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Chapter 7

Temporal relaxation of electron swarm in dc electric and magnetic fields crossed at arbitrary angle

7.1 Introduction

In this chapter, the hydrodynamic transport theory and Monte Carlo simulation code discussed in previous chapters are applied to investigate the temporal relaxation of electron swarms under the influence of dc electric and magnetic fields crossed at arbitrary angle in neutral gases. This work represents the first full treatment of the effect of non-conservative processes on the temporal relaxation of electron swarms in dc electric and magnetic fields crossed at arbitrary angle. In the previous chapter the steady-state results for electron transport parameters for certain model gases were presented. In this chapter we are entirely focused on transient behavior. Similar study has been recently published by Loffhagen and Winkler (1999). They employed the two-term approximation for solving the Boltzmann equation to consider the explicit effect of a magnetic field on the relaxation of electron swarms in atomic gases, focusing on spatially homogeneous (or spatially averaged) transport properties only (e.g. drift velocity or mean energy). The work presented in this thesis and in this chapter represents an extension of their earlier work by (i) considering transport properties of spatially inhomogeneous electron swarms (in particular diffusion tensor coefficients); (ii) considering the explicit effects of non-conservative collisions on the temporal relaxation process of electron swarms and (iii) using a multi-term solution of Boltzmann’s equation with the goal of overcoming the inherent inaccuracies associated with the two-term approximation. Our preliminary results which describe some quite general properties of the relaxation process of a swarm of charged particles in dc electric and magnetic fields have been recently presented by White et al. (2006) and Dujko et al. (2008a).

To investigate the temporal relaxation process under hydrodynamic conditions, we use certain model and real gases. The conservative inelastic Reid ramp model (Reid (1979)) is used to discuss the basic characteristics of the temporal relaxation process using physical arguments.
Some of the temporal profiles for electron transport coefficients obtained by a multi-term theory for solving the Boltzmann equation are tested using a fully kinetic MC simulation code. The excellent agreement between these two independent techniques validates both the basis of transport theory and numerical integrity of the codes used in this thesis. It must be emphasized that these comparisons may be viewed as another benchmark model for other plasma codes (both fluid and hybrid) in the limit of low electron density. Following the previous practice of many workers under steady-state conditions, we display the inadequacies of the two-term approximations for the Reid ramp model. The relaxation profiles obtained by the two-term approximations and multi-term theory for solving the Boltzmann equation are compared to each other during the early, intermediate and final stage of the relaxation process for various transport properties.

Through the use of the ionization model of Lucas and Saelee (Lucas and Saelee, 1975) and modified attachment model of Ness and Robson (1986) the effects of ionization and attachment respectively, on the temporal relaxation profiles of the transport properties are considered in detail. The relaxation process of both the bulk and flux data is monitored as a function of electric and magnetic field strengths and angle between the fields. Finally, in the last section of this chapter we consider the temporal relaxation of electrons in CO$_2$ and its mixture with noble gases. We do not propose this section to be a comprehensive investigation of the temporal relaxation in CO$_2$, but rather aim to demonstrate the applicability of the present theory and associated numerical code to a real system and to discuss some of the interesting relaxation phenomena. In particular, the so-called transient negative electron diffusivity, a phenomenon recently observed for the Reid ramp model in a crossed field configuration (White et al., 2008) has been observed for CO$_2$ in electric and magnetic fields crossed at arbitrary angles.

In order to analyze the temporal relaxation process and to find its characteristics parameters, like the relevant relaxation time, the following relaxation model is employed. The initial conditions represent the steady state magnetic free case where the electron swarm is acted on solely by a dc electric field. At time $t = 0$, magnetic field is switched on while the electric field is left to be unaltered. The relaxation properties of the electron swarm is monitored as a function of normalized time ($Nt$). The relaxation process is followed until the steady-state is reached. When considering the parallel effects of the electric and magnetic field strengths, the relaxation model is slightly modified. In such a case, at time $t = 0$, magnetic field is switched on while the electric field is increased.

### 7.2 Inelastic system: Reid ramp model

The Reid ramp model has been investigated by many workers under steady-state conditions for both the orthogonal (Ness (1994); White et al. (1997); Raspopović et al. (1999); Petrović et al. (2002)) and arbitrary field configuration (White et al. (1999)). The details of the model are given in the previous chapter (see Eq. (6.1)). Using a multi-term theory for solving the Boltzmann equation and Monte Carlo simulation technique, the temporal profiles of the transport coefficients are calculated. As already emphasized, the relaxation of electrons always starts
from the same initial conditions. These conditions represent the steady state magnetic field free case where the electron swarm is acted on solely by a dc electric field at $E/n_0 = 12$ Td. The relaxation properties of the swarm are monitored as a function of normalized time ($Nt$), magnetic field strengths ($B/n_0$) and angle ($\phi$) between the fields.

To understand the main aspects of the temporal relaxation of electron swarm in electric and magnetic fields, the characteristic timescales for momentum ($\tau_m$) and energy ($\tau_e$) relaxation must be compared with the gyration period ($\tau = \Omega^{-1}$, where $\Omega$ is the gyro frequency). These characteristic timescales for momentum and energy relaxation can be found from the corresponding momentum ($\nu_m$) and energy ($\nu_e$) dissipation frequencies:

$$\nu_m(\epsilon) = \sqrt{\frac{2}{m_e}} \epsilon^{1/2} \left( n_0 Q_m(\epsilon) + \sum_l n_0 Q_l^{tot}(\epsilon) \right),$$  \hspace{1cm} (7.1)

$$\nu_e(\epsilon) = \sqrt{\frac{2}{m_e}} \epsilon^{1/2} \left( \frac{2m_e}{M} n_0 Q_m(\epsilon) + \sum_l n_0 Q_l^{tot}(\epsilon) \right),$$  \hspace{1cm} (7.2)

where $m_e$, $M$, $\epsilon$, $n_0$, $Q_m$, $Q_l^{tot}$ denote the electron mass, the mass of neutral molecule, the electron energy, the gas number density, the momentum transfer cross section for elastic collisions and the total inelastic cross section. It is clearly evident from the definition of the dissipation frequencies that these characteristic timescales will vary by virtue of the variation of the swarm’s energy distribution. The characteristic timescales are shown in figure 7.1. We observe that the momentum dissipation occurs much faster than the energy dissipation particularly in the energy region where only elastic collisions occur. When inelastic collisions start to play a significant role the efficiency of energy dissipation becomes more pronounced. The application of magnetic field generally decreases the mean energy of the swarm and hence both $\tau_m$ and $\tau_e$ increase as a result. Figure 7.1 makes obvious (particularly when magnetic field is present) that the energy dependencies of $\tau_m$ and $\tau_e$ have to be taken into account in detail and cannot be replaced by some constant effective quantities. In general, these characteristic timescales are clearly evident in the relaxation profiles shown below.

In order to illustrate the effects of magnetic field on the relaxation process of the electron transport properties and having in mind that the impact of magnetic field is the most intensive for perpendicular fields, the first step in our study assumes a crossed field configuration. This choice also corresponds to the main experimental arrangements. In figure 7.2 we show the temporal relaxation profiles of the electron transport parameters for the Reid ramp model for various $B/n_0$ in a crossed field configuration. The results obtained by a multi-term theory for solving the Boltzmann equation are compared with those obtained by a Monte Carlo simulation technique. The excellent agreement between these two independent methods validates the basis of transport theory as well as numerical integrity of both the Boltzmann and Monte Carlo codes. The various transport properties display profiles that are either monotonic relaxation or damped period relaxation. For quantities like the $\epsilon$ and $n_0 D_{yy}$, relaxation is in general always monotonic and occurs on the timescale governed by $\tau_e$. In contrast, the relaxation profiles of the drift and diffusion along $E$ and $E \times B$ directions exhibit a damped oscillatory relaxation along a decaying profile for the $B/n_0$ considered. The period of oscillation is governed by the gyro-period $\tau$ while
Figure 7.1: Comparison of the reduced cyclotron period for various $B/n_0$ (solid horizontal lines) with the reduced timescales for momentum and energy relaxation as a function of energy.

the timescale for the envelope of the damped behavior is determined by $\tau_m$. The timescale for the long term relaxation is governed by $\tau_e$. The existence of the additional oscillatory behavior in the relaxation profiles is an imprint of the collective gyrations of the electrons damped by collisions that exchange momentum and energy.

