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The application of prediction markets to project prioritization in the not-for-profit sector

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Statement of the Contribution of Others

Nature of Assistance	Contribution	Names
Review and feedback of thesis chapters and underlying investigations	All thesis chapters and underlying investigations have been guided by review and feedback from my Advisory Panel	Associate Professor Sizhong Sun, Doctor Felecia Watkin-Lui, Professor Peter Case
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Daniel A Grainger

Abstract

The principal objectives and scope of the study

The goal of this thesis is to ultimately propose how prediction markets may be used to select the best possible portfolio of projects in a not-for-profit setting. The first logical principal objective is to identify a quality signal that demarcates a good from a bad market prediction (or decision). Upon doing so, the second principal objective of configuring prediction markets (into a decision market) to decide if a project should be in or out of the project portfolio, and importantly the probability that this selection is the best possible one, is achievable. This objective is achieved via a novel theoretical model introduced in this thesis and by three investigations to test the theory. The third and final principal objective is to consider synthesize the finding of this thesis to propose how to improve current prediction markets and create a new type of prediction and decision market that is augmented or embedded with the quality signal identified by the thesis theory and investigations.

Due to time and budget constraints the scope of this thesis does not extend to building a real-world implementation of the prediction and decision market types introduced by this thesis for real-world organizations. However, at the time of writing a joint venture arrangement between the university and other stakeholders is in development to implement this thesis' decision market for project selection in possibly two not-for-profit organizations.

The methodology employed

The keystone and novel contribution of this thesis is the identification of a quality signal that can be used to improve prediction and decision markets. The methodology to achieve this is to first establish a theoretical model that builds upon the work of previous research but that provides an original contribution in its focus on quality signals in prediction markets.

The idealized theoretical model suggests a possible quality signal candidate denoted as relevant information level i.e. the proportion of the market bids that are conditioned on private information. Given relevant information is arrived at theoretically, it is important to robustly test the hypotheses that increasing relevant information increases the probability of prediction and decision markets attaining the best possible (fully informed) predictions and decision respectively.

The first investigation is a computer simulated control-treatment setup in which each control and treatment prediction (and decision) market is identical in every way except for the relevant information level. The statistical significance of relevant information level is then analyzed. The benefit of computer simulations is the large number of prediction and decision market games that can be run. The weakness is that all traders are rational. Therefore, the second investigation incorporates, into the prediction and decision markets, human participants.

The second investigation involves a prediction market web-game created by the PhD candidate. Multiple prediction market web-games are run in a control-treatment setup in which relevant information is allowed to vary with all else being held the same for each control and its treatment. The statistical significance of relevant information level is then analyzed. The strength of this investigation is the incorporation of human idiosyncrasies; however, the weakness is this investigation remains within the confines of the laboratory setting. Hence the third and final investigation involves the analysis of real-world prediction market data.

The analysis of real-world prediction market data is undertaken to extend the testing of the hypotheses into a real-world setting. The strength of this is that the hygienic conditions of the laboratory are removed, however, the weakness is the potential endogeneity and confounding problems that can arise. To counter these potential problems, a control function approach is used to control for endogeneity and a fine strata continuous propensity score approach is used to control for confounding.

Summary of the results

The new theoretical model finds relevant information level to be a sufficient and necessary condition for prediction and decision markets to attain the best possible predictions and decision respectively. The computer simulations find that increasing relevant information level is a statistically significant effect that leads to improving the probability of prediction and decision markets attaining the best possible predictions and decisions. The prediction market web-games with human participants also confirms that relevant information is a statistically significant effect that when increased leads to increasing the probability of attaining the best possible prediction. Finally, the analysis of real-world prediction market data confirms that relevant information is a statistically significant effect that when increased leads to increasing the probability of attaining best possible decisions.

The principal conclusions

There are several conclusions able to be drawn from this thesis as follows:

Relevant information level is robustly tested in this thesis as a quality signal for prediction and decision markets to demarcate good prediction and decisions from bad ones. This is the key contribution by this thesis to research on prediction and decision markets. Prior to this work there existed no metric to assess the quality of a prediction or decision emanating from these markets. Without such a 'quality' metric, confidence in the associated predictions and decisions would remain logically unjustified. Hence 'relevant information level', as a quality signal, proposed and robustly tested by this thesis brings with it the ability to justly demarcate a good prediction or decision from a bad one.

Current real-world prediction markets may be simply augmented with a publicized measure of their relevant information level to improve markets in much the same way that Akerlof quality signals do so. That is, the implication of this thesis does not require radical changes to current real world prediction and decision markets. Rather, by simply publicising the 'relevant information level' metric, the confidence warranted in a prediction or decision market is revealed. In much the same way that Akerlof quality signals were embodied in small changes (e.g., second-hand car guarantees), 'relevant information level' as a quality signal is a small change i.e. publicising and therefore a guarantee of the efficacy of a prediction or decision market.

The new prediction and decision markets that were built in this thesis have the potential to select the best possible portfolio of projects. With high quality decision market selection comes the confidence that the best possible selection is made. The selection of the best possible portfolio of projects is a logical extension to this feature. Establishing a means of measuring how well prediction and decision markets select the best possible portfolio of projects is an important and novel contribution by this thesis. The ultimate goal of confidently guiding project selection so as to best leverage scare economic resources is made possible in this thesis.

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Chapter. 1 Introduction

"If you don't know where you are going, any road can take you there."

Charles Dodgson

Key Message of Chapter:

- **Research gaps in the literature are identified**
- **Research questions are formulated**
- **Research design to address research questions is justified**
- **Original contributions – theoretical, methodological and empirical - are summarized**
- **The thesis structure is presented**

Researchers and practitioners have long been trying to investigate and develop a pragmatic decision support tool that prioritizes projects optimally in not-for-profit settings. Contributing to this strand of research, this thesis investigates the development and application of high quality prediction and decision markets, one of the decision support tools, with an intent to introduce a means to reduce wasted resources due to poor project portfolios.

At the project management and portfolio of project management level, there exists a lack of a high-quality decision support tool, which is particularly noticed in not-for-profit project prioritization activities. The application of a multitude of decision support tools including Net Present Valuations (NPV), Real Options Analysis (ROA), and a family of Cost Benefit Analysis (CBA) tools cannot satisfactorily solve project prioritization problems. Project optimisation using CBA is not possible given that it considers in isolation the costs and benefits of a particular project and is not amenable to inter-project comparisons that would facilitate project ranking. Whilst NPV and ROA allow for inter-project comparison to facilitate ranking, they suffer from the underlying Capital Asset Pricing Model (CAPM) assumptions; whereby typically poor or absent project level comparables lead to poor beta estimates and therefore highly uncertain project valuations that would underpin the ranking exercise. These techniques ultimately share ambiguity driven by unmet assumptions. Importantly, they lack a quality signal to guarantee their efficacy.

It is important that a decision support tool incorporates a quality signal indicating the efficacy of its output. Similar to other tools, the prediction and decision markets currently lack a quality signal and as such their predictions and decisions remain questionable. In view of this, this thesis provides an original contribution in the form of identifying a quality signal for prediction and decision markets. The role of the quality signal is simply to demarcate a good prediction or decision market from a bad one. The quality signal identified in this thesis is called ‘relevant information level’. It is the proportion of traders in the prediction and decision markets that submit bids that express relevant information; where a bid is said to express relevant information if its value changes when the trader’s information changes.

The original proposal of this thesis is to augment prediction and decision markets with a quality signal (i.e., relevant information level) to indicate the credibility of predictions and decisions respectively. The ultimate aim is to determine how to build prediction and decision markets that are able to select, with a high probability, the best possible portfolio of projects in a not for profit setting. To this end, the central focus of this thesis is to determine the statistical significance of ‘relevant information level’ in prediction and decision markets.

The role of the original theoretical model developed in this thesis is simply to identify a possible quality signal. To this end, ‘relevant information level’ is theoretically identified as the quality signal. Because the theoretical model is idealized, relevant information level as the identified quality signal needs to be robustly validated in experiments and analysis. Hence, the statistically significant effect of relevant information level (as a quality signal) is tested in computer simulated prediction and decision market control-treatment experimental setups[1], it is then tested in prediction market webgames with human participants control-treatment experimental setups, and finally tested via analysis of real-world decision market data. This methodology represents a logical progression from tests with the rational computer simulated traders, to those beyond the rational trader assumption (i.e., a prediction market webgame incorporating human idiosyncrasies) in a laboratory setting, to a real-world setting incorporating human behavior beyond the confines of the laboratory. Together these three investigations represent a logically sequenced robust test of the theoretically identified quality signal (relevant information level).

The computer simulated experiments of prediction and decision markets considers a context in which the selection of a not for profit project is required to be determined as either in or out of a best possible project portfolio. The prediction market webgames with human participants consider whether the hypothetical not for profit dog-friendly-beach project is selected as either in or out of the Townsville Pet Society’s best possible project portfolio. The analysis of the implied decision market data for the Iowa Electronic Market’s 2008 U.S.

[1] A control and its respective treatment are identical in every way except for a possible difference in the value of relevant information level.

Presidential Election is a candidate selection problem that, in form, is a not for profit project selection problem to determine whether the project (i.e., candidate) is either in or out of best possible project portfolio. The important takeaway, is that all three tests apply prediction and decision markets to a portfolio of project selection problem in a not for profit setting.

The key findings and policy implications of this thesis are:

1. Increasing relevant information level in prediction and decision markets increases the probability of attaining best possible predictions and decisions

2. Augmenting current real-world prediction and decision markets with relevant information level is an immediately feasible policy change to improve these markets.

3. Building a real-world implementation of prediction and decision markets of the type proposed in this thesis (already having, by design, the embedded the relevant information level quality signal) to select, with high probability, the best portfolio of projects in a not for profit setting is both possible and in current demand. For example, at the time of writing, the PhD candidate has been presented with a post-doctoral opportunity in the form of a University-Indigenous Community health organization joint venture and is currently in the process of adapting the decision market (that he built for this thesis) to select the best possible portfolio of health projects. This real-world build, of the prediction and decision markets of the type advocated by this thesis, was a result of the stakeholders considering the quality signal an attractive feature; revealing to them how likely the best possible portfolio of health projects is selected by this (thesis' decision market) decision support tool.

This thesis initially mathematically proves that a relevant information level of one (i.e., all traders submit informed bids) is a sufficient and necessary condition for prediction markets to converge to best possible predictions. It also mathematically proves that increasing relevant information level leads to an increase in the probability of decision markets converging to the best possible decision. The thesis then tests these idealized findings and ultimately reveals the statistical significance of relevant information level in computer simulations of prediction and decision markets, in prediction market web-games with human participants, and in real-world prediction market data containing an implied decision market. In all settings, relevant information level is a statistically significant effect whereby increasing relevant information level leads to increasing the probability of prediction and decision markets converging to the best possible predictions and decisions respectively.

Ultimately, this thesis finds that high relevant information level is an important requirement for prediction and decision market as decision support tools. Prediction and decision markets with high relevant information level guarantee good prediction and decision performance. In practical terms, a good quality signal (high relevant information level) guarantees that it is highly probable that the prediction and decision markets correctly select and prioritize the best possible portfolio of projects. That is, a prediction and decision markets, as decision support tools accompanied by a good quality signal guarantee, delivers confidence to a manager or group of managers advocating the (prediction and decision market) derived selection and prioritization of the portfolio of projects to their project sponsors, project governance, or firm board. Accordingly, this thesis advocates the use of this type of (relevant information level augmented) prediction and decision market, as a decision support tool, to facilitate the difficult task of credibly prioritizing and identifying a firm's best possible portfolio of projects in a not-for-profit setting.

1.1 The Problem: How to Identify the Best Possible Prioritization and Portfolio of Not-For-Profit Projects?

Portfolio of project management is the dominant method to achieve desired outcomes of firms globally (Too and Weaver, 2014); including in small to medium enterprises (SMEs) which represent 90% of global firms (Inyang, 2013; Marcelino-Sádaba et al., 2014). The $12 trillion US dollars per annum expenditure on projects (estimated in 2013), the estimated 8% of global GDP in 2014 (Flyvbjerg, 2014), and the propensity of firms to use a portfolio of projects to achieve desired outcomes, has motivated the investigation of techniques to select the best possible portfolio of projects for a firm (Hall et al., 2015; Abbassi et al., 2014). Selection of projects into a firm's portfolio is complex, and ultimately requires the consideration of what portfolio of projects best share the scarce firm resources so as to increase the likelihood of the firm achieving its desired outcome (Martinsuo, 2013; Patanakul, 2015). Poor project prioritization and the selection of the wrong portfolio of projects, to implement in a not-for-profit setting, is still considered an open problem; requiring resolution in the 21st Century in order to reduce the misallocation of scare global resources (Lacerda et al., 2016; Silvius and Schipper, 2014; Sánchez, 2015). The selection of a portfolio of projects that align with corporate strategy is considered a key competitive advantage of the firm (Mikkola, 2001; Cooper et al., 2001; Kaiser et al., 2015); with non-commercial (not-for-profit) project portfolios also aligned to a firm's corporate strategy considered important for the economy and the firm's long-run sustainability (Martinsuo and Killen, 2014; Marcelino-Sádaba et al., 2015). At one extreme the social license to operate motivates some Australian firms to invest in not-for-profit projects (Lyons, 2016) and at the other extreme a moral obligation motivates Italian family owned and operated firms to invest in not-for-profit projects (Campopiano and De Massis, 2015).

In view of this, there is a practical need (namely gap) for the development of a decision support tool to select and prioritize a portfolio of projects in a not-for-profit setting.

Poor project prioritization metrics reduce potential benefits of not-for-profit conservation project investments by 30% to 50% (Pannell and Gibson, 2016). Of these investments, approximately \$20b per annum has been spent on projects to protect imperilled fauna and flora globally (Waldron et al., 2013). This implies an estimated \$6b to \$10b per annum in wasted resources. Furthermore, an estimated \$13.8t to \$23t per annum of resources are wasted by a portfolio projects attempting to maintain the global ecosystem (valued at \$46 trillion per annum as at 2007) (Costanza et al., 2014). This being the case, a 1% portfolio of project management improvement would save at least \$1t per annum of resources dedicated to maintaining the global ecosystem. Specifically, adequate prioritization and selection of the best possible portfolio of projects for each firm can improve the allocation of scarce and valuable resources (thereby reducing wasted resources) and ultimately increasing the likelihood of success, i.e., achieving the desired outcome (Teller et al., 2014).

Therefore, in addition to developing a decision support tool to select and prioritize a portfolio of projects in a not-for-profit setting, one will need to ensure the tool is effective, namely that the decision support tool, with high probability, selects and prioritizes the best possible portfolio of projects.

Portfolio theory has been applied in order to identify a project's 'in or out' status in relation to the firm's project portfolio (Heydari et al., 2016; Kaiser et al., 2015). Conventional valuation techniques, including discounted cash flow (DCF) and real options analysis (ROA), have been applied in an attempt to value portfolios of corporate social responsibility not-for-profit projects (Mooney and Lin, 2014). However, the need to consider both internal and external values to shareholders (in maximizing the firm's value) and stakeholders (with interests beyond maximization of the firm's value) respectively (Jensen and Meckling, 1976) is pivotal to sustainable project portfolio success; where success is not merely project efficiency and satisfied shareholders, but also satisfied stakeholders (Serrador and Turner, 2015). Notably, the external stakeholder's value is absent in conventional valuation techniques (Petro and Gardiner, 2015). Additionally, a more obvious and significant complication of conventional techniques is the difficulty to monetize benefits of not-for-profit projects (Mooney and Lin, 2014). The alternative Cost Benefit Analysis and scoring rule techniques that are used to estimate the value of these benefits can also suffer distortions, e.g., from the diverse background and expertise of the participants informing the estimations (Liao et al., 2015). Furthermore, in uncertain contexts with distributed information beyond the boundary of the firm - e.g., collaborative projects (vom Brocke and Lippe, 2015) - conventional methods fail to aggregate all pertinent information associated with the portfolio of projects (Petro and Gardiner, 2015).

Participatory approaches allowing the continuous revelation of information, over the implementation life of the decision support tool, to select the best possible portfolio of projects is considered invaluable (Nowak, 2013). Prediction and decision markets as decision support tools inherently operate in this way; allowing participation by a diversity of people and continuous revelation of information in its market game implementation (Malone et al., 2009). However, these market type decision support tools only work well if they are information efficient (Roth, 2008), e.g., the real-world Iowa Electronic (prediction) Market (IEM) is not always information efficient and as such not always accurate (Schmitz, 2011). Therefore, the conditions guaranteeing market efficiency are of principal interest (Jackman, 2015; Treynor, 1987; Goodell et al., 2015; Chen and Pennock, 2010) and specifically of central concern to the IEM, i.e., guaranteeing consistently accurate predictions by way of guaranteeing efficient aggregation of information (Berg and Rietz, 2006; Chen and Pennock, 2010; Cowgill et al., 2009). However, new decision support tools, such as the prediction and decision market type investigated in this thesis, can face uptake challenges driven by the absence of quality signals guaranteeing their effectiveness (Scott and Scott, 2016). Hence, a quality signal is both a monitor and guarantee of the accuracy of prediction and decision markets, and logically leads to their increased utilization as a decision support tool.

Therefore, in order to ensure the efficacy of prediction and decision markets, one needs to identify a quality signal that guarantees prediction and decision markets are performing well or not.

In summary, the research gaps that this thesis will address are:

1. The investigation of a quality signal to demarcate the good from bad prediction and decision markets.
2. The use of prediction and decision markets to, with a high probability, correctly select and prioritize the best possible portfolio of projects in a not-for-profit firm setting.

The research questions addressed by this thesis and directly associated with the research gaps are:

1. What is the quality signal for prediction markets?
2. How are decision markets to decide the best possible project portfolio and prioritization built using prediction markets?
3. Do prediction markets have the potential to be successfully applied as a high-quality decision support tool to prioritize and select the best possible portfolio of projects in a not-for-profit firm setting?

1.2 Research design and rationale

The research design and rationale addressing the three research questions (RQ) is presented as follows:

RQ1: What is the quality signal for prediction markets?

A new theoretical model is constructed by reflecting on literature on the theory of prediction and decision markets. This theoretical model identifies that a prediction and decision market characteristic, denoted as relevant information level, is a sufficient and necessary condition for convergence to the best possible prediction and decisions. Although the model setup is idealized, it logically motivates further investigation via computer simulations, games with human participants, and empirical analysis of real-world prediction market data.

RQ2: How are decision markets to decide the best possible project portfolio and prioritization built using prediction markets?

The theoretical model is tested initially in computer simulations (with Matlab) and then in games with human participants (using a fully functioning web-based prediction and decision market webgame built by the researcher). Given that the selection of the best possible portfolio of projects may be reduced to considering whether each project is in or out of the project portfolio, it was sufficient to consider an environment of one project in both the computer simulations and the games with humans. This analysis was then extended to a multi-project setting embodied in Presidential Candidates of the empirical study of this thesis. Finally, a hypothetical setting in the policy implications chapter directly addresses the application of prediction and decision markets to address the more general best possible portfolio project selection problem.

RQ3: Do prediction markets have the potential to be successfully applied as a high-quality decision support tool to prioritize and select the best possible portfolio of projects in a not-for-profit firm setting?

The quality signal (relevant information level) for prediction and decision markets is of central importance in this thesis. Firstly, a computer simulation runs thousands of prediction and decision markets to determine that the quality signal plays a statistically significant role in ensuring the markets achieve the optimal outcome (i.e. the best possible prediction and decision). Secondly, the human behavior element is introduced via multiple web-games with human and computer agents in a control and treatment experimental setup (where a control and treatment differs only in the quality signal value i.e. the relevant information level value); statistical significance again holds. Thirdly, empirical analysis of a real-world prediction market is undertaken to determine if the quality signal is still significant outside of the hygienic laboratory conditions; it is. Finally, a hypothetical scenario elucidates the potential application of prediction and decision markets that possess a good quality signal; providing both a step-by-step guide for implementation and a rigorous investigation of potential policy implications.

1.3 Possible solution and the contribution

This thesis provides an original contribution to research on prediction and decision markets as decision support tools. Specifically, *it provides the quality signal that enables a user to* *demarcate a good (well-functioning) prediction and decision market from a bad one*. This quality signal, called *relevant information level*, is the keystone of this thesis. In short, this thesis constructs a theoretical model and then validates the theoretically derived hypotheses, through experimental and empirical methods, that relevant information level is found to play a statistically significant role in achieving the best outcome and that increasing it also increases the probability of better predictions and decisions.

The thesis title is “the application of prediction markets to project prioritization decisions in the not-for-profit sector” which suggests that this thesis is an investigation into how prediction markets may be configured to prioritize projects in not-for-profit firms. With the review of literature in mind, a quality signal that ensures a well-functioning prediction market to prioritize projects is currently lacking. This thesis theorizes and validates a useful quality signal. Ultimately, high quality prediction markets in this thesis are configured to create a simple decision market to select whether a project is in or out of the best possible project portfolio; the best possible portfolio of projects being the collection of projects, when implemented by a firm, maximizes the probability of achieving the desired outcome of the firm’s shareholder or, depending on the outcome specified, its stakeholders (Jensen and Meckling, 1976).This thesis provides a means to improve current real-world prediction markets that exist in both the public arena and within firms. Specifically, it presents a way to measure the quality (relevant information level) of a prediction market that, when publicized, informs and changes trading behavior. The key desired outcome is, ultimately, to rid the world of poorly performing prediction markets; in much the same way as guarantees and warranties removed the lemons from the used-cars market (Akerlof, 1970). It also advocates the use of prediction markets having implied decision markets; given it is the conditional probability information that firms truly require. That is, a firm is primarily interested in the active selection of projects to maximize the chance of achieving outcomes; rather than passively predicting outcomes.

In general, this thesis suggests that prediction and decision markets benefit from high relevant information levels; which can be achieved by publicizing at regular intervals the relevant information level of a prediction and decision market, e.g., publicizing the daily relevant information level of the Iowa Electronic Market, or every 5 minutes in the case of the simple decision market that was built in this thesis investigation. For example, a publicized low relevant information level (of a prediction market) signals to traders that there exist arbitrage opportunities in the market that may be leveraged via their informed trading. This informed trading, in turn, increases the relevant information level; as arbitrage opportunities are exhausted.

Finally, a way to utilize the simple decision market game is demonstrated by way of a hypothetical example. The example provides a justification (building upon the validation of the significance of relevant information level undertaken in earlier chapters), and also serves as a simple and concrete step-by-step guide describing how to prioritize not-for-profit projects using the decision market game.

1.4 The Structure of the Thesis

This thesis comprises seven chapters. Chapter 2 is a literature review that provides the reader a concise foundational overview of relevant literature and elaborates further on the justification for the core research problems and subsequent theoretical and experimental investigations. Chapters 3 to 7 each provide the necessary and more comprehensive review of literature that precedes that chapter's specific investigation.

Chapter 3 sets out a theoretical model that *reveals relevant information level as a sufficient and necessary condition for well-function prediction and decision markets* of the specific type described in that chapter[2]. Importantly, it provided theoretical justification to pursue subsequent experimental and empirical analysis to validate the significance of relevant information level.

Chapter 4 describes the comprehensive series of computer simulations of prediction and decision markets of the type described in Chapter 3. It both circumvents the need to establish a difficult analytic solution and also simplifies statistical analysis via the very large number of prediction and decision markets runs. Ultimately, it establishes that relevant information level plays a statistically significant role.

Chapter 5 requires the development of a prediction market web-game to experimentally test the significance of relevant information level when humans participate. This game incorporates both algorithmic and human traders. The addition of human traders is the logical next step from the purely algorithmic setting of Chapter 4. 30 'control' games were paired

[2] This chapter was published in the peer reviewed *Journal of Prediction Markets*.

with 30 'treatment' games; the only difference between a control and treatment pairing being the relevant information level. The response of interest was the convergence of the control and treatment to the same prediction. If they differed, it was logically due to relevant information level. *Relevant information level was found to play a statistically significant role* in this series of games with human participants.

Chapter 6 is an analysis of data from a real-world prediction market. Because Chapters 4 and 5 are experiments in laboratory conditions, a real-world analysis is warranted to determine whether relevant information level also plays a statistically significant role in a real-world prediction market. The Iowa Electronic (prediction) Market (IEM) data on the 2008 US Presidential Elections are analyzed. Importantly, the data contained an implied decision market that unambiguously reported the best possible Presidential Candidate on each day prior to Election Day. Of interest is the probability of the correct implied decision as a function of the proxy for relevant information level. A logistic regression is performed that also includes other control variables informed by a literature review presented in the chapter. The best specification is achieved by employing forward and backward stepwise regression alongside the log likelihood ratio. Importantly, the best specification undergoes post-hoc tests (to ensure assumptions of normality hold and a continuity correction factor is not required) and also controlled for endogeneity and confounding. Ultimately, this chapter finds that *relevant information level in a real-world prediction market indeed plays a statistically significant role* and, in so doing, also *provides a means to measure relevant information level*.

Chapter 7 specifies the policy implications and conclusions of this thesis. By way of an example, in the form of a hypothetical not-for-profit Indigenous Australian firm, it provides comprehensive justification of a simple step-by-step approach to prioritizing the best possible portfolio of projects. This is done to transparently justify and facilitate the rollout of prediction and decision market games (of the type advocated by this thesis) as decision support tools in not-for-profit firm settings. Importantly, so long as high relevant information level exists, the decision market game identifies and prioritizes the best possible portfolio of projects for a given budget so as to maximize the likelihood of achieving the desired outcome.

By way of qualification, it should be noted that the prediction and decision market game of this thesis could be employed beyond the not-for-profit setting. It is ultimately a simple decision support tool to identify the best portfolio of projects *for any firm*. However, the interest and portfolio of project management work experience of the researcher lies primarily in the complex not for profit project selection space, the three investigations to test the theoretical model were deliberately contextualized as portfolio of project selection problems, and finally a post-doctoral implementation and evaluation of the prediction and decision markets of this thesis to portfolio of project selection in a real-world not for profit setting was sought by the researcher and at the time of writing exists in a University-Indigenous Community Health organization joint venture. Importantly, the prediction and decision market of this thesis is a decision support tool that provides a measure of the quality of its output in the form of the reported relevant information level. For example, if the prediction and decision market game is implemented in a firm and the highest relevant information level is achieved, then it is highly likely that the best possible prioritization and portfolio of projects has been identified by the game.

In conclusion, it is hoped that the quality measure (provided by this thesis) of the prediction and decision market as a decision support tool leads to an increase in their uptake; to credibly prioritize the best possible portfolio of not-for-profit projects and ultimately reduce wasted resources resulting from poor project selection.

Chapter. 2 Literature Review

"Prediction is very difficult, especially if it's about the future."
Niels Bohr

Key Message of Chapter:

- **Literature pertinent to this thesis is reviewed**
- **Subsequent chapters focus on particular research areas and extend the literature review to greater depths**
- **This chapter provides a concise and coherent consolidation of key ideas of thesis relevant literature in one place**

A concise review of literature relevant to the central problematic of the entire thesis is presented in this chapter. Subsequent chapters provide more in-depth, subject and research question specific, detail relating to their particular investigations.

The main ideas reviewed in the literature and relevant to this thesis are presented in this chapter as follows:

1. Resource misallocation and the problem of project portfolio management
2. Conventional decision support tools and their inapplicability to the not-for-profit setting
3. Prediction and decision markets and their applicability to the not-for-profit setting
4. Theoretical models to understand prediction and decision markets
5. Computer simulations, games with humans and real-world analysis to investigate prediction and decision markets
6. Policy parameters for prediction and decision markets

These six areas of research literature tie back to the problem this thesis is ultimately addressing and its associated research questions in Chapter 1.

Ultimately, this thesis is attempting to address the problem of resource wastage due to poor portfolio of project selection and prioritization. This thesis is basically advocating as a solution, high quality prediction and decision markets. To this end, a description of the problem of waste in portfolio of project management is required and delivered in section 2.1 of this chapter i.e., resource misallocation and the problem of project portfolio management. Because this thesis is advocating a specific prediction and decision market approach, other approaches are considered in section 2.2 of this chapter i.e., conventional decision support tools and their inapplicability to the not-for-profit setting. A review of research that has already utilized prediction and decision market in a not-for-profit setting is necessarily undertaken in section 2.3 of this chapter i.e., prediction and decision markets and their applicability to the not-for-profit setting. Of significant interest are the established theoretical models, that inform the new theoretical model of this thesis, and that are reviewed in section 2.4 of this chapter i.e., theoretical models to understand prediction and decision markets. This thesis tests the implied hypothesis of the new theoretical model (detailed in Chapter 3) by using computer-simulated experiments, games with human participants and real-world prediction market data analysis; hence section 2.5 of this chapter reviews the related research literature i.e., computer simulations, games with humans and real-world analysis to investigate prediction and decision markets. Ultimately, the useful mandate of the thesis is to suggest changes in policies of existing prediction and decision markets and to inform their future design. As such, a review on prediction and decision market policies is undertaken in section 2.6 of this chapter i.e., policy parameters for prediction and decision markets.

2.1 Resource Misallocation and The Problem of Project Portfolio Management in Not-For-Profit Firms

A review of research associated with 'resource misallocation and the problem of project portfolio management in not-for-profit firms' is important to undertake in order to establish the size and nature of the problem that this thesis addresses. This review connects directly to the research problem of suboptimal portfolio of project selection. Specifically, the enormity of wasted resources due to poor portfolio of project selection motivates the investigation of the new approach proposed in this thesis.

Portfolio of project management is the main way firms (globally) achieve outcomes (Too and Weaver, 2014); with small to medium enterprises (SMEs) representing 90% of global firms (Inyang, 2013; Marcelino-Sádaba et al., 2014). The per annum spend on projects was estimated at $12 trillion US dollars in 2013 and 8% of global GDP in 2014 (Flyvbjerg, 2014). An estimated $20b per annum has been spent on not-for-profit conservation projects globally (Waldron et al., 2013) to maintain the global ecosystem services valued at $46 trillion per annum (Costanza et al., 2014), but poor project prioritization has caused 30% to 50% of wasted economic value (Pannell and Gibson, 2016). The recognition of this wastage of scarce global resources resulting from poor project portfolio management (Lacerda et al., 2016; Sánchez, 2015; Hall et al., 2015; Abbassi et al., 2014) has led to a focus on how to prioritize and select the best possible portfolio of projects in not-for-profit settings (Lacerda et al., 2016; Silvius and Schipper, 2014; Martinsuo, 2013; Patanakul, 2015). Firms are incentivized to select a best possible portfolio of projects that aligns with its corporate strategy in order to secure a sustainable competitive advantage (Mikkola, 2001; Cooper et al., 2001; Kaiser et al., 2015; Martinsuo and Killen, 2014; Marcelino-Sádaba et al., 2015; Teller et al., 2014).

2.2 Conventional Decision Support Tools and Their Inapplicability to The Not-For-Profit Setting

A review of research associated with 'conventional decision support tools and their inapplicability to the not-for-profit setting' is important undertake given that the new prediction and decision market (proposed in this thesis) is envisaged to be utilized as a decision support tool instead of conventional ones in firms running not-for-profit projects. This ties back to the researcher's experience in managing projects and portfolio of projects where leveraging conventional decision support tools proved inadequate. The review of literature in this section provides insight beyond the researcher's experience and more broadly validates the inadequacies of conventional tools applied to not for profit project settings.

Decision support tools are defined as tools that integrate multiple pieces of information to assist in decision making (Bagstad et al., 2013); a subset of which are applicable to the not-for-profit setting (Eom et al., 1998). They can be categorized in many different ways depending on the criteria (or dimensions) defining each category (Ness et al., 2007). Complicating the categorization exercise further, they may be integrated to produce new families of integrated decision support tools (Oxley et al., 2004). To draw out thesis-relevant ideas in a concise way in this chapter, decision support tools are categorized into 3 broad categories: (1) ratio analysis tools; (2) monetary valuation tools; and (3) game theory tools.

Ratio analysis tools provide information comparing the quantity of something with something else. For example, DuPont analysis provides a means for management to consider and then change operational characteristics of interest in their firm (Soliman, 2008). A manager interested in increasing the gross profit to asset ratio (i.e., a return on asset measure), could choose to decompose this as the product of the gross profit to sales ratio and the sales to asset ratio; the former ratio being called the gross profit margin and the latter being called the asset turns ratio. Increasing either the gross profit margin or the asset turns are different strategies that lead to increasing the return on investment. In a not-for-profit organization the benefit to cost ratio is typically applied to inform decisions by firstly attempting to transform and aggregate multiple benefits and costs into monetary amounts (a somewhat difficult and subjective exercise), and then comparing the benefit to cost ratio of various alternative decisions to find the greatest one to implement (Ackerman and Heinzerling, 2002). The subjectivity involved in monetizing non-market benefits and costs have resulted in the complete removal of the monetization step whereby a management discussion of benefits and costs culminates in a consensus decision, e.g., SWOT analysis (Leigh and Pershing, 2006). However, the absence of the monetization step creates a vacuum that is filled by arbitrary weights that are used to construct an ambiguous ratio informing decisions (Leigh and Pershing, 2006).

