushed back to the lower energy part of the distribution. This is another demonstration of the cooling action associated with an increasing component of the magnetic field perpendicular to the electric field. Intuitively one might expect the strongest effect of the magnetic field for
Figure 8.25: The variation of the temporal profiles of the Hall diffusion coefficient with $B_0/n_0$ in a crossed field configuration for various $\omega/n_0$ for the Reid ramp model. The amplitude of electric field is 12 Td.

The phases when it reaches the maximal values (e.g., $\pi/2$ and/or $3\pi/2$ rad). However, as can be observed from figure 8.27 the opposite situation holds: the tail of the distribution function is more populated with high-energy electrons at different phases of the field. This indicates that the effect of magnetic field in domain of rf frequencies is temporally non-local.

Figure 8.26: Contour plots of the temporal variation of the spatially homogeneous energy distribution function with $B_0/n_0$ ((a) black line: $B_0/n_0 = 0$ Hx; red line: $B_0/n_0 = 200$ Hx and (b) blue line: $B_0/n_0 = 500$ Hx) in a crossed field configuration for Reid’s ramp model. The amplitude of electric field is 12 Td.
8.4.3 The effects of the phase-difference between the fields on the electron transport properties in ac electric and magnetic fields

In this section we investigate the influence of varying the phase difference between the fields on the electron transport properties for the Reid ramp model. We restrict our discussion and results to the influence of varying the phase difference \( \theta \) between the electric and magnetic fields and consider only a limited number of magnetic field strengths at the fixed electric field amplitude and field frequency. The reduced angular frequency \( \omega/n_0 \) is set to \( 1 \times 10^{-14} \text{ rad m}^{-3} \text{ s}^{-1} \).

![Figure 8.27: The 3-dimensional plots of the temporal profiles of the mean energy as a function of the phase-difference \( \theta \) between the fields for different \( B_0/n_0 \) for the Reid ramp model.](image)

The variation of the mean energy with the phase-difference \( \theta \) between the fields for four different values of \( B_0/n_0 \) is displayed in figure 8.27. The field frequency is set to \( 1 \times 10^{-14} \text{ rad m}^{-3} \text{ s}^{-1} \) while the amplitude of the electric field is set to 12 Td. In the limit of small phase differences, the magnetic field is large when electric field peaks and electrons cannot gain much energy from the electric field - the magnetic field cools the swarm. As a consequence, the mean energy is significantly reduced. As the phase difference increases, the magnetic cooling effects are reduced, particularly in phases where electric field peaks (and magnetic field magnitude is small) and we observe that the modulation amplitude and cycle-averaged value are increased. These effects become more apparent for an increasing \( B_0/n_0 \).

The variation of the longitudinal drift velocity component \( W_z \) with the phase difference \( \theta \)
between the fields for four different values of $B_0/n_0$ is displayed in figure 8.28. In the limit of low values of $B_0/n_0$, $W_z$ has limited variation with phase difference $\theta$ due to the fact that the collision frequency dominates the cyclotron frequency over all $\theta$. However, on careful inspection a small but noticeable reduction in $W_z$ can be observed, as the phase difference $\theta$ decreases. In addition, we may observe an increase of the phase delay between $W_z$ and electric field as $\theta$ decreases. Both of these properties have significant influence on the electron heating mechanism and must be considered when analyzing the temporal profiles of the mean energy. In the limit of high $B_0/n_0$ it is hard to elucidate the phase delay between $W_z$ and electric field since strong oscillations are induced due to the cyclotron rotation of the electron swarm. Nevertheless, the phase of the $W_z$-maximums are shifted towards phase where the electric field peaks. In contrast to the phase delay between $W_z$ and electric field, the behavior of the modulation amplitude is more obvious. For an increasing phase difference $\theta$ the modulation amplitude monotonically increases. In particular, in the limit of high $B_0/n_0$ and phase differences $\theta$ close to $\pi/2$ there is a period of rapid acceleration which originates at the phase where the electric field reach its maximal value. However, as the phase difference $\theta$ decreases, these periods of rapid accelerations are shifted towards higher phases due to the complex interplay between the explicit magnetic field effects and phase delay between $W_z$ and electric field.

Figure 8.29 shows the temporal variation of the transverse drift velocity component $W_x$, with the phase difference $\theta$ for four different values of $B_0/n_0$. A very specific and unexpected
Figure 8.29: The 3-dimensional plots of the transverse drift velocity component along the $\mathbf{E} \times \mathbf{B}$ direction as a function of the phase-difference $\theta$ between the fields for different $B_0/n_0$ for the Reid ramp model.

Feature of $W_x$ is the asymmetry in the temporal profiles for all $B_0/n_0$ considered. In the limit of low values of $B_0/n_0$, one may observe that the $W_x$-temporal profiles are minimally affected by the phase difference $\theta$ but the cycle-averaged value is. For a decreasing phase difference $\theta$, the cycle-averaged value of $W_x$ modifies from positive to a markedly negative value. In the limit of small phase differences between the fields, the $W_x$-temporal profiles are negative over all phases for all $B_0/n_0$ considered. This suggests that a phase difference $\theta$ may be used as an additional parameter to control a macroscopic drift in the $\mathbf{E} \times \mathbf{B}$ direction. Apart from an apparent asymmetry in the $W_x$-temporal profiles, we may observe that for an increasing phase difference $\theta$ the modulation amplitude increases while the peak values are shifted towards smaller phases.

As $B_0/n_0$ increases, strong oscillations are induced due to the cyclotron rotation of the electron swarm. As an illustrative example, the temporal profiles for $B_0/n_0$ of 1000 Hx show the same general trends with the phase difference $\theta$ as the $B_0/n_0 = 500$ Hx case, except they are narrower and the oscillatory feature in the limit of zero phase difference between the fields is significantly reduced.

In figures 8.30 - 8.32 we show the temporal variation of the diagonal elements of the diffusion tensor with the phase difference $\theta$ for four different values of $B_0/n_0$ while in figure 8.33 the variation of the temporal profiles of the Hall diffusion coefficient is shown. In contrast to the $\mathbf{E} \times \mathbf{B}$ drift velocity component (e.g. $W_x$) the $\mathbf{E} \times \mathbf{B}$ diagonal element of the diffusion tensor
$D_{xx}$ does not significantly change its magnitude, particularly for low values of $B_0/n_0$, but the phase of the maximum is shifted towards smaller phases, as the phase difference $\theta$ approaches...
0 for all $B_0/n_0$ considered. Similar behavior is shown in the longitudinal diffusion coefficient $n_0D_{zz}$. It is interesting to note that almost a perfect linear dependence between the phases of the peaks and phase difference $\theta$ exists. This result is not predictable from the behavior of the drift velocity components and its origin is not well understood. In the limit of high $B_0/n_0$ and for all phase differences, strong oscillations are induced due to the cyclotron rotation of the electron swarms which leads to the negative diffusion in the temporal profiles of both $n_0D_{xx}$ and $n_0D_{zz}$. This phenomenon becomes more evident as the phase difference $\theta$ decreases. The modulation amplitude of $n_0D_{xx}$ and $n_0D_{zz}$ is essentially unaffected by the variation of the phase difference $\theta$ for all $B_0/n_0$ considered. The only exception is behavior of $n_0D_{zz}$ in the limit of the lowest $B_0/n_0$ of 100 Hx where $n_0D_{zz}$ peaks at zero phase difference between the fields.

Figure 8.32: The 3-dimensional plots of the $n_0D_{zz}$ as a function of the phase-difference $\theta$ between the fields for different $B_0/n_0$ for the Reid ramp model.

From figure 8.31 we see that the transverse diffusion coefficient $n_0D_{yy}$ is a monotonically decreasing function in a cycle-averaged sense of the phase difference $\theta$ for all $B_0/n_0$ considered. The modulation amplitude also decreases as the phase difference $\theta$ decreases and in the limit of zero phase difference between the fields the modulation amplitude is strongly reduced. This is purely an effect of the changing effective collision frequency and mean energy. Thus the basic trends shown in this figure are very similar to those shown for the mean energy (see figure 8.27). It is interesting to note that in the limit of high $B_0/n_0$, the shape of the temporal profiles becomes more symmetric and rounded as the phase difference $\theta$ increases.