Let us consider firstly the relaxation of the mean energy and drift velocity components along the electric field and direction perpendicular to the electric and magnetic fields. In the early stage of the relaxation process we observe that the mean energy is only slightly affected by the action of the magnetic field while the drift velocity components instantaneously respond to the application of magnetic field. During this initial period of the relaxation process, the magnetic field causes a remarkable decrease in $W_z$ and thus the power input from the electric field is significantly affected as well. Less effective heating of the swarm leads to the depopulation of the high-energy electrons from the distribution due to the losses in inelastic collision processes. As already remarked in previous chapter, magnetic field orthogonal with respect to the electric field rotates the swarm from the $-E$ direction towards the $E \times B$ direction. Since the $E \times B$ component of the drift velocity does not exist prior to the application of a magnetic field, the change in this drift velocity component and its temporal evolution are clearly evident when the magnetic field is switched on. On the other hand, the change in $W_z$ is much less pronounced due to the fact that the electrons already have this drift velocity component before the application of a magnetic field. Further and in contrast to the drift velocity components, magnetic field cannot directly alter the swarm energy in the early stage of relaxation. This is true only when the condition $\tau < \tau_m$ is fulfilled. The magnetic field alters the mean energy indirectly through the action of the magnetic force $\mathbf{e} \mathbf{c} \times \mathbf{B}$, by turning electrons against or with the electric field force. For $B/n_0$ of 100 Hx, both drift velocity components display monotonic
relaxation profiles while for $B/n_0$ of 500 or 1000 Hx the relaxation profiles exhibit oscillatory behavior. This can be explained using the following physical arguments. When magnetic field is switched on, the condition $\tau < \tau_m$ is satisfied initially and the electrons on the average undergo gyrations about the magnetic field before they undergo collisions. For high $B/n_0$ the electrons may complete on average several cyclotron orbits between collisions and hence both drift velocity components can take both positive and negative values. In contrast, for lower $B/n_0$ the collective ‘collisionless’ gyro-orbiting swarm behavior cannot manifest itself since on average the electrons cannot complete orbits before undergoing collisions. During the early stage of the relaxation process, magnetic field induces a transfer of momentum from $W_z$ to $W_x$. This leads to the observed decrease of $W_z$ and the corresponding increase of $W_x$. As the relaxation process proceeds further, the oscillatory behavior is significantly reduced and both drift velocity components show slower and minor changes. After a suitable time, drift velocity components are decreased towards the steady state values as a consequence of momentum equilibration. In other words, directions of both drift velocity components are randomized owing to elastic collisions.

Figure 7.2: Temporal relaxation of the mean energy, drift velocity components and diagonal elements of the diffusion tensor for various applied magnetic fields in a crossed field configuration for Reid ramp model (solid black lines: Multi term Boltzmann code; dashed colored lines: Monte Carlo code)
In contrast, relaxation of the mean energy is much slower. The elastic energy loss, $(2m/M)\epsilon$ of the electron with energy $\epsilon$ is negligibly small and hence the relaxation time of the mean energy is predominantly determined by the dissipation frequency for inelastic collisions. In about the first 1 ns of the relaxation process, the mean energy is not affected by the action of magnetic field. As the relaxation process proceeds in time, however, the depopulation of the high-energy electrons results in a decrease of the mean energy.

Let us consider now the temporal relaxation of the diagonal elements of the diffusion tensor. From figure 7.2 we observe that $n_0D_{xx}$ and $n_0D_{zz}$ exhibit a damped oscillatory relaxation along a decaying profile for the $B/n_0$ considered. We note that the time for $n_0D_{xx}$ and $n_0D_{zz}$ to respond to the magnetic field appears to decrease monotonically with increasing $B/n_0$. In addition, as $B/n_0$ increases, the oscillatory feature in the profiles of these diffusion coefficients becomes more apparent. The amplitude of oscillatory feature is more pronounced for the diffusion coefficient along $E \times B$ direction ($n_0D_{xx}$) than that along $E$ direction ($n_0D_{zz}$). Most importantly, however, once the magnetic field reaches a certain threshold, the profiles of both $n_0D_{xx}$ and $n_0D_{zz}$ become transiently negative! For both the 500 Hx and 1000 Hx profiles, $n_0D_{xx}$ pass through zero and have the peak negative excursion occurring at approximately $\tau/2$. The threshold for $B/n_0$ where $n_0D_{zz}$ becomes transiently negative is greater than that of $n_0D_{xx}$. In addition, temporal relaxation for $n_0D_{zz}$ is more complex during the first few periods of relaxation.

Unlike $n_0D_{xx}$ and $n_0D_{zz}$, the profiles of $n_0D_{yy}$ do not exhibit the damped oscillatory type relaxation nature. This is due to the fact that the Lorentz force does not act explicitly in this direction for the crossed field configuration. In addition the timescale for response to the application of the magnetic field is longer than that of $n_0D_{xx}$ and $n_0D_{zz}$. Interestingly, however, and in contrast to $n_0D_{xx}$ and $n_0D_{zz}$, we note for this model that the initial response of $n_0D_{yy}$ to the application of an orthogonal field is to increase its value, leading to a transient peak structure dictated by the energy relaxation process and the nature of the cross-sections. In the long-time limit, i.e. the steady-state, an increase in $n_0D_{yy}$ is observed for low fields such as 100 Hx. For this model, the time to the maximum in the peak structure appears to decrease for increasing $B/n_0$.

7.2.1 Transient negative electron diffusivity

In this section we discuss the phenomenon of transient negative diffusivity using physical arguments. First let us consider diffusion in the $E \times B$-direction. If we consider the case of the highest $B/n_0$ shown, then when the magnetic field is switched on the condition $\tau < \tau_m$ is satisfied initially and the electrons on the average undergo gyrations about the $B$ field before they undergo collisions. Since the initial velocity distribution has rotational symmetry about the $E$-direction, the distribution of velocities in the positive and negative $E \times B$-directions are equal. This is clearly evident from the figure 7.3 where the temporal relaxation profiles of the diagonal temperature tensor elements are shown. If we consider pairing off all swarm particles which have $E \times B$ velocities that are equal in magnitude and opposite in sign, then it can be shown that the displacement in the $E \times B$-direction between these particles increases for the first
quarter-period of gyration and then decreases for the next half-period, thus oscillating in time. Hence, the rate of change of displacement between these particles in the $E \times B$-direction also oscillates in time. That is, we have the situation where collectively the electrons on the average are approaching each other in the $E \times B$-direction and hence the swarm’s diffusion coefficient in that direction becomes transiently negative. Of course for times greater than a few $\tau_m$, collisions begin to destroy this signature ‘collisionless’ behavior and the decay of the oscillations then follows. For lower $B/n_0$ the collective ‘collisionless’ gyro-orbiting swarm behavior cannot manifest itself since on average the electrons cannot complete orbits before undergoing collisions. Having said all this, we need to note that the effects of both fields are for most conditions only small perturbations on the otherwise chaotic particle trajectories. To support this we may note that the drift velocities are considerably smaller than the thermal velocities. Nevertheless, while the gyroscopic motion may be difficult to observe if one were to look at all trajectories, its imprint on the ensemble is distinguishable.

![Figure 7.3: Temporal relaxation of the diagonal elements of the temperature tensor for various applied magnetic fields in a crossed field configuration for Reid ramp model.](image)

Now let us consider diffusion in the $E$-direction. We can follow similar arguments to those used to describe diffusion in the $E \times B$-direction though they must be modified by virtue of the initial condition. At $n_0t = 0$, we have an anisotropic velocity distribution (i.e. the velocity distribution in the $E$-direction differs from that in the perpendicular directions) which is displaced in the $-E$-direction (figure 7.3 illustrates this point). There are more electrons traveling in the $-E$-field direction than those traveling against it. If we follow similar arguments used for the $E \times B$-direction we can pair off some (but not all) electrons with velocities in the $-E$-direction that are equal in magnitude but opposite in sign. We can monitor the displacement between each pair and again observe that this displacement oscillates in time. In contrast to the $E \times B$-direction, however, all electrons cannot be paired off in this case and the perturbations
to the classical damped oscillatory profiles then follow. For the highest $B/n_0$ considered, the condition $\tau < \tau_m$ is satisfied initially and this collisionless behavior can manifest itself.

7.2.2 The effect of angle between the fields on the temporal relaxation profiles

In this section we consider the influence of the angle between electric and magnetic fields on the temporal relaxation of electron swarm using a multi-term theory for solving the Boltzmann equation. In figure 7.4 we demonstrate the impact of angle between the fields on the mean energy, drift speed and $z$-component of the gradient energy vector. Similarly, in figure 7.5 we show the temporal evolution of the drift velocity components while figure 7.6 displays the temporal evolution of the diagonal elements of the diffusion tensor. The profiles are presented for $B/n_0$ of 100 Hx when the collisions dominate the electron transport and for $B/n_0$ of 1000 Hx when magnetic field controls the motion of the swarm.

![Figure 7.4](image)

From figures 7.4 - 7.6 we observe a remarkable change in the temporal relaxation profiles induced by the variation of the angle between the fields. From figure 7.4 we observe that the relaxation of the mean energy is always monotonic independently of the angle between the fields or $B/n_0$. Since deflection of the drift velocity from the electric field direction increases as $\psi$...
Figure 7.5: Temporal relaxation of the drift velocity components for the same conditions as in figure 7.4.

Figure 7.6: Temporal relaxation of the diagonal elements of the diffusion tensor for the same conditions as in figure 7.4.
is increased, the power dissipation in inelastic collisions of electrons with neutral gas molecules is much less effective causing a slower establishment of steady-state. Thus, as $\psi$ increases the important increase of the relaxation time with respect to the establishment of steady-state can be observed. This physical picture cannot be applied for the drift speed. The profiles show monotonic relaxation for $B/n_0$ of 100 Hx while for $B/n_0$ of 1000 Hx the profiles exhibit a damped oscillatory relaxation. We observe that the amplitude of oscillations increase with rising $\psi$. It is interesting to note that the positions of the extremes in drift speed do not change significantly for an increasing $\psi$. Another unusual result is the fact that drift speed reaches the steady-state in the limit of a crossed field configuration faster than for the non-orthogonal field configurations for $B/n_0$ of 1000 Hx. The origin of faster momentum relaxation for an orthogonal field configuration follows from an increase in the number of elastic collisions. The calculated rate coefficients for elastic collisions support this physical picture.