Valuation tools focus primarily on assigning monetary value to alternative decisions (Bagstad et al., 2013). A popular valuation tool used to monetize market tradable assets, albeit inconsistently applied, is the discounted cash flow (DCF) valuation (Oded and Michel, 2007). Extending the DCF technique further, non-market valuation tools are used to value assets not explicitly traded in a market place (Gómez-Baggethun et al., 2010). Such non-market valuation techniques include hedonic pricing, travel cost method, contingent valuation and choice experiments[3]. They all assume an underlying random utility model which, when applied to revealed or stated preference data, implies trade-offs and a valuation of the non-market traded good.

Theoretically, valuation is possible irrespective of whether assets are traded in a market or not; so long as their utility is able to be inferred (Arnold and Shockley, 2002). However, utility theory underpinning valuation theory suffers from logical inconsistencies when attempting to select the best decision for a society (Arrow, 1950). In response to this fundamental problem, the idea of Pareto optimality (and Kaldor-Hicks optimality) gained prominence (Coleman, 1980; Baujard, 2013). Pareto optimality entails making decisions that lead to an economic gain in value by at least one person without incurring an irreversible economic value loss to anyone else (Blaug, 2007). Despite the prevalent use of valuation, a key criticism of valuation tools has been that not all things can or should be monetized in an anthropocentric way (Brennan, 1992; Baujard, 2013), e.g., the value of ecosystem services

[3] See for example Gómez-Baggethun et al., (2010) for a historical review of non-market valuation and related concepts.

should not only be measured in terms of utility to humans (Gómez-Baggethun and Ruiz-Pérez, 2011).

Game theory may be seen as a generalization of valuing the best decisions; whereby value to both human and nature (Kadane and Larkey, 1982) can be considered, and money is only one of numerous scores to measure the impact of decisions (Von Neumann and Morgenstern, 2007; Nash, 1953; Nalebuff and Brandenburger, 1997). However, the sometimes arbitrary and subjective assignment of probabilities (Kadane and Larkey, 1982) and scores (or payoffs) remains a limitation of conventional game theory (Gneiting and Raftery, 2007; Guala, 2006). Game theory is used both to determine the best behavior (or strategy) for a player to enact to achieve a desired outcome in a given setting (Nash, 1953), or to design the best possible rules to incentivize specific player behaviors (Papadimitriou, 2001). The latter - called inverse game theory or mechanism design (Papadimitriou, 2001) - has been used to elicit and aggregate decision relevant information (Aumann, 1976; Harsanyi, 2004; Samuelson, 2004), e.g., as is implemented in (arguably non-conventional game theoretic) real-world prediction markets (Camerer and Fehr, 2006; Thompson, 2012; Chen and Pennock, 2010).

2.3 Prediction and Decision Markets and Their Applicability to The Not-For-Profit Setting

Whilst, prediction and decision markets were seen as having great potential to the researcher's projects' management work, the large project investments and the uncertain quality of prediction and decision markets was considered too much of a risk to implement. There is a deliberate line of connection with research question 1 (what is the quality signal for prediction markets?); for having the ability to determine the quality of a prediction and decision market output, project management will employ them with greater confidence. This section provides a view on the applicability of current prediction and decision markets to the not-for-profit project setting.

Real-world evidence suggests that behavioral assumptions in valuation and game theory decision support tools, being the maximization of individual utility and profit, do not typically hold (Kahneman, 2003; Van Den Bergh et al., 2000; Conitzer, 2009). However, prediction and decision markets are premised largely on mechanism design, in a market game, which elicits and aggregates information distributed across players (traders) to ultimately determine a best possible prediction or decision (Conitzer, 2009; Plott and Chen, 2002). Whereas prediction markets attempt to elicit the best possible predictions, decision markets attempt to elicit the best possible decisions (Hanson, 1999; Othman and Sandholm, 2010). Both are a generalization of the stock market and trade stocks on any random event (Wolfers and Zitzewitz, 2004); including predicted company performance (Plott and Chen, 2002) that the stock market is dedicated to.

The valuation approach has been utilized in an attempt to identify the best corporate social responsibility not-for-profit project portfolio (Mooney and Lin, 2014; Heydari et al., 2016; Kaiser et al., 2015; vom Brocke and Lippe, 2015; Petro and Gardiner, 2015), however, the need to consider internal and external values to shareholders and stakeholders (Serrador and Turner, 2015; Jensen and Meckling, 1976) are not catered for (Petro and Gardiner, 2015). The difficulty to quantify not-for-profit projects generally (Mooney and Lin, 2014) impairs the effectiveness of ratio methods and some game theory (e.g., scoring rule) approaches (Liao et al., 2015). In contrast, prediction and decision markets are decision support tools that encourage best practice participation of a diversity of people to continuously aggregate and elicit information to ultimately identify the best possible prioritization and project portfolio (Malone et al., 2009; Nowak, 2013; Hahn and Tetlock, 2005); including in a not-for-profit setting (Arrow et al., 2008). However, such market-based decision support tools fail dismally when market efficiency does not hold (Roth, 2008; Schmitz, 2011). Therefore, a (quality signal) guarantee of prediction and decision market efficiency is considered important (Jackman, 2015; Treynor, 1987; Goodell et al., 2015; Chen and Pennock, 2010; Berg and Rietz, 2006; Cowgill et al., 2009) and a quality signal (Akerlof, 1970) that would likely lead to their increased use (Scott and Scott, 2016).

2.4 Theoretical Models to Understand Prediction and Decision Markets

A review of research associated with ‘theoretical models to understand prediction and decision markets’ is important to ensure that the model proposed in this thesis is indeed new and also builds upon the strong foundations already laid in theoretical prediction and decision market research. There is a line of connection between this section and the first two research questions. Research question 1 (What is the quality signal for prediction markets?) is closely aligned to prior research on aggregation and accuracy in prediction and decision markets. Research question 2 (How are decision markets to decide the best possible project portfolio and prioritization built using prediction markets?) is adjacent to observations in reviewed research on implied decision markets in election candidate markets; effectively the candidate selection problem is a homomorphism of the project selection problem.

Publications on prediction and decision markets (also called information markets) has increased steadily since 1990 (Horn et al., 2014; Tziralis and Tatsiopoulos, 2012). Whilst there exists much experimental and real-world evidence of the effectiveness of prediction markets (Berg et al., 2008b; Berg et al., 2008a; Arnesen and Bergfjord, 2014), there was envisaged a need to provide a comprehensive theoretical foundation to reveal why they worked so effectively (Chen et al., 2006; Chen et al., 2004). Set theoretic representations of common knowledge and private information (Aumann, 1976) was modified to analyze a market setting (McKelvey and Page, 1986; Nielsen et al., 1990) with information certainty (Feigenbaum et al., 2003) and uncertainty (Chen et al., 2004). This culminated in a proof that independent and identical distribution across traders' private information is sufficient for a prediction market to attain the best possible prediction (Chen et al., 2004). The best possible prediction was associated with a market equilibrium - the direct communication equilibrium (DCE) - reached by the second round of trading; with DCE being the market equilibrium that would be obtained in the perfect scenario where all traders revealed private information to one another prior to bidding (Chen et al., 2004). However, the zero sum nature of such prediction market games means that no trade would have logically occurred in the first place (Milgrom and Stokey, 1982). As such, automatic market makers were introduced into prediction markets to ensure liquidity of trade by providing all other traders the opportunity of economic gain (Hanson, 2003; Chen et al., 2010); albeit at the bounded loss of the market maker (Hanson, 2003). Research into these automatic market makers (AMMs) developed market scoring rule prediction market models where all traders were required to trade with AMM's market maker algorithm (Hanson, 2012); instead of the usual stock market like continuous double auction theoretical prediction market models that did not require this (Chen et al., 2004). Both types of prediction and decision market models typically assumed rational, risk-neutral and myopic incentive compatible traders; an assumption justified by principles akin to complicated strategic reasoning outweighs the associated negligible strategic benefits (Chen et al., 2006). However, extending to strategic behavior models (Chen et al., 2007) whilst still eliciting truthful information was still attempted (Chen et al., 2010).

Decision markets are prediction markets that imply conditional probabilities for decision-making (Berg and Rietz, 2003), e.g., the 1996 Presidential Election Iowa Electronic Market whereby probabilities of the Presidential party winning given (or conditional on) each of the Party's Candidate was implied (Berg and Rietz, 2003). Theoretical models of decision markets (Chen and Kash, 2011) suffer from logical inconsistencies (Pennock and Sami, 2007), e.g., the market scoring rule decision market requires randomly choosing a decision even when the best decision is known (Chen et al., 2011; Chen et al., 2014). Of interest to this thesis is the alternative, joint elicitation decision market, in which "decision stocks" and "outcome and decision stocks" are traded to elicit conditional probabilities and therefore the best possible decision (Othman and Sandholm, 2010).

2.5 Computer Simulations, Games with Humans and Real-world Analysis to Investigate Prediction and Decision Markets

A review of research associated with 'computer simulations, games with humans and real-world analysis to investigate prediction and decision markets' is important to undertake; given that all three approaches are used to test the hypotheses derived by the new theoretical model proposed by this thesis. There is also a direct link with the three research questions.

Research question 1 (What is the quality signal for prediction markets?) is a hypothesis arising from the theoretical model i.e. relevant information level[4] is the quality signal. However, research question 1 and the statistical significance of the quality signal is tested in control treatment setups of both computer simulations (with thousands of markets run) and 60 prediction market web-games with human participants. Analyzing empirical data from a real-world prediction market also tests for the statistical significance of the quality signal. Research question 2 (How are decision markets to decide the best possible project portfolio and prioritization built using prediction markets?) is tested for in both computer simulations and real-world data. Research question 3 (Do prediction markets have the potential to be successfully applied as a high-quality decision support tool to prioritize and select the best possible portfolio of projects in a not-for-profit firm setting?) is considered in the analysis of a real-world political prediction market. The combination of computer simulations (allowing thousands of prediction and decision market runs), games with humans (divulging the exact effect of human idiosyncrasies due to controlled laboratory conditions), and the analysis of real-world empirical data (going beyond the laboratory confines to truly test the quality signal statistical significance), is logical, and in aggregate methodologically strong.

Testing the validity of the theoretical models may be undertaken via computer simulations, experimental games with human participants and by analyzing real-world prediction market data (Smith, 1989; Davis et al., 2007; Klingert and Meyer, 2012; Berg et al., 2008a).

[4] This will be defined comprehensively in chapter 3 but is simply the proportion of traders in a market whose bids incorporate their private information.

Computer simulations have found that the logarithmic market scoring rule (LMSR) prediction markets do not effectively adapt to market liquidity level changes (Brahma et al., 2010). Computer simulations with zero intelligence traders (i.e., those who buy and sell randomly) suggest that expert traders are not necessary in a prediction market (Othman, 2008). The artificial intelligence component intrinsic to prediction market computer simulations have encouraged the use of prediction market computer simulations as decision support tools; in a similar fashion to other automated algorithms, e.g., genetic algorithms (Jahedpari et al., 2014).

Decision support tools leveraging computer simulations are typically an interaction between human operators and computer algorithms; a hybridization that has been termed centaurs (Shrier et al., 2016). These hybrid human-machine systems are found to outperform human-only and machine-only systems (Chen et al., 2008b). Specifically, prediction markets have been considered the pinnacle embodiment of centaur-like systems (Nagar and Malone, 2011).

The stock market may also be termed a centaur system; given 80% of all stock market trades in 2010 were placed by computer algorithms (Lu, 2016). The Iowa Electronic (prediction) Market (IEM) is also a centaur mix of algorithmic and human traders (Schmitz, 2011). Contrary to previous beliefs developed for market settings, high frequency algorithmic traders have no competitive speed advantage over human traders (Moosa and Ramiah, 2015). In short, prediction markets are simply games, incentivizing play by algorithmic and human traders, to solve complex prediction and decision problems (Heiko et al., 2015). Importantly, prediction markets incentivize information sharing and learning amongst traders to ultimately aggregate important information (Balkenborg and Kaplan, 2010; Van der Wal et al., 2016). Prediction market games with humans and algorithms are considered a fruitful avenue for further study (Schlag et al., 2015), e.g., to answer such questions as why is it that prediction market web games provide excellent probabilistic forecasts (Pennock et al., 2001).

Mechanism design underlies the construction of a prediction market and deliberately incentivizes the elicitation of information (Maskin, 2008; Myerson, 2008; Conitzer, 2010); as such, understanding how to control for human effects is considered crucial (Fountain and Harrison, 2011). In short, it attempts to reduce misinformation introduced by strategic agents attempting to profit by manipulating the beliefs of others via bluffing (Conitzer, 2009), or that introduced by bias, e.g., Google's internal prediction market found an 'optimism and proximity' bias (Cowgill et al., 2009). Prediction markets that incentivize a dominant myopic (non-strategic) behavior appear to have reduced the number of strategic traders and their effects (Dimitrov and Sami, 2008). Despite the short term effects of strategic traders on market accuracy (Buckley and O'Brien, 2015) in the medium to long term the strategic mispricing are arbitrage opportunities that are corrected-for, resulting in increased liquidity and accuracy (Hanson et al., 2006). Real money increases the utility of humans (Fishburn, 1968) and was initially considered a key incentive for real money prediction markets (Servan-Schreiber et al., 2004); albeit illegal in some jurisdictions (Arrow et al., 2008). However, play money is found to be just as effective as real money in real-world public prediction markets (Servan-Schreiber et al., 2004) and in real-world internal corporate prediction markets (Siegel, 2009). Training humans to successfully play prediction markets is also important (Siegel, 2009), as is providing a simple web-game interface to prevent cognitive dissonance (Gaspoz and Pigneur, 2008). Thus, the balance of information transparency and human cognitive capacity needs to be found for an effective prediction market game with human participants (Yang et al., 2015; Kranz et al., 2015; Teschner et al., 2015).

Real-world prediction markets are populated by error prone human traders (Hansen et al., 2004). Despite the voter's paradox (Abrams, 1976) whereby any single vote or bet will make a negligible difference to the outcome (e.g., a single voter's effect on the U.S. Presidential outcome is calculated as approximately 1 in 10 million (Gelman et al., 2012)) and at best negligibly influences close elections (Strijbis et al., 2016), real-world prediction markets such as the IEM are very liquid, e.g., trading US Presidential Election stocks (Berg et al., 2008a). Additionally, many theoretical prediction market models have made the simplifying marginal trader hypothesis assumption (Forsythe et al., 1992) despite real-world empirical studies contradicting it (Blackwell and Pickford, 2011). Prediction markets are also theorized to be arbitrage free (Buckley and O'Brien, 2015; Hanson and Oprea, 2009); despite arbitrage opportunities observed in the IEM that possibly exist because of human limitations to perfectly exploit them (Schmitz, 2011). Market efficiency is also presumed in theoretical settings (Treynor, 1987) despite the inefficiencies observed in real-world settings (Herschberg, 2012). Specifically, the efficient market hypothesis does not ubiquitously hold across real-world prediction markets, e.g., the IEM cannot guarantee efficient aggregation of information for accurate predictions (Berg and Rietz, 2006; Manski, 2006). Moreover, illiquid (or thin) markets are rampant in real-world prediction markets; ultimately distorting prices and predictions (Chen and Pennock, 2012; Dudik et al., 2012). In short, the key observation is that, in contrast to idealized market models, in real-world markets humans are only approximately rational (McFadden, 2009; Ali, 1977).

Despite the differences between real and idealized prediction markets, real-world prediction markets are extremely accurate, e.g., the IEM has outperformed other prediction mechanisms - including polls and experts(Berg et al., 2008a) - most of the time (Berg et al., 2008b; Wang et al., 2015).

2.6 Policy Parameters for Prediction and Decision Markets

A review of research associated with ‘policy parameters for prediction and decision markets’ is of particular interest when considering the potential applications of this thesis’ findings. Importantly, literature associated with research question 3 (Do prediction markets have the potential to be successfully applied as a high-quality decision support tool to prioritize and select the best possible portfolio of projects in a not-for-profit firm setting?) informs the hypothetical scenario of Chapter 7. Specifically, the hypothetical application of prediction and decision markets of the type advocated in this thesis (i.e., those with high relevant information level as the quality signal) is made believable by leveraging literature relating to prediction and decision market policy implications.

Firms are the dominant force in global resource allocations; typically implemented through projects (Vishnevskiy et al., 2015). As such, project governance, prioritization, and selection, are key to efficient and effective management (Shleifer and Vishny, 1997; Porter, 1989; Brandenburger and Nalebuff, 1995; Crossan, 2005). In the Indigenous (Australian) setting SMEs are typically not-for-profit in nature (Berkes et al., 2000) and structured as either public-private-partnership, social enterprise, or impact investments, implementing a portfolio of projects to achieve multiple objectives (Lehner and Nicholls, 2014; Pathak and Dattani, 2014). In such difficult settings the usual rules of thumb (Loomes, 1998) are usefully complemented by multiple decision support tools (Heiko et al., 2015; Clemen, 1989). However, despite the burgeoning of computer enabled decision support tools, resulting from the cross-fertilization of economics and computer science (Heiko et al., 2015; Bonney et al., 1999), uptake of decision support tools is hindered by the lack of quality signals guaranteeing their output (Scott and Scott, 2016).

Firms have employed prediction markets as decision support tools to prioritize and select projects (Ho and Chen, 2007). However, barriers have been the misalignment with the culture of the firm (Buckley, 2016), inadequate prediction training on how to use prediction markets (Siegel, 2009), regulation prohibiting real money prediction markets (Arrow et al., 2008), and public repugnance of bets on certain prediction outcomes (Roth, 2007; Hanson, 2006). Prediction markets in firms also typically have a low number of traders and function better with unbiased and well-informed prediction market traders (Healy et al., 2010; Blackwell and Pickford, 2011; Milgrom and Weber, 1982; Repo, 1989; Chen and Pennock, 2010; Cowgill et al., 2009). Although prediction markets are renowned for their ability to predict epidemics (Li et al., 2016), electricity demand (Cramton and de Castro, 2009), market capitalization (Berg et al., 2009), inflation rate (Leigh and Wolfers, 2007), and weather (Shrier et al., 2016), of recent significant importance are prediction market games as decision support tools for project prioritization and portfolio management in firms (Gaspoz and Pigneur, 2008; O'Leary, 2011; Faghihi et al., 2015; Pennock et al., 2001; Schlag et al., 2015). However, like all new decision support tools, the benefit of the prediction markets over conventional methods needs to be considered on a case by case basis by each firm (Goel et al., 2010); even though their performance has exceeded pooled experts (Chen et al., 2005) and provide the added advantage of continuous updating to management (Goel et al., 2010).

Poorly performing prediction market decision support tools are possible (Fountain and Harrison, 2011) and those without conditional probabilities implying decisions are not relevant to the firm setting (Berg and Rietz, 2003; Hanson, 2006). Additionally, uncertainty and long time horizon not-for-profit projects complicate prediction markets (Pathak and Dattani, 2014). Hence, a user-friendly design (Kranz et al., 2015; Teschner et al., 2015; Van der Wal et al., 2016) that guarantees fast (Chen et al., 2015), and efficient aggregation of information is important (Myerson, 2008; Dimitrov and Sami, 2008). Of particular interest is how prediction markets, as decision support tools, can solve the combinatorial hard problem of prioritizing and selecting the best possible portfolio of projects for the firm (Feigenbaum et al., 2009; Chen et al., 2008c; Chen and Pennock, 2012; Xia and Pennock, 2011; Hanson, 2003). Real-world (Dudik et al., 2012) and experimental (Powell et al., 2013) evidence is only suggestive that prediction markets are able to solve such a difficult problem.

Although independent and identical distribution of information across the prediction market is theoretically sufficient for convergence of the prediction market to the best possible prediction (Chen et al., 2004; Surowiecki and Silverman, 2007), real-world prediction markets have not required independence to effectively converge to the best possible prediction (Escoffier and McKelvey, 2015). Market efficiency has served as the measure of the effectiveness of markets (Roth, 2008), e.g., there is concern that IEM is not information efficient (Berg and Rietz, 2006; Schmitz, 2011). Therefore, an investigation into a quality signal guaranteeing prediction market efficiency is logically motivated (Jackman, 2015; Goodell et al., 2015; Berg and Rietz, 2006).

Ultimately, the quest to deepen understanding of why prediction and decision markets (i.e., prediction markets eliciting conditional probability information) work, and how to measure their performance is considered necessary but complex (Chen et al., 2008a; Damnjanovic et al., 2012; Tetlock et al., 2005; Slamka et al., 2013). This thesis investigation into the application of prediction markets to project prioritization problems in not-for-profit firms, is motivated by the quest to address this core problematic. The researcher begins this journey by introducing a new prediction and decision market theoretical model; one that identifies a quality signal measure of prediction and decision market called relevant information level (Grainger et al., 2015). Subsequent steps entail validating the model via simulations and experimental and empirical evidence. The quest concludes with a consideration of the policy implications of the thesis' findings.

Chapter. 3 A Simple Decision Market Model

"When the facts change, I change my mind. What do you do, Sir?"
John Maynard Keynes

Key Message of Chapter:

- **A new theoretical model for prediction and decision markets is built**
- **If all traders, in a prediction market, express relevant information in their bids, then this is a sufficient and necessary condition for a prediction market to identify the "best possible prediction"**
- **Research question 1 is addressed by theorems motivating a hypothesis that relevant information is an important quality signal for real-world well-functioning prediction and decision markets**

This chapter was published in The Journal of Prediction Markets in 2015[5]. Fundamentally, it proposes a decision market built using multiple prediction markets with specific market rules that ensure the best possible decision is identified[6]. Importantly, it ties back to research question 1 and theoretically motivates a hypothesis that relevant information is the quality signal for well-functioning prediction and decision markets.

Economic modeling of decision markets has mainly considered the market scoring rule setup. Literature has made reference to the alternative, joint elicitation type decision market, but no in-depth analysis of it appears to have been published, to the best of my knowledge. This chapter develops a simple decision market model of the joint elicitation type, that provides a specific decision market nomenclature on which to base future analysis.

A generally accepted prediction market model is modified, by introducing two additional concepts: "proper information market" and "relevant information". This work then provides original contributions to the theoretical discourse on information markets, including finding the sufficient and necessary condition for convergence to the best possible prediction. It is shown in this new prediction market model that "all agents express relevant information" is a sufficient and necessary condition for convergence to the direct communication equilibrium in a proper information (prediction) market.

[5] Referenced as: GRAINGER, D., SUN, S., WATKIN-LUI, F. & CASE, P. 2015. A simple decision market. Journal of Prediction Markets, 9, 41-63.
[6] The specifics of this, including the definition of best possible decision, are contained in the following.

This new prediction market model is used to formulate a simple decision market model of the joint elicitation market type. It is shown that this decision market will select the best decision if a specific selection and payout rule is defined. Importantly, this new decision market model does not need to delay payment of any contracts to the observation of the desired outcome. Therefore, when dealing with long-term outcome projects, the new decision market does not need to be a long running market.

Future work will test for the statistical significance of relevant information (identified as important in this idealized new decision market model) in laboratory and real-world settings.

3.1 Introduction

This chapter reviews and extends the theoretical models of decision markets[7]. A generally accepted prediction market model is modified and a sufficient and necessary condition for it to provide a best possible prediction of future events is derived. This new prediction market model is used to build a simple decision market model that will always select the best possible decision for anyone using it as a decision support tool.

The research contained in this chapter provides three original contributions to the theoretical discourse on information markets: (1) formulating a prediction market model with a *sufficient and necessary* condition for a well-functioning prediction market, (2) creating a simple decision market model with a deterministic decision selection rule (as opposed to a mixed strategy decision selection rule required in a prominent decision market type discussed below) and (3) showing that this decision market does not need to operate for as long as the projects it analyses (and hence short term decision markets can assist in decision-making of long term projects).

The theoretical implications in this chapter assume idealized rational, risk-neutral and myopic traders populate information market models. This is not only done because it is the typical approach taken throughout literature when modeling such settings, but also because of the significant advantage gained. Just as the law of gravity assumes no air resistance, the novel information markets of this chapter assume no strategic or risk-averse traders exist. These idealized assumptions expedite the revelation of key insights for both gravity and information markets otherwise lost in intractable problems. That said, it is useful to test these insights

[7] Decision markets are a generalization of financial markets whereby random variable events are listed as assets and asset prices are used to derive conditional probabilities used in decision-making.

within settings that are consistent with real-world dynamics. This is done in subsequent Chapters 4 to 7.

The main theoretical contribution to the prediction market literature, made easy by the deliberate and novel choice of mathematical nomenclature, is placed in Appendices: Chapter 3 Appendix 1. The body of this paper then provides a narrative account of the ‘big ideas’ behind the mathematical formalism. This is done in an attempt to maximize the accessibility of this work to the broader readership; it also offers transparency and completeness by including the details of this original piece of research in the appendix. Section 3.2 reviews the related literature concerning theoretical information markets with an emphasis on decision markets. Section 3.3 reviews a generally accepted prediction market model that is modified to formulate the key results of this chapter. Section 3.4 introduces the concept of “proper market price” underpinning the definition of “proper information market”. The notion of “relevant information” is also introduced. Theorems 1 and 2 find that “all agents express relevant information” is a sufficient and necessary condition for convergence to the best possible prediction in a proper information (prediction) market. The new prediction market model is extended in Theorem 3 to a context with multiple stocks. With the multiple stocks context in hand a new simple decision market model is developed via Theorems 4 to 7. Section 3.5 discusses findings and section 3.6 concludes with suggested future research.

3.2 Related Work

Information markets continue to be of interest to researchers, with the number of articles published per year steadily increasing since 1990 (Tziralis and Tatsiopoulos, 2012). Whilst there exists much experimental and real-world evidence of the effectiveness of information markets, there was envisaged a need to provide a comprehensive theoretical foundation to reveal why they worked so effectively (Chen et al., 2006).

The following review of related work considers the early theoretical work on information markets. This leads naturally to the dominant Boolean models of prediction markets and the important issue of designing price formation mechanisms that are consistent with incumbent agent behavior. The current theoretical formulations of decision markets are surveyed and an impetus for this chapter, rigorously exploring joint elicitation decision markets, is revealed.

3.2.1 Early theoretical work on information markets

The seminal work of Aumann (1976) on knowledge and information arguably provides a pioneering formalism for information markets. Aumann defined an event as *common knowledge* if two agents are present at that event and they see each other present at that event (Aumann, 1976). This definition, made more rigorous by an associated set theoretic formalism, led to the analysis of multiple agents with private information trading in a marketplace. McKelvey and Page discovered that when a stochastically regular aggregate statistic is common knowledge, an equilibrium is reached after finite rounds of announcements and the posterior probabilities of all agents become identical (McKelvey and Page, 1986). These *information* and *equilibrium* ideas form the theoretical foundations of prediction (information) markets and decision (information) markets that reach the 'best prediction' equilibrium and 'best decision' equilibrium.

3.2.2 Boolean finite state space models of prediction markets

Theoretical prediction markets have been modeled as computational processes that aggregate and process distributed bits of information and hence the natural appeal to their representation as Boolean finite state space machines (Gao and Chen, 2010)

In Boolean finite state space prediction market models, with no aggregate uncertainty of information, the equilibrium price will be the correct forecast if the function denoting the prediction of interest can be represented as a weighted threshold function (Feigenbaum et al., 2003). If however, aggregate uncertainty is allowed into the model, then the market does not in general converge to the correct forecast; rather it can only ever approach what has been termed the best possible prediction (Chen et al., 2004).

A novel approach taken to further understand the complex nature of market information is the application of information theory concepts e.g. Shannon's entropy in the form of Talagrand's inequality applied to markets (Ronen and Wahrmann, 2005). Whilst applicable, and potentially the nucleus of market information, the entropy of an information market is arguably difficult to interpret.

In contrast, the complexities associated with framing market information in terms of information signals and entropy may be wholly sidestepped should simplifying *nice* conditions hold. For example, an important simplifying sufficient condition that has been proven for a prediction market model with aggregate uncertainty is *if there is an independent and identical distribution (IID) of agent information, then the information market will converge to its best possible prediction (called the direct communication equilibrium and simply denoted DCE)* (Chen et al., 2004). Importantly, the IID assumption simplifies the model setting so that other interesting features of prediction markets may be investigated.

3.2.3 Price formation mechanisms of prediction markets

A key feature of information market models is the price formation mechanism. Basically, research has considered two alternative setups. The first is simply assuming a double-sided auction exists at which buyers and sellers meet to bid and trade (as is the case in the stock market). The potential problem with this set-up is the risk of illiquidity in the market i.e. when no trade takes place for any one of the stocks. This situation potentially causes divergence from the ideal market price incorporating all market information. But, more alarmingly, there exists a logical certainty that no trade can ever take place if only rational players are allowed into a zero sum market game (Milgrom and Stokey, 1982). In contrast, the other price formation approach is to introduce a market maker that guarantees trading takes place and thus mitigates the illiquidity and mispricing risk; albeit at a financial loss to the market maker (Hanson, 2003).

3.2.3.1 Double-sided auction mechanism

Although it is utilized in the real-world, in the theoretical world the double-sided auction does not incentivize rational agents to reveal private information and so a liquidity problem arises in theoretical prediction markets as a direct result of *Milgrom's no trade theorem* (Chen et al., 2010). To overcome this significant theoretical problem, extra conditions were imposed upon theoretical information market models; ranging from noisy traders to forced trading. The underlying idea was either to create the potential for a rational trader to gain and therefore incentivize their participation, or simply create the law that traders must always bid. By participating in the market, noisy traders (who play a comparatively suboptimal strategy) create an opportunity for a rational player to gain by participating in the market. In one sense, the automatic market makers play a suboptimal strategy thereby incentivizing rational players to enter the market game and gain from the automatic market maker losses. Alternatively, forced trading such as that espoused in the prediction market of Chen (2004) directly removes the no-trade possibility. For example, a forced trading rule such as "each round traders must place an order quantity q between 0 and 100 and pay $\frac{q}{2}$ cents for it" guarantees trader participation, and is incentive compatible with the rational, risk neutral and myopic traders.

Game theory provides a useful means to analyze price formation in various double-sided auction settings (Shapley and Shubik, 1977). For example, in the Vickrey second price auction, in contrast to English and Dutch-style auctions, the dominant strategy is for agents to bid truthfully. As such, it is a model that captures the complexities of strategic bidding as well as incentivizing players to bid truthfully. However, Vickrey double-sided auctions assume risk neutral strategic agents. For risk averse agents, truth telling is no longer a dominant strategy in a Vickrey auction and information uncertainty exacerbates this even further; whereby a market price forms that is not based on true trader information (Sandholm, 1996).

3.2.3.2 Automatic Market Maker: Market scoring rule mechanism

Market scoring rules were originally designed to combine the benefits of scoring rules with the advantages of market efficient aggregation of information (Hanson, 2012). The benefits of a market maker with an appropriate market-scoring rule have become prominently advocated in existing literature. For example, Hanson's logarithmic market scoring rule (LMSR) market maker reduces the risk of thin illiquid markets to a bounded financial loss to the market maker when assuming rational, risk-neutral myopic agents trade within the marketplace (Hanson, 2012). The financial loss under these assumptions can be certainly bounded through the parameters of the LMSR given that the entropy of information distribution is directly related to the worst case loss (Hanson, 2003).

3.2.4 Strategic bidding in models

Information market models usually assume rational, risk-neutral and myopic incentive compatible traders and not strategic traders. Strategic behaviors such as reticence and bluffing have been explored with particular emphasis on aligning truthful betting with rational behavior, but this still anticipates a detailed development of non-myopic models (Chen et al., 2007). Of significant interest is the creation of incentives that encourage truthful betting amongst non-myopic agents (Chen et al., 2010). The need for a non-myopic agent model has been avoided altogether in some theoretical models that argue only myopic agent behavior remains in a market where the number of traders is large; since in such a market there will be negligible impact of strategic behavior on price and the cost of complicated strategic reasoning outweighs these negligible benefits (Chen et al., 2006).