The Hall diffusion coefficient exhibits a complex temporal variation with the phase difference $\theta$ (see figure 8.33.) Due to the complexity and interplay of various factors which influence the Hall diffusion coefficient it is hard to fully understand and elucidate even the basic trends in the
8.4.4 The effects of the field configuration on the electron transport properties in rf electric and magnetic fields

The primary aim of this section is to investigate the influence of varying the angle $\psi$ between the electric and magnetic fields on the electron transport properties. Two distinctively different scenarios are considered. First, we consider the electron transport under the influence of ac electric and magnetic fields and second, the electron transport properties under the influence of an ac electric field and dc magnetic field are investigated. Both of these scenarios are important from the viewpoint of applications in plasma processing. In both cases, we restrict our discussion and results to the influence of varying the angle between the electric and magnetic fields, and consider only a limited number of magnetic field strengths and fix the electric field. In this
section we present results only for $\psi$ between 0 and $\pi/2$ rad. Extension to other angles can be made through use of symmetry properties outlined by White et al. (1999a; 2002). The applied reduced angular frequency $\omega/n_0$ is set to $1 \times 10^{-14}$ rad m$^{-3}$ s$^{-1}$ while the electric and magnetic fields amplitude are $E_0/n_0 = 12$ Td and $B_0/n_0 = 1000$ Hx, respectively.

Figure 8.34: The 3-dimensional plots of the mean energy as a function of the angle between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

In figure 8.34 we show the variation of the temporal profiles of the mean energy with $\psi$ under the influence of ac electric and magnetic fields and under the influence of an ac electric field and dc magnetic field. In both cases, the mean energy monotonically decreases with $\psi$ which is a clear sign that a well known phenomenon of ‘magnetic cooling’ is directly carried over from dc electric and magnetic fields to combined ac electric and ac/dc magnetic fields. Comparison between the $\varepsilon$-temporal profiles in purely ac and in a combined ac/dc field reveals some quite general features. First, for parallel fields, there are no differences between the $\varepsilon$-temporal profiles. For parallel fields, on average the electrons are traveling in the direction of the electric and magnetic field and hence the magnetic field has no explicit effect regardless of its temporal dependence. However, as the angle between the fields increases, the cycle-averaged value of the mean energy is less for the combined ac/dc fields than that for pure ac fields. This behavior follows from the fact that in the combined ac electric and dc magnetic fields the magnetic field does not oscillate and remains unaltered during one cycle of the field. This means that magnetic field is large over all phases even when electric field peaks and electrons cannot gain much energy from the electric field.

Similar physical arguments should be used to explain the shape of the $\varepsilon$-temporal profiles and phase delay with respect to the applied electric field for an increasing $\psi$. For swarms in ac electric and magnetic fields, as $\psi$ increases the shape of the $\varepsilon$-temporal profiles becomes more triangular with a fast increase and slower decrease. The transition from sinusoidal to non-sinusoidal temporal profiles is clearly evident. In addition, the modulation amplitude is reduced. The complex time-dependence of the ratio between the cyclotron and collision frequencies may be associated with the asymmetry in the temporal profiles of the mean energy. At relatively low $B_0/n_0$, the peaks of the mean energy usually have enough time to relax before the field
is changed while the minimums do not have enough time to achieve the full relaxation. As a consequence, the profiles become asymmetric. In the asymptotic limit of strong $B_0/n_0$ in a crossed field configuration, we would observe ‘electron trapping’ whereby the energy of the swarm approaches its thermal value (Ness, 1994).

In combined ac electric and dc magnetic fields the synergism of the relaxation processes and explicit effects of the magnetic field affects the mean energy (and other transport properties) in a different manner. First, for an increasing $\psi$ keeping $B_0/n_0$ fixed, the shape of the $\varepsilon$-temporal profiles remains pretty much the same. In other words, the symmetry of the profiles is not affected by the variation of the angle between the fields. The profiles are simply shifted downwards with a small superimposed phase lag with respect to the applied electric field. While the absolute modulation amplitude (difference between the maximal and minimal value over all phases during one cycle of the field) is reduced, the relative modulation (absolute modulation over the mean value of the mean energy in a cycle-averaged sense) is essentially unaffected by the variation in $\psi$. As already remarked, the relaxation time for the mean energy is governed by $\tau_\varepsilon(t)$. By virtue of collision frequency increasing with energy for the Reid ramp model and time-dependence of the mean energy, the energy relaxation time for an increasing $B_0/n_0$ is increased. In other words, an ability to achieve the full relaxation of the energy is reduced as $\psi$ increases. Thus for an increasing $\psi$ the phase-lag of the $\varepsilon$ with respect to the applied electric field follows.

Figure 8.35: The 3-dimensional plots of the longitudinal drift velocity component $W_x$ as a function of the angle between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

Figures 8.35 - 8.37 display the variation of the temporal profiles of the drift velocity components with $\psi$ under the influence of ac electric and magnetic fields and under the influence of an ac electric field and dc magnetic field. As for the dc case, the drift velocity components $W_x$ and $W_y$ satisfy the following symmetry properties (White et al., 1999a): $W_x = W_y = 0$ for $\psi = 0^\circ$ and $W_y = 0$ for $\psi = 90^\circ$. The trends with $\psi$ in the temporal profiles of the drift velocity components are different between these two cases of the field configurations. In combined ac electric and dc magnetic field we see that for an increasing $\psi$ the modulation amplitude associated with the $W_x$, $W_y$ and $W_z$-temporal profiles monotonically increases, exhibits a maximal property
and monotonically decreases, respectively. In contrast to ac electric and magnetic fields, the $W_z$-temporal profiles are periodic oscillating at the field frequency. Further and in contrast to ac electric and magnetic fields, the $W_x$-temporal profiles are essentially symmetric and hence the macroscopic rms drift along the $E \times B$ does not exist. Note that for ac electric and magnetic fields the asymmetry in the $W_x$-temporal profiles is clearly evident and further enhanced for an increasing $\psi$. We also note a small phase delay with respect to the electric field in combined fields, a clear sign of an inability to achieve the full relaxation before the field is changed. The phase delay between the $W_z$-temporal profiles and electric field is much larger. This is indicative of different relaxation times associated with different drift velocity components for this particular field configuration. It is interesting to note that the temporal profiles of $W_y$ and/or $W_z$ on the one hand, and the temporal profile of $W_x$ on the other hand, are always out of phase. This illustrates how magnetic field redistributes the momentum from $y$ and/or $z$ to $x$ component.

![Figure 8.36](image1.png)  
**Figure 8.36:** The 3-dimensional plots of $W_y$ as a function of the angle between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

![Figure 8.37](image2.png)  
**Figure 8.37:** The 3-dimensional plots of $W_z$ as a function of the angle between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

In Figures 8.38 - 8.40 we show the variation of the temporal profiles of the diagonal elements
of the diffusion tensor with $\psi$ under the influence of ac electric and magnetic fields and under
the influence of an ac electric field and dc magnetic field. We observe that $n_0D_{xx}$ shows very
little sensitivity with respect to $\psi$ in ac electric and magnetic fields. For an increasing $\psi$ the
temporal profiles are pretty much the same. The modulation amplitude and the shape of the
temporal profiles are very little affected by the variation of $\psi$. For $n_0D_{xx}$ the negative excursion
occur in the temporal profiles for all field configurations. On the other hand, for an increasing
$\psi$ in combined ac electric and dc, $n_0D_{xx}$ monotonically decreases in a cycle averaged sense. The
modulation amplitude is significantly reduced while the sinusoidal shape of the temporal profiles
remain essentially unaffected by the variation of $\psi$. There are no imprinted oscillations in the
profiles and phase shifts with respect to the electric field.