Consideration of profiles associated with different drift velocity components in linear scale reveals that all drift velocity components relax to a steady-state with nearly the same time constant. There are small deviations between the relaxation times associated with the different drift velocity components and these mutual deviations are within 10 ns. Similar result has been found by Loffhagen and Winkler (1999). They have noticed that the period of momentum relaxation to a quasi-stationary state is about the same for the $z$- and $x$-component of the momentum and is nearly independent of $B/n_0$ in a crossed field configuration. However, it must be emphasized that their observation has been made for electrons in neon under conditions where elastic collisions dominate the inelastic collisions. However, it is more important to note that for a weak $B/n_0$ all drift velocity components show the monotonic relaxation while for a high $B/n_0$ the profiles exhibit a damped oscillatory relaxation. If we take a careful look at figure 7.5 we observe that the positions of the zeros and extremes in the profiles of various drift velocity components on the scale of normalized time can be linked to each other. This is a clear sign of the momentum redistribution between different drift velocity components in the early stage of the relaxation process. The variation of the angle between the fields does not alter the positions of the extremes in the profiles of the drift velocity components. For an increasing $\psi$ the oscillatory feature in the relaxation profiles becomes more obvious. While $W_x$ and $W_z$ oscillate and assume both negative and positive values, $W_y$ can take only positive values. Another striking phenomenon is that $W_y$ is comparable in magnitude to $W_x$ and $W_z$ but yet it is common in the literature for plasma modelers to fail to include this quantity in their models. An extension of standard plasma models to include the $(E \times B) \times B$ effects on electron transport may led to a better understanding of the power transfer to magnetically assisted/enhanced plasma reactors.

In figure 7.6 we show the temporal evolution of the diagonal elements of the diffusion tensor for various $\psi$ and $B/n_0$ of 100 and 1000 Hx. Again we observe a remarkable change in the relaxation profiles induced by the variation of angle between the fields. Quite generally, the relaxation profiles for all diagonal elements of the diffusion tensor show significant sensitivity with respect to angle between the fields. We observe that the oscillatory nature of the relaxation process for $n_0D_{xx}$ and $n_0D_{zz}$ is enhanced as the angle between the fields increases. In particular, for parallel fields ($\psi = 0^\circ$) the diffusion coefficient along the electric field $n_0D_{zz}$ is, as expected
Figure 7.7: Temporal relaxation of $n_0D_{xx}$ for the same conditions as in figure 7.4.

by symmetry, not altered by the presence of magnetic field and is equal to the magnetic field-free case value. However, as the angle between the fields increases for $B/n_0$ of 100 Hx, we may observe development of the additional oscillatory behavior in the relaxation profiles of $n_0D_{zz}$ damped by collisions that exchange momentum and energy. In the limit of high $B/n_0$ of 1000 Hx, these oscillations lead to the existence of transiently negative diffusivity. As can be seen from figure 7.6, $n_0D_{zz}$ decreases with increasing $\psi$ for both the collision-dominated regime ($B/n_0 = 100$ Hx) and magnetic field-controlled regime ($B/n_0 = 1000$ Hx).

Similar but not identical behavior shows diffusion coefficient along $E \times B$ direction ($n_0D_{xx}$). In the collision-dominated regime and in contrast to $n_0D_{zz}$, $n_0D_{xx}$ increases for an increasing $\psi$. Further and in contrast to $n_0D_{zz}$, for $B/n_0$ of 1000 Hx the amplitude of oscillatory feature is more pronounced than that of $n_0D_{zz}$. It should be noted that the amplitude of these oscillations increases when rising $\psi$ while the steady-state values are a decreasing function of $\psi$ (see figure 7.7). There is another distinct property associated with $n_0D_{xx}$ in the limit of high $B/n_0$: the transient negative diffusion exists for the whole range of $\psi$ considered in this work. Conversely, $n_0D_{zz}$ becomes transiently negative only in the limit of angles close to 90°.

Unlike $n_0D_{xx}$ and $n_0D_{zz}$, the temporal profiles of $n_0D_{yy}$ show entirely different nature when considering the effects of angle between the fields on the relaxation process. For parallel fields ($\psi = 0^\circ$), $n_0D_{yy}$ is essentially equal to $n_0D_{xx}$ due to symmetry properties outlined in the previous chapter. However, in contrast to other diagonal elements of the diffusion tensor, the oscillatory nature of $n_0D_{yy}$ is reduced as the angle between the fields is increased. In the limit of a crossed field configuration, these profiles are monotonic. For the crossed field configuration, the Lorentz force does not act in this direction and hence there are no imprinted oscillations on
the diffusion coefficient in this direction. On the other hand, for small angles between the fields, the electrons are under the action of Lorentz force producing the oscillatory relaxation profiles. As can be observed from figure 7.7, for \( B/n_0 \) of 1000 Hx and \( \psi \) of 0° and 30°, the Lorentz force produces the negative transient diffusivity. When considering the relaxation times, we observe the following quite general feature in the relaxation profiles: for both the collision and magnetic field controlled regime, the overall relaxation time is an increasing function of \( \psi \). To be more specific, the overall relaxation times for non-parallel fields are on the timescale of the energy relaxation. It should be noted that this appears to be more obvious for \( n_0D_{yy} \) comparing to \( n_0D_{xx} \) and \( n_0D_{zz} \). Another striking property is the fact that the steady-state values of \( n_0D_{yy} \) monotonically increase with increasing \( \psi \). However, one may expect a decrease in steady-state values of \( n_0D_{yy} \) for significantly higher \( B/n_0 \).

Another important aspect of the present discussion concerns the adequacy of the two-term approximation due to its known failure for the Reid ramp model. However, we do not attempt to analyze the validity of the two-term approximation in a comprehensive fashion, e.g. for all transport properties in various field configurations. Instead we demonstrate the inadequacy of the two-term approximation for the diagonal elements of the diffusion tensor for some particular field configurations. Apart from curiosity, the primary motivating factor behind this choice is the lack of such studies in the literature. As an illustrative example, Loffhagen and Winkler (1999) employed a multi-term approach to test the validity of the two-term approximation for the mean energy and drift velocity in helium, xenon and molecular nitrogen but for the \( E \)-only case and under spatially homogeneous conditions.

Figure 7.8 displays the temporal relaxation profiles of the diagonal elements of the diffusion tensor as a function of \( B/n_0 \) and \( \psi \). The results obtained by a multi-term theory for solving the Boltzmann equation are compared with those obtained by the two-term approximation. The inadequacy of the two-term approximation is clearly evident. In particular, significant deviations between temporal profiles in the early and intermediate stages of the relaxation process can be observed. This is a clear indication that the initial distribution function and its initial evolution deviates substantially from isotropy in velocity space. In general, however, as magnetic field and the angle between the fields increase, the deviations between the results obtained by the two-term approximation and multi-term theory are significantly diminished. This suggests that the magnetic field acts to destroy the anisotropy of the velocity distribution function, consequently inducing enhanced convergence in the \( l \)-index. A similar effect was observed in methane (White et al., 2006).

The corresponding steady-state results for the diagonal elements of the diffusion tensor are reported in tables 7.1 and 7.2. Large discrepancies between the two-term and multi-term results are found, particularly for \( B/n_0 \) of 100 Hx when collisions dominate the electron transport. We observe that the diagonal elements of the diffusion tensor can have errors of the order of 15%. Even larger errors of the order of 20% for the diagonal elements of the diffusion tensor have been reported by White et al. (1999a). As pointed out by White et al. (1999a), for transport parallel to the electric field the error associated with the two-term approximation is generally reduced.
Figure 7.8: Temporal relaxation of the diagonal elements of the diffusion tensor for $B/n_0$ of 100 and 1000 Hx and $\psi$ of 30° and 60° for Reid ramp model (color lines: multi-term calculations; black lines: two-term approximation).

with increasing $\psi$. In contrast, for transport perpendicular to the electric field, it appears in general that errors associated with the two-term approximation are enhanced with increasing $\psi$. However, this is true only when collisions dominate the electron transport. The comparison presented in Table 7.1 supports this observation. If magnetic field controls the behavior of the swarm, then the errors associated with both diffusion coefficients parallel and perpendicular to the electric field tend to decrease with increasing $\psi$ (see table 7.2). This appears to reflect the general trend to reduce anisotropy of the velocity distribution function in all directions as $\psi$ increases in the magnetic field-controlled regime. Plasma modellers should be aware of these
facts before employing the two-term approximation in their codes.

Table 7.1: Comparison between the two-term (TTA) and multi-term (MT) Boltzmann calculations of the diagonal elements of the diffusion tensor as a function of $\psi$ at $E/n_0$ of 12 Td and $B/n_0$ of 100 Hx (a) and 1000 Hx (b). $\Delta$ represents the percentage deviation between these two sets of the results.