3.2.5 Decision markets

Decision markets, also called conditional prediction markets, have been defined as an instrument to aggregate market information to reveal the best decision (Berg and Rietz, 2003). A real-world embodiment of a decision market, which was considered the only real-world example of a large conditional prediction market, was “The 1996 Presidential Election Iowa Electronic Market” (Berg and Rietz, 2003). Therein the conditional probability of the success of a party given a particular candidate was able to be expressed and could arguably be used for decision making as to which candidate maximized success. Othman & Sandholm (2010a) consider this prediction market - decision market link further and suggest that whilst current corporate prediction markets have been designed as “cameras” (that capture the prediction of the future outcome), they are in fact “engines” (where conditional predictions actually alter the decisions of the firm as it attempts to safeguard or avoid the predicted future outcome).

Theoretical models for decision markets still need to resolve a number of outstanding issues including but not limited to perverse incentives and inelegant rules (Pennock and Sami, 2007). The two theoretical decision markets that motivate the new theoretical model of this chapter are now reviewed.

3.2.5.1 Scoring rule decision markets

One theoretical formulation of a decision market has simply extended the market scoring rule prediction market model by incorporating into it a decision rule (Chen and Kash, 2011). These scoring rule decision markets are arguably the current popular form of theoretical decision market models. However, in this decision market formulation, a mixed strategy decision rule is required (Chen et al., 2014). Unfortunately, an inconsistency exists whereby the decision maker implements a mixed strategy and possibly selects a suboptimal decision despite knowing it as such (Chen et al., 2011).

3.2.5.2 Joint elicitation decision markets

A less popularized theoretical implementation of decision markets is the “joint elicitation market” which is defined as a market which trades contracts for “actions” and contracts for “outcomes and actions” and then simply uses the respective contract prices to calculate conditional probabilities (Othman and Sandholm, 2010). However, a potential problem of the joint elicitation market arises when an agent, knowing the exact moment that the final round is to occur, makes a strategic purchase of any arbitrary action whatsoever to achieve profit (Othman and Sandholm, 2010). The joint elicitation decision market is briefly mentioned in literature but no analysis of depth similar to the market scoring rule literature appears to have been undertaken. This chapter attempts to provide such an analysis. As in the market scoring rule literature, in the first instance, it assumes rational, risk-neutral and myopic traders. This simplifying assumption means that last round strategic play is no longer a problem.

3.3 Generally Accepted Prediction Market Model

Chen (2004) introduces, what is referred to by this chapter, a generally accepted prediction market model. In Chen’s model there exists one stock in the market with multiple agents engaged in multi-round bidding on that stock[8]. Each agent observes one bit of uncertain private information about that stock and market clearing prices from previous rounds that informs their bid. All agents must bid, and bids are aggregated into a market clearing price. At some future point in time the true value of the stock is revealed and an agent will profit depending on their position in the stock.

Chen (2004) takes “an axiomatic approach” and translates the above description into a rigorous formalism akin to the following:

Let $B = \{0,1\}$ and let $s \in B^N$ be a vector representing the possible state of the world where N is a positive integer. Let n traders (also called agents) observe a common prior probability distribution $P(s): B^N \to [0,1]$. One stock (contract) F is traded in this market. F pays money, rewarded at some future point in time, dependent on the value of a Boolean function $f(s): B^N \to B$ whose functional form is common knowledge. Specifically, F pays \$1 when $f(s) = 1$ and \$0 when $f(s) = 0$. Every round all agents submit a bid which is aggregated into a Shapley Shubik market clearing price $p = \frac{\sum b_i}{n}$, where b_i represents the bid of agent i (Feigenbaum et al., 2003). Agents submit rational risk-neutral myopic bids i.e. the expectation of $f(s)$ conditional on the one bit of information possessed by the agent ($x_i \in B$)

[8] Whilst there is a technical distinction between a stock (unit of ownership) and its contract (enforceable right due to ownership), these are used interchangeably in this chapter.

and the information the agent learns from the previous round market price *p* (and in general, the history of all market prices observed in previous rounds)[9].

The probability distribution of the information vector of all agents $Q(x|s): B^n \times B^N \rightarrow [0,1]$ is common knowledge where $x \in B^n$ represents the *n* traders' information bits. Notice that in round one arbitrary agent *i* possesses one bit of information only (namely its own bit x_i) and computes a bid based on that information $b_i = E[f(s)|x_i]$. All bidders calculate and submit their bids without directly communicating to each other. A n agent market clearing price $p = \frac{\sum b_i}{n}$ is then revealed to everyone at the end of round 1. In contrast, were all agents to *directly communicate* their private information to one another prior to bidding in round 1, then equilibrium would be reached at the end of the first round since all agents would bid conditioned on full information i.e. $E[f(s)|x]$; is the equilibrium attained and called *the direct communication equilibrium (DCE).* The DCE is the best possible prediction, since it incorporates all market information.

It can be shown, in this model, that if agent information is independent and identically distributed (IID), then this is a *sufficient condition* to ensure the information market will converge to the DCE price (Chen et al., 2004). A *necessary* condition was not identified.

In Chen (2004), a market attaining the DCE price does not also mean that all participating traders are fully informed[10]. In contrast, the theory developed in this chapter is interested in a market that is strictly identical to one in which traders directly communicate private information to one another. Thus, this chapter states that the DCE is attained only when all traders are fully informed and bid to form the DCE price.

3.4 New Prediction Market Model

This section develops a new prediction market model taking an axiomatic approach. The Chen (2004) model is modified to include two additional concepts, namely, *proper information market* and *relevant information*. These concepts are derived from two "proper market price" axioms. It is then shown that ***"all agents express relevant information"*** is a *sufficient and necessary* condition for convergence to the DCE in a proper information

[9] Note in the first round of bidding, agents do not observe a previous round market price.
[10] Fully informed means the private information of *all* other traders is known.

(prediction) market. Finally, a payout and selection rule is defined that links multiple prediction markets in order to construct a simple decision market model.

In this section, theorems are stated and the 'big ideas' behind them revealed to facilitate the accessibility of this work to the general reader. Mathematical proofs of theorems are provided in Appendix A: Chapter 3 Appendix 1 for completeness and transparency. Where mathematical statements are used in this section (to precisely describe the theoretical model), if they have not already been explained in previous sections, an explanatory narrative accompanies them.

3.4.1 Proper market price axioms

In Chen's model the market price p for n agents at the end of each round is simply calculated as the average of all bids i.e. $p = \frac{\sum b_i}{n}$, where b_i represents the bid of agent i. Agents learn information from market prices. These ideas inspire the following concepts:

At the start of the market, agents possess private information only. That is, agent i only knows one bit of information x_i and as such submits a bid $b_i = E[f(s)|x_i]$. Notice that the market price is simply $p = \frac{\sum b_i}{n} = \frac{\sum E[f(s)|x_i]}{n}$. This shall be referred to as the *private information stage*.

The market may reach a stage whereby all agents know their private information and the private information of all other agents. When agent i knows all bits of information x they submit bid $b_i = E[f(s)|x]$. Notice that the market price is simply $p = \frac{\sum b_i}{n} = \frac{\sum E[f(s)|x]}{n} = \frac{n \times E[f(s)|x]}{n} = E[f(s)|x]$. This shall be referred to as the *full information stage*.

An agent is said to learn a bit of information when the value of that information bit once uncertain becomes known with certainty.

An agent is said to unlearn a bit of information when the value of that information bit once known with certainty becomes uncertain.

Axioms (proper market price)

1. There does not exist a market price, resulting from any group or subgroup of the market traders with which an agent bids, whereby that agent learns absolutely no information at the *private information stage*.
2. There does not exist a market price where an agent unlearns information at the *full information stage*.

Axiom 1 and the private information stage property

Axiom 1 is motivated by the notion that a trader will ideally learn something from the first-round market price. Axiom 1 introduces the idea that, irrespective of the participating traders, no first-round market price is formed where an agent learns *not a single piece* of new information. As such the new model of this chapter requires the private information stage property: it is always possible to select any m of the n traders (where $m \leq n$) to form a market with a first-round market price that causes all of the m traders to learn something new.

Axiom 2 and the full information stage property

Axiom 2 is motivated by the notion that when all traders know all information bit values, the market price cannot cause any trader to unlearn their full information. When all traders are fully informed they know all private information x and will all bid $E[f(s)|x]$. As such the new model of this chapter requires the full information stage property: when all traders know all private information x there cannot exist a market price that causes agent i to become uncertain about (and therefore *unlearn*) the information bit value x_j of agent j. Notice that *unlearning* would occur if agent i sees that a market price p can be attained irrespective of the bit value of x_j. That is, the market price causes agent i to now consider that either $x_j = 1$ or $x_j = 0$ is possible.

If a market price possesses both the "*private information stage property*" and the "*full information stage property*" it is said to be a **proper market price**.

3.4.2 Definition (proper information market)

The new prediction market model of this chapter (which is simply the Chen (2004) model modified to require a proper market price) shall be called a *proper information market*.

3.4.3 Definition (relevant information)

The basic idea of relevant information is simply stated as follows. If an agent changes their bid when their information changes, it is said they express relevant information. If an agent does not change their bid when their information changes, it is said they do not express relevant information.

In the first round any agent i will bid $b_i = E[f(s)|x_i]$ or $b'_i = E[f(s)|x'_i]$, when their private information is x_i or x'_i respectively[11]. If $x_i \neq x'_i$ implies $b_i \neq b'_i$, it is said the agent expresses relevant information and that x_i is relevant information. Notice that if the first-round bid of agent i is known, then the value of their private information bit is able to be inferred. In contrast, if $b_i = b'_i$ when $x_i \neq x'_i$, then knowing the first-round bid of agent i does not allow their private information bit value to be inferred.

This idea may be generalized beyond the first round. For example, in rounds after round 1 an agent may know more than its one bit of private information. Say agent i knows x_j as well as Y (with Y representing other information including the agent's one bit of private information). Suppose this agent bids $b_i = E[f(s)|x_j, Y]$ then should the information bit x_j be changed to x'_j they would instead bid $b'_i = E[f(s)|x'_j, Y]$. If $x_i \neq x'_i$ implies $b_i \neq b'_i$ agent i is said to express relevant information and x_j is relevant information in this scenario.

For readers desiring a more formal treatment, Lemmas 1.1 and 1.2 in Appendix A: Chapter 3 Appendix 1 provide a mathematically rigorous definition of relevant information.

3.4.4 Main information market theorems

The 'big ideas' behind the theorems underpinning the new prediction market and decision market models of this chapter are now presented. For transparency and completeness, mathematical proofs are provided in Appendix A: Chapter 3 Appendix 1 for those readers wishing to review this work in greater detail.

3.4.4.1 Theorem 1 (Relevant information as a sufficient condition for DCE convergence)

"All agents express relevant information" is a sufficient condition for convergence to the direct communication equilibrium (DCE) in a proper information market.

The condition that "all agents express relevant information in a proper information market" is fundamental to the new prediction and decision market models in this chapter. Appendix A: Chapter 3 Appendix 1 contains the associated mathematical proof for Theorem 1; showing that when this condition holds a prediction market with one stock converges to the DCE in the second round of trade. The following provides the 'big ideas' associated with Theorem 1.

[11] x'_i denotes the opposite bit value of $x_i \in \{0,1\}$.

Because "all agents express relevant information", upon knowing the bid of any agent their private information bit value is also known. For example, if agent i has a private information bit value of '1' it bids \$0.70 and if it has a private information bit value of '0' it bids \$0.40. Now if a different agent was able to determine that agent i bid \$0.70, they would then know that the private information of agent i is 'x_i =1'. If on the other hand agent i bids were \$0.70 for both '1' and '0' bit values, knowing that agent i bid \$0.70 would not reveal agent $i's$ private information bit value to another agent. In effect, if agents do not express relevant information, their private information is hidden behind their bid.

Axiom 1 for the "proper information market" simply guarantees that any agent, trading with any group of other agents, will learn new information from the first-round market price that results. Given a group of n agents trading in a "proper information market", a new market for any m $(where\ m \leq n)$ of these agents may be constructed, and trading will result in a first-round market price that all m traders will learn from (by axiom 1).

Consider n agents trading in a proper information market where they all express relevant information. It is shown in the following that the first-round market price results from a unique arrangement of information bits across agents. Because it is unique, all agents may observe the first-round market price to learn the private information of all other agents. All agents then submit a fully informed bid, which results in the DCE at the end of round 2.

By way of justifying the uniqueness, assume agent i sees another arrangement of information bits across other agents that lead to the same first round market price. Let agent i compare two arrangements that lead to this same first round market price. Let agent i remove all other agents that had the same information in both arrangements and therefore contributed the same bid in both arrangements. Agent i is now left with two *new* arrangements that lead to a single *new* market price; simply because agent i has removed the same bid amounts from each of the original arrangements. It is important to notice that when agent i now compares the two new arrangements, other agents do not have the same information bit in each arrangement. Therefore agent i may consider the new market price but will not know with certainty the information bit of another agent. In short, it learns no information from the first-round market price of this new group of traders. This contradicts with axiom 1 which requires the agent to learn something from the first-round price of any group of traders, so the assumption that there is more than one arrangement that leads to the first-round market price of n traders must be false. That is, in a "proper information market where all agents express relevant information" all agents will observe a first-round market price, identify a unique arrangement of information bits that leads to it, and full information bids will establish the DCE in the second round. Thus "all agents express relevant information" is a sufficient condition for convergence to the DCE in a proper information market.

3.4.4.2 Theorem 2 (Relevant information as a necessary condition for DCE convergence)

"All agents express relevant information" is a necessary condition for convergence to the direct communication equilibrium (DCE) in a proper information market.

Consider a proper information market in which the DCE has been attained. This means that all agents have full information. Assume that there is at least one bit that is not relevant information. Of these bits consider private information bit x_j of agent j. Since x_j is not relevant information, all agents submit a bid that does not depend of the value of x_j. In turn a price p is formed (which is the simple average of all bids) that does not depend on the value of x_j. Price p is information that all agents receive to update their next round bids. Now price p is information that is basically stating that x_j could *probably* be '1' or '0'. In short, p not stating with certainty the value of x_j in effect places a modicum of doubt on the actual value of x_j. Therefore, all agents update their beliefs to a probable rather than a certain value of x_j. That is, p has caused the value of x_j to no longer be considered certain. In our terms, p has caused all agents to unlearn the value of x_j. But this contradicts axiom 2, which does not allow this *unlearning* to occur at the full information stage. Therefore, it must the case that all agents express relevant information. Hence "all agents express relevant information" is a necessary condition for convergence to the DCE.

The next logical step towards generalization by constructing a proper information (prediction) market with *multiple contracts* is addressed as follows.

3.4.4.3 Theorem 3 (Proper information market equilibrium with r stocks)

A proper information market with r stocks converges to a market equilibrium in which each stock attains its DCE when all agents express relevant information.

For simplicity, a market with r stocks may be thought of as r markets with one stock; whereby a trader participates in each of the r markets. That trader has 1 bit of private information per market; thus it has r bits of private information in total. In this model, the k^{th}

bit of private information represents all the information required by the agent to inform their first-round bid on the k^{th} stock. All agents express relevant information about the k^{th} stock and by Theorem 1 the DCE for the k^{th} stock is attained. In this way all stocks reach their respective DCE and the market of r stocks has reached equilibrium.

This proper information market with r stocks is now used to build a simple decision market model. In order to do so, a variation to the payout structure of the contracts is made and the implications of this explored.

3.4.4.4 Theorem 4 (derivative attains DCE)

A derivative on a stock attains its DCE in a proper information market "where all agents express relevant information".

A derivative in a stock market is in the most general sense a stock whose payout is dependent on another *underlying* stock. This *derivative* idea is used to create conditional probabilities; which are the bedrock of decision theory and this chapter's new decision market. In this simplistic model some stock k is called a derivative if it pays \$w when stock j pays \$1. In this market it may be shown that derivatives reach their DCE. Let all agents express relevant information in a proper information market. Firstly, notice that Theorems 1 and 2 hold independent of currency used and that a \$w payout is simply a $\mathfrak{L}1$ payout using some other currency denominated in $\mathfrak{L}$. Every agent will reason that "the probability of payout of the derivative given private information about that derivative" must be equal to "the probability of payout of the underlying stock given private information about that underlying stock". In essence, derivative stock k appears identical to underlying stock j in the first round with the exception that it is denominated in a different currency. Since Theorems 1 and 2 do not depend on currency denomination and given stock j converges to the DCE denominated in \$, the derivative stock k is equivalent to stock j converging to the DCE denominated in $\mathfrak{L}$. Say the derivative stock converges to a DCE of $\mathfrak{L}p$. Since $\mathfrak{L}1$ may be exchanged for \$w, the derivative's DCE is $\$p \times w$.

3.4.4.5 Theorem 5 (probability derivative)

For the DCE market price of derivative k (e.g. \$0.70) to directly reflect the probability of k being paid (e.g. 0.70), it requires a payout equal to "the market price of k divided by the market price of underlying stock j", in a proper information market where all agents express relevant information. This type of derivative k is called a probability derivative because it is a derivative with a market price that directly reflects the probability of it being paid.

When underlying stock j has a DCE market price of say $\$p = \$0.60$, this means that the probability of the event associated with stock j is $p = 0.60$. It is said that the market price $\$p$ directly reflects the probability p of stock j being paid. It is also said that the market price $\$p$ directly reflects the probability p of the event associated with stock j occurring. Say the payout of derivative k is $\$w$, then by Theorem 4 it will reach a DCE market price of $\$p \times w$. Let the probability of the event associated with stock k be q. For the DCE market price of k to directly reflect the probability q of the associated event then it is required that $\$p \times w = \$q$. This is rearranged to $w = \frac{q}{p}$. That is, a derivative payout equal to "the market price of k divided by the market price of underlying stock j" means that the DCE market price $\$q$ of k results when the probability of the event associated with k is q.

3.4.4.6 Theorem 6 (decision market contract payout structure)

Consider a proper information market where all agents express relevant information and in which probability derivative k (with DCE market price $\$q$) is associated with the event "O and P occurs", and the underlying stock j (with DCE market price $\$p$) is associated with the event "P occurs". Then the derivative payout (w) for k represents "the conditional probability of O given P".

Notice Theorem 5 implies that the payout for k is $w = \frac{q}{p}$. But this payout is simply "the probability that O and P occurs" divided by "the probability that P occurs"; which is "the conditional probability of O given P". For example, this may be used to express the probability of achieving some desired outcome O given project P is chosen.

With the previous example in mind, an organization may wish to choose the best project i.e. the project that when chosen maximizes the conditional probability of the desired outcome given the project. Theorem 7 provides a means to do this.

3.4.4.7 Theorem 7 (decision market selection rule)

Consider a proper information market where all agents express relevant information and in which probability derivative k_v (with DCE market price $\$q_v$) is associated with the event "O and P_v occurs", and the underlying stock j_v (with DCE market price $\$p_v$) is associated with the event “P_v occurs”; the derivative payout for k_v is represented by w_v. Consider a market filled with many such pairs of derivatives and underlying stock i.e. many different v values. Then the pair with the highest derivative payout (say w_u) means that event P_u maximizes the probability of event O occurring. P_u is said to be the best.

Notice that Theorem 6 implies that payout (w_v) for k_v represents “the conditional probability of O given P_v”. In a decision theory setting the best P_v is the one where “the conditional probability of O given P_v” is of largest value. In this market, the derivative payout w_u represents “the conditional probability of O given P_u”. Since w_u is the largest derivative payout it must be the case that that P_u maximizes the probability of event O occurring. That is, P_u is the best.

Notice that P_u may represent the best project that a firm may wish to invest in to maximize their desired Outcome (O) being achieved. The market in Theorem 7 would select Project P_u. This market shall be called a simple decision market.

3.5 Discussion

The strategy applied in this chapter has been to develop a well-defined prediction market model from which to formulate a simple decision market. After reviewing relevant literature, a generally accepted prediction market model in Chen (2004) is modified to create the new prediction market model in this chapter.

This work employs an axiomatic approach to provide the rigor and transparency of mathematical formalism. Well-defined axioms are used to develop theorems that explore the dynamics of the information markets presented in this chapter. Importantly, the sufficient and necessary condition for convergence to the direct communication equilibrium (DCE) in this new prediction market model is identified in Theorems 1 and 2. Upon identifying this condition, it is enforced in the new prediction market models in Theorems 3 to 7. A payout and selection rule is defined, that links multiple contracts in the new prediction market model, to build a simple decision market model depicted in Theorem 7. The investigation of the inherently more complex decision market setup is thus simplified by this modular approach.

A key theoretical contribution of this paper is the logical introduction of axioms that ensure “agent learning” from first round market prices and prevent “agent unlearning” from last round market prices[12]. This is explored in section 3.4 of the chapter where the proper market price axioms are defined. Simply put, "axiom 1 prevents an agent from learning absolutely nothing from the first-round market price (irrespective of the group or subgroup of market traders they participate with to generate that price)" and "axiom 2 prevents an agent with full information from unlearning information as a result of observing a market price". The proper market price axioms modify the prediction market model of Chen (2004) to form a new prediction market model. Importantly, this modification causes two mathematical properties to emerge that are central to the theorems. The new prediction market possessing these two properties is called "a proper information (prediction) market".

The proper information (prediction) market model setup provides the context to establish the sufficient and necessary conditions for convergence to the DCE. To this end relevant information is defined as an agent's information that, when changed, in turn changes the bid of that agent. Theorem 1 establishes that a sufficient condition for convergence to the DCE is that "all agents express relevant information" in a proper information market with one traded contract. Theorem 2 then shows that the condition "all agents express relevant information" is necessary for convergence to the DCE in a proper information market with one traded contract. Enforcing the sufficient and necessary "all agents express relevant information" condition in Theorems 3 to 7 ensures that their proper information (prediction) markets are well behaved and converge to the DCE.

The construction of a prediction market model with multiple stock contracts is an important precursor to the development of a decision market; which inherently requires multiple stocks to express conditional probabilities. Theorem 3 provides a formal statement that guarantees convergence to the equilibrium in a prediction market with multiple agents who trade multiple stocks. It should be noted that agents enacting strategic behavior are not considered, thus the market equilibrium attained in Theorem 3 is strictly not the rational expectations equilibrium; whereby rational expectations equilibrium has been defined as the market clearing price that does not cause strategically behaving agents to change their bid (Pennock and Sami, 2007). That is, the setup is simple and assumes rational, risk-neutral and myopic agents. There are two reasons for this. One, the new idealized model of this chapter can arguably still provide important insights about real-world phenomenon as do other idealizations, e.g., the model of gravity without air resistance still provides a useful

[12] The "last round market price" is the market price that is attained when all agents possess full information.

approximation to gravity dynamics near earth. Two, a game that is rational, risk-neutral and myopic incentive compatible rewards agents who play a dominant rational, risk-neutral and myopic strategy. In this way, the game arguably becomes a useful tool to efficiently aggregate distributed information and support decision making. Here the ultimate objective is not to model trader behavior in a real-world market, rather, it is to accentuate trader behavior within a decision market game. This dichotomy is simply game theory versus inverse game theory. Whereby the former is interested in modeling player behavior given game rules, whereas the latter is interested in designing game rules to elicit specific player behavior; in this case revealing private information in a rational, risk-neutral and myopic decision market game (Chen and Pennock, 2010).

Further modification of contracts in the new prediction market model of this chapter leads to a contract payout that is conditioned on another contract's payout. These new contracts with payouts that are dependent on an underlying stock being paid are called derivatives. Theorem 4 ensures that such derivatives converge to a DCE. From this point on, the new prediction market model in which "all agents express relevant information" about multiple contracts (be they stocks or derivatives) guarantees convergence of contracts to their respective DCE.

Theorem 5 takes the key step towards a formal simple (joint elicitation type) decision market model. Whereas Theorem 4 constructs a derivative whose market price depends on the market price of the underlying stock, the market price of derivative does not necessarily directly convey the probability of the associated event occurring. It is Theorem 5 that shows the necessary payout structure to ensure that the derivative market price also explicitly communicates the probability associated with the event that the derivative contract represents.

Theorem 6 builds upon Theorem 5 and defines the exact setup of a market in which there are "project stocks" and "project & outcome stocks" traded. What this chapter calls projects, previous literature calls actions or decisions. Hence the best project, best decision, or best action, have the same meaning here. Project applications are introduced in Theorems 6 and 7 with a view to applying this new and simple decision market to a real-world project selection setting in future research and as proposed in the policy implications Chapter 7. The information market for Theorem 7 acts to identify the (best) project that maximizes the likelihood of some desired outcome occurring. This information market is called a 'simple decision market'.

Theorems 6 and 7 show that the selection of the best project is consistent with the "project stock" paying $1 if selected and the associated "project & outcome stock" paying the derivative payout (which is simply the quotient of the "project & outcome stock" market price and the "project stock" market price). The selection rule is simply that the "project and outcome stock" with the highest derivative payout should be selected.

Theorem 7 considers a market of many pairs of "project contract" and "project & outcome contract". It formally shows that there exists a market of contract pairs, payouts and a selection rule that does indeed identify the best project. Theorem 7 models a group of agents that bid the expected values of "project contract" and "project & outcome contract" across all market contracts. The same rational, risk-neutral and myopic agent behavior assumptions found in a popular theoretical market scoring rule decision market also holds in the market of Theorem 7 (Chen et al., 2011). However, in contrast to that market scoring rule decision market, this new simple decision market has a deterministic selection rule rather than a mixed strategy selection rule. That is, this decision market model has a 100% probability of selecting the best decision, whereas the market scoring rule decision market does not. Therefore, these 'decision markets' built from prediction markets (making the 'best possible prediction') are theoretically guaranteed to make the 'best possible decision'. This motivates a hypothesis that relevant information is an important quality signal for well-functioning prediction and decision markets in the real-world. The selection decision of the best project is therefore theoretically guaranteed, which is important, because it is a precursor to selection and prioritization of the best possible portfolio of projects.

This particular implementation of a decision market, described by Theorems 6 and 7, requires the payment of both the "outcome & project" contract and the "project" contract *at the same time*. This contrasts with other decision market implementations found throughout literature whereby payment of some contracts is delayed to the observation of the outcome. Specifically, in this new model, payment of the "outcome & project" contract does not need to be delayed to the time at which the outcome is realized. This has implications in many real-world contexts. For example, large investments have been made in projects that are long running and have a desired outcome considered over a long-term, e.g., mining projects, health programs, infrastructure investments, etc. For succinctness, in this thesis, these shall be called "long-term outcome projects". Traders would arguably hesitate to participate in a decision market associated with a long-term outcome project where some contract payouts are delayed to the observation of the long-term outcome. In such a decision market, the time-value-of-money considerations would certainly become pertinent and arguably invalidate the risk neutral assumption of the decision market model. The new decision market, developed in this chapter, can arguably overcome this time-specific problem by simply running for a short time so that time effects become negligible. In short, this new decision market may be run over a week prior to project selection and cease trading up until project selection. In this form, the new decision market will be used to aggregate information for the firm to inform their project selection. It is important to note that this is not to say that the project selected by the decision market is the same as the project the firm selects[13]. However, it is likely that they are the same given that the decision market is arguably the best means to aggregate information for project selection purposes. In short, this new simple decision market is simply a short running game that is used at the start of a project to inform real-world decisions about project selection.

This simple decision market model assumes rational, risk-neutral and myopic traders. In this model, Milgrom's no-trade possibility has been circumvented via forced trading. A simple forced trading rule can be easily introduced into the decision market game to ensure liquidity and revelation of private information. For example, a rule such as "each round a trader must submit an order quantity q in the closed interval 0 to 100, receive this quantity at the end of the round and pay $\frac{q}{2}$ cents" would guarantee trading and also ensure a trader bids the expected value of the contract conditioned on all information they have, as required in the new decision market model.

What remains is extending this new model to cater for contexts beyond a rational, risk-neutral and myopic setting. This simple decision market model is populated by rational individual traders. However, it has been known for some time, that the rational trader assumption, in certain market settings, is false (Kahneman and Tversky, 1979). The new model, developed in this chapter, confines itself to risk neutral traders. It has been suggested that the risk neutral agent assumption causes no loss of generality because of the mathematical equivalence between risk neutral and risk averse models (Chen et al., 2006). This suggestion arguably aligns with the equivalence seen in real options analysis (Cox et al., 1979). If true, this would make the model findings more general, but to the best of the researcher's

[13] Traders are paid based on what the decision market selects, not what the firm selects. That is, traders simply play and are rewarded by the decision market game.

knowledge no such equivalence has as yet been rigorously proven for information markets. The new model of this chapter also limits its scope to myopic traders. Hence the impact of strategic play begs analysis.

Despite the previously stated limitations of this new simple decision market model, the central focus of this chapter concerns whether relevant information is important for well-functioning decision markets. This idealized decision market model and associated theorems clearly identify "relevant information" as important. As such, this theoretical finding compels further investigation of "relevant information" in laboratory and real-world contexts. Specifically, is "relevant information" statistically significant in these contexts? This motivates the experiments and empirical analysis of the next three chapters.

3.6 Conclusion & Future Work

A simple decision market model of the joint elicitation type has been developed in this chapter. A generally accepted prediction market model is modified to construct a new simple decision market model. This modification provides original contributions to the theoretical discourse on information markets. Notably, the sufficient and necessary condition for convergence to the DCE in the new prediction market model, developed in this chapter, is identified. The new prediction markets are then linked together, whilst enforcing the sufficient and necessary condition for convergence, in order to build a well-functioning simple decision market model that is unencumbered by mixed strategy selection rules. The development of the new theoretical models is made easy by the deliberate choice of mathematical formalism; a simple decision market nomenclature that is coherent, consistent and able to be utilized in future work. The implications, of the new simple decision market model, motivate further experimental and empirical investigation. Testing whether relevant information is statistically significant in a laboratory and real-world decision markets follows logically from the model developed in this chapter; which primarily identifies that *relevant information is important*.

In conclusion, a new theoretical model for prediction and decision markets is built. The focus is on private information that is incorporated into trader bids; denoted in this thesis as relevant information. The new theoretical model (comprehensively developed in this chapter) implies that if all traders (in a prediction market) express relevant information in their bids, then this is a sufficient and necessary condition for the prediction market to identify the 'best possible prediction'. Decision markets built from these new prediction markets (that are guaranteed to make the 'best possible prediction') are theoretically guaranteed to make the 'best possible decision'. This motivates a hypothesis that relevant information is important for well-functioning prediction and decision markets in the real-world; acting like a quality signal for well-function prediction and decision markets. The application of these new prediction and decision markets are natural to project selection and prioritization settings. Specifically, this chapter has shown that the selection decision of the best project from a group of projects is theoretically guaranteed, which is important, because it is a precursor to selection and prioritization of the best possible portfolio of projects. The significance of relevant information will be comprehensively investigated in the following Chapters (4 to 6) via experiments and real-world analysis. Chapter 7 then considers a hypothetical setting to explore potential implications of the thesis' findings.

Chapter. 4 Computer Simulation of a Simple Decision Market

"If you torture the data long enough, it will confess."
Ronald H Coase

Key Message of Chapter:

- **Computer simulated experiments of prediction and decision markets validate two hypotheses**
  - **Increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction**
  - **Increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision**

This chapter is a natural extension to the theoretical work of Chapter 3 as it sets out to experimentally test the statistical significance of relevant information by way of computer simulations. Whereas Chapter 3 theoretically identified that relevant information was an important ingredient to ensure that prediction and decision markets attained their respective best possible prediction and decision, this chapter tests the theoretical prediction in an experimental environment. This work addresses research question 1 by justifying that the quality signal for prediction and decision markets is the proportion of traders (in a market) that express relevant information in theirs bids. This proportion is denoted as relevant information level r. The hypotheses that increasing relevant information level in prediction and decision markets leads to increasing the probability of attaining the best possible prediction and decision respectively, are tested in a computer simulated control and treatment experimental setup. Research question 2 (pertaining to how to build prediction and decision markets that are able to determine the best possible portfolio of projects) is also addressed by the researcher's development of the computer-simulated market. Notably, the computer simulated experiments of prediction and decision markets considers a setting in which a not for profit project is determined as being either in or out of the best possible project portfolio.

Thousands of computer simulations of prediction and decision markets of the type proposed in Chapter 3 are built and run. The respective probabilities of convergence to best possible prediction and decision are recorded for each run as relevant information level is varied across simulations. Specifically, a control-treatment hypothesis testing method is utilized in both prediction market and decision market tests; where the control prediction/decision market always possesses a relevant information level of 1 (i.e. all agents express relevant information) and as such, the best possible prediction/decision. Each control market is paired with a treatment market identical in every way with the exception of the relevant information level (r). Whereas in the control market $r = 1$, in the treatment market $r \in [0,1]$. That is, in the computer simulations, the relevant information level of treatment market varies from 0 to 1; with 51 different values in prediction market simulations and 21 different values in decision market simulations. Therefore, any statistically significant difference between the control and the treatment is logically attributable to the variation of relevant information level.