Figure 8.38: The 3-dimensional plots of the diffusion coefficient $n_0D_{xx}$ as a function of the angle
between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

Temporal profiles of $n_0D_{yy}$ in ac electric and magnetic fields show a remarkable sensitivity
with respect to $\psi$. While the modulation amplitude is significantly reduced for an increasing
$\psi$, the cycle-averaged value is monotonically increased. In contrast to other diagonal elements
of the diffusion tensor, the oscillatory nature of the $n_0D_{yy}$-temporal profiles is reduced as the
angle between the fields is increased. In the limit of an orthogonal field configuration the $n_0D_{yy}$-
temporal profile is purely sinusoidal. In other words, there is a transition from non-sinusoidal to
sinusoidal temporal profiles as $\psi$ increases. This property can be fully understood by considering
the temporal relaxation profiles of $n_0D_{yy}$ in dc electric and magnetic fields (see Chapter 7). For
an orthogonal field configuration the Lorentz force does not act in this direction and hence there
are no imprinted oscillations on the diffusion coefficient in this direction. On the other hand, for
small angles between the fields, the electrons are under the action of Lorentz force producing the
oscillatory relaxation profiles. In the limit of small angles between the fields, the Lorentz force is
shown to induce the negative diffusivity. As can be observed from figure 8.38, this phenomenon
becomes most obvious for the parallel fields. In contrast to ac electric and magnetic fields,
the behavior of the $n_0D_{yy}$-temporal profiles in combined ac electric and dc magnetic fields is
entirely different. First, the cycle-averaged property displays a maximal property with $\psi$. One
of the most striking phenomena is an apparent inversion of the profiles as $\psi$ increases. There
are no imprinted oscillations in the profiles as $\psi$ increases and the shape of the temporal profiles remains sinusoidal for all field configurations.

Figure 8.39: The 3-dimensional plots of the diffusion coefficient $n_0D_{yy}$ as a function of the angle between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

In addition to the temporal profiles of $n_0D_{yy}$, the $n_0D_{zz}$-temporal profiles shows a significant sensitivity with respect to the angle between the fields. When increasing $\psi$ the cycle-averaged value is reduced while the modulation amplitude is increased. We also note that the oscillatory feature is enhanced and occasionally this diffusion coefficient becomes negative. In combined ac electric and dc magnetic fields, the variation of the $n_0D_{zz}$-temporal profiles with $\psi$ is quite striking, with an apparent decrease of the modulation amplitude. Likewise, we find the phase lag between the $n_0D_{zz}$-temporal profiles and electric field is increased with $\psi$. Our additional calculations at lower $B_0/n_0$ revealed an apparent inversion in the profiles.

Figure 8.40: The 3-dimensional plots of the diffusion coefficient $n_0D_{zz}$ as a function of the angle between the fields for ac magnetic field amplitude $B_0/n_0 = 1000$ Hx and dc magnetic field $B/n_0 = 1000$ Hx for the Reid ramp model.

Perhaps one of the most striking phenomena observed in the temporal profiles of the electron transport properties for swarms in combined ac electric and dc magnetic fields is the absence
of any signs of oscillations. First let us consider the development of oscillations in the temporal profiles of the electron transport properties in ac electric and magnetic fields. Generally speaking, in ac electric and magnetic fields the oscillations are imprints of the individual cyclotron motion on the collective averaged property under conditions when the cyclotron frequency exceeds the collision frequency. If the effects associated with the temporal non-locality of the magnetic field are neglected, then the oscillatory feature is usually enhanced for those phases of the field where magnetic field peaks and electric field goes through its zero value. By the virtue of constant orbital magnetic moment (Holt and Haskell, 1965), the radius of gyration is proportional to \((B/n_0)^{-1/2}\). It will therefore decrease as \(B/n_0\) increases. As a consequence, the amplitude of oscillations is reduced for an increasing \(B/n_0\). For lower \(B/n_0\), the collective ‘collisionless’ gyro-orbiting swarm behavior cannot manifest itself since on average the electrons cannot complete orbits before undergoing collisions. When the time-dependent electric field increases over certain phases of the field, the collision frequency is increased and collisions act to destroy the ‘collisionless’ behavior. As a consequence, the amplitude of oscillations is reduced and the decay of the oscillations then follows.

In combined ac electric and dc magnetic fields the radius of gyration is constant over all phases of the field. In addition, the cyclotron frequency is not a time-dependent quantity and it depends solely on a dc magnetic field strength. If the cyclotron frequency is greater than the collision frequency, the electrons gyrate about the magnetic field lines in a circle placed in \((E, E \times B)\) plane. Circular motion around a line of magnetic field force does not alter the magnitude of the electron velocity. In such a case the collective ‘collisionless’ gyro-orbiting swarm behavior cannot manifest itself since on average there are no variations in displacement and velocity of the electrons. Of course collisions always tend to destroy any signatures of the directed motion of electrons induced by the magnetic field. The absence of oscillations in the temporal profiles of the electron transport properties in combined ac electric and dc magnetic fields then follows.

### 8.4.5 The effects of non-conservative collisions on the electron transport properties in rf electric and magnetic fields

In this section we investigate the transport properties of an isolated swarm undergoing model ionization and attachment interactions with a series of gases under the influence of the electric and magnetic ac fields. The emphasis of this section is the observation and physical interpretation of the effects of non-conservative collisions on the instantaneous and cycle-averaged values of the electron transport properties in ac electric and magnetic fields.

#### 8.4.5.1 The ionization model of Lucas and Saelee

For the consideration of ionization processes we employ the benchmark model of Lucas and Saelee (6.2). The aim of using the ionization model of Lucas and Saelee is to isolate the explicit effects of ionization from effects introduced by various collisional processes present in real gases,
thus enabling us to illustrate the fundamental effects of the ionization processes on the tem-
poral profiles of the electron transport properties in ac electric and magnetic fields. It must be
emphasized that it is common in the literature on ac swarms to find ionization processes simply
treated as another inelastic process (see for example (Maeda and Makabe (1994a; 1994b); Goto
and Makabe (1990); Ferreira et al. (1991); Ferreira and Loureiro (1984; 1989))). These previous
publications are complemented by a comprehensive description of electron kinetics in ac electric
and magnetic field when ionization processes are present.

Figure 8.41: Temporal profiles of the mean energy as a function of the magnetic field amplitude
and phase difference between the fields for the ionization model of Lucas and Saelee. The
solid, dashed and dotted lines (black line: $B_0/n_0 = 0$ Hx; red line: $B_0/n_0 = 100$ Hx; green
line: $B_0/n_0 = 200$ Hx; blue line: $B_0/n_0 = 500$ Hx; pink line: $B_0/n_0 = 1000$ Hx) represent the
mean energy for $F = 0$, $F = 0.5$ and $F = 1$, respectively.

In figure 8.41 we demonstrate the influence of the ionization degree $F$ on the temporal
profiles of the mean energy for different magnetic field amplitudes and phase differences between
the fields for an applied reduced angular frequency $\omega/n_0 = 1 \times 10^{-16}$ rad m$^3$s$^{-1}$. From figure
8.42 it is clear that the phenomenon of ionization cooling of the swarm, observed in dc electric
field (Ness and Robson (1986); Nolan et al. (1997)) and dc electric and magnetic fields crossed
at arbitrary angle (see Chapter 6 of this thesis) is directly carried over to the ac electric
and magnetic fields. For the magnetic field-free case we note that increasing the ionization parameter
$F$ affects not only the magnitude but also the phase of the temporal profiles. We observe that
the phase lag between the mean energy and electric field decreases for an increasing ionization
degree $F$. As $B_0/n_0$ increases for a fixed phase difference between the fields, the phenomenon
of ionization cooling is reduced and in the limit of high $B_0/n_0$ it vanishes. This is a clear
sign that the Maxwellization of the high energy electrons significantly reduces the ionization degree. On the other hand, as the phase difference $\theta$ increases, the phenomenon of ionization cooling is further strengthened and becomes more obvious at lower $B_0/n_0$ and evident at higher $B_0/n_0$. This follows from the behavior of the ionization rate in ac electric and magnetic fields for this particular model. One would expect an increase in ionization rate as the phase difference between the fields increases. Figure 8.42 verifies this prediction.