### (a) $B/n_0 = 100$ Hx

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$n_0D_{xx}$</th>
<th>$n_0D_{yy}$</th>
<th>$n_0D_{zz}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(o)</td>
<td>$10^{23}$ m$^{-1}$ s$^{-1}$</td>
<td>$10^{23}$ m$^{-1}$ s$^{-1}$</td>
<td>$10^{23}$ m$^{-1}$ s$^{-1}$</td>
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<tr>
<td>TTA</td>
<td>7.8948</td>
<td>7.8948</td>
<td>5.0653</td>
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<tr>
<td>0</td>
<td>$\Delta$ (%)</td>
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<td>-4.4</td>
</tr>
<tr>
<td>MT</td>
<td>7.6655</td>
<td>8.2006</td>
<td>5.5309</td>
</tr>
<tr>
<td>30</td>
<td>$\Delta$ (%)</td>
<td>-2.8</td>
<td>-4.8</td>
</tr>
<tr>
<td>MT</td>
<td>7.6655</td>
<td>8.2006</td>
<td>5.5309</td>
</tr>
<tr>
<td>60</td>
<td>$\Delta$ (%)</td>
<td>2.6</td>
<td>-9.2</td>
</tr>
<tr>
<td>MT</td>
<td>8.0857</td>
<td>9.8770</td>
<td>5.2596</td>
</tr>
<tr>
<td>90</td>
<td>$\Delta$ (%)</td>
<td>7.9</td>
<td>-14.3</td>
</tr>
<tr>
<td>MT</td>
<td>8.6723</td>
<td>1.1471</td>
<td>5.2677</td>
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### (b) $B/n_0 = 1000$ Hx

<table>
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<tr>
<th>$\psi$</th>
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<th>$n_0D_{zz}$</th>
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<td>(o)</td>
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<td>2.8444</td>
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<td>0</td>
<td>$\Delta$ (%)</td>
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<td>MT</td>
<td>2.5113</td>
<td>2.5113</td>
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<td>$\Delta$ (%)</td>
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</tr>
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<td>$\Delta$ (%)</td>
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<td>6.1</td>
</tr>
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<td>MT</td>
<td>1.3465</td>
<td>5.130</td>
<td>1.7889</td>
</tr>
<tr>
<td>90</td>
<td>$\Delta$ (%)</td>
<td>3.0</td>
<td>-0.6</td>
</tr>
<tr>
<td>MT</td>
<td>0.39851</td>
<td>8.9059</td>
<td>0.056603</td>
</tr>
</tbody>
</table>

7.3 The ionization model of Lucas and Saelee

For the consideration of ionization processes on the temporal relaxation of the electron transport properties we use the ionization model of Lucas and Saelee (1975). This ionization model of Lucas and Saelee is defined in the previous chapter (6.2). It is useful here to emphasize once more that this model assumes the inelastic and ionization processes. The parameter $F$ controls the magnitude of the corresponding cross sections. We consider three different situations:

- $F = 0$ (Conservative case): The gas model is reduced to elastic and excitation cross sections and no ionization occurs;
• $F = 0.5$ (Non-conservative case): The cross sections for ionization and excitation have the same magnitude;

• $F = 1$ (Non-conservative case): The gas model is reduced to elastic and ionization cross sections and no excitation occurs;

Note that this model can be used to isolate and separate effects of inelastic and ionization collisions, respectively. This can be done through the variation of a parameter $F$. However, it is common in the literature on electron swarms to find ionization processes simply as another inelastic process (Winkler et al., 2002). If this scenario was used in our considerations there would be no variation in the calculated transport properties with respect to variation in the parameter $F$.

In this section the following strategy is applied: (i) First, we focus on the influence of the ionization process on the relaxation process of various transport properties; (ii) Second, we investigate the influence of the angle between electric and magnetic fields on the temporal evolution of the electron transport properties; and (iii) Third, we consider the consequences of the variation of $E/n_0$ on the temporal relaxation process. The relaxation of the electrons always starts from the same initial conditions. These conditions represent the steady state magnetic field free case where the electron swarm is acted on solely by a dc electric field at $E/n_0 = 10$ Td. The relaxation properties of the swarm are monitored as a function of normalized time ($Nt$), electric field strengths ($E/n_0$), magnetic field strengths ($B/n_0$) and angle ($\psi$) between the fields.

![Figure 7.9](image_url)

Figure 7.9: Comparison of the reduced cyclotron period for various $B/n_0$ (solid horizontal lines) with the reduced timescales for momentum and energy relaxation as a function of energy for the ionization model of Lucas and Saelee.

In figure 7.9 we show the characteristic timescales for momentum and energy relaxation for the ionization model of Lucas and Saelee. We note that both of the characteristic time
constants for momentum and energy relaxation are independent of the energy up to 15.6 eV due to the constant collision frequency for elastic collisions. We observe that the momentum dissipation occurs much faster than the energy dissipation particularly in the energy region where only elastic collisions occur. When inelastic collisions start to play a significant role there is an increase in the efficiency of the energy dissipation. Having in mind these facts, one may expect significantly faster relaxation of the drift velocity and/or its components with respect to the mean energy.

7.3.1 The effect of the ionization processes

In this section the influence of the ionization degree on the temporal relaxation process is investigated. In figure 7.10 we demonstrate the influence of the ionization degree \( F \) on the temporal evolution of the mean energy and ionization rate. As an illustrative example, the angle between the fields is fixed to 75° and the profiles of the mean energy and ionization rate are given for various \( B/n_0 \). We observe that the mean energy shows monotonic relaxation independently of \( B/n_0 \) and/or \( F \). Considering the effects of the ionization process on the relaxation profiles we observe the following interesting points. First, the mean energy decreases when increasing \( F \). This decrease is caused by the energy dilution effect associated with the ionization process (Robson and Ness (1988)). For an increasing \( B/n_0 \) this effect is significantly reduced. As can be observed, in the final stage of the relaxation process for \( B/n_0 \) of 500 and 1000 Hx the mean energies are not affected by the parameter \( F \) due to significant reduction in the ionization rate. This is clearly evident from the profiles of the ionization rate shown in figure 7.11. Second, from figure 7.10 we observe that the relaxation time of the mean energy depends on both \( F \) and \( B/n_0 \). For \( B/n_0 \) of 100 and 200 Hx when collisions dominate the effects of magnetic field, we may observe that the relaxation time of the mean energy decreases for an increasing \( F \). This is perhaps an evidence of the additional variation of the distribution function with both \( F \) and \( B/n_0 \). In general, however, the relaxation time increases when increasing \( B/n_0 \). This is indicative of the cooling action associated with an increasing component of the magnetic field perpendicular to the electric field (Ness (1994); White et al. (1999a; 1999d)).

In figures 7.12 - 7.14 we demonstrate the influence of the ionization degree \( F \) on the temporal relaxation of the drift velocity components. For the ionization model of Lucas and Saelee, the implicit effect of ionization on the drift velocity and its components is weak, and hence the flux components for \( F = 0.5 \) and \( F = 1 \) cases are essentially equal to the \( F = 0 \) profile. The black curves represent these flux components while the red and blue curves represent the bulk components for \( F = 0.5 \) and \( F = 1 \), respectively. Since the effect of the ionization process on \( W_x \) is also very weak, we show only the temporal profiles of the flux components.

As can be observed from figures 7.12 - 7.14 the oscillatory feature of the relaxation profiles is enhanced as the \( B/n_0 \) is increased. We observe that the amplitude of the oscillations increases with rising \( B/n_0 \) while the positions of the extremes are not affected by the parameter \( F \). The steady-state values for this particular field configuration behave as follows: both \( W_x \) and \( W_z \) monotonically decrease while \( W_y \) monotonically increases with increasing \( B/n_0 \). The positions of
the extremes during the first stage of the relaxation process follow directly from the collisionless behavior of the swarm and may be associated with the gyro-frequency. More about this aspect of the relaxation process will be given in the next section. More importantly, however, we
Figure 7.12: Temporal relaxation of $W_x$ as a function of $B/n_0$ and fixed $\psi$ of 75° for the ionization model of Lucas and Saelee.

Figure 7.13: Temporal relaxation of $W_y$ as a function of $B/n_0$ and fixed $\psi$ of 75° for the ionization model of Lucas and Saelee (black line: $F = 0$, red line: $F = 0.5$, blue line: $F = 1$).

Note a significant difference in the relaxation profiles between the bulk and flux components. In the early stage of the relaxation process there is no distinction between the bulk and flux values of $W_y$ indicating there is no spatial variation of the average electron energy in this direction for all $B/n_0$ considered. On the other hand, there is a clear distinction between the bulk and flux components of $W_z$. This suggests a strong variation of the average energy along
the swarm in this direction induced by the electric field only being present. However, as the relaxation process proceeds in time, the synergism of magnetic field and ionization collisions on the temporal relaxation profiles becomes clearly evident. While the distinction between the bulk and flux values for $W_y$ is more pronounced, the same distinction for $W_z$ is firstly diminished during the intermediate stage of the relaxation process and then upon reaching the steady state there are limited differences between the bulk and flux values. These results suggest that due to complex interplay of magnetic field and dissipation of the energy and momentum in collisions, a complicated redistribution of high energetic electrons occurs. In general, the differences between the bulk and flux components of the drift velocity components are dictated by the relaxation processes of the corresponding energy gradient vector components. The temporal evolution of these transport properties are shown in figures 7.15 - 7.17. From figure 7.15 we observe a little spatial variation in the average electron energy along the $x$-axis which is consistent with low sensitivity of the $W_x$-profiles to the ionization process. The steady-state value of $\gamma_x$ is positive and hence in the opposite direction to the drift in that direction. Conversely, the steady-state values of $\gamma_y$ and $\gamma_z$ are negative for all $B/n_0$ considered, indicating the average energy increases through the swarm in these directions in the direction that the swarm is drifting. Starting from its initial value, $\gamma_y$ remains nearly unchanged in the first about 10 ns at the gas pressure of 1 Torr for all $B/n_0$. The other two components of the energy gradient vector, $\gamma_x$ and $\gamma_z$ response much faster. In particular, as the relaxation process proceeds further in time, $\gamma_y$ increases for $B/n_0$ of 500 and 1000 Hx, reaching the peak, and then it starts to decrease until the steady-state is reached. This is consistent with the transient peaks observed in the relaxation profiles of the bulk component of $W_y$ for $B/n_0$ of 500 and 1000 Hx.