Because of the large number of simulations run at each relevant information level (i.e., 2500 pairs of control and treatment per r value), the law of large numbers may be invoked to accurately estimate the functional form of the probability distribution. Additionally, the dichotomous nature of the treatment market, converging or not to the control market, lends itself to a binary logit analysis of the simulated experimental results; in order to determine the statistical significance of r . The hypotheses: "increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction" and "increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision" are found to hold at the 1% significance level; utilizing the logit analysis. In addition, the probability of selecting the best possible decision in a decision market linearly increases with increasing relevant information level, which is consistent with theoretical expectations.

Given the ultimate resolution of the (thesis) problem of reducing wasted project resources, by building high quality decision markets for real-world selection and prioritization of the best possible portfolio of projects, further research that extends beyond the computer simulations of this chapter and into markets with human participants is suggested and addressed in Chapters 5 (in the laboratory) and 6 (in the real-world). However, this chapter is a necessary precursory stepping-stone; to investigate the statistical significance of relevant information level in a multitude of markets only made possible in a computer-simulated world.

4.1 Introduction

The following investigation provides original contributions to the literature on prediction and decision markets via computer simulations of the rational, myopic, and risk neutral, incentive compatible prediction and decision market models defined in Chapter 3. This chapter anticipates the work on prediction market games with human participants in Chapter 5. It has been suggested that such (human participant) markets appear to work well so long as appropriate incentives exist and training of human agents is undertaken prior to the market game (Klingert and Meyer, 2012). The games with human participants (in Chapter 5) test the statistical significance of relevant information level in prediction markets incorporating humans. The prediction market games with human participants may also address an open research problem, i.e., "little is known about private information aggregation by markets with risk averse rational traders" (Iyer et al., 2010). In particular, does their private information aggregate into a best possible market decision? However, prior to attempting to solve such big questions complicated by human behavior, a robust computer simulated foundation is provided in this chapter. A key question resolved in this chapter is "what minimum condition implies that the computer simulated equilibrium reached is the DCE?" (Pennock and Sami, 2007).

This chapter analyzes an extremely large number (in the order of tens of thousands) of computer simulated prediction and decision markets to test the theoretical model of Chapter 3. This chapter also extends the theory of Chapter 3 and finds that the proportion of relevant information in a market (relevant information level r) is statistically significant in computer-simulated experiments. The theoretical foundation of this chapter and of Chapter 3 inspires two prediction and decision market hypotheses to be tested. The tests are undertaken, firstly, in the computer-simulated setting (of this chapter), secondly, in a human participant setting in laboratory (Chapter 5), and thirdly, in the empirical analysis of real-world prediction market data (Chapter 6).

The two hypotheses are:

"Increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction".

"Increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision".

In the current chapter, these hypotheses are tested in computer simulations of prediction and decision markets that implement the proper information market[14] setup described in Chapter 3. Specifically, computationally adept agents (also called traders) inhabit these markets, trading stocks with state dependent payouts. In each round, an agent submits a bid based on their private (known only to that agent) information and public information (in the form of commonly known previous round market prices, and probability distributions). An environment of information uncertainty exists wherein agents' receipt of information about the world is stochastically related to the actual state of the world. Furthermore, at the aggregate level there is information uncertainty, i.e., only a best possible prediction or best possible decision is feasible even when a trader knows all market information. Importantly, probabilistic dependence across information can exist in these simulations. This provides greater generality than the information independence assumptions typically applied to simplify analysis. In particular, it is theorized in Chen (2004) that independent and identically distributed (IID) trader information leads to a best possible prediction (market) price. In short, this chapter *also tests* markets in which the IID assumption does not necessarily hold. As such, this chapter aligns more closely with real-world markets, within which interaction amongst traders inevitably causes probabilistic dependence across trader information (Treynor, 1987).

The prediction market simulation is coded in Matlab. The decision market simulation is built using four of these simulated prediction markets that are interlinked via specific payout and decision rules consistent with the theoretical specifications of Chapter 3. The extension of theoretical model of Chapter 3 to a corollary proved in Appendix A: Chapter 3 appendix 3 (i.e., the probability of a decision market selecting the best possible decision increases linearly with relevant information level), is then tested in decision market simulations. The prediction and decision market Matlab computer simulated control-treatment experiments are facilitated by time efficient concurrent processing techniques utilizing the University's[15] high performance computing cluster. However, irrespective of the programming platform, a key requirement implemented in these computer simulated experiments is that the simulated prediction and decision markets are consistent with the theorized *proper information prediction and decision market models* (Grainger et al., 2015). In short, it is important to

[14] Recall from chapter 3 that a proper information market is a prediction or decision market (built from prediction markets) in which all traders must learn something from the first round market price and have no cause to doubt should they become fully informed at some future round.
[15] James Cook University Townsville Australia

simulate proper information (prediction and decision) markets with stochastic states of the world, which in turn generate stochastic agent information conditioned on that world state. Computer simulated agents (traders) are rational, risk neutral, myopic and submit bids based upon their private information and what information they learn through commonly known previous market prices and probability distributions for states of the world and agent information. The bids are aggregated into a Shapley-Shubik market price; in this case the average of trader bids. The market price is ultimately compared to the direct communication equilibrium (DCE) price, i.e., the equilibrium price that would result if all traders prior to bidding knew all market information.

Constructing a proper information market is central to testing the two hypotheses. A *proper information market* is a market in which proper market prices hold and where none, some, or all traders express relevant information. A *proper market price* is one in which "an agent must learn something from the first-round market price about other agents' information" and "after the first round, the market price cannot cause an agent to unlearn something if ever that agent attains full information about other agents' information". A trader is said to express *relevant information* if their bid changes when their information bits change. If their bid is unchanged then they are said not to express relevant information (in their bid); in effect their information is not discernible from their bid and is hidden. *Relevant information level* is the proportion of traders in a market that express relevant information. In the simulated markets of this chapter, proper market prices generally hold, and relevant information level is allowed to vary across prediction and decision markets.

Proper market prices and relevant information are the most significant points of departure of this thesis from previous literature. Proper market prices are restricting conditions that define what market prices may be observed in the prediction and decision markets considered in Chapter 3. Relevant information is a characteristic of the underlying bids expressed by traders either revealing or hiding their private information. The centrality of these concepts to the theoretical model motivate efforts to ensure that proper market prices hold almost everywhere [16] in all the computer-simulated markets of this chapter. Varying relevant information level then provides a means to statistically test the two hypotheses.

In all computer-simulated prediction and decision markets, a control market is paired with a treatment market. The only difference between the control and the treatment market is a

[16] Ensuring that proper market prices hold almost everywhere is clarified in detail in appendix 3 of this chapter.

possibly different value of relevant information level; all else is identical. All control markets possess a relevant information level value of 1; meaning a best possible prediction and decision is attained in that market[17]. The treatment market, each control is paired with, possesses a relevant information level ranging from 0 to 1. Of prime interest is observing any divergence, of the treatment from the control's best possible prediction or decision, which results from the variation of relevant information level.

The price of a stock in a prediction or decision market elicits the probability of an associated real-world event. For example, an "it will rain tomorrow" stock, with a stock price of $0.65, means the event "it will rain tomorrow" has a 65% chance of occurring. Best possible prediction simply means the market price (and predicted probability of an event) that would form if all agents revealed their private information to one another prior to bidding; the associated *fully informed* equilibrium market price that is attained is referred to as the *direct communication equilibrium* (DCE) (Chen et al., 2004). *Best possible decision* (*action*, or *project*) simply means the decision (action, or project) that is implied by the decision market built using prediction markets that have attained DCE prices. The best possible decision is consequently the one that maximizes the probability of an outcome being achieved when compared to other possible decisions; where all constituent prediction markets have attained the DCE.

The proportion of simulated prediction and decision treatment markets converging to the DCE-based best possible prediction and best possible decision (attained in the associated control markets) are recorded. Given that 2500 market (control-treatment) pairs are run at each relevant information level in the prediction and decision market simulations, the law of large numbers implies the functional form of the probability distribution functions associated with prediction and decision markets. For the prediction market simulation data, relevant information level has 51 uniformly distributed possible values from 0 to 1. The prediction market experiment's null hypothesis ("increasing the proportion of relevant information (relevant information level r) in a prediction market does not increase the probability of attaining the best possible prediction") is rejected at the 1% significance level. For the decision market simulation data, relevant information level has 21 uniformly distributed possible values from 0 to 1. The decision market experiment null hypothesis ("increasing the

[17] Grainger et al. (2015) provides a mathematical proof that all traders expressing relevant information (relevant information level of 1) in proper information markets implies that prediction and decision markets provide the best possible predictions and decisions.

proportion of relevant information (relevant information level r) in a decision market does not increase the probability of attaining the best possible decision") is rejected at the 1% significance level. Moreover, a linear regression of "the proportion of decision markets converging to the best possible decision" on "relevant information level" is constructed to reveal a strong linear relationship (with a high coefficient of determination) and statistically significant positive gradient. As a result of the law of large numbers, this provides a credible *linear* functional form for the probability distribution function; i.e., the likelihood of the best decision being selected *linearly* increases as relevant information level increases.

The format of this chapter is as follows. A review of related literature is undertaken in section 4.2. The hypotheses for the investigations in this chapter are considered in section 4.3. The methodology and computer simulations constructed for this study are described in section 4.4 in order to provide appropriate context and communicate important details of the experimental setups. Section 4.5 sets out the main results and principal findings with annotations to assist interpretation. The analysis performed in section 4.6 explores principal findings in greater detail. Section 4.7 discusses the implications of the analysis while section 4.8 concludes the chapter and suggests future research directions.

4.2 Related Literature

This section reviews the literature related computer simulated prediction and decision market experiments of this chapter.

4.2.1 Computer simulated prediction and decision markets

Prediction markets are profusely utilized in firms (Cowgill and Zitzewitz, 2015). However, firms arguably require decision markets (trading action stocks) instead of prediction markets (trading outcome stocks) given they fundamentally wish to choose the best possible decision (or action, or project) that maximizes the probability of some desired outcome (Hanson, 1999). Both are considered useful and the quest to discern the conditions to ensure they properly function remains an open questions and hence justifies the investigation of this chapter (Cowgill and Zitzewitz, 2015; Buckley, 2016).

Computer simulated decision markets may be divided into three categories propose in literature i.e. conditional elicitation, market scoring elicitation and joint elicitation of probabilities market types (Othman and Sandholm, 2010). Market scoring elicitation decision markets that utilize scoring rules is considered an important area of research (Chen et al., 2011). The Market Scoring Rule (MSR) price formation process underpinning market scoring elicitation contrasts with the Continuous Double Auction (CDA) price formation mechanism typically found in the stock market and the Iowa Electronic (prediction) Market (Pennock, 2004)[18].

Because of the dominance of the market scoring rule approach as a prediction and decision market computer algorithm, it becomes important to review the MSR in order to clarify the point of failure in the MSR decision market setting computer algorithm.

4.2.2 Computer algorithm Market Scoring Rule (MSR)

MSR price formation is achieved via a computer algorithm that traders sequentially arrive at, submit a stock order quantity, and receive a cost (and therefore an implied market). Early literature on proper scoring rules investigated how to incentivize desired behavior (Savage, 1971). Market scoring rules utilized this work as a basis to design rational, risk-neutral, and myopic incentivizing sequential scoring rules to act as market makers (processing market orders and setting market prices) in prediction markets (Hanson, 2012). The market maker was always available to trade with; enhancing MSR prediction market liquidity. However, attempts to apply market-scoring rules in decision markets have been confounded at the theoretical level. It has been revealed that logical inconsistencies are unavoidable in applying the rules, i.e., a mixed strategy decision selection rule is required even when the decision maker knows with certainty the best decision to choose (Chen et al., 2014). This is explored in greater detail in the following subsection (4.2.2.1) in order to highlight exactly where the MSR extension from prediction market to decision market fails, and why this thesis' extension from prediction to decision market succeeds.

4.2.2.1 Computer algorithm Logarithmic Market Scoring Rule (LMSR) for prediction markets

The first key MSR idea is the notion of 'proper scoring rule' (Gneiting and Raftery, 2007). A proper scoring rule is one that incentivizes the agent to report their beliefs truthfully (Gneiting and Raftery, 2007). For example, consider a logarithmic scoring rule with a payout score $S(p) = b \times ln(p); b\ constant$. That is, if the player (or trader, or agent) reports that the probability of an event occurring is p then they will be paid score $S(p)$ if that event

[18] It should also be noted that CDA is arguably implied in both the conditional and joint elicitation market types of Othman and Sandholm (2010a).

occurs or they will be paid $S(1-p)$ if that event does not occur. It may be the case that the player reports p but believes q. Thus they would calculate an expected score $E[S(p)] = q \times b \times ln(p) + (1-q) \times b \times ln(1-p)$. They would maximize (in a rational, risk neutral, and myopic way) this expected score by submitting a report p that maximises $E[S(p)]$ thus:

$$\frac{dE[S(p)]}{dp} = \frac{bq}{p} - \frac{b(1-q)}{1-p} = 0$$

which simplifies to:

$$p = q$$

Also

$$\frac{d^2E[S(p)]}{dp^2} = -\frac{bq}{p^2} - \frac{b(1-q)}{(1-p)^2} < 0$$

so the score is a maximized when $p = q$.

That is, the best strategy for the player is to truthfully report the probability q.

The logarithmic market scoring rule (LMSR) is considered to be the most popular market scoring rule (Guo and Pennock, 2009). LMSR is very different to a stock market type price formation mechanism and has been termed an inventory based market maker (Brahma et al., 2012). In summary, LMSR records an inventory of shares in the market, and utilizes this along with a trader order to arrive at the cost for that order.

Hanson (2003) pioneered the use of proper scoring rules in a prediction market setting and provided the pivotal step of implementing a sequential market maker with a logarithmic cost function; i.e., traders arrive in a queue at a logarithmic proper scoring rule (LMSR cost function) that is used to determine stock prices. This insight heralded the birth of the LMSR. The LMSR cost function simply keeps track of the quantity (inventory) of stocks outstanding (trading in the market) and generates a cost for any quantity order request a trader has of it. Specifically, the LMSR cost function is:

$$C(q) = b \times ln\left(\sum_i e^{\frac{q_i}{b}}\right)$$

where q_i (an element of the q quantity vector) records the outstanding inventory number/quantity of stocks of asset i (Goel et al., 2008).

The intuition behind the LMSR cost function may be simply stated. When the event associated with asset i occurs, the LMSR market maker (MM) pays \$1 on each outstanding stock, i.e., the MM pays $q_i \times \$1$. The MM is effectively also rewarding the market with a proper score for providing a probability report p_i; i.e., a proper scoring rule $S(p_i) = b \times ln(p_i)$ ensures truthful reporting by a rational, risk neutral and myopic (agent) trader. That is, the MM effectively rewards the market with a score $S(p_i) = \$q_i = b \times ln(p_i)$. This may be rearranged to obtain $p_i = e^{\frac{q_i}{b}}$. Now ideally, $\sum_i p_i = 1$ when all mutually exclusive probability events (indexed by i) are traded in the market. Therefore, $p_i = e^{\frac{q_i}{b}} = \frac{e^{\frac{q_i}{b}}}{1} = \frac{e^{\frac{q_i}{b}}}{\sum_i p_i} = \frac{e^{\frac{q_i}{b}}}{\sum_i e^{\frac{q_i}{b}}}$. Since $p_i = \frac{dC(q)}{dq_i}$, integrating arrives at the LMSR cost function $C(q) = b \times ln\left(\sum_i e^{\frac{q_i}{b}}\right)$.

A trader may wish to trade a number of different stocks and so arrives at the logarithmic MM (cost function) with order quantity Δq. This trade would change the outstanding stocks from q to $q^* = q + \Delta q$. The trader is required to pay $[C(q^*) - C(q)]$[19].

At some future point in time the market game ends and settlement occurs. In general, the market game starts with q_0 shares already outstanding, and at the game end there are q' shares outstanding. If the event associated with q' occurs, then the LMSR (market maker) needs to pay traders \$1 for every share (or stock, or unit) it has provided traders since the start of the game, i.e., $q' - q_0$. Payments the market maker has received since the start of the game is simply:

$$C(q') \ldots - C(q^*)] + [C(q^*) - C(q)] + C(q) \ldots - C(q_0) = C(q') - C(q_0)$$

Therefore the net Loss (payment made) by the logarithmic market maker to all traders is $Loss = \$1 \times (q' - q_0) - (C(q') - C(q_0))$.

The maximum possible loss may be calculated as follows:

[19] Note: negative quantities of trader payments represents that the trader is selling the quantity of stocks.

$$Loss = q' - q_0 - (C(q') - C(q_0))$$
$$= b \times ln\left(e^{\frac{q'}{b}}\right) - b \times ln\left(e^{\frac{q_0}{b}}\right) - (b \times ln\left(\sum_i e^{\frac{q'_i}{b}}\right) - b \times ln\left(\sum_i e^{\frac{q_{oi}}{b}}\right))$$
$$= b \times ln\left(\frac{e^{\frac{q'}{b}}}{\sum_i e^{\frac{q'_i}{b}}}\right) - b \times ln\left(\frac{e^{\frac{q_0}{b}}}{\sum_i e^{\frac{q_{oi}}{b}}}\right) \leq b \times ln\left(\frac{\sum_i e^{\frac{q_{oi}}{b}}}{e^{\frac{q_0}{b}}}\right)$$

Now $b \times ln\left(\frac{\sum_i e^{\frac{q_{oi}}{b}}}{e^{\frac{q_0}{b}}}\right)$ is the maximum possible loss and it is also an entropy type term where constant b and the initial outstanding assets govern the extent of the maximum loss. Note that there is a MM tradeoff problem inherent in the value of b, whereby a large value attracts traders by providing greater liquidity for the desired (stock) position of the trader (i.e., large change in quantity leads to small price change), but a low value minimizes the market maker's loss.

4.2.2.2 Computer algorithm Market Scoring Rules (MSRs) for decision markets

Chen (2014) considers a general form market-scoring rule for prediction markets (which includes the LMSR) and takes the next logical step of applying MSRs to build decision markets. However, Chen (2014) finds that this leads to a logical inconsistency. The details of this are important to understand and as such an attempt is made to explain it in an accessible way in the following.

General MSR prediction markets

Let $S(O, p)$ represent the score for the agent reported prediction probability vector p associated with outcome O. Then it is a proper MSR if $\sum q \times S(O, q) \geq \sum q \times S(O, p)$; where q is the probability the agent truly believes, i.e., the agent truthfully reporting q is the best strategy to maximize the trader's expected score.

General MSR decision markets

A MSR decision market is related to a MSR prediction market via a MSR decision rule. Consider the MSR decision rule as a mixed strategy (decision rule) d over all possible actions (denoted $d(a, C)$); where action a is selected with probability $d(a, C)$ given the set of all conditional probabilities C (i.e., the probability of outcome O conditioned on each possible action a).

Let $S(O, p", a, C)$ represent the score for agent reported conditional probability $p"$ of outcome O given action a is selected.

Then in general, it is a proper MSR if:

$$\sum_{a} d(a, C) \sum_{q} q \times S(O, q, a, C) \ \geq \sum_{a} d(a, C) \sum_{p"} q \times S(O, p", a, C)$$

i.e., the truthful probability of outcome conditioned on action (reported conditional probability q) is the best (reporting) strategy for the rational, risk-neutral and myopic trader.

Given a strictly proper scoring rule $S(O, p)$ associated with a prediction market for outcome O, to ensure logical consistency, the decision market scoring rule is specified as follows:

$$S(O, p", a, C) = \begin{cases} \dfrac{1}{d(a, C)} S(O, p") \ iff \ d \ selects \ action \ a \\ 0 \qquad\qquad\qquad\qquad otherwise \end{cases}$$

The expected score of an agent would then therefore be:

$$\begin{aligned} &\sum_{a} d(a, C) \sum_{p"} q \times S(O, p", a, C) \\ &= \sum_{p"} d(a, C) \times q \times \frac{1}{d(a, C)} S(O, p") \\ &= \sum_{p"} q \times S(O, p") \end{aligned}$$

Since $S(O, p)$ is a strictly proper scoring rule then $p" = q$, i.e., this specification ensures a decision market with truthful reporting by traders exists.

However, the term $\frac{1}{d(a,C)} S(O, p")$ requires that $d(a, C) \neq 0$, i.e., all actions have a non-zero probability of selection; a condition called full support.

Therefore, whilst truthful reporting is guaranteed, there exists a problem with MSR decision markets built in this way. Specifically, the decision maker may come to know the best action to choose towards the end of the market game, however they are still required to randomly select an action in accordance with decision rule $d(a, C) \neq 0$ in order to guarantee truthful reporting.

4.2.3 Related computer simulation research

Ideally, the mechanism design of information markets (prediction markets and decision markets) aim to incentivize the elicitation of private probability information from trading agents (Myerson, 2008). However, the assumption of myopic incentive agents is false in 'real-world' LMSR markets where the capacity for strategic play leads to suboptimal equilibria (Dimitrov and Sami, 2008). Despite this, idealized LMSR and other market models can provide useful general insights on possible real-world market dynamics.

Computer simulations provide a useful means of testing idealized dynamics of information markets under multitudinous varying scenarios; something that would otherwise be impractically time-consuming to prove in people-based trading experimental settings. For example, computer simulations have usefully revealed that LMSR markets do not adapt well to changes in market liquidity levels (Brahma et al., 2010). Other computer simulated information markets have been investigated to explore more abstract settings that potentially reveal dynamics underpinning all markets. For example, the abstracted zero intelligence simulated agent (who does not learn or seek profit and in one sense buys and sells randomly), arguably provides evidence that experts in an information market do not influence its performance (Othman, 2008). At the extreme, simulated prediction markets are not constructed with the intention of understanding human prediction markets but, instead, are employed as an artificial intelligence tool to solve problems in much the same way that agent based algorithms (e.g., genetic algorithms) are applied (Jahedpari et al., 2014).

The DCE is the best possible (prediction) price that would be attained by a market if all agents revealed their private information to one another prior to bidding (Chen et al., 2004)[20]. This chapter ultimately uses computer simulations to test the significance of relevant information level r on convergence of prediction and decision markets to the best possible DCE-based predictions and decisions. The following subsection (4.3) considers the key theoretical concepts that the computer simulations of this chapter are intended to validate.

4.3 Hypotheses for the Computer Simulated Experiments

Theoretically, a simple decision market may be built using multiple prediction markets with proper market prices (Grainger et al., 2015). The Grainger et al., (2015) prediction market

[20] This thesis defines a market as *functioning well* when the direct communication equilibrium (DCE) is attained in finite rounds.

and decision market theoretical model concepts are reviewed in this section so as to highlight the key concepts underlying the computer simulation investigation of this chapter. The theoretical model describing the prediction market with proper market prices is set out in subsection 4.3.1 with key theorems provided in subsection 4.3.2.

4.3.1 Theoretical model: prediction market with proper market prices

There exist exogenously defined states of the world denoted by Boolean vector $s \in \{0,1\}^N$. There exists a commonly known probability distribution $P(s)$. There is one stock $F^{(A)}$ traded in this market that pays at some future point in time: it pays \$1 when a function $f^{(A)}(s) = 1$ and \$0 otherwise. Noting that $f^{(A)}(s) = 1$ when event A occurs, and is zero otherwise. There are n agents (or traders, or players) that submit a bid in each round of the market game. Agent i receives a private bit of information $x_i^{(A)}$ pertaining to the value of $f^{(A)}(s)$. Information bits received signal to each agent the probable value of the function[21] e.g. $x_i^{(A)} = 1$ may signal to agent i that $f^{(A)}(s) = 1$ is more likely than $f^{(A)}(s) = 0$. There is a commonly known probability distribution $Q(x^{(A)}|s)$; where $x^{(A)} \in \{0,1\}^n$ and $x_i^{(A)}$ is the i^{th} element of the agents' information vector $x^{(A)}$. All agents are rational, myopic and risk neutral. In the first-round agent i therefore submits a bid $b_i^{(A)} = E\left[f^{(A)}(s)|x_i^{(A)}\right]$, i.e., the expected value of the function $f^{(A)}(s)$ conditioned on their private information $x_i^{(A)}$. At the end of round 1 the stock's market price (Shapley-Shubik function) is revealed to all agents as $p_1^{(A)} = \frac{1}{n}\sum_{i=1}^{n} E\left[f^{(A)}(s)|x_i^{(A)}\right]$. In the second round, agent i submits a bid based upon all previous round market prices $P^{(A)}$ and its private information bit $x_i^{(A)}$; thus its revised bid is $b_i^{(A)} = E\left[f^{(A)}(s)|x_i^{(A)}, P^{(A)}\right]$. For example, at the start of round 2 there is only one previous round market price $P^{(A)} = \left\{p_1^{(A)}\right\}$ and therefore $b_i^{(A)} = E\left[f^{(A)}(s)|x_i^{(A)}, P^{(A)}\right] = E\left[f^{(A)}(s)|x_i^{(A)}, p_1^{(A)}\right]$. Bidding continues each round in a similar way. If all agents learn full information $x^{(A)}$, as a result of learning via $x_i^{(A)}$ and $P^{(A)}$, they will all bid $E\left[f^{(A)}(s)|x^{(A)}\right]$. Thus the market price on the stock will be $E\left[f^{(A)}(s)|x^{(A)}\right]$ called the direct communication equilibrium (DCE) or best possible prediction price; since this is the price that would be

[21] Aggregate uncertainty of information exists, i.e., the true value of the function is unknown even if all agents aggregate information.

attained if all agents directly communicated their private information to one another resulting in all agents bidding in a best possible (fully informed) way.

All market prices in the prediction market model of Grainger et.al. (2015) are 'proper market prices'; a fundamental concept that is formally defined as follows.

'Proper market prices' are simply a criterion on the stock's market price at the private information stage (first round where agents know only their private information) and full information stage (where all agents know all information bits across the market); whereby in the former, agents must learn something about other information bits (i.e., they must learn something from the first round price formed by any subgroup of m traders; where $m \leq n$ and n is the total number of agents trading in the market) and in the latter, agents with full information cannot unlearn information (as a result of the market price).

Formally, in the first round $p_1^{(A)} = \frac{1}{m}\sum_{i=1}^{m} E\left[f^{(A)}(s)|x_i^{(A)}\right] \neq \frac{1}{m}\sum_{i=1}^{m} E\left[f^{(A)}(s)|y_i^{(A)}\right]$ and for arbitrary agent k, $y_i^{(A)} = \begin{cases} x_i^{(A)} \; if \; i = k \\ x'^{(A)}_i \; if \; i \neq k \end{cases}$ where $x_i^{(A)} = 1 - x'^{(A)}_i \in \{0,1\}$.

Formally, in the full information (last round) for agent k, $p_{full\,info}^{(A)} = E\left[f^{(A)}(s)|x_i^{(A)}, x_{-i}^{(A)}\right] \neq E\left[f^{(A)}(s)|x'^{(A)}_i, x_{-i}^{(A)}\right] \; for \; all \; i \neq k.$

Each agent in this market may or may not express relevant information in their bids.

'Relevant information' is expressed by an agent if the bids that they submit change when their private bit of information that they possess changes, i.e., for agent i, $b_i^{(A)} = E\left[f^{(A)}(s)|x_i^{(A)}, P^{(A)}\right] \neq E\left[f^{(A)}(s)|x'^{(A)}_i, P^{(A)}\right]$.

'Relevant information level' is the proportion of agents in a market that express relevant information in their bids; denoted $r \in [0,1]$.

4.3.1.1 Theoretical model: decision market with proper market prices

Let there exist multiple prediction markets with proper market prices. Specifically, there exists $2M$ prediction markets, i.e., stocks $F^{(A_j)}$ and $F^{(OA_j)}$where $j = 1 \; to \; M$ are traded. These markets are linked by a payout rule such that "stocks $F^{(A_g)}$ and $F^{(OA_g)}$ are paid \$1 and

$\$\frac{price(F^{(OA_g)})}{price(F^{(A_g)})}$ respectively if $g = argmax_j\left[\frac{price(F^{(OA_j)})}{price(F^{(A_j)})}\right]$ at some future decision-making point in time, else they are both paid \$0". Note that the maximum at g means that the decision market selects the action A_g and the probability of the outcome O given this action A_g is a maximum across all actions. If all agents have full information then the DCE price is reached (in all prediction markets such that all agents are fully informed) and the decision market selects the best possible (fully informed) decision (action, project). Here **'Best possible decision'** is the game theoretic argmax function (Othman and Sandholm, 2010). It is the maximum of the conditional probabilities of outcome given an action (vis. project) considered across all actions. This is implicitly a joint elicitation decision market (Othman and Sandholm, 2010) using prediction markets that trade the join contracts 'outcome and action' and also the 'action' contracts. The respective contract prices when combined (i.e., the quotient of their prices) imply conditional probabilities, and when at the DCE (where all agents express full information) also imply the best possible decision (action, project).

4.3.2 Theorems implied by the theoretical model

Two key theorems arise from the theoretical model in section 4.3.1. Specifically, the prediction market and the decision market, Theorem 1 and Theorem 2 emerge respectively (Grainger et al., 2015).

Theorem 1:

"All agents expressing relevant information" is a sufficient and necessary condition for convergence to the direct communication equilibrium in a prediction market with proper market prices.

Theorem 2:

If $F^{(OP)}$ and $F^{(P)}$ represent prediction market stocks associated with events OP (Outcome O occurs and project P (vis. P as a decision or action) is selected) and P (Project P is selected), and a particular decision rule (pay \$1 if P occurs (\$0 otherwise) and $\$w = \frac{\mathrm{price}(F^{(OP)})}{\mathrm{price}(F^{(P)})}$ if OP occurs (\$0 otherwise), and select P if w is the maximum value across all projects), and all agents express relevant information in the constituent prediction markets with proper market prices (i.e., DCE prices are attained), then P is the best possible project (decision) if w is the maximum across all projects.

Theorem 2 suggests that full relevant information implies that the decision market selects the best possible decision. However, Theorem 2 does not define how the probability of 'selecting the best decision' varies with the proportion of traders in the market expressing relevant information i.e., the 'relevant information level'. The following paragraph describes this logical extension from Theorem 2; with a more formal corollary and proof provided in Appendix B: Chapter 4 Appendix 1.

With intent to convey this idea to the broader readership, there is a simple reason for expecting a positive gradient linear relationship of the probability of convergence versus relevant information level. In a decision market, an $r \times 100\%$ *increase in relevant information above no relevant information leads to an* $r \times 100\%$ *of 'possible improvement' in market selection performance; where 'possible improvement' is the difference between the full information and no relevant information probability of selecting the best possible decision. For example, if the decision market having no relevant information, selects the best decision 60% of the time, then the maximum possible improvement for the market is the remaining 40% of decisions it incorrectly selects. If all the market traders expressed relevant information then the full 40% improvement would be realized (i.e., the market would always select the best possible DCE based decision). In contrast, if no market trader expressed relevant information then this would mean the situation remains unchanged (at 60% correct selection by the market), i.e., 0% of possible improvement is realized. Now, consider an alternative setting where all traders expressed relevant information in 50% of market games and in the other 50% of market games no trader expressed relevant information, then two things may be observed. Firstly, on average there is a 20% of possible improvement achieved (half of the time 40% and the other half of the time 0%) across all markets run. Secondly, the relevant information level averaged across all markets is 0.5 (1 for half the market runs and 0 for the other half). The key concepts in this thought experiment are the notion of 'average market' and 'average relevant information level' with the 'equivalence principle' to be shortly applied. Consider two situations:*

Situation 1: the average relevant information level across many markets with fixed relevant information level of r

Situation 2: the average relevant information level across many markets whereby fully informed markets run $r \times 100\%$ *of the time and no relevant information level markets run the rest of the time*

If relevant information level is related to (decision market) performance, then average relevant information level is related to the average performance. Consider an idealized observer who identifies two decision markets as performing as well as one another if and only if they have the same average relevant information level. The 'average market' in situation 1 and situation 2 has the same 'average relevant information level r*'. Such an observer could not observe both situations as different probabilities of selecting the best possible decision. Therefore, the average markets of situation 1 and situation 2 are equivalent and the probability of selecting the best possible decision given* r *is:*

$P[D|r] = r \times P[D|r = 1] + (1 - r) \times P[D|r = 0]$ *; where* D *is the event "the best possible decision is selected by the decision market".*

A concise and mathematically rigorous foundation is provided as a corollary in Appendix B: Chapter 4 Appendix 1. But the take-away here is that this thought experiment, extending the theoretical model, suggests that if relevant information level is statistically significant to the probability of a decision market selecting the best possible decision, then it must be a linear relationship. This may be tested for in the computer simulations.