![3-dimensional plot of the ionization rate as a function of the phase difference \(\theta\) for \(B_0/n_0\) of 200 Hx for the ionization model of Lucas and Saelee. The parameter \(F\) is set to 0.5.](image)

**Figure 8.42:** The 3-dimensional plot of the ionization rate as a function of the phase difference \(\theta\) for \(B_0/n_0\) of 200 Hx for the ionization model of Lucas and Saelee. The parameter \(F\) is set to 0.5.

In figure 8.43 we show the variation of the temporal profiles of the bulk and flux longitudinal drift velocity components with the ionization degree $F$ for four different phase differences between the fields $\theta$ and various magnetic field amplitudes $B_0/n_0$. For the ionization model of Lucas and Saelee, the implicit effect of ionization on the temporal profiles of $W_z$ is weak, and the flux components for both $F = 0.5$ and $F = 1$ are essentially equal to the $F = 0$ temporal profiles. As expected, for an increasing $F$ the differences between the bulk and flux values are enhanced. From figure 8.44 it is clear that there are no differences between the bulk and flux values for the phases where the ionization rate is weak independently of $B_0/n_0$ and/or $\theta$. We note that increasing the magnetic field amplitude $B_0/n_0$ for a fixed phase difference $\theta$ reduces the explicit effects of ionization and as a consequence the bulk coincide with the flux profiles. However as the phase difference $\theta$ increases the explicit effects associated with the ionization processes are enhanced and distinction between the bulk and flux values becomes more evident. This follows directly from the variation of the ionization rate with the phase difference $\theta$ shown in figure 8.42.

Among the many interesting phenomena observed in the $W_z$-temporal profiles, it is important to consider the phase lag between the bulk and flux components with respect to the electric
Figure 8.43: Temporal profiles of the bulk and flux longitudinal drift velocity components $W_z$ as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee. The solid, dashed and dotted lines (black line: $B_0/n_0 = 0$ Hx; red line: $B_0/n_0 = 100$ Hx; green line: $B_0/n_0 = 200$ Hx; blue line: $B_0/n_0 = 500$ Hx; pink line: $B_0/n_0 = 1000$ Hx) represent the flux $W_z$-temporal profiles, the bulk $W_z$-temporal profiles for $F = 0.5$ and the bulk $W_z$-temporal profiles for $F = 1$, respectively.

For the magnetic field free case one may observe different phase lags between the bulk and flux profiles with respect to the electric field. The phase lag of the ionization rate with respect to the electric field explains this property. The ionization rate peaks at the same phase as the bulk drift velocity component. On the other hand, there are no appreciable phase delays between the flux drift velocity and electric field. This is due to a large number of elastic collisions which favor the fast relaxation of the momentum. The application of the magnetic field introduces a new complexity associated with the explicit effects of the ionization processes on the longitudinal drift velocity component. We observe that for some phase difference $\theta$ and for an increasing $B_0/n_0$ the phase delay of the ionization rate with respect to the electric field increases. At the same time, the magnitude of the ionization rate is a monotonically decreasing function of $B_0/n_0$ independently of the phase difference $\theta$. However, the phase shift of the ionization rate is not the same for various phase differences $\theta$. As can be observed there are no phase shifts between the ionization rate and electric field in the limit of $\pi/2$ phase difference between the fields. As a consequence, the distribution of the bulk and flux values are identical over the cycle with respect to the electric field. Both drift velocity components peaks at the same phase which correspond to the phase where electric field goes through its maximum. The only difference is associated with the instantaneous values of the bulk and flux components which follows from the behavior
of the $z$-component of the energy gradient vector $\gamma_z$ (see figure 8.45). The phase shifting of the ionization rate with the phase difference $\theta$ can be understood by considering the temporal profiles of the mean energy. The phases of peak values in ionization rate are found to correspond to those of the mean energy, independently of the phase difference $\theta$ and/or $B_0/n_0$.

Figure 8.44: Temporal profiles of the flux drift velocity component $W_x$ as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee (red line: $B_0/n_0 = 100$ Hx; $B_0/n_0 = 200$ Hx; $B_0/n_0 = 500$ Hx; $B_0/n_0 = 1000$ Hx).

The variation of the temporal profiles of the transverse drift velocity component along the $E \times B$ direction $W_x$ with the phase difference $\theta$ and $B_0/n_0$ is displayed in figure 8.44. In contrast to $W_z$, there are no differences between the bulk and flux temporal profiles for all phase differences $\theta$ and $B_0/n_0$ considered. For clarity, only the flux $W_x$-temporal profiles are presented. The absence of the explicit effects of ionization on the $W_x$-temporal profiles suggests a very little spatial variation in the average energy along the $E \times B$ direction. On the other hand, the appreciable distinction between the bulk and flux $W_z$-temporal profiles indicates an increase in the average energy along the swarm in the $z$-direction. This is verified on figures 8.45 and 8.46, shown below. In addition, all phenomena observed in figure 8.44 including the asymmetry in the profiles can be qualitatively understood using similar arguments to those used in section 8.4.2 for the Reid ramp model.

Figures 8.45 and 8.46 show the temporal variation of the $z$ and $x$ components of the energy gradient vector with the ionization degree $F$ for four different phase differences between the fields $\theta$ and various magnetic field amplitudes $B_0/n_0$. Consideration of the temporal variation of the gradient energy vector components is of a key importance for physical understanding.
of the effects of non-conservative collisions on the electron transport coefficients. As remarked previously, there is very little spatial variation in the average energy along the $E \times B$ direction. On the other hand, the spatial variation in the average energy along the direction of electric field is much larger. From figure 8.45 we note that the ionization processes act to reduce the spatial variation in the average energy along the direction of electric field independently of $B_0/n_0$ and/or phase difference $\theta$. In addition to the inelastic nature of the ionization collisions, the phenomenon of ionization cooling also decreases the average energy along the same direction. For this model, ionization takes place predominantly at the front of the swarm and the reduction in the spatial variation in the average energy along the electric field direction follows. Therefore, as can be observed from figure 8.45, for an increasing ionization degree $F$, $\gamma_z$ is reduced. We also note that for an increasing ionization degree $F$, the shape of the temporal profiles is significantly affected.

The transition to spatial uniformity in the average energy along the electric field direction as $B_0/n_0$ increases is clearly evident. From figure 8.45 we observe that this phenomenon is strengthened for a decreasing phase difference $\theta$. In contrast to $\gamma_z$, the $x$-component of the energy gradient vector displays a maximal property with $B_0/n_0$ independently of the phase difference $\theta$ between the fields. We also note that for an increasing ionization degree $F$ the spatial variation in the average energy along the $E \times B$ direction is enhanced independently of $B_0/n_0$ and/or phase difference between the fields $\theta$.

Temporal variation of the flux longitudinal diffusion coefficient $n_0D_{zz}$ with the ionization
Figure 8.46: Temporal profiles of the $x$-component of the gradient energy vector as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee for the same conditions as in figure 8.42.

degree $F$ for four different phase differences between the fields $\theta$ and various magnetic field amplitudes $B_0/n_0$ is displayed in figure 8.47. The variation of the transverse diffusion coefficient along the magnetic field direction $n_0D_{yy}$ is shown in figure 8.48. In contrast to the drift velocity components, the implicit effects of the ionization processes on the diagonal elements of the diffusion tensor are quite strong. As a consequence, one may observe different profiles for different ionization degrees $F$. As expected, increasing $B_0/n_0$ for a fixed phase difference $\theta$ the implicit effects of ionization processes are reduced. In general, the flux diffusion coefficients essentially follow the same variation with the phase difference $\theta$ and with the ionization degree $F$ as that shown for the mean energy. This can be expected since large energies and velocities favor the rise of the diffusion coefficients. In particular, the shape of the $n_0D_{yy}$-temporal profiles is identical to those of the mean energy, a well-known fact observed previously for the Reid ramp model (Raspopović (1999); White et al. (2002)) and for some real gases Dujko (2004; 2005). The relaxation profiles of this transport quantity in a crossed dc electric and magnetic fields are monotonic and hence there are no imprinted oscillations in the $n_0D_{yy}$-temporal profiles in ac electric and magnetic fields.