Considering the relaxation time for the bulk and flux drift velocity components, it becomes
obvious that the temporal relaxation of the bulk components take much longer than the flux components. We observe that the relaxation time of the bulk components is on the timescale for the energy relaxation. The relaxation time of the bulk components of $W_y$ and $W_z$ increases for an increasing $B/n_0$. This is determined by a careful examination using the linear scale for the normalized time. In contrast, as can be observed from figures 7.12 - 7.14, the relaxation time
Figure 7.17: Temporal relaxation of $\gamma_z$ as a function of $B/n_0$ and fixed $\psi$ of 75° for the ionization model of Lucas and Saelee (black line: $F = 0$, red line: $F = 0.5$, blue line: $F = 1$).

of the flux components is not significantly affected by an increase in $B/n_0$. The origin of this property may be associated with the fact that the characteristic time for momentum relaxation to a stationary value is about the same for all momentum components and is nearly independent of $B/n_0$. The number of elastic collisions is not affected by the variation of $B/n_0$ by the virtue of the constant collision frequency for elastic collisions. This has important ramifications for other field configurations, as shown in the next section. When considering the relaxation time for the energy gradient vector, the picture is much simpler. The relaxation time of the energy gradient vector components is essentially determined by the time constant for the long term energy relaxation. As can be observed from figures 7.15 - 7.17, the relaxation time of $\gamma_x$, $\gamma_y$ and $\gamma_z$ is an increasing function of $B/n_0$.

In what follows the influence of the ionization process on the relaxation process associated with the diagonal elements of the diffusion tensor is analyzed. In figures 7.18 - 7.20 we show the temporal relaxation of these transport properties as a function of the parameter $F$ and $B/n_0$ for fixed $\psi$ of 75°. The implicit effects of the ionization processes on the diagonal elements of the diffusion tensor are quite strong and markedly different profiles for different ionization degrees $F$ can be observed. As expected, these effects are more pronounced for the magnetic field controlled regime, e.g. for lower $B/n_0$ and/or during the early stage of the relaxation process. We observe that the diagonal elements of the diffusion tensor tend to decrease as the ionization degree $F$ increases. The explicit effects of the ionization process result in a clear distinction between the bulk and flux relaxation profiles. These distinctions are quite different during various relaxation stages. At the beginning of the relaxation process, these differences are quite large (up to 40%) and as the relaxation process proceeds further, their evolution depend on $B/n_0$. For lower $B/n_0$, the differences between the bulk and flux values upon reaching the
steady-state are still remarkable while for higher $B/n_0$ the differences between the bulk and flux values are significantly reduced. This is another example how magnetic field controls the action of the ionization processes.

We observe that the oscillatory feature of the relaxation profiles is enhanced as the $B/n_0$ is
increased. These oscillations induce a transient negative diffusivity observed in the flux and bulk relaxation profiles of $n_0D_{xx}$ and $n_0D_{zz}$. Note that for this particular field configuration, $n_0D_{yy}$ is not the subject of this phenomenon. It should be emphasized that this is the first theoretical observation of the negative transient electron diffusivity associated with both the bulk and flux components of the same diffusion coefficient. In order to understand the negative transient electron diffusivity associated with the bulk components of the diffusion tensor, two distinctively physical pictures can be applied. On the first hand, for a strict consideration of the negative transient electron diffusivity associated with the bulk components we need to consider the rate of change of displacement between individual particles in configuration space. On the second hand, having in mind that the bulk component is in this case just a perturbation of the corresponding flux component induced by the action of the ionization process, it is entirely appropriate to interpret the negative transient electron diffusivity associated with the bulk component of the diffusion tensor using physical arguments addressed previously for the flux components. In any case, the link between the positions of the extremes in profiles of both the bulk and flux components and gyro-frequency may be clearly established. As an illustrative example, for $B/n_0$ of 500 Hx $n_0D_{xx}$ passes through zero at the end of the first quarter-period of gyrations, reaching a minimum at the end of the first half-period of gyrations and then continuously increases reaching a peak at the end of the period of gyration. As the relaxation process proceeds further and the collisions start to play a significant role, this oscillatory behavior is firstly diminished and then entirely suppressed. In contrast to $n_0D_{xx}$ and $n_0D_{zz}$ for this particular field configuration, there is no negative excursion in the profile of $n_0D_{yy}$. Though for an increasing $B/n_0$ the oscillatory feature of the relaxation profiles is enhanced, we note in general a clear transition from the oscillatory to monotonic relaxation.

![Figure 7.20: Temporal relaxation of $n_0D_{zz}$ as a function of $B/n_0$ and fixed $\psi$ of 75° for the ionization model of Lucas and Saelee (full lines: flux; dashed lines: bulk).](image-url)
7.3.2 The effect of the angle between the fields on the temporal relaxation profiles

Within this section the effect of the angle between the fields on the temporal relaxation profiles is investigated. Figures 7.21 (a) and (b) show the influence of $\psi$ on the temporal relaxation process associated with the mean energy and ionization rate, respectively. The mean energy exhibits a monotonic relaxation for all $\psi$ considered. Note as $\psi$ increases both the mean energy and ionization rate decreases markedly. In fact, an increase in $\psi$ induces a dramatic variation of the ionization rate up to almost 10 orders of magnitude. As already remarked, the origin of this behavior follows directly from the magnetic field cooling effects. Further, when increasing the angle between the fields the relaxation time can be enlarged by two orders of magnitude for $\psi$ considered in this work. Quite generally, the relaxation time of the ionization rate is on the timescale for the energy relaxation.

Figure 7.21: Temporal relaxation of the mean energy (a) and ionization rate (b) as a function of $\psi$ and fixed $B/n_0$ of 500 Hx for the ionization model of Lucas and Saelee.

Figures 7.22 - 7.24 display the influence of the angle between the fields on the temporal relaxation profiles of the drift velocity and energy gradient vector components. Both the flux and bulk components of the drift velocity components are presented. We use the full lines to represent the flux components while the dashed lines represent the bulk components. There are no observed differences between the bulk and flux components of $W_x$ indicating a small spatial variation in the average energy along the $x$-axis. The temporal relaxation profiles of $\gamma_x$ as a function of $\psi$ shown in figure 7.16 confirms and validates this observation. Interestingly, during the early stage of the relaxation process the $x$-component of the drift velocity initially rapidly increases, reaching a peak, and then it starts to decrease, reaching a zero again for all values of $\psi$ considered. Consideration of profiles for additional and higher $B/n_0$ reveals that steady-state values of $W_x$ display a maximal property while during the early stage of the relaxation process
the profiles assume both the positive and negative values. This is an evidence of the essentially collisionless behavior. When the magnetic field is switched on the condition $\tau < \tau_m$ is satisfied initially and the electrons on the average undergo gyrations about the magnetic field before they undergo collisions. The first peak and minimum correspond to the first quarter- and first half-period of gyrations respectively. On the other hand, $W_y$ peaks at the first half-period while $W_z$ reaches the minimum. Note that the relaxation times for $W_x$ are not significantly affected by the variation of $\psi$.

Figure 7.22: Temporal relaxation of $W_x$ and $\gamma_x$ as a function of $\psi$ and fixed $B/n_0$ of 500 Hx for the ionization model of Lucas and Saelee.

Figure 7.23: Temporal relaxation of $W_y$ and $\gamma_y$ as a function of $\psi$ and fixed $B/n_0$ of 500 Hx for the ionization model of Lucas and Saelee.
The temporal evolution of $W_y$ and $\gamma_y$ are shown in figures 7.23 (a) and (b), respectively. As already remarked, the $\psi$-dependence of the profiles associated with the steady-state flux components go as $\sin 2\psi$. By the virtue of the energy independent elastic collision frequency, the overlapping between some temporal profiles for the flux components for certain field configurations follows directly from the symmetry properties outlined by White et al. (1999a). However, as can be observed, this symmetry property does not hold for the bulk components. If inelastic collisions play more important role, then this symmetry property does not hold for the flux components as well. Both the bulk and flux components display a maximal property. The relaxation time for the flux components is not significantly affected by the variation of $\psi$ and fall within the range of 0.1-1 $\mu$s at the gas pressure of 1 Torr. The bulk components need longer time to reach the steady-state and as already noted these relaxation times are on the timescale for the energy relaxation.