The theoretical (mathematical) prediction and decision market model of this thesis is a deliberately simple and idealized representation of the real-world; only ever intended to serve as a guide providing insights about the real-world. As such, the theorems and corollary suggest that relevant information level is potentially important in well-functioning simulated and real-world prediction and decision markets. This logically motivates the investigation of the statistical significance of the proportion of relevant information in markets (i.e., relevant information level r). Thus, the testable hypotheses in a computer-simulated setting are as follows:

Hypothesis 1: "Increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction".

Hypothesis 2: "Increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision".

Moreover, by the Appendix B: Chapter 4 Appendix 1 corollary, ***if hypothesis 2 holds then there is a linear relationship*** between relevant information level and the probability of the best possible decision being selected by the decision market.

4.4 Experimental Design and Methodology

This section is intended to: (1) describe the design of the computer simulation experiments; (2) outline the experimental methodology; (3) consider in detail the key elements of the simulated markets; and, (4) specify the experimental methods that are employed in the analysis.

4.4.1 Computer simulation experimental design

In order to test hypotheses 1 and 2, many (tens of thousands) computer simulated experiments of prediction and decision markets are run in Matlab.

For the prediction market and for each relevant information level, firstly, the Matlab simulation randomly chooses a state of the world; secondly, the agents' information vector conditioned on the world state is generated; thirdly, the agents' bid vector is generated to form a market price; fourthly, all agents use this market price information and their private bit to infer a best estimate of the agents' information vector; and, finally, all agents submit another bid using this best estimate to form an equilibrium market (prediction) price. Multiple control-treatment pairs of market games are run. A pair, consisting of a control and treatment market, differs only in relevant information level; the former (control market) always at relevant information level 1 and the latter at some relevant information level in the closed interval 0 to 1. The proportion of market games for which the computer simulated control and treatment market equilibrium prices are equal is recorded. This dichotomous outcome lends itself to binary logit analysis. That is, the statistical significance of relevant information level on the probability of attaining the best possible prediction is determined. Given that 2500 games are run at each of the 51 relevant information levels, the law of large numbers suggests that the variance associated with the proportion is extremely small.

For the decision market and for each relevant information level, four prediction markets are run whereby each attain an equilibrium price that when combined imply a computer simulated market decision. Multiple control-treatment pairs of decision market simulations (games) are run. Here too, a pair differs only in relevant information level; whereby the control (decision) market has relevant information level of 1 and the treatment relevant information level varies from 0 to 1. The proportion of decision market game pairs for which the computer-simulated control and treatment market decisions are the same is recorded. This dichotomous outcome lends itself to binary logit analysis. That is, the statistical significance of relevant information level on the probability of attaining the best possible decision is determined. Given that 2500 games are run at each of the 21 relevant information levels, the law of large numbers suggests that the variance associated with the proportion is extremely small. Given a theoretically expected linear relationship (of proportion increasing with relevant information level increasing), a linear regression is performed with a very high correlation of determination and a statistical geometric interpretation (Saville and Wood, 2012; Bryant, 1984; Siniksaran, 2005) used to assess whether the functional form is linear.

4.4.2 Experimental methodology

The validity of two theoretically derived hypotheses; one relating to prediction markets and one relating to decision markets is tested using computer simulations[22].

Hypothesis 1: "Increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction".

Hypothesis 2: "Increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision".

In both the prediction market and decision market settings, relevant information level is varied in the treatment market and the ability of the treatment market to attain the computer simulated 'best possible prediction' or 'best possible decision' occurring in the control market is observed. This control-treatment experimental setup (Aldrich, 2007) is justified given that it is possible to construct a computer simulation of two markets differing only in one variable; in this case relevant information level r. This contrasts with real-world settings

[22] NB: proper market prices hold in the computer simulated markets as described in the sections below.

that are confounded by the inability to perfectly ascribe the same values (bar the one being varied across experiments) to a control and its treatment. Thus, because this is a computer-simulated experiment, a control-treatment setup may be perfectly performed, and the statistical significance of relevant information level accurately assessed for this setting.

For the prediction market hypothesis, the results (outcomes) under analysis are dichotomous; 1 indicating control and treatment attain the best possible prediction, or 0 indicating otherwise. The dichotomous outcomes (Y), as a function of relevant information level r, lend itself naturally to a binary logit response analysis (Trexler and Travis, 1993); i.e., $P[Y = 1|r] = expit(\beta_r r + \beta_0)$. Of interest is determining whether increasing r is associated with increasing $P[Y = 1|r]$ in a statistically significant way. This is satisfied for this series of computer simulations when $\beta_r > 0$, and when $\beta_r = 0$ is rejected at a 1% significance level i.e., the hypothesis testing method is justifiably applied. In words, the null hypothesis to be rejected at the 1% significance level is "increasing the proportion of relevant information (relevant information level r) in a prediction market does not increase the probability of attaining the best possible prediction".

Additionally, and because of the enormity of data accrued via computer simulated prediction markets, the probability distribution functional form can be graphed. This is a direct result of the law of large numbers. Specifically, at each relevant information level there are 2500 prediction market experiments run. Because the outcome $Y \in \{0,1\}$ is binomial, the observed average is $P[Y = 1|r]$ and the estimate for the standard deviation standard deviation $\sigma(r)$ is $\sqrt{\frac{P[Y=1|r](1-P[Y=1|r])}{2500}} = \frac{\sqrt{P[Y=1|r](1-P[Y=1|r])}}{50}$. By the Markov and Chebychev inequality $P[|P[Y = 1|r] - \mu(r)| > k\sigma(r)] < \frac{1}{k^2}$ and if $k = 10$ then $P\left[|P[Y = 1|r] - \mu(r)| > 10\frac{\sqrt{P[Y=1|r](1-P[Y=1|r])}}{50}.\right] < 1\%$ i.e., there is a 1% chance of observing $P[Y = 1|r]$ more than 10 standard deviations from the true mean. Noteworthy is that due to the large number of experiments (2500) at each r the observed value $P[Y = 1|r]$ is extremely close to the true mean at the 1% significance level; at worst when, $P[Y = 1|r] = 0.5$, it has a 10-percentage points distance from the true mean. Alternatively, a 99% confidence interval may be established at each r where the true mean $\mu(r)$ has a 99% chance of being in the interval:

$$\left[P[Y = 1|r] - 10\frac{\sqrt{P[Y=1|r](1-P[Y=1|r])}}{50}, P[Y = 1|r] + 10\frac{\sqrt{P[Y=1|r](1-P[Y=1|r])}}{50}\right].$$

In summary, due to the large number of experiments and probability distribution functional form can be closely estimated.

The decision market methodology is similar to the aforementioned prediction market approach. For the decision market hypothesis, the results (outcomes) under analysis are dichotomous; 1 indicating control and treatment attain the best possible decision, or 0 indicating otherwise. The dichotomous outcomes (Y), as a function of relevant information level r, lend itself naturally to a binary logit response analysis (Trexler and Travis, 1993); i.e., $P[Y = 1|r] = expit(\beta_r r + \beta_0)$. Of interest is determining whether increasing r is associated with increasing $P[Y = 1|r]$ in a statistically significant way. This is satisfied for this series of computer simulations when $\beta_r > 0$, and when $\beta_r = 0$ is rejected at a 1% significance level i.e., the hypothesis testing method is justifiably applied. In words, the null hypothesis to be rejected at the 1% significance level is "increasing the proportion of relevant information (relevant information level r) in a decision market does not increase the probability of attaining the best possible decision".

Because of the corollary in Appendix B: Chapter 4 Appendix 1, a linear relationship is anticipated. Moreover, because of the large number of computer-simulated experiments run at each r (i.e., 2500), an identical argument to the prediction market experiments, that the probability distribution function may be confidently graphed due to the law of large numbers, holds. Therefore, after graphing the results an ordinary linear regression method is justified to discern the goodness of fit to the theoretically expected linear relationship. To this end, a suitably high coefficient of determination (R^2) along with a geometric interpretation i.e. $cos^2(\theta) = R^2$; where θ is the angle between the observation vector and the model space (that is assumed linear) is utilized (Saville and Wood, 2012). In this geometric sense, a high coefficient of determination (R^2) represents a high goodness of fit to a linear model. The coefficient of determination and F-test are also related via $F_{1,n-2} = \frac{R^2(n-2)}{(1-R^2)}$ and utilizing this, the null hypothesis (gradient of line is zero i.e., $\beta_r = 0$) can be rejected at a 1% significance level. In summary, the binary logit analysis and the geometric statistics goodness of fit approach is justified for the series of decision market computer-simulated experiments in this chapter.

4.4.3 Key elements of the computer simulated markets

The main elements of the computer simulation are:

1. State of the world
2. Agent information conditioned on the state of the world
3. Agent bidding and stock flow mechanism
4. Ensuring proper market prices hold
5. Comparing control and treatment markets.

Each of these elements will be considered in subsections 4.4.3.1 to 4.4.3.5.

4.4.3.1 State of the world

For each traded stock in the computer simulation, there are two states of the world $s \in \{0,1\}$. The prior probability distribution $P(s = 1)$ is common knowledge. For the prediction market, there is one stock that pays \$1 when a binary function $f(s) = 1$ at some future point in time and \$0 otherwise. For simplicity and without loss of generality $f(s) = s$ in the computer simulated experiments. That is, the stock pays \$1 when $s = 1$ and \$0 when $s = 0$. In the actual Matlab simulation, this 'state' function has been defined accordingly.

4.4.3.2 Agent information conditioned on the state of the world

Central to the entire simulation is the ability to *quickly* generate a single agent probability information distribution as well as the DCE probability information distribution. Without this feature, the simulation is arguably computationally intractable. This is not a simple task and, as such, further explanation of the approach is offered in Appendix B: Chapter 4 Appendix 2.

Upon obtaining the value for the state of the world, the agent information vector is then drawn from the probability distribution $Q(x|s)$. In the simulation, each agent expresses 1 information bit and there are n agents. Given that Chen (2004) suggests 'independent and identically distributed (IID) information' implies relevant information and hence DCE convergence, there is a need to also consider the 'dependence across information' setting. This will ensure that relevant information is truly the cause of DCE convergence and not simply a result of IID agents' information. Furthermore, dependence across information is arguably what is observed in the real-world as agents express information based upon information sourced from the world, which inherently includes other agents' information (Treynor, 1987). In summary, it is unlikely in the real-world that an agent's information is independent of other agents' information (for all agents).

To establish the required probability distribution for large n agents is a computationally difficult task in a dependence context. Given each agent expresses 1 information bit and there are n agents, then there are 2^n simple outcomes to be assigned a probability value in the probability distribution $\mathrm{Q}(\mathrm{x}|\mathrm{s})$; where each simple outcome is a specific point in the vector space $\{0,1\}^n$. To keep things simple the binomial theorem is utilized. In so doing, the generation of information vectors (with or without dependence) becomes computationally tractable. The formal treatment of this issue is detailed in Appendix B: Chapter 4 Appendix 2.

4.4.3.3 Agent bidding and stockflow mechanism

There are n agents in the market. The theorems assume all agents possess common knowledge over the agents' information probability distribution $\mathrm{Q}(\mathrm{x}|\mathrm{s})$ as well as being endowed with the significant processing ability to exhaustively check all possible information vectors x to determine which one led to the first-round market price p. The computer-simulated implementation of these informed bids and the market price formation mechanism is considered in the following subsections.

How uncertain information affects bids

There exists uncertainty about the information (about the state of the world) that Agent i receives and expresses (via their bidding). Agent i knows how the information bit it expresses depends on the state of the world by considering commonly known probability distribution $\mathrm{Q}(\mathrm{x_i}|\mathrm{s})$[23]. Now agent information $\mathrm{x_i} = 1$ may be interpreted as the agent believing that $s = 1$ with uncertainty; given $\mathrm{Q}(\mathrm{x_i}|\mathrm{s})$ is known to that agent. Computer simulated agent i is rational, risk neutral and myopic and thus submits a first-round bid that incorporates this uncertain information $\mathrm{b_i} = \mathrm{E}[\mathrm{f}(\mathrm{s})|\mathrm{x_i}] = \frac{\mathrm{Q}(\mathrm{x_i}|\mathrm{s})\mathrm{P}(\mathrm{f}(\mathrm{s})=1)}{\mathrm{Q}(\mathrm{x_i})}$.

How the market price formation mechanism is modelled

The aggregation of bids is modeled by a Shapley Shubik price $\mathrm{p} = \frac{\sum \mathrm{b_i}}{\mathrm{n}}$. This may simply be thought of as a computer simulated market game in which there is a rule that all agents are

[23] It is trivial to see that knowing $\mathrm{Q}(\mathrm{x}|\mathrm{s})$ implies knowing $\mathrm{Q}(\mathrm{x_i}|\mathrm{s})$.

required to submit bids and if their bid is greater than p they receive one stock and pay $\mathrm{b_i} - p$[24]. Notice that the sum of all payments is:

$$\sum(\mathrm{b_i} - p) = \sum b_i - np = np - np = 0.$$

Moreover, this rule forces trading to occur and therefore overcomes the theoretical no-trade problems without the need for the market institution to subsidize trading[25].

Agent learning (from market price)

In the second round, computer simulated agents are able to learn via the first-round market price and their own private information bit. That is, they submit bids that depend on their private information bit and also the relevant information contained in the first-round market price.

Agent learning occurs as follows. In the first round computer simulated agent i bids $\mathrm{b_i} = \mathrm{E[f(s)|x_i] = P[s = 1|x_i]} = \frac{\mathrm{P[x_i|s=1]P[s=1]}}{\mathrm{P[x_i]}}$; since $f(s) = s \in \{0,1\}$ in the simulations. $\mathrm{P(s = 1)}$ is the prior probability that the stock will pay $1. It is also conceivable that this prior probability distribution is informed via prior market games and simply the market price from the last round of a previous market game. Any market price p not only reflects the probability of the prediction, but also contains both the private information of agent i and also other agents' information.

The theoretical model agent has significant computational resources (being able to consider all possible information bits across all agents) and discovers all relevant information bits. For computational tractability the computer-simulated agents in the study leverages the binomial technique of Appendix B: Chapter 4 Appendix 2 to ensure dependence across agents' information so as to properly test the hypotheses. Thus, without loss of generality, for any agent i, $P[x_\mathrm{i}|s] = u$ and $P[x'_\mathrm{i}|s] = d$; where u and d are constants e.g. $u = 0.8, d = 0.2$. In this simulated setting, agent i need only know the number of agents (n) in the market and how many (λ) of these are relevant information bidders, so as to learn the value of other agents' information bits. Specifically agent i would reason as follows: $(n - \lambda)$ agents provide

[24] If their bid is less than the market price then they sell one stock and receive this price/bid monetary difference.
[25] It should be noted that other rules exist to avoid Milgrom's no-trade problem. In the games study with human participants, a different trading rule incentivizes trading, guarantees market liquidity, and is also rational, myopic, risk neutral compatible.

bids that do not express relevant information, i.e., they all submit the same bid $E[f(s)] = P(s) = e$ (in effect their information bits are hidden behind the bid). Now, of the remaining (relevant information) agents, some bids are conditioned on an information bit of 1 and others on 0. Those l agents who bid conditioned on 1 will bid $E[f(s)|x_{\mathrm{i}} = 1] = \frac{P[x_{\mathrm{i}} = 1|s]P(s)}{P(x_{\mathrm{i}}=1)} = g$ and those who bid conditioned on 0 will bid $E[f(s)|x'_{\mathrm{i}}] = h$.

Agent i then reasons that the first-round market price p was formed via the market price formation (mechanism) equation $p = \frac{(n-\lambda)e+lg+(\lambda-l)h}{n}$. The only unknown being l, agent i solves this equation and therefore knows all relevant information bids that are in the market[26]. As such, in the second-round agent i submits a bid based on all the relevant information bits in the market. Agent i also realizes that by definition, it will never know the information bits that are not relevant information ones and thus bids the same in all following rounds. Therefore, the computer simulated market equilibrium is reached at the end of the second round.

4.4.3.4 Ensuring proper market prices hold

A qualifying condition of all the theorems is that proper market prices hold. The theoretical model requires that proper market prices hold at the private and full information stage, i.e., in the first and last[27] round of the simulation respectively.

Proper market prices are a reasonable assumption for real-world markets (Grainger et al., 2015). In the real-world, it is unlikely to find a market in which the first-round market price does not reveal new information to an agent. It is also unlikely, that when all agents know that they possess full information about the market, that a market price would cause them to doubt their information.

Similarly, it is unlikely that the computer-simulated agents in this chapter do not learn from the first-round market price. It is also unlikely that a market price will cause a fully informed computer-simulated agent to doubt (unlearn) information. That is, proper market prices effectively hold in the computer simulations. Appendix B: Chapter 4 Appendix 3 provides a

[26] If all bids express relevant information ($i.e. \lambda = n$) then agent i attains full information, i.e., all bids may be known and no bid hides an information bit.
[27] Per the previous section, the last round in these simulations may be considered as round 2; whereby at the end of it the computer simulated equilibrium price is reached.

more formal mathematical treatment explaining how proper market prices hold in the simulations.

4.4.3.5 Comparing control and treatment markets.

In the experiments, simulated treatment markets are compared to their control market counterpart; with the only difference between them being the relevant information level. All simulated control markets have a relevant information level of 1, i.e., all agents express relevant information and the equilibrium price in each control market always attains the DCE price for that market[28].

The variation of relevant information level ($r \in [0,1]$) in the simulated treatment markets is of interest. Specifically, of interest is the treatment (prediction or decision) market's ability to converge to the associated control (prediction or decision) market's DCE (based prediction or decision). The relevant information level is the only parameter that differs between each treatment and control market pair. Hence any differences between the control and treatment market is logically due to relevant information level.

The prediction markets contain 50 traders and relevant information level can vary from 0 agents with relevant information to all 50 agents with relevant information; i.e., 51 relevant information levels being $r = \frac{k}{50}; k = 0, 1, 2, \dots, 49, 50$.

The decision markets contain 20 traders and as such relevant information may vary from 0 agents with relevant information to all 20 agents with relevant information; i.e., 21 relevant information levels being $r = \frac{k}{50}; k = 0, 1, 2, \dots, 19, 20$. Where each decision market in the simulation market is implemented using 4 prediction markets in which all 20 agents trade.

For both prediction market and decision market simulations, at each relevant information level, there are 2500 simulations run; which in the following sections aids in simplifying analysis by allowing the invocation of the law of large numbers.

The large number of simulations is computationally demanding and on average the full decision market and prediction market experimental runs are measured in days (and in several cases, weeks) of Matlab processing time. To efficiently utilize time, code was

[28] Where the DCE price in each control market instance depends on the actual state of the world and the private information of all agents in that market.

modularized (i.e., broken into logical pieces), concurrently processed across a high-performance computing cluster, and reassembled to reveal the experimental results. By way of example, the key piece of code used for the decision market simulation is exhibited in Appendix B: Chapter 4 Appendix 4. The simulation results are provided in graphical form in the results section of this chapter.

4.4.4 Specifics of the experimental methods

For the prediction market simulations the relative frequency that "the treatment market equilibrium (prediction) price is equal to the control market (prediction) DCE price" is recorded. For the decision market simulations the relative frequency that "the treatment (decision) market selects the project that is selected in the control (decision) market based on DCE prices" is recorded.

In the prediction market and decision market experiments the hypothesis testing, binary logit analysis and law of large numbers are employed.

4.4.4.1 Specifics of the methods for prediction market simulations

The stochastic nature of the simulated setting and arguably the real-world setting means that there is a chance that a prediction market equilibrium price associated with "a market where some (and not all) agents expressing relevant information" converges to the DCE. The intent is to reject this possibility at a suitable significance level, i.e., 1%.

The number of agents in each prediction market simulation is $n = 50$. The "number of traders n_{rel} with relevant information" is varied from 0 to n. At each relevant information level $r = \frac{n_{rel}}{n}$, 2500 markets are run to determine "the proportion ρ_r of simulated treatment prediction markets (with relevant information level r) that converge to the associated control market DCE price". The law of large numbers and independence across simulations run imply that "proportion ρ_r" is close to the true mean of the distribution and as such an estimate of the functional form of the probability distribution function may be graphed.

The hypothesis test to determine the statistical significance of r is defined as follows:

Null Hypothesis H_0: *"Increasing the proportion of relevant information (relevant information level r) in a prediction market does not increase the probability of attaining the best possible prediction".*

Alternative Hypothesis H_1*: "Increasing the proportion of relevant information (relevant information level r) in a prediction market does increase the probability of attaining the best possible prediction".*

Significance level: *1%*

4.4.4.2 Specifics of the method for decision market experiment

Each simulated decision market is constructed using four prediction markets and linked via a decision and payout rule. A treatment and its associated control decision market will differ only in the relevant information level; whereby all agents in the control decision market express relevant information, and none/some/all agents in the treatment decision market express relevant information. There exist four contracts (stocks) in each (treatment and control) decision market: "project A", "outcome & project A", "project B", "outcome & project B". The ultimate purpose of the decision market is to identify which project maximizes the chance of the outcome.

Formally stated:

State of the world $\mathrm{s} \in \{0,1\}^2$; denote $\mathrm{s} = (\mathrm{s}_1, \mathrm{s}_2)$; $\mathrm{s}_1, \mathrm{s}_2 \in \{0,1\}$

Commonly known probability distribution $\mathrm{P(s)}$ is defined such that:

$$
\begin{aligned}
&\mathrm{P[A]} = \mathrm{P}[\mathrm{s}_1 = 1] \\
&\mathrm{P[A']} = \mathrm{P}[\mathrm{s}_1 = 0] \\
&\mathrm{P[OA]} = \mathrm{P}[\mathrm{s}_1 = 1, \mathrm{s}_2 = 1] \\
&\mathrm{P[OA']}\mathrm{P}[\mathrm{s}_1 = 0, \mathrm{s}_2 = 1]
\end{aligned}
$$

Events "Project **A**", "**O**utcome & project **A**", "project **A'**", "**O**utcome & project **A'**" occur, when their respective functions $\boldsymbol{f}^{(\mathbf{A})}(\boldsymbol{s})$, $\boldsymbol{f}^{(\mathbf{OA})}(\boldsymbol{s})$, $\boldsymbol{f}^{(\mathbf{A'})}(\boldsymbol{s})$, $\boldsymbol{f}^{(\mathbf{OA'})}(\boldsymbol{s})$, equal 1.

The above-mentioned functions are defined as follows:

$$\boldsymbol{f^{(A)}(s_1, s_2) = \begin{cases} 1 \; iff \; s_1 = 1 \\ 0 \; otherwise \end{cases}}$$
$$\boldsymbol{f^{(A')}(s_1, s_2) = \begin{cases} 1 \; iff \; s_1 = 0 \\ 0 \; otherwise \end{cases}}$$

$$\boldsymbol{f^{(OA)}(s_1, s_2) = \begin{cases} 1 \; iff \; s_1 \times s_2 = 1 \\ 0 \; otherwise \end{cases}}$$

$$\boldsymbol{f^{(OA')}(s_1, s_2) = \begin{cases} 1 \; iff \; (1 - s_1) \times s_2 = 1 \\ 0 \; otherwise \end{cases}}$$

To clarify this further for the reader, the following table is one way to represent these functions:

	S_2=0	S_2=1
S_1=0	A'	A' OA'
S_1=1	A	A OA

For example, the cell in the above table where $s_1 = 1$ and $s_2 = 1$, shows that $f^{(A)}(s_1, s_2) = 1$ and $f^{(OA)}(s_1, s_2) = 1$; meaning that “Project **A**” and “**O**utcome & project **A**” events occur and their respective stocks paid in accordance with the *payout and decision rule* discussed below.

Four separate prediction markets are run; one for each event.

For example, in the prediction market with “project stock A”:

$x_i^{(A)}$ is the private information agent i receives about stock A

Commonly known conditional probability distribution $Q(x^{(A)}|s)$ exists.

Round 1: agent i bids $b_i = E\left[f^{(A)}(s_1, s_2)\middle|x_i^{(A)}\right]$

Other rounds: agent i bids $b_i = E\left[f^{(A)}(s_1, s_2)\middle|x_i^{(A)}, \text{previous round market prices}\right]$

Payout and decision rule (that links four prediction markets into a decision market)

A payout and decision rule exists that acts to link the four prediction markets and elicit conditional probability (decision market) information.

On the final round, contracts (stocks) associated with the four prediction markets will have market prices; $\boldsymbol{p_A}$, $\boldsymbol{p_{O\&A}}$, $\boldsymbol{p_{A'}}$ and $\boldsymbol{p_{O\&A'}}$. At the end of the final round, if "project A and "Outcome & project A" contracts are trading at $\boldsymbol{p_A}$ and $\boldsymbol{p_{O\&A}}$ respectively and $\frac{\boldsymbol{p_{O\&A}}}{\boldsymbol{p_A}} > \frac{\boldsymbol{p_{O\&A'}}}{\boldsymbol{p_{A'}}}$ then "project A and "Outcome & project A" contracts are paid \$1 and \$ $\frac{\boldsymbol{p_{O\&A}}}{\boldsymbol{p_A}}$ respectively and all other contract paid \$0.

For the purposes of the experiment each control and treatment simulation pair outcome will be recorded as follows:

$$S_{treatment} = \begin{cases} 1 \; if \; \frac{\boldsymbol{p_{O\&A}}}{\boldsymbol{p_A}} > \frac{\boldsymbol{p_{O\&A'}}}{\boldsymbol{p_{A'}}}; 0 \le r \le 1 \\ \quad 0 \;\; otherwise \end{cases}$$
$$S_{control} = \begin{cases} 1 \; if \; \frac{\boldsymbol{p_{O\&A}}}{\boldsymbol{p_A}} > \frac{\boldsymbol{p_{O\&A'}}}{\boldsymbol{p_{A'}}}; r = 1 \\ \quad 0 \;\; otherwise \end{cases}$$

For each decision market simulation, at each level of relevant information, the treatment market is compared to the control market.

At each relevant information level r, 2500 markets are run and the indicator function[29]

$$J[r] = \begin{cases} 1 \; if \; S_{treatment}[r] = S_{control} \\ 0 \; if \; S_{treatment}[r] \neq S_{control} \end{cases}$$

is used to calculate $\rho[r] = Average[J[r]]$.

The law of large numbers and independence across simulations run imply that "proportion $\rho[r]$" is close to the true mean of the distribution and as such an estimate of the functional form of the probability distribution function may be graphed. A linear regression on the ρ versus r plot is performed, and the coefficient of determination is utilized as a goodness of fit measure for linearity.

The hypothesis test to determine the statistical significance of r is defined as follows:

<u>Null Hypothesis H_0</u>: "increasing the proportion of relevant information (relevant information level r) in a decision market does not increase the probability of attaining the best possible decision".

[29] This indicator function is also central to the corollary in Appendix 1.

Alternative Hypothesis H_1: "increasing the proportion of relevant information (relevant information level r) in a decision market does increase the probability of attaining the best possible decision".

Significance level: 1%

4.5 Results

In this section the experimental results are presented and annotated to emphasize key features. The impact of changes in relevant information on the ability of the computer simulated prediction and decision market (of the type advocated in this thesis) to correctly attain the respective DCE-based best possible prediction and best possible decision is revealed. The figures below typify the results.

4.5.1 Prediction markets with dependence across information

Figure 4.1 is representative of the prediction market computer simulation results.

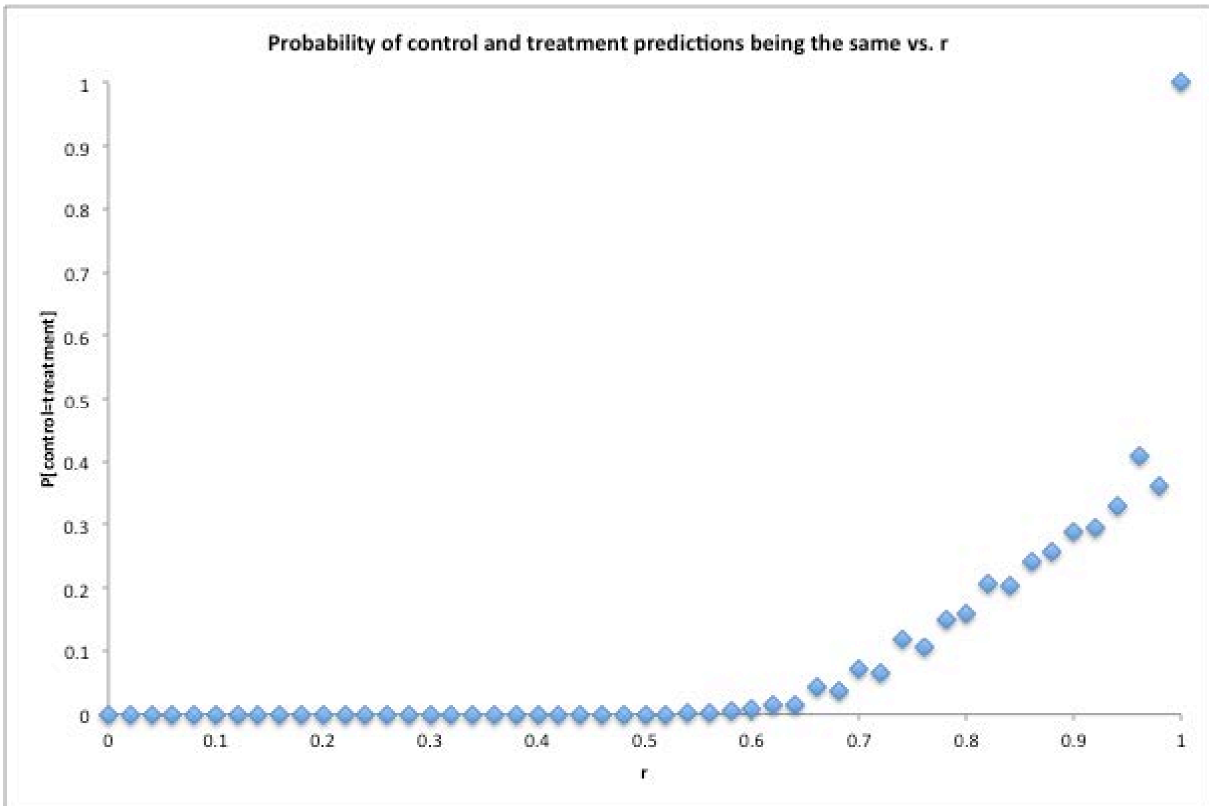

Figure 4.1 Probability of control and treatment prediction markets converge to the same prediction price vs. r .
(N.B. each point represents the average of the results of 2500 simulated "control and treatment" experiments).

The horizontal axis represents the relevant information level $r \in [0,1]$. All prediction markets run contained 50 simulated traders with dependence across information. If, for example, only 5 traders expressed relevant information, then $r = \frac{5}{50} = 0.1$. The vertical axis represents the proportion ρ of treatment (prediction) markets that converged to the control (prediction market), i.e., convergence to the DCE. At each relevant information level there are 2500 market simulations run. For example, when $r = 0.1$ the figure indicates that $\rho = 0$. By the law of large numbers, 2500 independent simulation runs and $\rho = 0$ at 10% relevant information level imply a population mean $\mu = \rho = 0$ and a population standard deviation of $\sigma = \frac{\sqrt{\mu(1-\mu)}}{50} = 0$. Therefore, at a 1% significance level and y the law of large numbers it may be stated that no prediction markets converge to the DCE at relevant information level $r = 0.1$.

In Figure 4.1 $\rho = 0$ until approximately $r = 0.55$ where the curve begins to gradually rise. There is also a sudden (possibly discontinuous) increase at $r = 1$ to $\rho = 1$. This implies that the standard deviation varies at each r. Thus the functional form depicted in Figure 4.1 can only be described as a reasonably close estimate to the true probability distribution function[30].

[30] The law of large numbers is the advantage brought by a computer-simulated approach.

4.5.2 Decision markets with dependence across information

Figure 4.2 is representative of the decision market computer simulation results.