The variation of the bulk and flux temporal profiles of the longitudinal $n_0D_{zz}$ and transverse $n_0D_{xx}$ diffusion coefficients with the ionization degree $F$ for four different phase differences between the fields $\theta$ and various magnetic field amplitudes $B_0/n_0$ is displayed in figures 8.49 and 8.50, respectively. For the magnetic field-free case, bulk longitudinal and bulk transverse diffusion coefficients are enhanced in phases where significant ionization occurs. The appearance
Figure 8.47: Temporal profiles of the flux longitudinal diffusion coefficient $n_0 D_{zz}$ as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee for the same conditions as in figure 8.42.

Figure 8.48: Temporal profiles of the flux transverse diffusion coefficient $n_0 D_{yy}$ as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee for the same conditions as in figure 8.42.
of a spike in the bulk longitudinal diffusion profiles is indicative of an inability of the transport property to relax in combination with a non-monotonically relaxing transport property (White et al., 1999b). The application of the magnetic field significantly affects the symmetry of both the bulk and flux components. We also note that increasing $B_0/n_0$ for a fixed phase difference $\theta$, the distinction between the bulk and flux components is reduced. On the other hand, as the phase difference $\theta$ increases, the explicit effects of ionization become more evident since the ionization rate monotonically increases with $\theta$. In general, the flux and bulk diffusion coefficients can vary substantially from one another not only in magnitude but also in the phase lags of the temporal profiles.

![Temporal profiles of the bulk and flux longitudinal diffusion coefficient $n_0D_{zz}$ as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee.](image)

In conclusion, the study of spatially inhomogeneous swarm transport coefficients in ac electric and magnetic fields remained entirely unexplored until now. It should be emphasized that the variations of the bulk diffusion coefficients are associated with not only first order spatial variation of the average energy (e.g. the energy gradient vector components) but also with a second order expansion of the average energy (see Eq. 2.124). This is beyond the scope of this thesis and in conclusion we note the following generic features observed in the bulk diffusion coefficients in ac electric and magnetic fields for the ionization model of Lucas and Saelee: (i) an enhancement in the cycle-averaged value as the ionization degree $F$ increases; (ii) the modulation amplitude of all profiles is remarkably reduced in the limit of high $B_0/n_0$; (iii) the
instantaneous bulk diffusion coefficients approach the instantaneous flux diffusion coefficients as $B_0/n_0$ is increased; (iv) the effects of ionization become more evident as phase difference between the fields increases. All in all, much remains to be understood with regards to the behavior of the diffusion coefficient in ac electric and magnetic fields when electron transport is greatly influenced by the ionization processes.

![Figure 8.50: Temporal profiles of the bulk and flux transverse diffusion coefficient $n_0D_{xx}$ as a function of the magnetic field amplitude and phase difference between the fields for the ionization model of Lucas and Saelee for the same conditions as in figure 8.49.](image)

8.4.5.2 The modified attachment model of Ness and Robson

For the consideration of attachment processes on the temporal profiles of the electron transport properties we employ the modified attachment model of Ness and Robson (6.4). The emphasis of this section is the observation and physical interpretation of the effects associated with electron attachment on the temporal profiles of transport coefficients. Following the dc studies in Chapter 6, we employ the following models: (i) the attachment cross section is directly proportional to the electron velocity ($p = 0.5$: the so-called ‘attachment cooling’ model); (ii) the attachment collision frequency is independent of energy ($p = -0.5$); and (iii) the attachment cross section is inversely proportional to the electron energy ($p = -1$: the so-called ‘attachment heating’ model). When the attachment collision frequency is independent of energy, all transport coefficients and transport properties were found to be independent of the attachment amplitude independently of $B_0/n_0$ and/or phase difference $\theta$. Likewise, the bulk transport coefficients were found to be equal to the flux coefficients. This supports the numerical integrity of the present code in the
presence of attachment processes. For clarity, we display the electron transport properties only for power laws \( p = 0.5 \) and \( p = -1 \).

![Figure 8.51: Temporal profiles of the mean energy as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson. The solid, dashed and dotted lines (black line: \( B_0/n_0 = 0 \) Hx; red line: \( B_0/n_0 = 100 \) Hx; green line: \( B_0/n_0 = 200 \) Hx; blue line: \( B_0/n_0 = 500 \) Hx; pink line: \( B_0/n_0 = 1000 \) Hx) represent the mean energy for conservative (no attachment) case, \( p = 0.5 \) and \( p = -1 \), respectively.]

In figures 8.51 - 8.57 we display the variation of the temporal profiles of the transport properties with the phase difference \( \theta \) and \( B_0/n_0 \) in the model attaching gas. For \( p = 0.5 \) the attachment collision frequency monotonically increases with energy. The attachment processes occur predominately at the front of the swarm and hence the higher energy electrons are preferentially consumed. This gives rise to the phenomenon of attachment cooling (i.e., the reduction in the mean energy due to attachment). On the other hand, for \( p = -1 \), the attachment collision frequency monotonically decreases with energy and predominant removal of the lower energy electrons result in an increase in the mean energy (i.e., the so-called ‘attachment heating’). For \( p = 0.5 \), as \( B_0/n_0 \) increases for a fixed phase difference between the fields, the attachment cooling phenomenon is significantly reduced. This is more obvious in the limit of small phase differences than in the limiting case where the fields are \( \pi/2 \) out of phase. For a decreasing phase difference \( \theta \) the ratio between the cyclotron and collision frequencies increases, the ability of the electric field to pump energy into the swarm is reduced and the attachment collision frequency reduces. This is clearly evident from figure 8.53 where the variation of the attachment rate with the phase difference \( \theta \) and \( B_0/n_0 \) is displayed.
Figure 8.52: Temporal profiles of the attachment rate as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson when the attachment cross section is directly proportional to the electron velocity ($p = 0.5$) (black line: $B_0/n_0 = 0$ Hx; red line: $B_0/n_0 = 100$ Hx; green line: $B_0/n_0 = 200$ Hx; blue line: $B_0/n_0 = 500$ Hx; pink line: $B_0/n_0 = 1000$ Hx).

For power law $p = -1$, the effects associated with the attachment processes are clearly evident in the temporal profiles of the mean energy even at the highest $B_0/n_0$ considered, independently of the phase difference $\theta$. In fact, the heating action is strengthened with increasing $B_0/n_0$ for a fixed phase difference $\theta$. This follows from the variation of the attachment rate for this particular model. In figure 8.52 we show the temporal variation of the attachment rate with the phase difference $\theta$ and $B_0/n_0$ for the attachment heating model. As can be seen, the attachment rate is a monotonically increasing function of $B_0/n_0$ for a fixed phase difference $\theta$ and monotonically decreasing function of $\theta$ for a fixed $B_0/n_0$.

When considering the amplitude of oscillations and phase lag with power law, the following interesting points can be observed. For the magnetic field-free case we note small variations in both the amplitude and phase lag with respect to power law due to the variation in the total energy transfer collision frequency with $p$. For power law $p = 0.5$ this feature is the most obvious when $B_0/n_0 = 0$ Hx, while for the attachment heating model the opposite situation holds: as $B_0/n_0$ increases the phase lag and amplitude of oscillations are increased.

The variation of the bulk and flux temporal profiles of the longitudinal drift velocity component is displayed in figure 8.54. For both power laws, over the range of $B_0/n_0$ and phase differences considered here, there are no variations in the flux temporal profiles with $p$, indicat-
Figure 8.53: Temporal profiles of the attachment rate as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson when the attachment cross section is inversely proportional to the electron energy \( p = -1 \) (black line: \( B_0/n_0 = 0 \) Hx; red line: \( B_0/n_0 = 100 \) Hx; green line: \( B_0/n_0 = 200 \) Hx; blue line: \( B_0/n_0 = 500 \) Hx; pink line: \( B_0/n_0 = 1000 \) Hx).