The temporal relaxation of $W_z$ and $\gamma_z$ is shown in figures 7.24 (a) and (b). Both the bulk and flux steady-state values of $W_z$ are decreasing functions of $\psi$. For small $\psi$ and during the early stage of the relaxation process both the bulk and flux components of $W_z$ assume only positive values. This means that the initial velocity in this direction acts to prevent the situation where $W_z$ becomes negative. However, the oscillatory feature of $W_z$ is enhanced as $\psi$ is increased and a new physical situation occurs. In the limit of large angles both the bulk and flux components can be negative or the flux is negative while the bulk is positive. This suggests that the magnetic field is strong enough to rotate the flux component of $W_z$ in velocity space around the lines of the magnetic field. On the other hand, the transiently negative bulk component is indicative of a small displacement along the $z$-axis in the configuration space and intensive rotation around the lines of a magnetic field. These conditions correspond to the situation when the gyro-frequency is much larger than the collision frequency and many gyrations will on the average be completed.
between the collisions. In other words, the electrons are held tight by the magnetic field lines.

From figure 7.24 (a) we observe that differences between the bulk and flux components of $W_z$ are different during various relaxation stages. During the early stage of the relaxation process these differences are quite large indicating a strong influence of the ionization processes. Considering the relaxation times in figure 7.24 (a), it becomes obvious that the temporal relaxation of the bulk components take much longer than the relaxation of the corresponding flux components. The differences in the relaxation times between the bulk and flux components can be up to two orders of magnitude. Consideration of profiles in linear scale reveals that the relaxation times of the flux component are not significantly affected by the variation of $\psi$ while the relaxation times for the bulk components generally increase when increasing $\psi$.

![Figure 7.25](image)

Figure 7.25: Temporal relaxation of $D_{yy}$ and $D_{zz}$ as a function of $\psi$ and fixed $B/n_0$ of 500 Hx for the ionization model of Lucas and Saelee.

In figures 7.25 (a) and (b) we investigate the influence of $\psi$ on the temporal relaxation profiles of $D_{yy}$ and $D_{zz}$, respectively. The previous study associated with the Reid ramp model unearthed some generic features in the temporal relaxation profiles of the diagonal elements of the diffusion tensor which are again observed for the ionization model of Lucas and Saelee. We highlight these features and focus on the effects of non-conservative collisions. The same physical arguments given above for the Reid ramp model can be used again to explain the basic trends in the profiles. The following interesting points are observed in the profiles of $D_{yy}$ and $D_{zz}$ in figure 7.25:

- The temporal profiles of both $D_{yy}$ and $D_{zz}$ show a remarkable sensitivity with respect to $\psi$;
- For an increasing $\psi$ the oscillatory feature of $D_{zz}$ is enhanced while the oscillatory feature of $D_{yy}$ is diminished.
• For perpendicular fields $D_{yy}$ exhibit a monotonic relaxation;

• While $D_{yy}$ becomes transiently negative for small $\psi$, the negative excursion in the profiles of $D_{zz}$ may be observed for significantly larger $\psi$;

• The relaxation time of the bulk and flux components of both $D_{yy}$ and $D_{zz}$ is a monotonically increasing function of $\psi$;

• The difference between the bulk and flux components of both $D_{yy}$ and $D_{zz}$ can differ by 40% and distinctively depends on $\psi$;

7.3.3 The influence of the reduced electric field on the temporal relaxation profiles

In this section we highlight some important implications on the temporal relaxation of various transport properties associated with the action of the reduced electric field. The action of the reduced electric field is an important process with respect to the production of anisotropy in the velocity distribution function during the relaxation process. To illustrate the influence of $E/n_0$ for the ionization model of Lucas and Saelee, we have performed the calculations of the temporal relaxation profiles of various transport properties for a range of $E/n_0$. In order to illustrate all important aspects of these effects, the relaxation model is modified. In this particular case, the initial conditions represent the steady-state where the electron swarm is acted on solely by a dc electric field. We apply $E/n_0$ of 5 Td. At time $t = 0$, the magnetic field is switched on while the electric field is increased. We assume $B/n_0$ of 500 Hx and $\psi$ of 60° while the increased $E/n_0$ assume the values of 10, 20, 30 and 40 Td. The aim of using modified relaxation model is to consider and elucidate the parallel effects of $E/n_0$ and $B/n_0$.

In figures 7.26 (a) and (b) the temporal relaxation profiles of the mean energy and ionization rate are presented. As expected, both the mean energy and ionization rate increase with increasing $E/n_0$ and exhibit monotonic relaxation. We observe that as $E/n_0$ increases, the establishment of the steady-state occurs much faster. In fact, the relaxation time of reaching the steady-state can differ by several orders of magnitude and distinctly depends on $E/n_0$ and thus on the energy dissipation efficiency of the electrons in collisions (see figure 7.7). When approaching steady-state at the lower $E/n_0$ the power loss by elastic collisions dominates while at the higher $E/n_0$ the inelastic collisions start to play a significant role and make the dominant contribution to the power loss. This ultimately speeds up the relaxation process and leads to a better energy exchange in the system.

In figures 7.27 (a) and (b) and figure 7.28 we show the influence of $E/n_0$ on the temporal relaxation profiles of the drift velocity components. The full lines represent the flux values while the dashed lines represent the bulk values. There are no significant differences between the bulk and flux components of $W_x$ and hence only the flux values are presented. This is verified in the relaxation profiles of the $x$-component of the gradient energy vector shown below. On the other hand, there is a clear distinction between the temporal relaxation profiles of the bulk and
Figure 7.26: Temporal relaxation of the mean energy and ionization rate as a function of $E/n_0$ for $B/n_0$ of 500 Hx and $\psi$ of 60° for the ionization model of Lucas and Saelee (black line: 10 Td; red line: 20 Td; green line: 30 Td; blue line: 40 Td).

flux components associated with the $y$- and $z$-drift velocity components. Both of these drift velocity components show a remarkable sensitivity to the ionization processes. We see that the distinction between the bulk and flux temporal relaxation profiles is enhanced as the $E/n_0$ is increased. It is interesting to note that the position of the extremes in the relaxation profiles of the flux components are only slightly affected by the variation of $E/n_0$. The amplitude of oscillations increases with rising $E/n_0$ and the steady-state values (both the bulk and flux) are the increasing function of $E/n_0$. Further, we observe that both the bulk and flux components of $W_z$ overshoot and oscillate about the steady-state values for $E/n_0$ of 20, 30 and 40 Td. In contrast, for $E/n_0$ of 10 Td, $W_z$ does not overshoot its steady-state value which is a clear sign that the magnetic field is still too strong to allow the rapid acceleration in the $z$ direction of the initial electrons during the early stage of the relaxation process. After a suitable time, the flux drift velocity components are decreased towards the steady-state values as a consequence of momentum equilibration. The overshooting of the drift velocity for the $E$-only case was observed by Shizgal and MacMahon (1985) in electron thermalization at low $E/n_0$ and by Kondo et al. (1993; 1994) in rare gases. As pointed out by Kondo et al. (1993; 1994), overshoot occurs as a result of the smaller collision loss of the elastically scattered electrons which are released with low energy in the low $E/n_0$ region and the deceleration by the large inelastic collision loss after acceleration. This physical picture can be used for the electron swarms under the influence of electric and magnetic fields but only when collisions dominate the electron transport. If the swarm is under the action of a strong magnetic field, overshooting the steady-state is prevented by the virtue of the deflection of the drift velocity from the electric field direction. In general, overshoot is a combined effect of the electric field and collisions while the oscillatory behavior is a consequence of the action of a magnetic field.

Another striking property observed in the relaxation profiles of the drift velocity components
Figure 7.27: Temporal relaxation of $W_x$ and $W_y$ as a function of $E/n_0$ for fixed $B/n_0$ of 500 Hx and for fixed $\psi$ of 60° for the ionization model of Lucas and Saelee (black line: 10 Td; red line: 20 Td; green line: 30 Td; blue line: 40 Td).

Figure 7.28: Temporal relaxation of $W_z$ as a function of $E/n_0$ for fixed $B/n_0$ of 500 Hx and for fixed $\psi$ of 60° for the ionization model of Lucas and Saelee (black line: 10 Td; red line: 20 Td; green line: 30 Td; blue line: 40 Td).

is behavior of the bulk components. As remarked previously, the difference between the bulk and flux drift velocity components can be attributed to the effects of the spatial non-uniformity of the local ionization rate resulting from a spatial non-uniformity of average electron energies within the swarm. In the final stage of the relaxation process, if the bulk is greater than the corresponding flux component of any drift velocity component, then the local ionization rate ahead of the centroid of the swarm is larger than at behind the centroid. Figure 7.29 displays
the temporal evolution of the energy gradient vector components as a function of \( E/n_0 \) and for fixed \( B/n_0 \) and \( \psi \). While the distinction between the bulk and flux components of \( W_z \) in the early stage of relaxation is clearly evident for all \( E/n_0 \), the same distinction does not exist for \( W_y \). This is consistent with the temporal evolution of the \( z \)- and \( y \)-gradient energy vector components during the early stage of the relaxation process (see figure 7.29). We note that the relaxation time of the gradient energy vector components decreases when increasing \( E/n_0 \). One can understand this behavior using the fact that as \( E/n_0 \) increases the collision frequency for inelastic collisions is increased and hence faster establishment of steady-state follows. Concerning the temporal evolution of \( z \)- and \( y \)-energy gradient vector components, it is observed from figure 7.29 that these transport properties do not monotonically increase with \( E/n_0 \). Instead \( \gamma_y \) increases showing a maximal property in the steady-state values while \( \gamma_z \) decreases when increasing \( E/n_0 \). Hence in order to analyze the effects of the ionization processes on the \( y \)- and \( z \)-drift velocity components, the spatial variation in the average energy and magnitude of the average velocity components must be considered on an equal footing. Large spatial variation in the average energy along the swarm is not a guarantee for stronger effects of the ionization processes as it was for the previous relaxation model. In any case, from figures 7.27 (a) and (b) and figure 7.28 we observe that the difference between the bulk and flux components increases when increasing \( E/n_0 \). When increasing \( E/n_0 \), the separation between the flux and bulk profiles of \( W_y \) occurs much faster. The rapid rise in the bulk components of both \( W_y \) and \( W_z \) during the intermediate stage of the relaxation process coincides with the rapid rise of the mean energy. Interestingly, in the same stage of the relaxation process, \( \gamma_y \) increases while \( \gamma_z \) rapidly decreases.