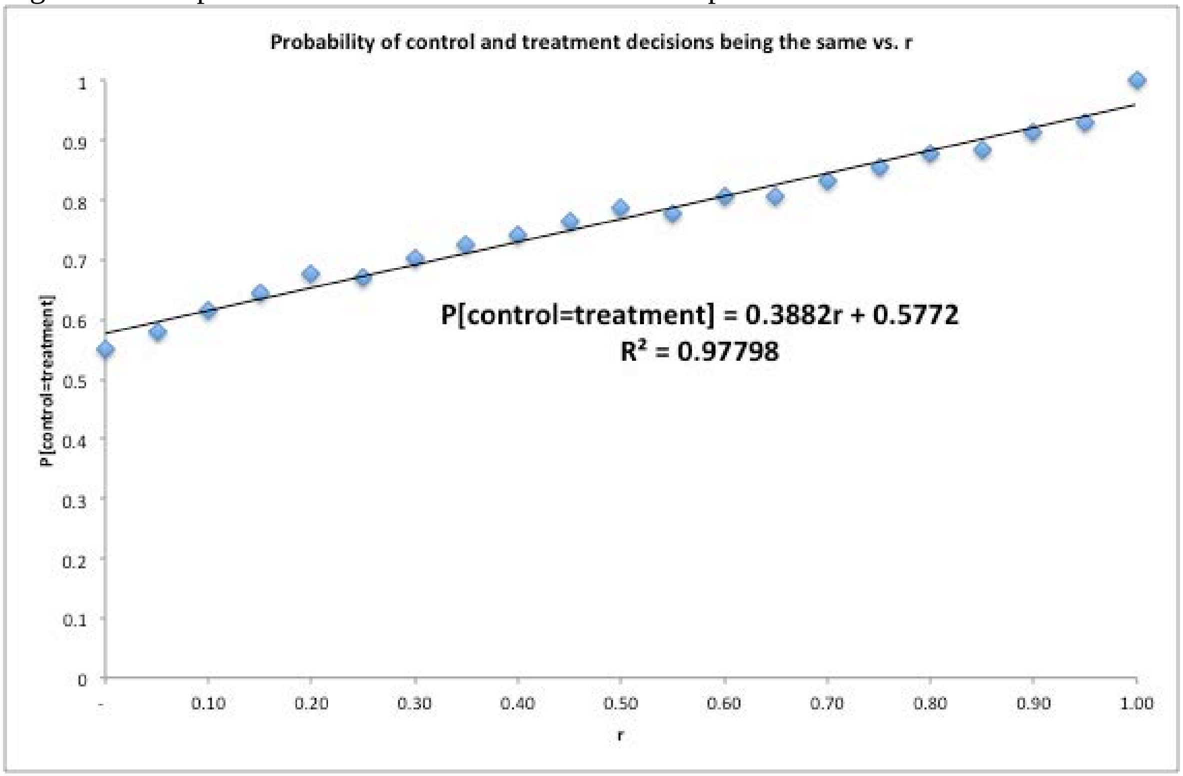

Figure 4.2 Probability of control and treatment decision markets selecting the same decision vs. r. (N.B. each point represents the average of the results of 2500 simulated "control and treatment" experiments).

The horizontal axis represents the relevant information level $r \in [0,1]$. All decision markets run contained 20 simulated traders each participating in 4 prediction markets with dependence across information. If, for example, only 5 traders expressed relevant information, then $r = \frac{5}{20} = 0.25$. The vertical axis represents the proportion ρ of the treatment (decision) markets that converged to the control (decision) market's DCE-based best decision. At each relevant information level there are 2500 markets run. For example, when $r = 0.25$ the figure indicates that $\rho = 0.7$. That is, 70% of the treatment decision markets converge to the same decision as their associated control decision markets (i.e. the decision based on DCE based fully informed prediction markets) at the 25% relevant information level.

In the above figure ρ increases linearly with increasing r to a maximum value of $\rho = 1$ at $r = 1$. The 0.978 coefficient of determination justifies a high goodness of fit claim to a linear function. As such, it can be stated that the curve in Figure 4.2 likely conforms to the theoretically expected linearity. Additionally, the F-value being related to coefficient of determination R^2 by $F_{1,n-2} = \frac{R^2(n-2)}{(1-R^2)} = \frac{0.978\ ^2(21-2)}{(1-0.978\ ^2)} = 417.6$ indicates that the positive gradient (and therefore relevant information level) is significant at a 1%.

4.6 Analysis

In this section an analysis of the data presented and annotated in the previous results section is provided. The control for both the prediction and decision market simulations is fixed at relevant information level $r = 1$. In contrast, the treatment's relevant information is allowed to vary such that $r \in [0,1]$. All other characteristics are the same in the control and treatment. Thus, given a treatment market relevant information level r' any difference observed between the treatment market and the control market ($r = 1$) is logically attributed to relevant information level r'.

4.6.1 Prediction market simulations

The analysis of prediction market data is kept deliberately simple. The objective is the efficient utilization of sample data to reveal the true population characteristics (Fisher, 1960).

Given that 2500 markets are run at each relevant information level r, the law of large numbers implies that the statistic $\rho(r)$ converges to the true mean of the population; since each of the 2500 markets are independent of each other.

Formally, for each simulation run the simulation outcome $J(r_t)$ is recorded and indicates whether the treatment (with relevant information level r_t) converges to the control ($J(r_t) = 1$) or not ($J(r_t) = 0$). 2500 runs at r_t are used to calculate a sample mean $\rho(r_t) = \rho_t$.

The law of large numbers suggests that at relevant information level r_t, the true mean $\mu_t = E[J(r_t)] = \rho_t(1) + (1 - \rho_t)(0) = \rho_t$. Similarly, the second moment about the mean for the population (at relevant information level r_t) is $\sigma^2 = \frac{E[(J(r_t)-\mu_t)^2]}{2500} = \frac{\rho_t(1-\mu_t)^2+(1-\rho_t)(0-\mu_t)^2}{2500} = \frac{\mu_t(1-\mu_t)}{2500}$; $since\ \rho_t = \mu_t\ due\ to\ the\ law\ of\ large\ numbers.$

For all $r < 0.5$, $\rho = 0$. Therefore, for all $r < 0.5$, $\mu = \rho = 0$ and $\sigma = 0$. That is, for each relevant information level $r < 0.5$ a probability distribution with a mean of zero and a variation about that mean of zero exists. Hence Figure 4.1provides an accurate description of the functional form of the probability distribution function for $r < 0.5$. However, for $r > 0.5$, $\rho > 0$ implies a standard deviation greater than 0. At a 99% confidence level, the true mean may be 10% points from ρ. Therefore, Figure 4.1 can only be described as a good graphical depiction of the true probability distribution function.

Determining the statistical significance of relevant information level in the computer-simulated prediction markets is of most interest. A binary logit response performed on the prediction market results is depicted in Table 4.1.

Table 4.1
Binary logit analysis of same treatment and control prediction market prices in the second round of bidding.

Variable	B coefficient	SE standard error	exp(B)
constant	-11.317***	0.101	
r	11.796***	0. 114	132720
df (degrees of freedom)		1	
$\chi 2$		31335.23	
p-value		0.000	
number of observations		127500	
proportion of observations with dependent equal to '1'		8.6%	

Note: Response of 1 indicates that the control and treatment prediction markets converged to the same price in the second round of bidding; otherwise the response is 0. Other covariates including information vector and state of the world are randomised. *p<0.10 **p<0.05 *** p<0.01

Table 4.1 depicts that r is a statistically significant positive value at the 1% level. More formally, given:

Null Hypothesis H_0: *"Increasing the proportion of relevant information (relevant information level r) in a prediction market does not increase the probability of attaining the best possible prediction".*

Alternative Hypothesis H_1: *"Increasing the proportion of relevant information (relevant information level r) in a prediction market does increase the probability of attaining the best possible prediction".*

Significance level: *1%*

The null hypothesis is rejected.

Hence experimental/simulation data support the hypothesis "Increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction".

4.6.2 Decision market simulations

The decision market results depicted in Figure 4.2 appear to adhere to a linear form. The geometric statistical interpretation of linear regression (Bryant, 1984) implies a high goodness of fit to the linear model onto which the observations are projected. Specifically, the small angle between the experimental (simulated) n dimensional sample vector and its projection on the $n-1$ dimensional linear model subspace is indicative of a high goodness of fit of the data generating process being explained by a linear model. This interpretation typically aids in visualizing how close experimental (simulation results) are to the linear model, and is one typically attributed to R.A. Fisher (Saville and Wood, 2012). If the experimental results vector lies very close to the linear model hyper-plane then that linear model has a high goodness of fit with the experimental data; and as such a good model. Using this interpretation, the coefficient of determination R^2 in this work is the square of the cosine of the angle between the linear model (with single independent variate relevant information level r) and the experimental results vector.

The experimentally derived linear regression finds $R^2 = 0.97798$. The angle between the linear model (with single variate relevant information level r) and the experimental results vector is therefore $\alpha = cos^{-1}(R) = 8.5\ degrees$, i.e., the vector lies extremely close to the linear model plane. Thus, this geometric interpretation suggests that there is a high goodness of fit to a linear relationship.

The F-test $F_{1,n-2} = \frac{R^2(n-2)}{(1-R^2)} = \frac{0.978\ ^2(21-2)}{(1-0.978\ ^2)} = 417.6; where\ n = 21$ suggests that the possibility of a negative or zero gradient is rejected at a 1% significance level.

Hence, a high goodness of fit and a positive gradient linear relationship model is supported by Figure 4.2.

Closer examination of the results via a binary logit analysis is depicted in Table 4.2.

Table 4.2
Binary logit analysis of same treatment and control decision market decisions in the second round of bidding.

Variable	B coefficient	SE standard error	exp(B)
constant	0.167***	0.018	
r	2.323***	0.038	10.206
df (degrees of freedom)		1	
χ2		4121.70	
p-value		0.000	
number of observations		52500	
proportion of observations with dependent equal to '1'		76.9%	

Note: Response of 1 indicates that the control and treatment decision markets converged selected the decision in the second round of bidding; otherwise the response is 0. Other covariates including information vector and state of the world are randomised. *p<0.10 **p<0.05 *** p<0.01

Table 4.2 depicts that r is a statistically significant positive value at the 1% level. More formally, given:

Null Hypothesis H_0: *"increasing the proportion of relevant information (relevant information level r) in a decision market does not increase the probability of attaining the best possible decision".*

Alternative Hypothesis H_1*: "increasing the proportion of relevant information (relevant information level r) in a decision market does increase the probability of attaining the best possible decision".*

Significance level: *1%*

The null hypothesis is rejected.

Hence experimental/simulation data support the hypothesis *"Increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision".*

4.7 Discussion

Computer simulations performed to emulate “prediction markets with proper market prices” and “decision markets with proper market prices” support the two hypotheses.

The implications of these experimentally validated hypotheses shall be discussed in the following subsections.

4.7.1 Prediction market simulation and implications

The analysis and results suggest (as depicted in the prediction market Figure 4.1 and Table 4.1), that increasing relevant information level increases the probability of prediction markets to correctly converge to the best possible (DCE) prediction.

Firms typically employ prediction markets to support their decision making (Thompson, 2012). The findings suggest it wise for the firm to institute a policy that ensures traders express relevant information in those markets; else the predictions will not be the best DCE predictions to the potential detriment of the firm. For example, a market such as the Iowa Electronic Market does not disallow computerized traders (Berg and Rietz, 2006). It is conceivable that these computerized traders bring no private information whatsoever and simply condition their bids on previous round market prices alone. By definition, such computerized traders do not express relevant information and as such reduce relevant information level and the likelihood of the prediction market to attain the best (DCE) predictions. The size of the reduction in prediction market performance can be marked as can be seen in Figure 4.1.

4.7.2 Decision market simulation and implications

For decision markets, the analysis shows that increasing relevant information level improves the likelihood of selecting the DCE-based best possible decision. The results depicted in Figure 4.2 also suggest that a decrease in relevant information level from $r = 1$ in a decision market is less marked than in a prediction market with respect to the likelihood of DCE being attained, i.e., a gradual linear reduction in the decision market Figure 4.2 contrasts the punctuated step change seen the prediction market Figure 4.1.

Arguably, firms should implement decision markets instead of prediction markets. A firm’s objective is to choose a project A that maximizes the probability of the desired outcome O

and hence it should elicit conditional probability information $P[O|A]$ (Othman and Sandholm, 2010). If a firm implements a prediction market that trades only projects (with the project with the highest prediction market price selected by the firm), it is conceivable that selecting this project does not maximize the probability of achieving the desired outcome. Such a market setup does not elicit conditional probability information; and thus neglects a key piece of management information. For example, imagine that a desired outcome O is known to have $P[O] = 0.74$ and two projects A and B are traded in the prediction market. In the final round the prediction market elicits probabilities $P[A] = 0.60$ and $P[B] = 0.40$. Management then selects project A given it has the greater probability. However, notice that it is possible that the prediction market may not be able to reveal the true conditionals $P[O|A] = 0.7$ and $P[O|B] = 0.8$ meaning that management should have instead chosen to invest in project B.

Another formulation of the prediction market has been to trade outcomes only instead of projects, e.g., the initial prediction markets for printer sales in HP (Ho and Chen, 2007). This formulation risks a self-fulfilling prophecy condition. Specifically, imagine that management has a choice between project A and some mutually exclusive complementary action project A'. They then observe some probability of a desired outcome $P[O] = 0.62$ and mobilize their project resource investments with this information in mind. It is possible that this may simply be a case of the market believing that management has a particular interest in 'pet' project A (selecting it with $P[A] = 0.8$) even though $P[O|A] = 0.6$ and $P[O|A'] = 0.7$, i.e., A' is the superior project. But, the market aggregates this information into a market price $P[O] = P[O|A]\mathrm{P[A]} + P[O|A']\mathrm{P[A']} = 0.62$. That is, the prediction market has not guided management to mitigate the risk of erroneous project investment and instead calculated the probability of the outcome based on how it believed management would respond to the prediction market prices.

4.7.3 The big ideas inspired by this research

The computer-simulated experiments of this chapter imply that relevant information level is important to well-functioning prediction and decision markets. This investigation provides compelling evidence that validate the theoretical model, which states, the greater the relevant information level in a prediction/decision market, the better it will perform.

The ultimate objective is to design a credible decision-making tool to add to the manager's decision support toolkit. That is, typically a manager in a firm currently utilizes other forms of decision-making tools; e.g., discounted cash flow (DCF). The decision market at the very least complements decision support tools. At the very best, it holds out the prospect of automated decision-making.

The link between prediction market and decision markets is subtle. In the work to date, a decision market is built utilizing prediction markets. However, building a prediction market using decision markets has not been considered. Such an undertaking should not be lightly disregarded. It would provide a deeper level of understanding of the important connection between prediction and decision markets; similar to investigations on the relationship between real options analysis and the DCF that led to a greater depth of understanding of the important connections between those management decision support tools (Arnold and Shockley, 2002).

In the simulation work on decision markets, two projects (a project and its complement) are traded. This is in one sense building a decision market that is able to prioritize a group of two projects. The benefit of considering two projects in the simulation work is pragmatic given the limited computation time, but it can also be generalized given that the findings for the two-project setting logically extend to the multi-project setting. A simplistic sketch of proof to convince the reader is: Prioritization of projects can be thought of as a problem of membership of a project p to the nth priority set P_n. p is either in P_n, or out of P_n. That is, the problem of prioritization is reduced to a decision market of two contracts[31]: 'p is in P_n' and 'p is NOT in P_n'. That is, this approach is applicable to selection and prioritization of a portfolio of projects.

The computer-simulated experiments of this chapter do not incorporate the effects due to human interaction in the market. However, understanding the impact of relevant information level in computer agent prediction and decision markets with proper market prices is a logical prerequisite to considering more complex human-based settings; particularly the notable complexity of bounded rationality (Simon, 1972). The next step is to construct prediction/decision market games with human participants; a task undertaken in Chapter 5.

[31] There is obviously a combinatorial issue here, whereby in practice the number of contracts required in such a decision market increases exponentially with the number of projects to be prioritized. This will be dealt with in chapter 7.

Upon doing so, appropriate policies to enhance prediction/decision markets containing human agents will likely be revealed. The implication of including or excluding humans in each study is comprehensively discussed in chapters 5 to 7 of this thesis. Notably, this methodological approach is justified in chapter 1 as a means to build towards a robust testing of the theoretical model and associated hypothesis whilst generalising beyond only-human settings.

It is important to re-emphasize at this point the deliberate separation, in this thesis, of research on markets without humans (in this chapter) and those with humans (in chapters 5 and 6). The hypothesis arising from the theoretical model of chapter 3 is under test. It is a model that simplifies the world by assuming rational traders amongst other assumptions. It is therefore important, as a starting point, to investigate whether the hypothesis based on rational traders holds in a rational world; such as the one simulated in this chapter. If it did not, then further analysis in more complex human based settings would be premature, as the hypothesis under test would have already failed in the simulated world. Whilst the hypothesis is found to hold in the simulated world of this chapter, this simply gives motivation to investigate the more complex laboratory based and then real world based human settings. The computer simulated study of this chapter is in one sense akin to a financial option whereby it is purchased at a relatively small cost and given it has found the hypothesis to hold, the option to now exercise the relatively higher cost human based studies is logically justified.

4.8 Conclusion & Future Research

In summary, the experimental results conform to theoretical expectations. Specifically, the computer simulation results and analyses are consistent with the two hypotheses "Increasing the proportion of relevant information (relevant information level r) in a prediction market increases the probability of attaining the best possible prediction" and "Increasing the proportion of relevant information (relevant information level r) in a decision market increases the probability of attaining the best possible decision".

Of prime concern to the overarching work in this thesis is the construction of a high-quality decision market. Importantly, these computer simulations provide compelling evidence that the probability of decision markets selecting the best decision linearly increases with increasing relevant information level.

The immediate implications of this work suggest policies that encourage high relevant information levels in prediction and decision markets. For example, any algorithmic trading program that bids the previous round market price, by definition fails to express relevant information and hence reduces the overall ability of the market to converge to the best possible DCE-based prediction or decision. As such, this type of algorithmic market participant should be disallowed from decision markets.

This chapter motivates Chapter 5, 6 and 7 i.e, decision markets with human participants, real-world analysis (of conditional prediction markets), and the policy implications, respectively.

4.8.1.1 Decision markets with human participants

The next phase of the series of experiments is decision market games with human participants (Chapter 5). Again, of interest is varying relevant information level, but this time in a prediction/decision market setting allowing human participants. Specifically, the traders shall include computer and human agents in each prediction/decision market. The results of these games with human participants shall inform future decision market design and policy and importantly determine if relevant information level is still statistically significant when humans participate.

4.8.1.2 Real-world analysis (conditional prediction markets)

Both computer simulation investigations and prediction/decision market games with human participants investigations of this thesis reside in controlled laboratory conditions. Real-world analysis (in Chapter 6) relaxes this sanitized context and exposes the model to the dynamics of the real-world. There does not currently exist a 'real word' decision market of the type proposed in this thesis. However, there does exist instances of stocks conditional on others within 'real-world' prediction markets. These instances have been called conditional prediction markets (Chen and Kash, 2011). The aim of Chapter 6 shall be to determine if relevant information level is statistically significant in a real-world conditional prediction market (decision market).

4.8.1.3 The policy implications

The overall objective of this thesis is to design a decision market decision support tool and specific policies that increase the likelihood of selection and prioritization of the best possible portfolio of projects. The policies shall be explored in Chapter 7.

For different problems, discounted cash flow (DCF), non-market valuations, and decision market (DM) methods bring different advantages and disadvantages[32]. In concert these decision support tools provide guidance for, and act as resources to be called upon by, managers tasked with solving real-world problems.

Decision and prediction markets currently fail to guarantee that their decisions and predictions are accurate. This chapter contributes an original metric to research on prediction and decision markets in the form of relevant information level; measuring how good prediction and decision markets are. Importantly, with such a prediction and decision market quality signal in hand, managers are able to assess the credibility of the predictions and decisions that result.

[32] Specifically, the question of how best to use decision/prediction markets will be considered in the chapter 7 on policy implications.

Chapter. 5 Prediction Market Games with Human Participants

"…the vox populi [voice of the people] is correct to within 1 per cent of the real value…"
Sir Francis Galton, 1907

Key Message of Chapter:

- **Prediction market games with humans under controlled conditions show that 'relevant information level' is statistically significant.**
- **The findings motivate that 'relevant information level' is statistically significant in real-world markets.**
- **Given relevant information level is important to attain the best possible prediction, then it is likely that real-world decision markets built from prediction markets attain the best possible decision.**

The research in this chapter ties directly to research questions 1 and 3. That is, it investigates and finds that the quality signal called relevant information level (being the proportion of traders in a market conditioning their bids on their private information) is statistically significant in a particular type of prediction market that the researcher has built for this research. Specifically, prediction market games with human participants are run and 'relevant information level' is found to be statistically significant for convergence to 'the best possible prediction'. Notably, the prediction market webgames with human participants consider whether the hypothetical not for profit dog-friendly-beach project is selected as in or out of the Townsville Pet Society's project portfolio.

In each prediction market game, either no information or uncertain private information is provided to a trader, and in conjunction with public information (in the form of previous round 'average quantity of stock ordered'), that trader submits quantity orders over 5 rounds. At the end of the fifth round only one of the two stocks in the game will payout, and the trader with the most 'game money' declared the winner of the game.

The number of human participants, the number of algorithmic traders, the distribution of information bits, and the stock that pays out, characterize a prediction market game. For each 'treatment game' (treated with a relevant information level ranging from 0 to 1) there is a 'control game' identical in every way to the 'treatment game' except that the relevant information level of the 'control game' is 1.

A control and treatment may converge to the same ‘average quantity of stock ordered’ in the final round of the game. The ‘average quantity of stock ordered’ in the control game in the final round is called ‘the best possible prediction’ owing to the fact that it may be considered a prediction probability of fully informed traders. Convergence is recorded as a 1, and 0 is recorded otherwise. ‘Relevant information level’ r is found to be statistically significant for convergence in this series of prediction market games.

5.1 Introduction

This chapter extends the investigation of ‘relevant information’ as an important ingredient in well-functioning prediction and decision markets to a context in which human traders are added. Confining the setting to a simple prediction market game, the ‘relevant information level’ as it relates to ‘the best possible prediction’ is confirmed in this chapter. That is, *when humans participate in prediction market games, increasing ‘relevant information level’ is still*[33] *associated with an increased probability of the prediction market making the ‘best possible prediction’*.

The work discussed in this chapter is motivated by the question “Does relevant information level play a statistically significant role in well-functioning[34] prediction and decision markets inhabited by human traders?” This builds upon insights of previous chapters that found:

In theoretical prediction and decision markets where all traders express relevant information convergence to the best possible predictions and decisions is guaranteed

Computer simulated prediction and decision markets increase their probability of attaining the best possible prediction and decision as relevant information level r increases.

The chapter unfolds in the following sequence. After related literature is reviewed in section 5.2, the specific details of the experimental setup are described in section 5.3. Section 5.4 annotates the results and section 5.5 analyzes them. Section 5.6 provides a discussion of the key findings and section 5.7 concludes with suggestions for future research.

[33] ‘Relevant information level’ was also previously seen to be significant for a prediction market to converge to the ‘best possible prediction’ in the theoretical model and computer simulations.
[34] Well-functioning prediction and decision markets being those that make the best possible prediction and select the best possible decision respectively.

5.2 Related Literature

Literature that relates to games with human participants is presented in this section. Specifically, a short background of prediction markets used to elicit information from human participants is provided, the rational human assumption pervading such economic modeling of markets is considered, the benefits of augmenting humans and machines via prediction markets is highlighted, previous studies on games with humans is reviewed, and finally the impetus for this study on prediction market games with human participants is justified.

5.2.1 Background of prediction markets with human participants

All real-world prediction markets have human participants. These prediction markets range from the pioneering political Iowa Electronic (prediction) Market (IEM) (Berg et al., 2008a), to prediction markets for entertainment in the Hollywood stock exchange (HSX) (Wolfers and Zitzewitz, 2004), to prediction market for sporting results (e.g. Sports Trade (Wen et al., 2016)), and to internal prediction markets residing in firms (e.g. Google and Hewlett Packard (Cowgill et al., 2009; Cowgill and Zitzewitz, 2015; Plott and Chen, 2002)). Prediction markets typically provide a web-based interface which trades off complexity for user-friendliness, e.g., the web-based prediction market for disease (Li et al., 2016). However, irrespective of application and design of prediction markets, as with any speculative market populated by humans, biases occur to move market prices to incorrect equilibria; as such a deeper analysis of preconditions for market efficiency has been considered important (Treynor, 1987). The investigation of relevant information level is simply an investigation of how information converts into a best possible prediction equilibria (Grainger et al., 2015). Specifically, it is an investigation of the required conditions for market efficiency in prediction markets.

5.2.2 The rational human assumption

Markets of economic theory are populated by rational traders; with rationality defined as actions consistent with unchanging preferences (McCubbins et al., 2012). These ideas emerge from utility theory whereby rationality is logical consistency given a context of constraints, preferences and actions (Hammond, 1997). Utility theory was necessitated by the Saint Petersburg Paradox (Bernoulli, 1954) and has since evolved into a sophisticated normative theory utilizing preference sets and the calculus (Leonard, 1995). In short, it is a tool to measure individual or social ‘satisfaction’ derived from ‘things’; including money (Fishburn, 1968). Whilst utility ideas are more than a century old (Bernoulli, 1954), the exponential growth of utility theory applied to predicting human behaviour has only occurred since World War II (Simon, 1976).

Cardinal and ordinal types of utility theory exist, broadly championed by psychologists and economists respectively (Harsanyi, 1953; Lewin, 1996). Whilst ordinal utility has dominated economic theory, Harsanyi's conception of empathetic preferences as the bridge between two individuals that makes their utilities comparable is suggestive that cardinal utilities should be applied instead (Binmore, 1998). However, expected utility theory (based on ordinal utility and 'subjective probabilities'[35] (Fishburn, 1968) has dominated economics. This dominant hegemony was challenged and found incompatible with real-world data; notably that real-world human choices depend on how those choices are framed (Tversky and Kahneman, 1985). For example, if a question is framed in terms of a loss instead of a gain, different choices may result; a phenomenon called the endowment effect which is simply a trader's hesitation to part with what they own (Kahneman, 2003).

Herbert Simons (1972) introduced the idea of *bounded rationality*; the important idea that human cognition has limits (Simon, 1972). Subsequent variants of expected utility theory, to incorporate this experimentally revealed limitation was at best considered arbitrary modifications (Schoemaker, 1982), and bounded rationality's core message was that the expected utility theory assumptions of perfect cognition in individuals was fundamentally wrong (Simon, 1986). Instead of perfect cognition, humans use "rules of thumb" (Loomes, 1998). Contemporary psychology and economics cannot but recognize the existence of "cognitive anomalies" and the use of rules of thumb in contravention to the assumed rational economic human (McFadden et al., 1999). However, although human behavior is currently considered not one of perfect rationality, e.g., there is always arbitrage in a real-world market (Herschberg, 2012) it does approximate rational behavior in certain settings, e.g., in environments of information uncertainty wherein information is valuable and logically acted upon (McFadden, 2009). Paradigm shifts away from utility theory by incorporating real-world observations have been suggested, e.g., a model whereby "preferences adapt to decisions rather than the other way around" given preferences in the real-world are observed as dynamic (Van Den Bergh et al., 2000). In short, humans are not rational in most settings,

[35] i.e., an individual's measure of confidence

and utility models of individual and social behavior in general fail to predict human behavior (Kahneman, 2003).

A comprehensive review of behavioural economics, as it relates to this thesis, is performed in chapter 2 in addition to this section. It should however be noted that this thesis is not an investigation of individual level behaviours such as that of the endowment effect and the ultimatum game of behavioural economics. Rather, the key message of this thesis relates to aggregate level information in price signals (e.g., bids); be they placed by algorithms or humans. This thesis augments behavioural economics' research by informing research on aggregate level market game behaviour; which is arguably thin in behavioural economics research. Moreover, the behavioural observation experiments of the type of Kahnemann and Tversky are not of central interest. Specifically, this thesis generalises the idea of price formation beyond human to non-human inhabited markets.

Literature to this point merits being synthesized and applied within this study. That is, the study in this chapter accepts these research literature findings and in response constructs a specific 'uncertainty of information' prediction market (Grainger et al., 2015) game that incentivizes rational human behavior. It then tests the hypothesis that relevant information level plays a significant role in increasing the probability of converging to the best prediction (DCE) outcome. If found to be significant, it may then be said that relevant information level is significant in a game with human participants; rationally behaving or otherwise in response to the rational incentives. With this in hand, it is then logical that the maximum relevant information level will elicit full information from all market traders (human or otherwise). That is, modeling human behavior is not of central interest; rather, identifying the tool (relevant information level) that elicits full market information is of utmost importance.

5.2.3 Centaurs are humans usefully augmented with machines

A recently constructed type of decision support tool are the Hybrid human-machine (or sometimes denoted human-algorithm) systems; which have been found to outperform other (human-only and machine-only) systems (Chen et al., 2008b). This combination of machine and human intelligence has been termed centaurs (Shrier et al., 2016). Prediction markets, moreover, have been considered adept at combining human and machine intelligence and a natural realm for the development of centaurs (Nagar and Malone, 2011).

The typically limited number of employees participating in company prediction markets creates a potential liquidity problem (Yang et al., 2015). Automatic (algorithmic/machine) Market Makers (AMMs) have been designed to resolve such prediction market liquidity problems (Chen and Pennock, 2012) and also resolve the potential liquidity problems arising in combinatorial settings (Jumadinova and Dasgupta, 2015); an AMM being simply a machine trader always willing to trade with human traders to guarantee market liquidity (Slamka et al., 2013).

Greater than 80% of all stock market trades in 2010 were placed by computer algorithms (Lu, 2016). Hence stock markets also satisfy the centaur definition allowing machines and humans to trade and elicit, at the aggregate level, implied predictions and decisions. The construction of a centaur type prediction market is explicitly achieved in the Iowa Electronic Market (IEM) prediction market (Schmitz, 2011). The prediction market studied in this chapter is consistent with these centaur markets by having the proportion of machine trading ranging from 90% or 95%; with specific details of their construction described in Appendix C: Chapter 5 Appendix 1.

It was initially assumed that algorithmic traders had the speed of trade advantage over humans, but studies have shown that holding period is in no way correlated with profitability and therefore such high frequency algorithmic traders have no such speed of trade advantage over human traders (Moosa and Ramiah, 2015). In short, centaur prediction markets are a natural evolution from human only prediction markets, and AMMs are simply the embodiment of the machine part of the centaur prediction market.

5.2.4 Games with humans

The prediction market may also be seen as a distributed game with humans. In general, a distributed game with humans is one in which human players are geographically diverse and as such bring a diversity of information that may be utilized to solve complex problems (Heiko et al., 2015). Prediction markets incentivize information sharing (Balkenborg and Kaplan, 2010) which leads to social learning that elicits valuable information at the aggregate level (Van der Wal et al., 2016). As such, distributed games (e.g., prediction and decision market games) are considered an important decision support tool worthy of further study (Schlag et al., 2015); eliciting and aggregating information to solve complex problems. For example, prediction market web games provide excellent probabilistic forecasts by sourcing decision relevant information from geographically diverse, and motivated players (Pennock et al., 2001).

Mechanism design (or inverse game theory) is key to efficient information elicitation and aggregation, and achieved by 'designing' rules that incentivize behaviors; in contrast to game theory which attempts to predict behaviors for 'given' rules (Maskin, 2008). Incentive compatibility is the central idea underlying mechanism design (Myerson, 2008); simply being the design of incentives (or rules) that are compatible (or logically consistent) with the behavior the game maker wishes to encourage. The typical purpose of mechanism design has been to incentivize truth telling (Conitzer, 2010) and given the existence of strategic agents attempting to manipulate the beliefs of others (Conitzer, 2009), game theoretic techniques have been used to reduce the opportunity for 'bluffing (or lies)', e.g., introducing "a discounted market scoring rule" into a prediction market to incentivize myopic (non-strategic) behavior (Dimitrov and Sami, 2008). However, strategic traders attempting to manipulate market prices in experimental prediction market settings are found to benefit the market; improving equilibrium price accuracy as they introduce arbitrage opportunities that, in turn, increase liquidity of markets (Hanson et al., 2006). But strategic manipulations do have a short term effect on market accuracy (Buckley and O'Brien, 2015). Thus, understanding how mechanism design reduces strategic and other unwanted human trader effects on prediction markets is crucial, given biased prediction markets are easily produced (Fountain and Harrison, 2011). For example, Google's internal prediction market found an 'optimism and proximity' bias (Cowgill et al., 2009).

Since real money prediction markets are illegal in some jurisdictions (Arrow et al., 2008) game money becomes useful in prediction market games. As previously mentioned real money has utility (Fishburn, 1968), so there may be an issue of substituting play money in its stead. Experiments have found that play money in internal corporate prediction markets allows the effective elicitation of predictions (Siegel, 2009). Furthermore, analysis of real-world markets evidence that play money markets perform as well as real money prediction markets, however, real money may provide better motivation for traders to profit and play money may incentivize better information elicitation (Servan-Schreiber et al., 2004).