The implicit effect of attachment on the drift velocity is weak for this model. From figure 8.55 we observe that for attachment cooling \( (p = 0) \) the instantaneous bulk drift velocity has a lower magnitude as compared with the flux drift velocity component. The opposite situation holds for attachment heating. The origin of this behavior is well known in dc steady state systems under the influence of electric field only (Nolan et al., 1997) and electric and magnetic fields crossed at arbitrary angle (see Chapter 6 of this thesis), and is directly carried over to the ac electric and magnetic fields. In general, differences between the bulk and flux values are due to the increase in the average energy through the swarm in the direction of the drift which results in a preferential spatial attachment of electrons from the front/tail of the swarm for attachment cooling/heating, respectively (Ness and Robson (1986); (Nolan et al., 1997); White et al. (1999b)). This in turn reduces/enhances the bulk drift velocity component along the electric field direction. As \( B_0/n_0 \) increases for a fixed phase difference \( \theta \), aside from the effect on the magnitude of the drift velocity, the resulting effects on the phases of the profiles are clearly evident. For attachment cooling \( (p = 0) \) the phase shift of the bulk \( W_z \)-temporal profiles with respect to the flux profiles is reduced for an increasing \( B_0/n_0 \) (keeping phase difference \( \theta \) fixed). The converse applies for attachment heating. In such a case the phase shift of the bulk profiles is enhanced for an increasing \( B_0/n_0 \), keeping phase difference \( \theta \) fixed. The origin of this behavior lies in the spatial variation of the average energy and its variation with \( B_0/n_0 \) and/or \( \theta \).
Figure 8.54: Temporal variation of the bulk and flux longitudinal drift velocity component as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson. The solid, dashed and dotted lines (black line: $B_0/n_0 = 0$ Hx; red line: $B_0/n_0 = 100$ Hx; green line: $B_0/n_0 = 200$ Hx; blue line: $B_0/n_0 = 500$ Hx; pink line: $B_0/n_0 = 1000$ Hx) represent the flux $W_z$-temporal profiles, the bulk $W_z$-temporal profiles for $p = 0.5$ and the bulk $W_z$-temporal profiles for $p = -1$, respectively.

To conclude our discussion of attachment processes in ac electric and magnetic fields, we comment on the temporal profiles of the diagonal elements of the diffusion tensor. In figure 8.55 we show the variation of the flux temporal profiles of the longitudinal diffusion coefficient $n_0D_{zz}$ as a function of $B_0/n_0$ and phase difference $\theta$. In contrast to the longitudinal drift velocity component, the implicit effects of attachment on this transport property are quite strong. As a consequence, we observe different profiles for different models. We also note that there are appreciable differences in amplitude and phase of the profiles for different models. The phase difference $\theta$ between the fields can be used as an additional parameter to control these properties. As already pointed out for the ionization model of Lucas and Saelee, the origin of this behavior is associated with the variation of the mean energy. In general, for attachment cooling ($p = 0$) the implicit effects of attachment are reduced as $B_0/n_0$ increases while for attachment heating ($p = -1$) the implicit effects are not weaken, in fact they can be strengthened. Similar behavior was found for the transverse diffusion coefficients.

In figure 8.56 we show the variation of the bulk and flux transverse diffusion coefficient $n_0D_{xx}$ with $B_0/n_0$ and phase difference $\theta$ for attachment heating ($p = -1$). For this model, we note an enhancement in the cycle-averaged value of the bulk $n_0D_{xx}$-temporal profiles in comparison with the flux, independently of $B_0/n_0$ and/or phase difference $\theta$. A very specific feature of attachment
Figure 8.55: Temporal variation of the flux longitudinal diffusion coefficient $n_0D_{zz}$ as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson for the same conditions as those in figure 8.51.

Figure 8.56: Temporal variation of the bulk and flux transverse diffusion coefficient $n_0D_{xx}$ as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson. The solid and dashed lines (black line: $B_0/n_0 = 0$ Hx; red line: $B_0/n_0 = 100$ Hx; green line: $B_0/n_0 = 200$ Hx; blue line: $B_0/n_0 = 500$ Hx; pink line: $B_0/n_0 = 1000$ Hx) represent the flux profiles and the bulk profiles for $p = -1$, respectively.
Figure 8.57: Temporal variation of the bulk and flux transverse diffusion coefficient $n_0D_{yy}$ as a function of the magnetic field amplitude and phase difference between the fields for the modified attachment model of Ness and Robson. The solid and dashed lines (black line: $B_0/n_0 = 0 \text{ Hx}$; red line: $B_0/n_0 = 100 \text{ Hx}$; green line: $B_0/n_0 = 200 \text{ Hx}$; blue line: $B_0/n_0 = 500 \text{ Hx}$; pink line: $B_0/n_0 = 1000 \text{ Hx}$) represent the flux profiles and the bulk profiles for $p = 0.5$, respectively.

heating is that the instantaneous bulk diffusion coefficients do not approach the instantaneous flux diffusion coefficients as $B_0/n_0$ increases. In fact, the relative difference between the bulk and flux values is enlarged. This follows from the fact that the attachment rate coefficient monotonically increases with $B_0/n_0$ for a fixed phase difference $\theta$. On the other hand, as phase difference $\theta$ increases for a fixed $B_0/n_0$ we note an enhancement in the cycle-averaged value of the bulk profiles. The variation of the mean energy with phase difference $\theta$ should be used as a physical argument to understand this behavior of the bulk $n_0D_{xx}$-temporal profiles. As already remarked, large energies and velocities favor the rise of the diffusion coefficient.

In figure 8.57 we show the variation of the bulk and flux transverse diffusion coefficient $n_0D_{yy}$ with $B_0/n_0$ and phase difference $\theta$ for attachment cooling ($p = 0.5$). For this model, we note a reduction in the cycle-averaged value of the bulk $n_0D_{xx}$-temporal profiles in comparison with the flux, independently of $B_0/n_0$ and/or phase difference $\theta$. In general, the effects of attachment are reduced as $B_0/n_0$ increases keeping phase difference $\theta$ fixed and enhanced as phase difference $\theta$ increases keeping $B_0/n_0$ fixed. As a consequence and in contrast to attachment heating, the instantaneous bulk diffusion coefficients approach the instantaneous flux diffusion coefficients as $B_0/n_0$ increases.
In this section the collisional heating mechanism of the electrons in ac electric and magnetic fields in neutral gases is analyzed. Similar studies have been done for some realistic plasma devices (see for example Minea and Bretagne (2003) and reference therein for the rf magnetrons or Cunge et al. (2001) and reference therein for an ICP plasma discharge) under conditions of spatially and temporally non-uniform electric and magnetic fields in complex geometries without detailed qualitative and quantitative understanding. We believe that one of the most critical steps in plasma modeling is testing and verification of these models and interpretations against swarm type models and spatially uniform fields.

First, we consider the interaction of electrons with a spatially uniform oscillating electric field, \( E = E_0 \cos \omega t \), \( E_0 = \text{const} \). Let us assume that an electron moves without collisions, an assumption that is meaningful if the electron performs a large number of oscillations between two successive collisions. If we integrate the equations of collisionless motion

\[
\frac{dv}{dt} = -eE_0 \cos \omega t , \\
\frac{dr}{dt} = v ,
\]

then the following equations for the velocity and displacement follow:

\[
v = \frac{eE_0}{m\omega} \sin \omega t + v_0 , \\
r = \frac{eE_0}{m\omega^2} \cos \omega t + v_0 t + r_0 .
\]

From Eqs. (8.4) and (8.5) we see that an electron oscillates at the frequency of the field. The displacement is in the phase with the field while the velocity is out of phase by \( \pi/2 \). Thus in the limiting case of ‘collisionless’ oscillations, or in other words, if collisions do not occur, then the field does no work, on the average, on an electron. Eqs. (8.4) and (8.5) implies that

\[
\langle -eE \cdot v \rangle = -\frac{eE_0^2}{m\omega} \langle \cos \omega t \sin \omega t \rangle - eE_0 \cdot v_0 \langle \sin \omega t \rangle = 0 ,
\]

where angle brackets denote time averaging. Therefore, if collisions do not occur, the energy gained during one half of the field cycle is returned to the field in the other half of the cycle, and no energy can be transferred.