![Figure 7.29: Temporal relaxation of the gradient energy vector components as a function of \( E/n_0 \) for fixed \( B/n_0 \) of 500 Hx and for fixed \( \psi \) of 60° for the ionization model of Lucas and Saelee (black line: 10 Td; red line: 20 Td; green line: 30 Td; blue line: 40 Td).](image)

Another interesting aspect is behavior of the flux components. Surprisingly, as \( E/n_0 \) in-
creases, the flux components of the drift velocity components need a longer time to reach the steady-state. This property is the most obvious for $W_y$. This is perhaps an evidence of additional variation of the velocity distribution function with $E/n_0$.

In figure 7.30 and figures 7.31 (a) and (b) we show the influence of $E/n_0$ on the temporal relaxation of the diagonal elements of the diffusion tensor. We observe that as $E/n_0$ increases, the faster establishment of steady-state of both the bulk and flux components follows. We note that unlike $n_0D_{yy}$ in the long-time limit, i.e. the steady state, a decrease in both $n_0D_{xx}$ and $n_0D_{zz}$ can be observed. The reason for this is the so-called magnetic anisotropy effect: for relatively high $B/n_0$ of 500 Hx the diffusion is significantly reduced since the explicit orbital effect acts to inhibit diffusion in a plane perpendicular to the magnetic field. This effect is the strongest in the limit of an orthogonal field configuration. For the particular field configuration considered in this case where the angle between the fields is set to $60^\circ$, the explicit orbital effect on $n_0D_{yy}$ is relatively small and diffusion along the $y$-direction is almost purely thermal. In addition, we may observe that the oscillatory relaxation nature is also significantly diminished although all diagonal elements of the diffusion tensor respond to the application of magnetic field on the same timescale.

![Figure 7.30: Temporal relaxation of $D_{xx}$ as a function of $E/n_0$ for fixed $B/n_0$ of 500 Hx and for fixed $\psi$ of 60° for the ionization model of Lucas and Saelee (black line: 10 Td; red line: 20 Td; green line: 30 Td; blue line: 40 Td).](image)

In the early stage of the relaxation process, the deviations between the bulk and flux components for all diagonal elements of the diffusion tensor can be observed. This is a clear indication that the initial distribution function is significantly affected by the ionization processes. However, as the relaxation process proceeds in time, the differences between the bulk and flux components are distinctively dependent on $E/n_0$. For example, for $E/n_0$ of 10 Td, the differences between
the bulk and flux components are firstly diminished during the intermediate stage of the relaxation process and then completely removed in the long-time limit, i.e. when steady-state is achieved. The reason for this lies with the fact that magnetic field of 500 Hx depopulates the highly energetic electrons from the tail of the distribution function and reduces the effects of the ionization processes. At the same time the spatial variations of the average energy along the $x$- and $z$-directions are significantly reduced. However, as can be observed from figure 7.29, in contrast to $\gamma_x$ and $\gamma_z$, $\gamma_y$ is significantly increased in the long-time limit. Note that for $E/n_0$ of 10 and 20 Td, respectively, there are no differences in $\gamma_y$. At the same time, the distinction between the bulk and flux components of $n_0D_{yy}$ for $E/n_0$ of 20 Td is clearly evident. This is a clear indication that one must use the second order variations of the average energy through the swarm in order to explain the behavior of the bulk diagonal elements of the diffusion tensor.

An additional interesting feature in the relaxation profiles of the bulk components of the $n_0D_{xx}$ and $n_0D_{zz}$ is the development of a transient peak structure during the intermediate stage of the relaxation process for the highest values of $E/n_0$ considered in this work. As $E/n_0$ increases, the peak grows in magnitude and becomes the dominant feature in the temporal profile. A rapid increase of the mean energy and ionization rate (see figures 7.26 (a) and (b)) during the intermediate stage of the relaxation process can be assigned with the development of this transient peak structure. It is interesting to note that the spatial variations in the average energy along the $z$-direction rapidly decrease during the same stage of the relaxation process. This is another indication that one must use the second order variations of the average energy through the swarm in order to understand the influence of non-conservative collisions on the diffusion coefficients.
7.4 Temporal relaxation of electrons in real gases

7.4.1 Preliminaries

In this section we investigate the temporal relaxation of the electron transport properties in carbon dioxide (CO\textsubscript{2}) and its mixtures with noble gases. CO\textsubscript{2} and its mixtures with noble gases are often used in modern particle detectors. As an illustrative example, the originally proposed gas mixture Ar : CH\textsubscript{4} : N\textsubscript{2} (91:5:4)% for the ATLAS Monitored Drift Tube system has been replaced recently by a mixture Ar:CO\textsubscript{2} due to its better resistance to ageing (Kirchner et al., 2001). However, we do not propose this to be a comprehensive investigation of electron transport in each gas and its mixture, but rather aim to demonstrate the applicability of the present theory and code to real systems and to discuss some of the relaxation phenomena that arise from them. Another important aspect of the present discussion concerns the adequacy of the two-term approximation and validity of Legendre polynomial expansions for solving the Boltzmann equation in the context of the temporal relaxation problem for real gases.

![Electron impact cross-sections for (a) carbon dioxide (Bulos and Phelps, 1976) and (b) argon ((Hayashi, 1992); Ness and Makabe (2000)) used in this study.](image)

Figure 7.32: Electron impact cross-sections for (a) carbon dioxide (Bulos and Phelps, 1976) and (b) argon ((Hayashi, 1992); Ness and Makabe (2000)) used in this study.

The momentum transfer and 12 inelastic cross sections including cross sections for ionization and electron attachment of Bulos and Phelps (1976) are used in this study and displayed in figure 7.32 (a). Two-term approximation for solving the Boltzmann equation was used in deriving their cross sections. Ness and Robson (1986) and Braglia et al. (1981) employed the same set of cross sections for CO\textsubscript{2} to investigate the electron transport under steady-state conditions. More recently, however, White et al. (2006) used also the set of cross sections for CO\textsubscript{2} developed by Bulos and Phelps to illustrate the main aspects of the temporal relaxation of electron swarm in electric and magnetic fields in a crossed field configuration. The cross sections for electrons in argon is shown in figure 7.32 (b). We employ the momentum transfer cross section of Hayashi (Hayashi, 1992) while the cross sections for inelastic collisions (including the cross section for
ionization) were taken from the recently published paper of Ness and Makabe (2000). Other available sets of cross sections for argon separate electronic excitation in different groups of cross sections with higher and lower thresholds. These cross sections are required if one needs to calculate the specific channels of excitation or specific line intensities.

Figure 7.33: Comparison of the reduced cyclotron period for various $B/n_0$ (solid horizontal lines) with the reduced timescales for momentum and energy relaxation as a function of energy for (a) carbon dioxide and (b) argon.