Training is also critical when implementing effective prediction markets with human participants (Siegel, 2009). The learning curve for prediction and decision market interfaces is steep, but can be reduced through simplifications, e.g., "hiding excessive financial aspects of the marketplace" (Gaspoz and Pigneur, 2008). That is, there is an issue of balancing information transparency with human cognitive capacity in prediction market interfaces (Yang et al., 2015). Typically, a simple prediction market interface is key to eliciting rational trading behavior (Kranz et al., 2015) with enough information to provide informed trading, but not so much that cognitive overload impairs traders (Teschner et al., 2015). Theoretically, informed trading becomes valuable in high uncertainty of information settings and information (being valuable) is certainly elicited to ultimately improve market price accuracy (Hanson and Oprea, 2009).

Whilst certain prediction markets in laboratory settings are able to aggregate disparate and uncertain information about the state of the world (Deck et al., 2015), some such as the logarithmic market scoring rule (LMSR) prediction market suffer inaccuracies in contexts with uncertainty of information (Slamka et al., 2013). "Expressiveness" (i.e., combinatorial settings of many stocks 'expressing' different state dependent payouts) further compounds these inaccuracies and also leads to computationally intractable combinatorial problems (Feigenbaum et al., 2009). As such, the simple prediction market proposed in this thesis is specifically designed to cope with uncertain real-world applications in a computationally tractable way. This has been achieved by using simple (less expressive stocks) and a simple market mechanisms (e.g. the simple average of bids); since more complicated and expressive stock prediction markets do not outperform simple stock prediction markets in experimental settings (Powell et al., 2013), and the simple average of bids perform at least as well as unweighted averages in practice (Sun et al., 2012). That is, the simple prediction market model (Grainger et al., 2015), considered in great detail in previous chapters, will be implemented. Therefore, the model will now be reviewed with the intention of teasing out the pertinent details applicable to this study on prediction market games with human participants.

5.2.5 Grainger's theoretical prediction market model to be tested

The Grainger (2015) simple prediction market model is inspired by the Chen (2004) and Feigenbaum et al. (2003) models; the former investigating prediction markets in contexts with uncertainty of information at the aggregate level (Chen et al., 2004) and the latter investigating distribution of information and computational tractability issues in prediction markets (Feigenbaum et al., 2003).

The Grainger (2015) simple prediction market model is ultimately used to build a simple decision market model (Grainger et al., 2015). There are two important reasons for this build. Firstly, decision markets are more suitable for decision making in firms (Hanson, 1999); which is the goal of this thesis, i.e., developing a high quality decision support tool. Secondly, the decision implied by only using a prediction market is ambiguous, e.g., if a prediction market predicts an 80% chance of rain, was the prediction good if it does not rain and was the decision to take an umbrella a good one (Damnjanovic et al., 2012)? In contrast, if a decision market 'decides' to take an umbrella and it does rain then the decision is unambiguously good.

Whilst a simple decision market built using prediction markets is not an original idea (Othman and Sandholm, 2010), the capability of the Grainger (2015) simple decision market to always achieve the best possible decision is novel. This is achieved by introducing two new concepts: relevant information level and the proper market prices condition (Grainger et al., 2015). Simply put, relevant information level is the proportion of traders submitting bids conditioned on private information, and the proper market prices condition is the requirement that the underlying prediction market is structured such that a trader learning nothing from a first-round market price does not occur and a market price cannot cause a trader to unlearn information if they are fully informed. As with all economic models the setup is idealized, but the insight provided is sufficiently significant to warrant laboratory and real-world investigation within this thesis. That is, given rational, myopic and risk-neutral traders in an ideal prediction market with proper market prices then the only way to attain the best possible prediction is if all traders express relevant information (Grainger et al., 2015). Additionally, the probability of achieving the best possible decision in an ideal decision market built using this type of prediction market increases linearly (to 100%) with increasing relevant information level. The computer simulations of the previous chapter validate these findings; but computer agents behave rationally and humans are not guaranteed to do so. Thus, this compels an investigation into a (proper market prices) prediction market with human participants and how relevant information level affects the probability of achieving the best possible prediction.

The model assumes rational, myopic and risk-neutral traders. However, humans do not act rationally, and in contexts with information uncertainty are not risk-neutral; taking on risk to 'prospect' for information in order to profit (Kahneman and Tversky, 1979), e.g., race track bettors are “risk lovers” (Ali, 1977). Whereas individuals are not always rational and risk neutral they have been found to be myopic especially in setting of high information uncertainty (Kahneman, 1994); since, in a complex distributive decision support setting (such as prediction and decision markets), although information asymmetry increases strategic manipulative tendency, this is countered by the complex calculus requiring resolution by the potential strategic manipulator (Malekovic et al., 2016). Shorter running prediction market games are considered desirable, being less cognitively demanding, but are typically rare with long run predicted outcomes being of interest (Chen et al., 2015). This burden is removed in the Grainger (2015) prediction and decision market design; as the predicted outcome is not a long run real-world event, rather, it is a short run market state outcome inherently tied to the real-world (Grainger et al., 2015). To elucidate further, it is similar to the betting problem considered by the renowned mathematicians Pascal and Fermat in which payouts could be made prior to the completion of a betting game and when probability concepts were suitably utilized (Ore, 1960). That is, just as a long betting game can be stopped after a short time and payouts made, so too can the prediction and decision market game be stopped sooner and suitable payouts made.

Relevant information is simply traders bidding using their private information; a concept considered but not called this in previous research. That is, private information provides the advantage of profiting in trade (Milgrom and Weber, 1982) and traders with “valuable information” are incentivized to participate in prediction markets (Dudik et al., 2012). Previous literature also hints at relevant information level inducing the best possible price. For example, a small group of well-informed traders maintain stock prices at their true level has been called the marginal trader hypothesis (Blackwell and Pickford, 2011). It should be noted that this thesis does not entirely subscribe to this interpretation, since not trading is also valuable information to aggregate and a trader’s signal to the market. This may also be interpreted as a bid of zero resulting from a trader conditioning on their private information.

Real prediction market structures such as information efficiency in the IEM have been studied (Schmitz, 2011). Proper market prices ensure a trader learns relevant information from market prices (Grainger et al., 2015). Whereas the usual Bayesian updating (of the algorithmic traders in this study) allows traders to tractably process small amounts of information (Gelman and Shalizi, 2013), it is proper market prices that guarantees learning takes place.

The effect of variation of relevant information in prediction markets with proper market prices contributes to the study on what is required for market efficiency; as market efficiency has been considered a conclusion with the preconditions requiring investigation (Treynor, 1987). In essence the probability of the best possible decision being attained is an embodiment of market efficiency. But it is more than the market efficiency concept of old (Malkiel and Fama, 1970); for it provides a measure of market efficiency by using the probability of best possible decision as a function of relevant information level. For example, a market may have 80% of its traders revealing private information that is converted into the market correctly deciding 90% of the time.

Ultimately the question “why are corporate prediction markets not popular?” (Cowgill and Zitzewitz, 2015) may have a simple answer. That is, prediction markets may simply lack a quality signal to differentiate a lemon market from a peach market (Akerlof, 1970). The central contribution of this thesis is that relevant information level is the quality signal to differentiate lemon from peach prediction markets; thereby fixing the potential market failure that has prevented the popular use of prediction and decision markets. But relevant information needs to be credibly grounded. Thus far it is found credible in theory (Grainger et al., 2015) and comprehensive computer simulations. The next logical step taken in this chapter is to evaluate the significance of relevant information level in prediction markets with human participants.

5.3 Experimental Setup

The purpose of this investigation is to test whether relevant information level plays a statistically significant role in a prediction market inhabited by humans. This investigation is required because it is possible that human traders annul the effect of relevant information on the probability of prediction markets making the best possible prediction. Only prediction markets are examined because decision markets will make the best possible decision if their constituent prediction markets make the best possible predictions. Because the study involves human participants ethics approval was sought and granted by James Cook University Townsville Australia’s Human Research Ethics Committee (HREC); reference number H6263.

5.3.1 Hypothesis under test and methodological justification

The null hypothesis for this experiment is H_0: "relevant information level does not significantly affect the probability of the convergence of the prediction market to the best possible prediction"; the aim being to reject the null at a significance level of 5%.

In order to test the hypothesis, a control treatment experiment setup is implemented, in which relevant information level is the only variable having a different value when comparing a control with its treatment. Variables other than relevant information level are randomly varied across a sufficient number of control and treatment pairs. Randomization and sufficient sample size is considered the key features for robust experimental design (List et al., 2011), remains the "gold standard" (Berry, 2015), and provides a means to infer causality (Antonakis et al., 2010). Specifically, with relevant information level being the only difference between a control and its treatment and with other variables randomized, then any statistical difference detected is arguably 'caused' by relevant information level (Antonakis et al., 2010). This contrasts with the empirical setting which suffers from the propensity of the applied treatment (relevant information level) covarying with other variables (Rosenbaum and Rubin, 1983b) and complicating the test for statistical significance of relevant information level. A sufficient sample size provides for a high power of test, i.e., a sample size which allows for a high probability of correct rejection of the null hypothesis (Lachin, 1981). It should be noted that the null hypothesis and significance testing method has been criticized on theoretical grounds. Specifically, it can only ever provide the opportunity of rejecting the null, and certainly not proving the null holds true as some researcher have presumed; however, as a method, it is embraced as pragmatic (Krueger, 2001). The prudent use of such a method requires post hoc tests to ensure that the normality assumption is satisfied (Bera et al., 1984) and that no continuity correction factor is required (Guillen, 2014). Greater details of these post hoc test considerations are provided in Appendix C: Chapter 5 Appendix 2.

5.3.2 Justification of experiment design and method to analyze results

The following sections describe a priori calculations, simulations and pilots to inform and justify a final experimental design. Approximately 9 months was required for design, coding, cycles of testing & modification, and implementation of the actual experiment and analysis. This investment was required given the need to test the statistical significance of a specific prediction market characteristic (i.e., relevant information level) in the most efficient way.

The final experimental design was a simple prediction market trading two stocks of which only one stock pays at the end of the fifth round of the game. A control-treatment setup is utilized in which a control and a treatment are identical in every way with the possible exception of relevant information level (which takes on a value from 0 to 1 in 0.05 steps). A control and its treatment may or may not converge to the same 'best possible prediction' (represented in the average quantity ordered, e.g., an average order quantity of 30 indicates that the prediction market believes there is a 30% chance of the event associated with that stock occurring); this dichotomous random variate is recorded as 1 (for converging to the same) or 0 (otherwise). To such a setting, the maximum likelihood estimation (MLE) of the binary logit model is applied. The assumption of independence between games is also made and checked post-hoc. Post-hoc analysis also verifies that a continuity correction factor is not required. These being the case, the MLE (which is implemented in STATA and makes the independence across games assumption and assumes a continuity correction factor is not required) provides valid estimates of coefficients.

5.3.2.1 Experimental design and power of test

Efficient experimental design is a trade-off between resources and the power of test[36]. Considering time and budget an a priori theoretical design for the experiment was devised to maximize the power of test and minimize human participant playing hours and money invested in the game. This is comprehensively described in Appendix C: Chapter 5 Appendix 3. The key takeaway of Appendix 3 is that under very conservative assumptions approximately 110 human participants would guarantee a 90% chance of correctly rejecting the null hypothesis. The actual experiment required 92 human participants to reject the null with a very low p-value. To minimize the monetary cost of the web-game the researcher designed and coded the game in php, and built a webserver by reconfiguring a laptop, router and freeware. Participants accessed the server to play the game by typing the IP number of the internet-facing router, and entering their username and password provided by the researcher.

5.3.2.2 Simulation to 'double check' the power of test calculation

In addition to the power of test calculations a simulation of the experiment was performed in a spreadsheet and is discussed in detail in Appendix C: Chapter 5 Appendix 4. This

[36] Probability of correctly rejecting the null hypothesis.

simulation was performed as a double check to the power of test calculations. In short, the probability of correctly rejecting the null was considered sufficiently high.

5.3.2.3 Three pilot tests run prior to actual experiment

Three pilot tests were run prior to the actual experiment; as described in Appendix C: Chapter 5 Appendix 5. In short, the first was run to test the interface & functionality of the game. It was videotaped and shared with an advisory panel for comment. It informed refinements to the game interface and validated the existence of no obvious coding bugs[37]. An important point to note was that the researcher initially constructed the game so as not to be connected to the real-world, e.g., a prediction market stock was called “Project A will be selected”. However, a suggestion to make the stock less abstract and more concrete was incorporated into a modification of the game; a hypothetical Australian federal election scenario was constructed. The second pilot game (with this modification) ran four-prediction-markets appropriately constructed to form a decision market to choose the best possible Australian federal election candidate. A coincidental problem arose in that concurrent to this pilot an actual federal election was taking place and the chance of bias introduced into the game was considered too high. Therefore a third and final pilot game was developed as a concrete but hypothetical scenario with the same decision market web-game structure. It was a story concrete enough to be easily visualized but with a low risk of introducing external uncontrollable bias. A refinement ‘to keep the experiment as simple as possible’ was suggested and the game was reduced to a prediction market of two complementary prediction-market stocks; a “build a dog beach” stock and a “not build a dog beach” stock. The game was then played in a pilot mode and confirmed as ready for the actual experiment.

5.3.3 The final web-based game used for the actual experiment

The final web-based game was a two-stock prediction market game designed and coded in php by the researcher. The game allowed the researcher to control the variations of a specific variable of the market (i.e., relevant information level) with other variables randomized.

The specific prediction market setup was one in which human traders were presented with a web-based screen accessible by phone, tablet, or computer. A link to a training ‘how to play’ video was available to the player upon logging on with their unique username and password.

[37] The modular build in which stubs and harnesses tested each new program module lowered the likelihood of coding errors.

Each prediction market contained 20 traders comprising one or two human traders with the remainder being algorithmic traders. On the web-based screen human traders were presented with private information (with uncertainty over the 5 rounds of the game) in the form of the probability of an event or its complement occurring, or instead receive no private information at all (for the entire 5 rounds). In this game stocks associated with two complementary events were traded; including "a Dog Friendly Beach (DFB) is built" stock and the complement "a Dog Friendly Beach (Not-DFB) is not built" stock. The trader was also presented with two user-friendly sliders showing quantity and cost and allowing traders to place a quantity order of DFB stocks and Not-DFB stocks respectively. Only buy orders were allowed to simplify analysis, but this did not limit the trader's ability to express preferences given that they were able to bid on an event or its complement. Traders were also presented with a table containing private and public information including the average quantity of stock ordered in the previous round. Appendix C: Chapter 5 Appendix 6 provides specific details and screenshots of the webgame interface.

Pilot test revealed that 5 rounds of the webgame were sufficient for the convergence of prediction markets to a prediction. Thus, in the actual experiment, the game halted at the end of the fifth round at which point it was revealed that only one of the events occurred and the associated stock paid $1 per stock held by the trader. It should be noted that the revealed event was randomly chosen but hidden to all at the commencement of the game, which in turn affected the private information received by traders; the private information being informative as to what event 'probably' occurs.

Traders were also provided with the same initial amount of game money ($250) to be used throughout the game to pay for orders they made in each round. To simplify analysis, it was a non-binding constraint; given it was not able to be exhausted over the 5 rounds of game play. All orders were 'buy' orders and the larger the order placed the greater the cost per unit of stock purchased. This was done to ensure a simple rational, risk-neutral, myopic incentive compatible game (please refer to Appendix C: Chapter 5 Appendix 8 for details) and was communicated to the human traders as a simple demand function for scarce stocks. In short, if an order of q stocks was placed for DFB stocks, then the price per stock would be $\frac{q}{2}$ cents, and the total cost requiring payment for DFB stocks for that round would be $q \times \frac{q}{2}$ cents. The maximum quantity of a particular stock that could be purchased in any round was 100 units.

Traders were able to contemplate previous round information, their private information (if received by them), remaining game money and amount of DFB and Not-DFB stocks owned to inform the orders they placed with sliders. The ultimate objective was to be the trader with the highest total game money (i.e. remaining money plus $1 per stock they owned that paid) in their prediction market game. The traders with the highest total game money across all games (including algorithmic traders) would go into a draw to win a prize.

For specific details and screenshots of the actual web-based prediction market game please refer to Appendix C: Chapter 5 Appendix 6.

5.3.4 The actual experiment

For the actual experiment, 60 prediction market web-games were played in total; 30 treatments and 30 controls. Five rounds for each game were played with the first round of every game facilitated by a PowerPoint 'how to play' scaffold with verbal cues from the researcher to ensure consistency across games[38]. 82 humans participated across 60 games. These human participants were drawn from 5 University tutorial classes in environmental economics, business economics and statistics subjects. Each control was paired with a treatment (which is termed here a 'control-treatment pair of games') so that they were identical in every way with the possible exception of relevant information. Whereas the relevant information level retained a value of 1 in the control the relevant information level in the treatment was assigned from the closed interval 0 to 1. Humans were randomly assigned to games with either 1 or 2 humans in an instance of a prediction market game; that is, a control-treatment pair of games would require either 2 or 4 humans. The total number of human and algorithmic traders in any game was 20. Therefore, in a game there was either 2 humans and 18 algorithmic traders or 1 human and 19 algorithmic traders. Owing to the fact that at least 84% of all stock market trades in 2010 were placed by computer algorithms (Lu, 2016) and with algorithmic trading is considered on the rise (Chaboud et al., 2014), the ratio of humans to algorithmic traders in the prediction market games of this study is considered reasonable. The algorithmic traders in this study are kept simple and update bids in a Bayesian manner on receiving publicly available price information. The specific details of algorithmic traders are provided in Appendix C: Chapter 5 Appendix 1. Noteworthy, is the analysis in Appendix 1 which justifies that proper market prices hold in a market filled with

[38] In addition, it was ensured that traders played one game only; so as to control for learning affects across games.

such traders; theoretically such a market would result in relevant information being the sufficient and necessary condition for convergence to the best possible prediction (Grainger et al., 2015). In short, across this series of games, the proportion of traders approximates the real-world algorithmic trader proportions, and the nature of algorithmic traders (being consistent with proper market prices) remains true to the theoretical model.

Each game was randomly assigned a stock that would pay out, and uncertain information concerning the chance of each prediction market stock paying-out was randomly distributed across algorithmic and human traders. There were 5 rounds played in each game (whereby in each round a bid was placed by all traders) and at the end of the fifth round the stock paying-out was revealed. The algorithmic or human trader with the highest game money was considered winner of that game and was eligible for the opportunity to win the $200 prize. Appendix C: Chapter 5 Appendix 6 provides specific details and web-game screenshots of the actual Prediction Market Game.

It should be noted that designing a rational, risk-neutral, and myopic incentive compatible prediction market game was an important consideration. This was undertaken so as to be consistent with the theoretical model and computer simulation work already completed. The finding of this series of prediction market games with human participants may then be logically combined with insights of the theoretical and computer simulation work. Appendix C: Chapter 5 Appendix 8 provides details as to how these prediction market games satisfy rational, risk-neutral, and myopic incentive compatibility.

A control-treatment pair of games either converges or not to the same prediction probability (represented in the average quantity ordered, e.g., an average order quantity of 30 indicates that the prediction market believes there is a 30% chance of the event associated with that stock occurring). A treatment and control was considered having converged if at the end of the final round they were within 5 quantity units of one another; the maximum possible distance being 100 units[39]. Appendix C: Chapter 5 Appendix 7 provides a theoretical overview of the method used to analyze the results of this particular experimental setup (i.e MLE of the binary logit model). More complex details pertaining to the distribution of coefficients and how the likelihood ratio relates to the chi-squared distribution may be found

[39] The five units region was chosen because each trader is one twentieth of the total number of traders and the maximum effect they may exert is a quantity order of 100 units of stock; which implies five units of movement to the average quantity ordered. Thus a distance between control and treatment average quantity orders of more than five units constitutes more than one trader’s influence.

in statistical texts[40]. These statistical concepts are used to analyze results with STATA employed to ease analysis. The results and annotated details of the STATA analysis are provided in the following sections.

5.4 Results

In total, 60 prediction market web-games were played; 30 treatments and 30 controls. These games involved 82 human traders across 5 tutorial classes. The number of humans in a game was randomly chosen as one or two humans. The number of algorithmic traders was included into the game such that the total number of traders was 20. The state s was randomly assigned a value of 0 or 1; 1 indicating that the DFB (Build Dog Friendly Beach) event occurred and all associated DFB stocks paid, and 0 indicating that the Not-DFB event occurred and all associated stocks are paid. Relevant information r was suitably varied across all games. The details of which are captured in Table 5.1.

[40] Suffice to say that coefficients are considered normally distributed and the log of the likelihood ratio is proportional to a chi-squared distribution.

Table 5.1
Results of 60 games depicting control and treatment prediction markets converging (1) or not (0) for the two stocks traded (DFB and Not-DFB). For detailed notes relating to each experiment, please refer to Appendix C: Chapter 5 Appendix 10.

						control		treatment		Control and treatment converge to the same prediction (within 5 units of each other)		
	control game	treatment game	number of humans	s	r	DFB	Not DFB	DFB	Not DFB	r	DFB (PM)	Not DFB (PM)
	1	2	1	0	0	16.8	84	61.3	38.7	0	0	0
expt1	3	4	1	1	0.3	90.6	9.4	81.5	18.6	0.3	0	0
	5	6	1	0	0.5	9.3	85.7	19.1	78.2	0.5	0	0
	7	8	1	1	0.9	79.7	20.3	67.7	30.4	0.9	0	0
	9	10	1	0	0.95	4.9	95.9	3.7	96.4	0.95	1	1
	11	12	1	0	1	10.8	86.5	11.4	86.4	1	1	1
expt2	13	14	2	1	0	84.9	15.1	62.8	37.1	0	1	1
	15	16	2	1	0.3	95.1	6.4	60.9	37.3	0.3	1	1
	17	18	2	1	0.5	90.4	9.2	86.7	12.6	0.5	1	1
	19	20	2	1	0.9	84.4	15.2	91.7	8.3	0.9	1	1
	21	22	2	1	0.95	92.8	7.7	87.6	8.7	0.95	1	1
	23	24	2	0	1	22.1	80	25.8	76.2	1	1	1
	25	26	1	0	0	9	91	43.5	50.9	0	1	1
expt3	27	28	1	1	0.1	95	5.7	68	32.5	0.1	1	1
	29	30	1	0	0.3	4.1	96	28.6	67.9	0.3	1	1
	31	32	1	0	0.85	37.3	59.3	47.9	53.4	0.85	1	1
	33	34	1	0	0.95	14.4	90.6	17	83	0.95	1	1
	35	36	1	1	1	79.9	19.7	80.2	19.8	1	1	1
	1	2	1	0	0	14.9	85.1	56.6	38.7	0	1	1
	3	4	1	1	0.3	84.7	14.6	84.5	19.5	0.3	1	1
	5	6	1	0	0.5	10.1	89.9	23.8	80.4	0.5	1	1
expt4	15	16	2	1	0.3	96.6	3	54.3	39.6	0.3	1	1
	17	18	2	1	0.5	80.8	10.5	88.2	12	0.5	1	1
	19	20	2	1	0.9	89.9	9	96.6	3	0.9	1	1
	21	22	2	1	0.95	54.3	39.6	80.8	10.5	0.95	1	1
	23	24	2	0	1	88.2	12	89.9	9	1	1	1
	1	2	1	0	0	14.3	85.7	57.7	39.5	0	1	1
expt5	3	4	1	1	0.3	90.6	9.4	72	19.9	0.3	1	1
	5	6	1	0	0.5	9.1	90.9	21.6	72	0.5	1	1
	7	8	1	1	0.9	68.6	27.7	74.7	25.3	0.9	1	1

Table 5.1 also highlights in yellow ‘the end of fifth round average quantity ordered in control and treatment markets for both DFB and Not-DFB stocks’. Additionally, the convergence of the average quantity ordered of both control and treatment for both the DFB stock and Not-DFB; whereby 1 indicates the average quantity ordered for control and treatments in the final round are within 5 units of each other and considered as converged, whereas 0 indicates otherwise. This is highlighted in orange in Table 5.1. For detailed notes relating to the five experiments please refer to Appendix C: Chapter 5 Appendix 10.

5.5 Analysis

To investigate whether the relevant information level r significantly and positively affects the probability of convergence of treatment to control prediction market, I estimate a Logit model. The STATA code is presented in Appendix C: Chapter 5 Appendix 11. The results are depicted in Table 5.2.

Table 5.2
Binary logistic regression of the convergence of control and treatment prediction markets for the DFB stock vs. relevant information level $\boldsymbol{r}$.

Variable	B coefficient	SE standard error	exp(B)
constant	-3.480**	1.370	
r (relevant information level)	3.687**	1.666	39.940
df (degrees of freedom)		1	
χ2		7.24	
p-value		0.007	
number of observations		30	
proportion of observations with dependent equal to '1'		27%	

Note: Controls are state of the world paid, vector of private information, and probability distributions of both of these.
*p<0.10 **p<0.05 *** p<0.01

Notice that coefficient of r and also the constant term are statistically significant at the 5% significance level. Also notice, that the hypothesis that the model is explained by chance alone is strongly rejected given the very small p-value (0.0071) for the chi-squared distribution of the log of the likelihood ratio (Buse, 1982).

Post hoc tests in Appendix C: Chapter 5 Appendix 2 ensure that the normality assumption holds; which is an important qualification given that maximum likelihood estimation is inconsistent under non-normality (Bera et al., 1984). Additionally, a continuity correction factor is not required and hence no complicated adjustment needs to be applied in order for the normal distribution to be a reliable approximation for the actual binomial distribution setting (Guillen, 2014).

Therefore, the required conditions for a binary logit analysis are in place; the functional form properly specified, and relevant information level is found to be statistically significant at the 5% level.

5.6 Discussion

The statistical significance of relevant information level is of prime interest. This is because previous chapters (including a new theoretical model and computer simulations) have identified relevant information as an important ingredient for prediction markets and decision markets to make the best possible predictions and decisions respectively. However, since prior work did not incorporate human behavior, this study (incorporating human traders) was the next logical step.

Off the shelf prediction and decision markets do not exist that both ensure proper market prices hold (Grainger et al., 2015) and that also allow the deliberate manipulation of relevant information level. As such, the researcher built a prediction and decision market that allowed the manipulation of relevant information level in a stochastic prediction market setting in which proper market prices likely held. This logically provided a means to test the effect of relevant information level on convergence of the prediction and decision markets to the best possible predictions and decisions respectively.

A priori design, simulations, pilots and the analysis of the game and experiment provided for an experiment with a high power of test with a carefully constructed game to test the hypothesis under investigation. That is, this tailor-made prediction and decision market game was a means to, with a high probability, correctly reject the null hypothesis; the null hypothesis being ‘relevant information level is not statistically significant’.

A control and treatment experiment setup was chosen which was structured such that the only possible difference between any control-treatment pair was the prediction market’s relevant information level. Therefore, any difference in the dynamics of control and treatment markets across all games could only be attributed to the relevant information level. The dynamic of interest being the convergence of the prediction and decision markets to the best possible prediction and decision are dichotomous outcomes that justified the binary logit model.

Simple prediction market games were played in the actual experiment given they are the basic building block of decision markets (Grainger et al., 2015). If relevant information level was found to be statistically significant in the prediction market games then it logically follows that both prediction and decision markets and their ability to converge to the best prediction and decision are affected by relevant information level.

The control prediction market game was always fixed at the highest relevant information level $r = 1$ across all games, meaning all incumbent traders are fully informed, implies that the control market equilibrium serves as proxy for the best possible prediction. The prediction market game predictions are represented in the average quantity order (in a similar manner to the logarithmic market scoring rule prediction markets (Hanson, 2012)). For example, if on the final round the average quantity order is 80, then this means the market believes there is an 80% chance of the stock paying out \$1; which in turn means there is an 80% chance of the associated event occurring.

The treatment and control markets were considered to have converged to the same 'prediction' if they were within 5 units of one another at the end of the final round; this was recorded as a 1, and not converging was recorded as 0. For example, if at the end of the final round the average order quantity for treatment and control prediction markets were 60 and 63, then the markets were considered as predicting the same and considered converged (so a 1 was recorded).

The dichotomous 'convergence' outcome (i.e., 1 or 0) with an explanatory relevant information level continuous variable typically lends itself to analysis via a binary logistic regression. Upon doing so the coefficient of relevant information level was found to be positive at the 5% significance level. That is, increasing relevant information level is associated with increasing probability of convergence of the treatment market to the best possible prediction (i.e., the control market average order quantity) irrespective of the variation in number of humans for this specific prediction market. Post-hoc tests were performed to reassure that these finding were valid by testing the normality assumption was valid and that no continuity correction factor was required.

The specific setup of the prediction market game is important. It ensures liquidity, bounded market loss, a sufficient number of traders, and is a rational, risk-neutral, and myopic incentive compatible game. Liquidity is ensured, as there is no bidder to seller matching problem, and instead stocks are available using a simple slider that submits a buy order and also calculates the cost of order. In one sense the slider serves as an automatic market maker. The market loss is bounded by the value of prize on offer as winners of each game go into a lottery to win that prize. Simple algorithmic traders ensure a sufficient number of traders reside in the market so that no one trader can move the prediction markedly. However, in this series of games, the algorithmic traders and human traders were assigned uncertain information (conditioned on the stock that would pay out) at the start of the game, whereas in the real-world human traders would source their own uncertain information and a satisfactory mechanism would need to be devised for algorithmic traders to source useful uncertain information (which is likely achieved by algorithmic traders in markets such as the Iowa Electronic (prediction) Market (Schmitz, 2011)). A rational, risk-neutral, and myopic incentive compatible game is ensured by the rules of this specific prediction market as is described in Appendix C: Chapter 5 Appendix 8. That is, the game will elicit the desired information from a rational, risk-neutral, and myopic trader. Extending the scope to strategic traders, Appendix C: Chapter 5 Appendix 9 provides a utility-based proof that a rational strategic trader will trade in the same way as a rational, risk-neutral and myopic trader given this specific prediction market web game setting and if the utility maximizing bid falls in the (allowed quantity order bid) open interval $(0,100)$. However, as explained in Appendix C: Chapter 5 Appendix 9, it is conceivable that bids of 0 or 100 maximize utility and are not rational, risk neutral and myopic bids. Hence as a market mechanism that incentivizes a specific (rational, risk neutral and myopic) behavior, in even strategic and rational traders, it can be considered holding almost everywhere. However, this kind of utility calculus presumes a rational trader and this is utility theory's Achilles heel. Numerous studies have revealed that human behavior is not that of rational automatons found in utility theory (Kahneman and Tversky, 1979). Therefore, neither of the mathematical proofs provided in Appendices 8 and 9, respectively, mean that humans will play in a rational, risk-neutral, and myopic way. But, it does mean that by doing so the chance of winning this specific prediction market game may be maximized. Thus by the end of the game, one may argue that the dominant traders remaining will likely bid in a rational, risk-neutral and myopic way; in many ways a refinement similar to the genetic algorithm approach (Holland and Reitman, 1977). In short, the market undergoes a ‘natural selection’ and ‘evolves’ into one with human behaviors identical to those traders in the theoretical model and computer simulations of previous chapters. Which, in turn, implies that the best possible predictions and decisions are likely elicited. This has extremely interesting implications. It suggests that instead of a paradigm attempting to discern unknown human behavior and the effect on prediction market predictions, one should instead shift the paradigm and *design prediction market rules that then ensure a particular set of known behaviors evolve and dominate in the remaining population of human traders*.

5.7 Conclusion & Future Research

‘Relevant information level’ is found to be statistically significant at the 5% level in the control and treatment experimental setup using prediction market games with human participants. Therefore, to this point the significance of relevant information is justified by theory, computer simulations, and in the controlled laboratory setting with human participants described in this chapter. What remains now is to investigate a real-world prediction market setting in order to determine if relevant information level is significant there as well. This will be undertaken in the real-world analysis chapter of this thesis.

The prediction and decision market games built in this study may find utility in a real-world context. Whilst application and testing in that domain is beyond the scope of this thesis, such investigation may be pursued through post-doctoral studies. However, these prediction and decision market games differ from prediction market software currently in the marketplace; mainly because of the emphasis placed on the importance of high relevant information levels in prediction and decision markets. Relevant information level is simply an alternative measure of prediction and decision market efficiency. This idea will be comprehensively explored in the policy implication chapter of this thesis.

The idea of using prediction and decision market games to inform decisions will be explored in the policy implications chapter. The ultimate goal of this thesis is to research using prediction markets (of high quality, i.e., high relevant information level) to build decision markets that may be usefully applied to project portfolio management. Specifically, the decision market is required to provide a highly probable solution to the combinatorial hard problem i.e. what projects are in or out of the best possible portfolio of projects. The ability of the type of decision market constructed in this thesis to select and prioritize the best portfolio of projects, in a not-for-profit setting, will be investigated in the policy implications chapter.