For power absorption to occur there must be some randomization mechanism that break the regularity of the electron motion and the \( \pi/2 \) phase shift between the velocity and electric field. At relatively high pressure, the required phase mixing is due to collisions with the neutral background gas. Collisions between the electrons and neutral molecules ‘throw off’ the phase, thereby disturbing the purely harmonic course of the electron’s oscillations.

The time-average power absorbed by the swarm (or plasma or any active medium), \( P_{\text{abs}} \), is given by (Liebermann and Lichtenberg, 1994):

\[
P_{\text{abs}} = \frac{1}{T} \int_0^T J(t) \cdot E(t) dt ,
\]

(8.7)
where $T = \frac{2\pi}{\omega}$ is the period and $J(t)$ is current density. For swarms or in the ‘cold plasma’ approximation the current density is proportional to the drift velocity. From Eq. (8.7) we see that in the time intervals when the current and electric field have the same sign, the instantaneously power is positive and the electric field pumps the energy into the system. Conversely, when the current and electric field have the opposite signs the instantaneously power is negative and the energy is transferred from an active medium to the external circuit. This suggests that a phase difference between the current density (or drift velocity for swarms) and electric field controls the power absorption of the electrons. This is illustrated schematically in figure 8.58.

Figure 8.58: Schematic diagrams of power absorption in the plasma: (a) no phase difference between the current density and electric field; (b) the phase difference of $2\pi/5$ between the current density and electric field.

Generally speaking, for swarms under the influence of an ac electric field in low-frequency regime, the effective relaxation times are sufficiently small over all phases of the field, that full relaxation applies and drift velocity follows the field in a quasi-stationary manner. In such a case, the time-averaged power absorbed by the swarm depends on the magnitude of the drift velocity and shape of the drift velocity temporal profile. As already remarked, the time-resolved negative differential conductivity in the temporal profiles of the drift velocity can enhance/reduce the overlap between the drift velocity and electric field. As a consequence, the power absorption by the swarm could be increased/reduced. On the other hand, for an increasing field frequency the phase difference between the drift velocity and electric field is increased and a reduction of the power absorption follows.

Let us consider now the electrons in ac electric and magnetic fields in the collision free case. For simplicity, we will consider the case in which the electric and magnetic fields are out of phase by $\pi/2$ in a crossed field configuration. In such a case, magnetic field rotates electrons which have the circular orbits in $(E, E \times B)$ plane and the motion of electrons has components at both the cyclotron frequency $\Omega$ and at the frequency of the fields $\omega$. The major characteristics of the orbits are dependent on the ratio $\Omega/\omega$. In particular, when $\Omega = \omega$ the
single electron move in circles of ever increasing radii. During this spiral motion the velocity of electron continually increases. Since its kinetic energy increases an electron absorbs energy from the ac field. This is the well-known cyclotron resonance effect - the resonant absorption of energy from a radio-frequency or microwave-frequency electromagnetic field by electrons in a uniform dc/rf magnetic field when the frequency of the electromagnetic field equals the cyclotron frequency of the electrons. In what follows, we will consider the implications of this phenomenon for swarms in rf electric and magnetic fields. For illustrative purposes we employ the Reid ramp model and CF₄.

Figure 8.59: The variation of the cycle-averaged mean energy with $B_0/n_0$ for the Reid ramp model for various (a) field frequencies and (b) phase differences between the fields.

In figure 8.59 (a) we show the cycle-averaged mean energy as a function of the magnetic field amplitude for various field frequencies for the Reid ramp model. The crossed electric and magnetic field are $\pi/2$ out of phase and have the following forms $E(t)/n_0 = 10\sqrt{2}\cos(2\pi ft)$ Td and $B(t)/n_0 = B_0/n_0 \sin(2\pi ft)$ Hx, respectively, where $f$ is the field frequency. The neutral gas number density is fixed to $3.54 \times 10^{22}$ m$^{-3}$. The cycle-averaged mean energy can be used as a ‘measure’ of the absorbed power by the swarm. The positions of the extremes in cycle-averaged mean energy are found to correspond to those of the power absorption. For the field frequency of 50 MHz, the cycle-averaged mean energy is a monotonically decreasing function of magnetic field. However, as the field frequency is increased, the oscillatory-type behavior is clearly evident. This indicates that under specific conditions the magnetic field can efficiently pump the energy into the system. Note that for swarms in dc electric and magnetic fields in a crossed field configuration, the mean energy always monotonically decreases with $B/n_0$ independently of the energy dependence of the cross sections (see figures 6.1 and 6.13). Figure 8.59 (b) displays the variation of the cycle-averaged mean energy with the magnetic field amplitude $B_0/n_0$ for different phase differences $\theta$ between the fields. For an increasing phase difference $\theta$, the cycle-averaged mean energy is increased. Thus the field frequency, magnetic field amplitude and phase
difference between the fields can be tuned to exploit/control the power absorption of the swarm.
In what follows we wish to give physical arguments to describe the resonant absorption of energy
for swarms in high frequency electric and magnetic fields.

In low-frequency regime when all transport properties have enough time to relax, the physical
mechanism of the magnetic cooling in dc electric and magnetic fields (White et al., 1999d)
is directly carried over to the ac fields. However, for an increasing field frequency, the phase-lag
between the drift velocity and electric field is enhanced. In other words, the number of electrons
traveling against the field is significantly increased. In such a case, if magnetic field is not too
strong, then the action of the component of the magnetic field perpendicular to the electric field
is to turn those electrons traveling against the electric field to travel with the electric field. This
indicates that the magnetic field rotates electrons in such a manner to reduce their inertia in
high frequency field. The signature of these effects is clearly evident in figure 8.58. For example,
when electric and magnetic fields are $\pi/2$ out of phase, the cycle-averaged mean energy mono-
tonically increases with $B_0/n_0$ until the period of gyration reaches one half of the period of the
field. For that particular value of $B_0/n_0$, the majority of electrons traveling against the electric
field are rotated to travel with electric field. As $B_0/n_0$ is further increased, the cycle-averaged
mean energy decreases and the minimum is reached when on average the electrons complete one
full gyration during each cycle of the field. As $B_0/n_0$ again increases, there is another region of
an apparent rise in the cycle-averaged mean energy until the new cyclotron resonance condition
is achieved. It should be noted that due to the temporal non-locality of the effects associated
with the magnetic field it is hard to elucidate the cyclotron resonance conditions. We also note
that as $B_0/n_0$ increases, the positions of the extremes are moved downwards and to the right,
since as $B_0/n_0$ increases the electrons on average may complete many gyrations per collision.
In the asymptotic limit of strong $B_0/n_0$, the reduction of the Larmor radius associated with an
increasing $B_0/n_0$ acts to reduce both the longitudinal and transverse drift velocity components,
because electrons are ‘held’ by the magnetic field lines. As a consequence, the cycle-averaged
mean energy monotonically decreases with $B_0/n_0$ towards thermal energy.

![Figure 8.60: The variation of the temporal profiles of the longitudinal drift velocity component
with $B_0/n_0$ for the Reid ramp model. The electric and magnetic fields are $\pi/2$ out of phase.](image)
With this behavior of the cycle-averaged mean energy as background, we may expect a
decrease of the phase delay of the longitudinal drift velocity component with respect to the
applied electric field while transverse component along the $E \times B$ should peak in phases where
electric field changes the sign. This prediction is verified in figure 8.60 where the variation of
the temporal profiles of the longitudinal drift velocity component with $B_0/n_0$ is displayed. Thus
from Eq. (8.7) an increase in the absorbed power over the range of magnetic field amplitudes
0-400 Hx then follows.

![Graph showing variation of cycle-averaged mean energy with $B_0/n_0$]

Figure 8.61: The variation of the temporal profiles of the cycle-averaged mean energy with
$B_0/n_0$ for CF$_4$. The electric and magnetic fields are $\pi/2$ out of phase.