In figures 7.33 (a) and (b) the characteristic timescales for momentum and energy relaxation for carbon dioxide and argon are compared with the reduced cyclotron period for various $B/n_0$. The large cross sections for vibrational excitations in CO$_2$ induce faster energy relaxation than that in argon. Even larger differences between the timescales for momentum relaxation in CO$_2$ and argon at the low $E/n_0$ range are clearly evident. The reason for this lies with the fact that the cross section for momentum transfer in elastic collisions in CO$_2$ is much larger than that in argon. In the limit of high $E/n_0$, however, these differences are significantly reduced since inelastic collisions start to play significant role. Under these high $E/n_0$ conditions, the collision frequency for inelastic collisions generally exceeds the collision frequency for momentum transfer in elastic collisions. In addition, for both gases the differences between the characteristic timescales for momentum and energy relaxation in the limit of high $E/n_0$ are significantly diminished. On the other hand, at the low $E/n_0$ range, one may observe large difference between the timescales for momentum and energy relaxation in argon. The energy loses in elastic collisions are proportional to the mass ratio $m/M$ and thus very small compared with those in inelastic collisions. As a consequence, the timescale for energy relaxation assumes very large values in those energy regions where only elastic collisions occur and significantly smaller values at those energies where inelastic collisions start to play more important role. Having in mind these facts, one may expect that by introducing a small amount of molecular gas admixture (e.g. carbon dioxide into inert gas argon), the relaxation process of the electrons will occur much faster. This is clearly illustrated at the figures shown below.
7.4.2 Temporal relaxation of electrons in CO$_2$

The large cross sections for vibrational excitations in CO$_2$ produce a large asymmetry in velocity space which makes the two-term approximation inadequate for the analysis of transient behavior of various transport parameters from the initial to the final time where all transport parameters have reached their steady-state values. $l_{\text{max}} = 4$ was required in order to achieve the full convergence of various transport properties. In figures 7.34 (a) and (b) we show the temporal relaxation of the mean energy and longitudinal drift velocity component as a function of $\psi$ for fixed $B/n_0$ of 500 Hx and $E/n_0$ of 12 Td. In the relaxation profiles of the mean energy and longitudinal drift velocity component we may observe similar trends to those observed for the Reid ramp model. Temporal profiles obtained by the two-term approximation are compared with those obtained by a multi-term theory for solving the Boltzmann equation. The significant deviations between temporal profiles obtained by the two-term approximation and multi-term theory in the early and intermediate stages of the relaxation process can be observed. It should be noted that these deviations are the largest in the limit of a magnetic field free case. As the angle between the fields increases, one may observe a decrease in the error of the two-term approximation. For $E/n_0$ of 12 Td, the application of a magnetic field acts to destroy the anisotropy of the velocity distribution function, consequently inducing enhanced convergence in the $l$-index. Similar behavior of the velocity distribution function is previously observed for the Reid ramp model. However, due to the complex energy dependence of the cross sections for inelastic collisions in molecular gases over a wide range of $E/n_0$ one must be careful with the application of a two-term theory. It would be easy to show examples where the opposite situation holds, e.g. the magnetic field can act to enhance the anisotropy of the velocity distribution function.

Figure 7.34: Temporal relaxation of the mean energy (a) and longitudinal drift velocity component (b) for various angles between the fields for electrons in CO$_2$ at $E/n_0$ of 12 Td and $B/n_0$ 500 Hx. (dashed lines: two-term approximation; full lines: multi-term theory)
In figures 7.35 (a) and (b) the temporal relaxation profiles of the $n_0D_{yy}$ and $n_0D_{zz}$ are displayed as a function of $\psi$ for fixed $B/n_0$ of 500 Hx and $E/n_0$ of 12 Td. We may observe some generic features in the temporal profiles of these two transport coefficients observed previously for both the Reid ramp model and an ionization model of Lucas and Saelee. For example, high sensitivity of the temporal profiles to the angle between the fields is clearly evident. In contrast to $n_0D_{zz}$, the oscillatory nature of $n_0D_{yy}$ is reduced as the angle between the fields is increased. In the limit of crossed field configuration, the Lorentz force does not act along the $y$-direction and hence there are no imprinted oscillations on the diffusion coefficient in this direction. The temporal profile of $n_0D_{zz}$ for an orthogonal field configuration becomes transiently negative. This is a clear sign that transiently negative diffusivity is not an inherent property of model gases. The physical arguments outlined previously for the Reid ramp model can be used again to understand the development of this phenomenon in CO$_2$. On the other hand, when considering the deviations between the temporal profiles obtained by two-term approximation and multi-term theory, it is interesting to note that $n_0D_{yy}$ stands in contrast to $n_0D_{zz}$. In the long-time limit and for an orthogonal field configuration, the term approximation appears to be very accurate for $n_0D_{zz}$. Outside of these limits, the accuracy of the two-term approximation deteriorates. In contrast, for $n_0D_{yy}$ it appears that errors associated with the two-term approximation are even further enhanced with increasing the angle between the fields. This is a clear sign that the anisotropy of the velocity distribution function must be enhanced in the planes perpendicular to the electric field vector. Note that the two-term approximation generally tends to underestimate the results obtained by a multi-term theory.

In order to demonstrate the temporal relaxation of the off-diagonal elements of the diffusion tensor, in figures 7.36 (a) and (b) the temporal relaxation profiles of $n_0D_{zx}$ and $n_0D_{yz}$ are displayed as a function of $\psi$ for fixed $B/n_0$ of 100 Hx and $E/n_0$ of 12 Td. For a selected set of
conditions, the relaxation is monotonic independently of the angle between the fields. However, as \( B/n_0 \) increases the damped oscillatory type relaxation will be developed (not shown here). Temporal profiles of \( n_0D_{zx} \) and \( n_0D_{yz} \) obtained by the two-term approximation are compared with those obtained by a multi-term theory. The inadequacy of the two-term approximation for both \( n_0D_{zx} \) and \( n_0D_{yz} \) is clearly evident. In the early stage of the relaxation process the errors associated with the two-term approximation are small but as the relaxation process proceeds further in time these errors are significantly enlarged. On the other hand, our calculations revealed a decrease in the error of the two-term approximation for an increasing \( B/n_0 \). This is consistent with previous calculations done for certain model gases.

### 7.4.3 Temporal relaxation of electrons in CO\(_2\)-argon mixtures

In this section we investigate the temporal relaxation of the electron transport properties in CO\(_2\)-argon mixtures. A four term approximation was in general needed in the \( l \)-index for all mixtures. A value of \( \nu_{\text{max}}^{\text{max}} = 80 \) was set throughout. Convergence in this index was poor under conditions of high magnetic fields for low fractions of CO\(_2\).

Figures 7.37 - 7.40 display the temporal relaxation of certain transport properties for various CO\(_2\)-argon mixtures at \( E/n_0 \) of 12 Td, \( B/n_0 \) of 1000 Hx and \( \psi \) of 60°. As expected, the relaxation of the mean energy is always monotonic independently of the gas mixture. From figure 7.37 (a) we observe with an increasing fraction of argon that the initial and steady-state values of the mean energy are increased. In particular, we observe that that the initial values of the mean energy are more sensitive to adding an atomic admixture than that of steady-state. Similar behavior exhibit drift velocity components (see figure 7.37 (b) and figures 7.38 (a) and (b)). As the fraction of argon increases, the amplitude of oscillations are enhanced. In the early
stage of the relaxation process, this may lead to the situation where both $W_x$ and $W_z$ assumes positive and negative values. Conversely, for pure CO$_2$ (and for low fractions of argon) both $W_x$ and $W_z$ do not change the sign. Quite generally, it can be seen that, among all possible mixtures of CO$_2$ and argon considered here, the relaxation process is the fastest for the pure CO$_2$ and slowest for (50%)Ar/CO$_2$ mixture.

Figure 7.37: Temporal relaxation of the mean energy (a) and longitudinal drift velocity component (b) for various angles between the fields for electrons in CO$_2$-argon mixtures at $E/n_0$ of 12 Td and $B/n_0$ 500 Hx.

Figure 7.38: Temporal relaxation of $W_x$ (a) and $W_y$ (b) for various angles between the fields for electrons in CO$_2$-argon mixtures at $E/n_0$ of 12 Td and $B/n_0$ 500 Hx.

These phenomena are all attributable to the decreasing contribution of CO$_2$’s collisional process. Recall that the existence of the additional oscillatory behavior in the relaxation profile
is an imprint of the collective gyrations of the electrons damped by collisions that exchange momentum and energy. By adding an atomic admixture, the importance of the CO$_2$ collisional processes such as vibrational excitation processes are significantly suppressed. In addition, under conditions considered here, the average energy of the swarm is in the vicinity of the Ramsauer-Townsend minimum in the argon elastic cross section, meaning that the electron interact less strongly with the argon atoms. Therefore, as the argon gas concentration is increased a slower establishment of the steady-state follows. At the same time, the oscillatory behavior in the relaxation profiles becomes more pronounced.

Figure 7.39: Temporal relaxation of $n_0D_{yy}$ (a) and $n_0D_{zz}$ (b) for various angles between the fields for electrons in CO$_2$-argon mixtures at $E/n_0$ of 12 Td and $B/n_0$ 500 Hx.

Figure 7.40: Temporal relaxation of $n_0D_{xx}$ for various angles between the fields for electrons in CO$_2$-argon mixtures at $E/n_0$ of 12 Td and $B/n_0$ 500 Hx.
In figures 7.39 (a) and (b) and figure 7.40 we show the temporal profiles of the diagonal elements of the diffusion tensor for various CO$_2$-argon mixtures. Generally speaking, as the argon gas concentration increases, the oscillatory nature of the profiles is enhanced. While $n_0D_{xx}$ and $n_0D_{zz}$ become transiently negative in the early stage of the relaxation for the mixtures under consideration, $n_0D_{yy}$ does not exhibit the same effect. As can be seen, the effect of the transiently negative diffusivity is controlled by the concentration of an atomic admixture. As the argon gas concentration increases, the negative excursion in the temporal profiles for $n_0D_{xx}$ becomes more evident. For $n_0D_{zz}$, however, as the argon gas concentration increases certain non-linearities can be observed. First, as the argon gas concentration increases, $n_0D_{zz}$ reaches the maximal negative value and then as the argon gas concentration is further increased, the effect of transiently negative diffusivity becomes less apparent. Another interesting property lies with the fact as the argon gas concentration increases, the steady-state values for $n_0D_{yy}$ and $n_0D_{zz}$ are increased. On the other hand, the steady-state values of $n_0D_{xx}$ are a monotonically decreasing function of the argon gas concentration.