In order to demonstrate this phenomenon for real gases, in figure 8.61 we show the variation
of the cycle-averaged mean energy with $B_0/n_0$ for three different phase differences between
the field for CF$_4$. The crossed electric and magnetic field have the following forms $E(t)/n_0 = \sqrt(2) \cos(2\pi ft)$ Td and $B(t)/n_0 = B_0/n_0 \sin(2\pi ft + \theta)$ Hx, respectively, where $\theta$ is the phase
difference between the fields while the field frequency $f$ is set to 500 MHz. The neutral gas
number density is fixed to $3.54 \times 10^{22}$ m$^{-3}$. We observe that the cycle-averaged mean energy
is greatly affected by the magnetic field. The first peak is for factor 6 greater than the cycle-
averaged mean energy for the magnetic field-free case. It must be emphasized that varying the
magnetic field amplitude will also affect other transport properties. In the context of plasma
maintenance, the variation of the ionization rate and/or rates for dissociation in neutral/excited
states is of particular note. One may expect that the positions of the extremes in the cycle-
averaged effective ionization rate would correspond to those of the cycle-averaged mean energy.
Thus an optimum plasma conditions at which the processes of ionization and dissociation are
the most efficient can be controlled by the variation of $B_0/n_0$. Of course these predications are
valid for plasmas in the ‘swarm’ limit.
8.4.7 The validity of the two-term approximation in rf electric and magnetic fields

In this section we investigate the accuracy of the two-term approximation for electrons in ac electric and magnetic fields undergoing both conservative and non-conservative collisions, taking as examples the Reid ramp model and CF₄. The former has become the standard test for both the Boltzmann equation analysis and Monte Carlo simulation technique, particularly due to its well-known failure for swarms under the influence of a dc electric field (Reid (1979); Ness and Robson (1986)) and for swarms under the influence of dc electric and magnetic fields (Ness (1994); White et al. (1997; 1999a)). The latter is particularly interesting, since it has large cross sections for vibrational excitations in low-energy region, and in general requires a distinctively multi-term analysis. In figures 8.62-8.64 we display the 3-dimensional plots of

![Figure 8.62](image)

Figure 8.62: The 3-dimensional plots of the percentage difference between the two-term and multi-term solution of the Boltzmann equation in \(n_0D_{xx}\) as a function of the phase difference \(\theta\) between the fields for two applied electric field amplitudes of 12 and 24 Td for the Reid ramp model.

![Figure 8.63](image)

Figure 8.63: The 3-dimensional plots of the percentage difference between the two-term and multi-term solution of the Boltzmann equation in \(n_0D_{yy}\) as a function of the phase difference \(\theta\) between the fields for two applied electric field amplitudes of 12 and 24 Td for the Reid ramp model.
the percentage deviations between the two-term and fully converged multi-term results for the diagonal elements of the diffusion tensor over a range of the phase differences between the fields. The magnetic field amplitude is set to $B_0/n_0 = 200$ Hz and we consider two different values of the electric field amplitudes of 12 and 24 Td. The applied reduced angular frequency $\omega/n_0$ is set to $1 \times 10^{-15}$ rad m$^3$s$^{-1}$ and we consider an orthogonal field configuration. The inadequacy of the two-term approximation for all diagonal elements of the diffusion tensor is clearly evident. As for an ac electric field only, the two-term approximation predicts the temporal profiles that are wrong in magnitude and out of phase with respect to those obtained by a multi-term theory. For a chosen set of conditions the errors are of the order of 40%. The mean energy and drift velocity components have the errors of the order of 10% (not shown). From figures 8.62-8.64 we observe that increasing $E_0/n_0$ deteriorates the accuracy of the two-term approximation. Intuitively we expect this, since for an increasing $E_0/n_0$ and fixed $B_0/n_0$ the inelastic collisions start to play more significant role for this model and an enhancement in the asymmetry of the electron velocity distribution function in velocity space follows. In addition to the electric field strength, the phase difference between the fields affects the accuracy of the two-term approximation. In general, for a decreasing phase difference between the fields the two-term approximation increases in accuracy. We also note that the largest deviations between the two-term and multi-term results occur for those phases of the field where electric field reaches its maximal value. Interestingly, the two-term approximation generally tend to underestimate the converged multi-term results for $n_0D_{yy}$ for all phases of the field independently of the phase difference between the field and/or electric field amplitude $E_0/n_0$.

![Figure 8.64: The 3-dimensional plots of the percentage difference between the two-term and multi-term solution of the Boltzmann equation in $n_0D_{zz}$ as a function of the phase difference $\theta$ between the fields for two applied electric field amplitudes of 12 and 24 Td for the Reid ramp model.](image)

Figures 8.65-8.67 display the comparison between the two-term and fully converged multi-term results for the diagonal elements of the diffusion tensor for CF$_4$. We consider the reduced magnetic field amplitudes $B_0/n_0$ of 0, 100, 500 and 1000 Hz, the electric field amplitude $E_0/n_0$ is set to 100 Td and the field frequencies are 10, 20, 100 and 200 MHz at the gas pressure of 1 Torr. There exist some interesting phenomena in these plots but they are peripheral to the focus
of the present section and we defer the detailed consideration of these phenomena to a future work. Immediately we see that the two-term approximation can be in serious errors for CF<sub>4</sub> since it predicts the significantly different shapes of the temporal profiles. Obtaining the basic trends associated with the accuracy of the two-term approximation with \( B_0/n_0 \) and/or applied field frequency is a laborious task due to the complex energy dependence of the cross sections for inelastic collisions. For example, on the one hand, it is clear that for an increasing \( B_0/n_0 \) the two-term approximation increases in accuracy for \( n_0D_{zz} \) and/or \( n_0D_{xx} \) for an increasing field frequency. On the other hand, this is not true for \( n_0D_{yy} \). This is a clear sign that a detailed consideration of the spatially inhomogeneous velocity distribution function components in ac electric and magnetic fields is inevitable to fully understand the origin of errors associated with the two-term approximation for swarms in CF<sub>4</sub>.

Figure 8.65: Comparison between the temporal profiles of \( n_0D_{xx} \) obtained by the two-term approximation (dashed lines) and multi-term theory (full lines). Temporal profiles are given as a function of \( B_0/n_0 \) (black line: 0 Hx; red line: 100 Hx; green line: 200 Hx; blue line: 500 Hx; pink line: 1000 Hx) and field frequencies ((a) 10 MHz; (b) 20 MHz; (c) 100 MHz; (d) 200 MHz).

Along similar lines we also note that the two-term approximation sometimes gets even the basic physics wrong. As an illustrative example, one may observe that for \( B_0/n_0 = 1000 \text{ Hx} \) and frequency \( f = 200 \text{ MHz} \) the two-term approximation predicts the negative excursion in the \( n_0D_{xx} \)-temporal profile. It is clear from corresponding multi-term results that this is wrong. Another example is anomalous behavior of the longitudinal diffusion coefficient for \( B_0/n_0 = 200 \text{ Hx} \) at 20 MHz. While the multi-term theory provides both aspects of anomalous behavior, e.g. the longitudinal diffusion peaks when electric field change the sign and the magnitude of \( n_0D_{zz} \) exceeds the magnitude of both transverse diffusion coefficients, the two-term approximation fails to predict all aspects associated with the anomaly. These examples clearly show that the inadequacies of the two-term approximation can plague the swarm studies in ac electric and
magnetic fields. All in all, for the accurate calculation of transport properties in molecular gases under the influence of ac electric and magnetic fields it appears that a multi-term theory and/or Monte Carlo simulation is inevitable.

Figure 8.66: Comparison between the temporal profiles of $n_0 D_{yy}$ obtained by the two-term approximation (dashed lines) and multi-term theory (full lines) for the same conditions as in Fig. 8.64.

Figure 8.67: Comparison between the temporal profiles of $n_0 D_{zz}$ obtained by the two-term approximation (dashed lines) and multi-term theory (full lines) for the same conditions as in Fig. 8.64.