Chapter. 6 The Iowa Electronic Market Data for the 2008 U.S. Presidential Election: Real-world Analysis

"As far as the laws of mathematics refer to reality, they are not certain, and as far as they are certain, they do not refer to reality."

Albert Einstein

Key Message of Chapter:

- **Analysis of the Iowa Electronic Market data finds relevant information level plays a statistically significant role;**
- **The probability of an implied decision market (implied in stocks conditional on other stock) increases with relevant information level;**
- **The analysis controls for possible endogeneity issue and other confounding factors.**

This chapter connects to research questions one and two of this thesis. That is, the analysis of empirical data finds that relevant information level plays a statistically significant role, and the reason why an implied decision market exists within the prediction market is fully explained. Specifically, empirical data in the form of 3 consecutive months of liquid trading in the Iowa Electronic Market 2008 Presidential Election are analyzed. The relationship of the implied daily decision market proxy for correctness (i.e. whether it chooses Obama), as the response variable, to the day's theoretical informed proxy for relevant information level, as an explanatory variable, is examined. Notably, the analysis of the implied decision market data for the Iowa Electronic Market's 2008 U.S. Presidential Election is a candidate selection problem that, in form, is a not for profit project selection problem of determining whether a project (candidate) is in or out of the best possible project portfolio.

A multivariate (MV) logistic regression incorporating variables informed by a review of literature finds that the coefficient of relevant information level is statistically significant. To deal with the possible endogeneity issue, instrumental variables (IVs) are used in the estimation. Specifically, for this dichotomous response setting, a control function IV approach is applied to control for endogeneity. The risk of confounding factors is controlled for by the fine strata propensity score analysis (PSA); which simply put, uses PSA to balance (i.e. create what is effectively a randomized setting of known covariates) a stratum that is fine enough to minimize the likelihood of variations of unknown confounders but sufficiently large enough to ultimately demonstrate the statistical significance of relevant information level.

The combination of MV, IV and PSA, provides compelling evidence that relevant information level is statistically significant in real-world prediction markets. This bodes well for the argument running throughout this thesis; that relevant information level is important for well-functioning decision markets. With a view to application in selecting the best portfolio of projects within the not-for-profit sector, the next and final chapter explores the policy implications for internal corporate decision markets.

6.1 Introduction

Both “computer simulation” and “prediction market games with human participants” have found relevant information level to play a statistically significant role in convergence to the DCE. However, both reside in controlled laboratory conditions. “Real-world analysis” relaxes the sanitized laboratory context and exposes the theoretical model to explaining dynamics in the real-world. Relevant information level is found to play a statistically significant role in the analysis of real-world data from the Iowa Electronic Market (IEM) 2008 Presidential Election.

Specifically, the significance of relevant information level (explanatory) to the probability of the correctness of the IEM implied daily decision market (response) is determined. Suitable proxies are defined for the explanatory and response variables. A binomial logistic multivariate regression (MV) of the response versus the explanatory variable alongside a selection of other covariates (guided by a review of literature) is performed. Instrumental Variables (IVs) to construct a control function that controls for endogeneity finds no endogeneity problem and reveals relevant information level as statistically significant. The propensity score analysis (PSA) balances known and potentially confounding variables; with the fine strata PSA method controlling for the possibility of the unknown and potentially confounding variables. In the fine partition of PSA balanced data, relevant information level is found to play a statistically significant role in convergence to the DCE.

There does not currently exist a real-world implementation of the decision market of the type proposed in Chapter 4 of this thesis. There does however exist real-world prediction market stocks conditioned on other stocks in prediction markets; called conditional prediction markets (Chen and Kash, 2011). Conditional probabilities revealed in these markets imply decisions; thus conditional prediction markets may also be called implied decision markets. The statistical significance of relevant information level in the real-world IEM implied decision market provides additional evidence[41] that high relevant information levels entail quality decision markets. These quality decision markets will be considered further in the next chapter on policy implications.

A review of related literature is undertaken in section 6.2. The empirical real-world IEM data and methodology of analysis are justified in section 6.3. Section 6.4 provides the analysis of the data. The analysis is further discussed in section 6.5 and a conclusion that suggests further research is provided in section 6.6. Appendix D: Chapter 6 Appendices provide comprehensive details referred to by these sections.

6.2 Related Literature

Literature is reviewed that contrasts theoretical and real-world (prediction and decision) markets, investigates the determinants of the IEM 2008 US Presidential Election prediction market so as to identify the covariates for analysis, and provides the justifying methodology for methods used to analyze real-world prediction market empirical data.

6.2.1 Theoretical versus real-world prediction and decision markets

Real-world political stock (prediction) markets diverge from idealized theoretical models with the former populated by error prone traders and the latter by rational traders (Hansen et al., 2004).

In theory, truth-telling rational traders provide for simpler models, however, rational traders who attempt to strategically manipulate the market to their benefit improve prediction accuracy as a result of the added liquidity generated via the mispricing manipulations (Hanson and Oprea, 2009). But within these idealized political prediction market models, the rational trader when considering how people will vote, will reason that not a single (rational) person should vote; given the negligible impact of any single vote on election outcomes (Gelman et al., 2012). The probability that any single voter will make a difference to the presidential outcome is calculated as approximately 1 in 10 million (Gelman et al., 2012) and marginally increases in close elections (Strijbis et al., 2016). Despite this theory, the real-world IEM prediction market has been considered extremely liquid when trading US Presidential Election stocks (Berg et al., 2008a). This begs the voter’s paradox (Abrams, 1976) question: are those who vote in the real-world truly rational?

[41] Other evidence being computer simulations and games with human participants that are discussed in previous chapters.

The concept of a fully informed market whereby a market price attained is as if everyone revealed (directly communicating) their private information to each other is a key idea (Camerer, 1998). This has idea has come to be called the direct communication equilibrium (DCE) (Chen et al., 2004). Theoretical work assuming risk-neutral-price-taking traders *finds* that the DCE price *likely* remains unchanged when traders use it to revise their beliefs (Manski, 2006). Whereas the proper market price axiom (Grainger et al., 2015) *requires* that the DCE price does not cause a trader to change their information (beliefs); and therefore the equilibrium price inextricably remains unchanged. The difference is subtle. Grainger et al’s model *enforces* that at equilibrium prices, traders are fully informed and remain that way. In contrast, the Manski (2006) work *observes* that, at equilibrium prices, traders may not be fully informed. In short, whilst the idea of ‘equilibrium price remaining the same if only market prices are used by traders’ (Manski, 2006) is closely related to the ‘not relevant information’ concept proposed in Grainger et al’s theoretical model, the *premise* of Grainger et al’s theoretical model on prediction markets is not consistent with the *conclusion* of Manski’s theoretical model on prediction markets. However, it is not unusual for theoretical models to differ. What matters is the trade-off between how well idealized models ‘simply explain’ and how well they ‘completely explain’ real-world data. For example, many theoretical models of real-world data have made a simplifying assumption called the marginal trader hypothesis (Blackwell and Pickford, 2011) i.e. the marginal trader sets the market price. However, real-world empirical studies have found evidence contradicting the marginal trader hypothesis (Blackwell and Pickford, 2011). Whilst this hypothesis is found wanting in completely explaining real-world data, the marginal trader hypothesis provides a pedagogically elegant and simple explanation of how idealized political stock market prices form (Forsythe et al., 1992).

In economics, there has been a tension between formalism and empiricism, with the latter considered a necessary means to validate “blackboard economics” (Blaug, 1998). For example, prediction markets are widely considered to be arbitrage free (Buckley and O’Brien, 2015; Hanson and Oprea, 2009). However, arbitrage opportunities have been observed in the Iowa Electronic Market (Schmitz, 2011), and arguably remain in such real-world markets because of human limitations to perfectly exploit them (Herschberg, 2012).

Theoretically, the justification of the market efficiency assumption is an area of research considered worthy of closer analysis (Treynor, 1987). Furthermore, the efficient (prediction) market hypothesis does not ubiquitously hold across (prediction) market designs (Manski, 2006). As such, equilibrium market price will not always efficiently aggregate all private information. In fact, the IEM cannot guarantee efficient aggregation of information for accurate predictions (Berg and Rietz, 2006). Part of the problem is that there must be private information to be elicited otherwise predictions and decisions will be wrong (Chen and Pennock, 2010). The other part is that thin markets, in many real-world betting markets, distort prices (Chen and Pennock, 2012). For example, a combinatorial prediction market for the 2012 US Presidential election run to investigate the problem of thin markets in a combinatorial settings was not always accurate (Dudik et al., 2012).

Whilst speculative markets, such as race track betting, contain traders best described as "risk lovers" (Ali, 1977), such humans behave in an approximately rational way when information is uncertain (McFadden, 2009). In this context, idealized economic models of individual behavior become useful estimates for the real-world (Arrow, 1986). The focus of investigation then becomes how information transparency is related to prediction (and decision) market performance (Yang et al., 2015). The probability of the correct decision as a function of the proportion of traders revealing their private information through their bids is therefore logical to investigate (Grainger et al., 2015). It is found to be a linear relationship in both a theoretical model (Grainger et al., 2015), computer simulations of a previous chapter and in this chapter's analysis of IEM real-world empirical data. The IEM 1996 US Presidential prediction market was a conditional prediction (decision) market, and said able to *decide* the best possible candidate (Berg and Rietz, 2003). This ability of a conditional prediction market to decide the best possible (fully informed and traded) candidate is analyzed in this chapter in a more recent conditional prediction market i.e. the IEM 2008 US Presidential prediction market.

6.2.2 The IEM 2008 US presidential election prediction market, determinants and covariates for analysis

Presidential prediction markets have a history of outstanding prediction performance; in particular the Iowa Electronic Market (IEM) outperforms other prediction mechanisms most of the time (Berg et al., 2008b). The IEM is found to regularly outperform reputable polls (Berg et al., 2008a); polls being the contemporary mechanism employed for predicting election outcomes. Election polls typically require large cost and time investments and have needed statistical adjustments to control for non-representative samples (Wang et al., 2015).

In contrast to polls, the IEM is considered consistently accurate in its continuously updated predictions (Berg et al., 2008a; Berg et al., 2008b).

There were a number of determinants identified for the 2008 presidential election outcome. A determinant for the 2008 election Obama success was his selection of Joseph Biden as a running mate; who was well-versed in foreign affairs and national security (Saldin, 2008). However, McCain's Selection of Palin and Obama's selection of Biden over Clinton induced significant media coverage; as such sex was considered a key explanatory variable in the presidential prediction market (Saldin, 2008).

Hope and race were also considered determinants in the 2008 elections (Finn and Glaser, 2010). Voter turnout is a function of inspiration and demographic group (Harder and Krosnick, 2008). Therefore, it is logical to consider the socioeconomic status of candidates; including race which was considered a significant factor in the 2008 elections (Finn and Glaser, 2010). Racism in the 2008 U.S. Presidential election was found to be statistically significant in reducing Obama's share of votes (Pasek et al., 2009). The multinomial logit regressions performed in psychological studies undertaken on the 2008 US Presidential Election suggest that, despite an Obama victory, anti-African-American racism may have been the most significant determinant reducing Obama's share of votes (Pasek et al., 2009). Other election outcome determinants identified via psychological studies were the alignment of personality and ideology of the candidate with the voter (Jost et al., 2009), "onerous [voter] registration procedures" and the mere act of "interviewing people" which reduces and increases voter turnout respectively (Harder and Krosnick, 2008).

The comparison of two political stock markets for the Berlin 1999 state elections considered media coverage as an explanation for prediction differences (Hansen et al., 2004). Arguably, campaign receipts and disbursement may proxy for this and other forms of coverage e.g. social media. Obama party's 'savvy' sourcing and use of campaign funds (i.e. receipts and disbursements) and use of social media was considered a major reason for the 2008 US presidential election success (Wattal et al., 2010).

The state of the economy has been found to be a determinant in presidential elections with voter's punishing poor economic performance of the incumbent (Lewis-Beck and Stegmaier, 2000). Abramowitz's "time for change framework (Abramowitz, 2008)" suggests that the electorate makes a choice of whether to continue or discontinue with the incumbent government's economic policies (Sweezey, 2013). Voters reward or punish the government depending on economic performance (Lewis-Beck and Stegmaier, 2000). Notably, a revisionist perspective championing a reverse causality where politics causes voter's view of the economy was found to be wrong (Lewis-Beck et al., 2008). In short, the economic performance of a party affects the voter's political choice.

Political economy ideas and specific circumstances of the 2008 US Presidential elections provide some guidance on possible determinants and associated covariates to analyze in this chapter. Political economic research suggests that voters reward or punish Presidential Candidates (Nadeau et al., 2013). As such the Dow Jones Industrial Average (DJIA) is used as a measure of US economic health. Of interest is whether the relevant information level treatment is in some way related to the DJIA; if it is then this needs to be controlled for. The 2008 election saw the use of social media to entice the younger voters who typically do not vote (Vaccari, 2010). It also saw the issue of sexism ultimately expressed in the strategic Vice-Presidential choice of Republican Sarah Palin in response to Hillary Clinton not being chosen for the role by the Democrats (Dwyer et al., 2009). Therefore a dichotomous dummy variable is used where 1 represents female and 0 male to control for its impact on the probability of Obama being elected. Racism in the guise of Anti-Afro-Americanism towards Obama featured heavily and is also captured in a dummy covariate with 1 for Afro-American and 0 otherwise (Pasek et al., 2009). The ability to lobby for finance is also considered a determinant of election outcome, depending on the context (McKay, 2012); it is captured in Federal Electoral Commission record of campaign receipts (Federal Electoral Commission, 2008). The ability to market the political message is also considered a determinant of election outcome, depending on the context (McKay, 2012); it is captured in Federal Electoral Commission record of campaign disbursements (Federal Electoral Commission, 2008).

6.2.3 Methodology and methods in literature to analyze real-world empirical data

The literature on methodology and methods pertinent to guiding the analysis of the empirical data for this chapter are reviewed in the following subsections.

6.2.3.1 Methodology in literature

Real-world empirical data differ from experimental data. Whereas in experimental settings all other covariates are randomized and independent of the application of the treatment, in empirical settings there is a propensity for other variables to systematically covary with the treatment (Cochran and Rubin, 1973). In short, the underlying methodological strategy is to avoid the possibility of the response affecting the treatment (i.e. the endogenous problem) (Larcker and Rusticus, 2010) and to guarantee that other variables in the empirical setting are effectively randomized (i.e. to avoid confounding problems) when applying the treatment as in the experimental setting (Lee, 2013).

Multivariate regression (MV) typically applied to link a response variable to explanatory variables assumes no endogeneity is present (Johnston et al., 2008). MV is justified by the simplicity of its application and prima facie insights. However, the statistical significance of a link between relevant information level (treatment) and the probability of decision market correctness (response) in analyzing IEM empirical data may suffer from endogeneity. Simply put, the MV not considering endogenous effects will not be able to guarantee that the treatment is not a function of the response. For example, it is possible that the proportion of informed trading (relevant information level) is a function of accurate prices (that relate to the correctness of the decision market).

The possibility of endogeneity and the dichotomous response justifies control function instrumental variable (for succinctness denoted as IV) approach (Lewbel et al., 2012); whereby the IV is not correlated with the error terms, rather it is correlated with the treatment which in turn affects the response, and the response is not able to directly affect the IV (Zohoori and Savitz, 1997).

Additionally, a confounding variable may be the underlying cause of the variation of both the response and treatment (Caliendo and Kopeinig, 2008). This possibility justifies the use of propensity score analysis (PSA) to control for this by effectively creating randomized data partitions that emulate a randomized experimental setting (Zohoori and Savitz, 1997; Caliendo and Kopeinig, 2008; Crown, 2014). Fine enough partitions (fine stratified PSAs) reduce the chance of observed and unobserved confounder effects (Lin et al., 1998).

In short, the contemporary methodology is to control for endogeneity in the MV by utilizing an IV and also control for possible observed and unobserved confounders by utilizing a fine stratified PSA (Keele, 2015).

6.2.3.2 Multivariate regression (MV) method in literature

Typically, analysis of empirical political data (such as the data set analyzed in this chapter) has utilized propensity score analysis (Keele, 2015), but Multivariate regression (MV) is considered just as powerful; so long as post-hoc tests validate that the underlying assumptions of the regression holds (Brazauskas and Logan, 2016).

Post hoc tests should validate the normality assumption holds given that MV is inconsistent under non-normality (Bera et al., 1984). In MVs that inherently utilize a binomial distributed format (such as the binary logit model), the rejection of the need for a continuity correction factor is also important to test (Guillen, 2014).

Explanatory variables that the researcher wishes to test may not be explicitly contained in the data. Therefore, a readily observable proxy for an unobservable or latent explanatory variable, which is implied by the theory, is constructed instead; the proxy whilst technically different is often called the explanatory variable (Bollen, 2002). Explanatory variables in the MV may be continuous or dummy variables; with the latter being useful in representing set membership information (Garavaglia and Sharma, 1998) e.g. race is a dummy variable with a value of 1 indicating the Presidential candidate is Afro-American, and a value of 0 indicating otherwise.

The log of the likelihood ratio of two hypothesized MV models is proportional to a chi-squared distribution and may be used to determine the significance of explanatory variables (Siniksaran, 2005). Given that the Wald, Lagrange Multiplier and Likelihood Ratio (LR) and F tests are related, the use of the LR test is sufficient to find the best MV specification (Siniksaran, 2005).

6.2.3.3 Instrumental variable (IV) method in literature

An instrumental variable (IV) is typically used when an explanatory variable in regression is endogenous (Winship and Morgan, 1999; Bascle, 2008). The instrumental variable is assumed to only influence the response via the mediating explanatory variable with no response to instrument causality (Pearl, 1995). For example, the endogenous problem of determining whether ‘consuming sugar causes tooth decay in people’ or ‘people with tooth decay consume sugar’ may be resolved by using ‘a tax on sugar’ as the IV. Two things may be immediately seen. Firstly, the IV directly affects consumption of sugar and is mediated through this consumption to tooth decay. Secondly, tooth decay has no causal effect on the sugar tax. Therefore, and statistically significant correlation between the IV and tooth decay provides evidence that the mediating variable is also significant (Pearl, 1995).

The sugar tax IV example is trivial. Typically settings are complex and the identification of a logically consistent IV is crucial and not immediately obvious (Heckman and Pinto, 2015). In dichotomous response settings (such as the one investigated in this chapter) a control function IV approach is applicable (Lewbel et al., 2012). The key idea underlying the control function is to use IVs to ultimately construct a model specification whereby the error term does not covary with any explanatory variables. It is a two-step approach. Consider the model $M(X, E) = 0$ that has exogenous variables X that are orthogonal to potentially endogenous variables E. In the first step, the potential endogenous variables E are linearly regressed onto IVs Z. The resulting regression of $E\ on\ Z$ (i.e. $F(E, Z) = 0$) and observations are used to produce an estimate e of the error vector; where e is assumed as normally distributed with a mean of zero. Notice that no endogeneity problem exists in regression $F(E, Z) = 0$ since Z are exogenous instruments. In the second step, the error vector ε in model M is assumed normally distributed with a mean of zero but with a potential endogeneity problem in E. Because both e and ε have a mean of zero and are normally distributed then linear regression $\varepsilon = ae + \varepsilon'$ implies that a is proportional to the magnitude of vector ε projected onto vector e, and that ε' is independent of $\varepsilon\ and\ e$. Therefore, a new model $M'(X, E, e) = 0$ is such that $X, E, and\ e$ are independent of one another and the coefficient a for e is statistically significant only if E covaries with ε; otherwise there is no endogeneity problem in the original model M. Importantly, the error term ε' suffers no endogeneity issues. Whilst this IV approach controls for endogeneity, in typically complex real-world settings, confounding variables may exist; a setting not resolvable by the IV alone (Wunsch et al., 2006). There is no way to statistically test for confounding in empirical data (Pearl, 2011). As such the following section provides a means to control for a potential confounding problem.

6.2.3.4 Propensity score analysis (PSA) method in literature

Propensity scores analysis (PSA) is either used to match (control and treatment observations) or to create strata (in which variables are effectively randomized) to control for confounding variables (Winship and Morgan, 1999). The fundamental idea is to extract empirical data from the sample that appears as if generated from a randomized experiment (D'Agostino, 2007); a situation that does not typically exist in real-world data (Holland, 1986).

Randomizing in this way is considered the "gold standard" (Rubin, 2007) and also a key step to inferring causality (Antonakis et al., 2010).

Stratified propensity score analysis is a means to create strata of randomized known variables (conditioned on a balancing score), but potentially suffers from the unknown confounding variables (Joffe and Rosenbaum, 1999). However, fine strata PSA effectively reduces the possible variation of unknown confounders in the strata data (Lin et al., 1998), but the reduced sample size increases the chance of a type 1 error i.e. not rejecting the null when it should be rejected (Williamson and Forbes, 2014).

PSA is easily applied to dichotomous treatment settings have also been applied to continuous treatment settings and called generalized propensity scores (Kluve et al., 2012). Generalized PSA effectively partitions a continuous treatment (Kluve et al., 2012; Bia and Mattei, 2008; Egger and Von Ehrlich, 2013). This can be achieved by treating unknown thresholds (that demarcate partitions) as parameters in a maximum likelihood estimation (Hirano and Imbens, 2004).

6.3 The Data and Methodology

The empirical data are described, the hypothesis under test specified, the explanatory and response proxies constructed, and the methods of analysis comprehensively detailed. Specifically, as is typically performed, in this section the proxy variables for the treatment (relevant information level) and response (correct implied decision market) are derived from theory and a regression that controls for possible endogenous and confounding problems is constructed (Heckman and Pinto, 2015; Pearl, 2011; Pearl, 1995; Pearl, 2013).

6.3.1 Implied decision market data within the IEM 2008 presidential election prediction market

In this empirical IEM 2008 Presidential Election prediction market data, conditional probabilities can be calculated and hence an implied decision market exists. The data consist of three consecutive months (May, June, July 2008) of liquid trading in Democratic Candidate (Obama, Clinton, Edwards, Other) and Democratic Party close of day stock prices. In total there are 89 days implying 356 daily stock price observations; associated with Democratic Candidates and 89 daily stock prices associated with the Democratic Party stock. Stock prices are stochastic by their nature and of interest is the covariation between daily stock prices of a Democratic Candidate with the Democratic Party. Intuitively, this covariation suggests an element of Party outcome conditional on Candidate outcome. This will be more rigorously explored in the following sections.

6.3.2 Hypothesis under test

Finding of previous chapters suggests that the probability of the correct decision made by a decision market increases linearly in relevant information level. The hypothesis under test in those chapters was whether relevant information level was statistically significant in a computer simulation and in games with humans. Both found relevant information level to be statistically significant but both also took place in laboratory conditions. The real-world empirical setting provides the final test in this thesis i.e. is relevant information level significant in a context beyond the laboratory?

Information on the proportion of traders expressing their private information in bids (relevant information) is hidden and hence proxies for this and for a correct (decision market) decision are constructed; being logically implied by theory. Therefore, formally, the null hypothesis anticipated to be rejected upon analysis of the data is:

The daily proxy for relevant information level is not a statistically significant explanatory variable of the daily proxy for the correctness of the (implied decision market) decision response variable.

6.3.3 Proxy for the response of the implied decision market

The fundamental idea for the response variable is as follows. For each correct decision made by the implied decision market on a particular day, $y = 1$ is recorded; otherwise $y = 0$ is recorded. For example, if on a particular day the implied decision market for Obama and Clinton decides that they will be selected as President, then $y = 1$ is recorded for the Obama stock and $y = 0$ will be recorded for the Clinton stock for that day.

A more thorough treatment is as follows.

There are only daily close of prediction market stock prices available. However, this data contains sufficient conditional probability information. Therefore a daily implied decision market decision of the Candidate that will be President is possible to calculate.

Specifically, the price of the Democrat Party stock (denoted by D), Democrat Candidate stock (denoted by C) and all other Democrat Candidate (denoted by the complement C') for two adjacent days (day $t-1$ and day t) have respective close of day prediction market probabilities (directly implied by prediction market prices) $P[D_{t-1}]$, $P[D_t]$, $P[C_{t-1}]$ and $P[C_t]$. This information is sufficient to construct a relationship for implied conditional probabilities $P_t[D|C]$ and $P_t[D|C']$ when both are assumed constant across day t. That is, the simplifying assumption is that the decision for that day is inherent in these daily conditional probabilities that are treated as constant from the closing time of one day ($t-1$) to the next day (t) closing time. It follows that if $P_t[D|C] > P_t[D|C']$ then the probability of the Democrat party winning is maximized if the decision to nominate Democrat Candidate C is made.

More formally:

At close of day $t-1$: $P[D_{t-1}] = P_t[D|C]P[C_{t-1}] + P_t[D|C'] \, (1 - P[C_{t-1}])$ -- eqn1
And
At close of day t: $P[D_t] = P_t[D|C]P[C_t] + P_t[D|C'] \, (1 - P[C_t])$ -- eqn2

NB: $P[C']$ is the complement of C and therefore the sum of all other candidate probabilities in the market; $P[C']$ is only equal to $1 - P[C]$ if the market is arbitrage free. This specification is important for the following.

Subtract eqn2 from eqn1 to obtain:

$$P[D_{t-1}] - P[D_t] = P_t[D|C](P[C_{t-1}] - P[C_t]) + P_t[D|C'](P[C_t] - P[C_{t-1}])$$

and rearrange this to obtain

$P_t[D|C] = P_t[D|C'] + \frac{P[D_{t-1}] - P[D_t]}{P[C_{t-1}] - P[C_t]}$ -- eqn 3

Notice that $P_t[D|C] > P_t[D|C']$ when $\frac{P[D_{t-1}] - P[D_t]}{P[C_{t-1}] - P[C_t]} > 0$.

Simply put, when the price change associated with $P[D]$ and $P[C]$ over the interval of time (from close price of one day to the close price of the next day) move in the same direction, then $P_t[D|C] > P_t[D|C']$ i.e. the daily decision market is effectively deciding that candidate C is to be selected; which may be a correct or incorrect decision. The actual election outcome determines the correctness of the daily decision. The actual outcome is dichotomous i.e. Candidate C is either "elected as President" or "NOT elected as President". Thus the *decision market is also dichotomous (right or wrong) and the correctness of the decision market is represented by:*

$$y(C) = \begin{cases} 1 \; if \; the \; impled \; (day's) decision \; market \; reflects \; candidate \; C's \; "actual \; election \; outcome" \\ 0 \; otherwise \end{cases}$$

For example, if the (day's) decision market implies that Obama will win the election then $y(Obama) = 1$; because he was actually elected as President and the decision market is right on this occasion. If, however, the (day's) decision market implies that Obama will not win the election then $y(Obama) = 0$; because he was actually elected as President and the decision market is wrong on this occasion. To elucidate further, if the (day's) decision market implies that Clinton will win the election then $y(Clinton) = 0$; because she was not actually elected as President and the decision market is wrong on this occasion. If, however, the (day's) decision market implies that Clinton will not win the election then $y(Clinton) = 1$; because she was actually not elected as President and the decision market is right on this occasion.

Combining eqn3 and the definition of $y(C)$ results in the proxy for correctness and response variable:

$$y(C) = \begin{cases} 1 \; if \; \dfrac{P[D_{t-1}] - P[D_t]}{P[C_{t-1}] - P[C_t]} > 0 \\ 0 \; otherwise \end{cases}$$

6.3.4 Proxy for relevant information level (explanatory variable) on a daily basis

The proxy for the latent relevant information level explanatory variable is also embedded in the empirical data.

The theoretical model is necessarily required to determine the form of this proxy variable.

Theoretically, the probability of the best possible decision (i.e. those based on direct communication equilibrium (DCE) prices) is linear in the relevant information level $r \in [0,1]$; whereby relevant information level r is simply the proportion of traders in the market whose bids are conditioned on their private information.

Therefore, assume on day t that there exist the proportion $1 - r$ of bidders that do not condition their bids on private information i.e. they simply bid the previous day's closing price $P[C_{t-1}]$ for candidate C (and $P[C'_{t-1}]$ for the other candidates. Notice that $P[C_{t-1}] \neq 1 - P[C_{t-1}]$ is possible given that these traders are simply mirroring the previous day's prices and not necessarily correcting for arbitrage opportunities. The other proportion r of the market of bidders express relevant information and submit informed arbitrage free bids $b(C)$ and $b(C') = 1 - b(C)$; given they act rationally.

The resulting market prices formed are:

$$P[C_t] = (1-r)P[C_{t-1}] + rb(C)$$
$$P[C'_t] = (1-r)P[C'_{t-1}] + rb(C')$$

Adding these two equations:

$$P[C_t] + P[C'_t] = (1-r)(P[C_{t-1}] + P[C'_{t-1}]) + r(b(C) + b(C'))$$

Simplifying to:

$$P[C_t] + P[C'_t] = (1-r)(P[C_{t-1}] + P[C'_{t-1}]) + r(1)$$

Further simplification to:

$$P[C_t] + P[C'_t] - 1 = (1-r)(P[C_{t-1}] + P[C'_{t-1}] - 1)$$

Rearranging to arrive at the proxy:

$$r = \frac{(P[C_t] + P[C'_t]) - (P[C_{t-1}] + P[C'_{t-1}])}{1 - (P[C_{t-1}] + P[C'_{t-1}])}$$

In theory the proportion $r \in [0,1]$ with any r outside the interval [0,1] having no 'proportion' meaning. However, given real-world close of day stock prices are stochastic, it is possible for the calculated value of r to lie outside of this interval. Therefore, to ensure that the empirical analysis retains meaning, those meaningless r (outside the interval [0,1]) are discarded. Upon doing so 144 empirical observations are retained for further analysis.

6.3.5 The methodology

A mix of multivariate regression (MV), instrumental variable (IV), and propensity scoring analysis (PSA) provides different insights on the same empirical dataset (Biondi-Zoccai et al., 2011). IV and PSA are typically applied to reveal insights deeper than MV alone;

including discerning causality in empirical data via the widely accepted albeit philosophically contentious counterfactual framework (Winship and Morgan, 1999; Brand and Xie, 2007). The calculus of causality ultimately rests upon the concept of conditional independence i.e. a subset of empirical data collected with the same (conditioned on the) balancing (propensity) score appears statistically independent of one another (Pearl, 2013; Imbens and Rubin, 2015; Pearl, 2010).

In the following analysis section, tables providing the usual statistics (Peng et al., 2002; DeMaris, 1995) alongside graphs that highlight key messages to aid the reader. The analysis is comprehensive; including the necessary post hoc testing e.g. validating the underlying assumption that the binomially distributed empirical dataset is well approximated by normal distributions so as to undertake significance testing in the usual way (Schader and Schmid, 1989).

6.3.5.1 Multivariate regression and analysis method

Multiple MVs in the form of binomial logit regressions differing in the explanatory variables they include are undertaken. The best specification is determined by comparing these different MVs using the likelihood ratio (LR) test. For example, the log of the likelihood ratio of two hypothesized models having all but one variable in common is proportional to the chi-squared distribution and therefore can be used to determine the significance of that one variable (Siniksaran, 2005). Due to the Wald, Lagrange Multiplier, Likelihood Ratio (LR) and F tests all being (geometrically) related, the use of the LR test to determine the best regression specification is sufficient (Siniksaran, 2005).

Specifically, the MV will be day's decision market correctness y regressed against covariates race (R), past and current state of the economy (previous day DJIA denoted DJIAprev and current day's DJIA denoted DJIA), age of candidate (A), sex of candidate (S), campaign effectiveness in marketing messages (represented campaign receipts (Re), and Disbursements (D)) and proportion of traders who bid in an informed manner (relevant information level r with higher order effect r^2). A binary logit model is used and the initial regression has the form:

$$\begin{aligned}P[y = 1|R, DJIA, DJIAprev, A, S, Re, D, r] \\ = expit(\beta_R R + \beta_{DJIA} DJIA + \beta_{DJIAprev} DJIAprev + \beta_A A + \beta_S\, S + \beta_{Re}\, Re \\ + \beta_D D + \beta_r r + \beta_{r2} r^2 + \beta_0)\end{aligned}$$

Backward and forward stepwise regression utilizing multicollinearity and likelihood ratio tests are applied to identify the best specification.

The likelihood ratio test is central in determining what variables to included or exclude in the forward and backward stepwise regressions. The likelihood ratio (LR) is the ratio of the null hypothesis model to the model restricted by the Maximum Likelihood Estimation (MLE) requirement (that the unknown parameters maximize the likelihood of observing the data). The idea underlying may be described quite simply. The binary logit MLE can be considered to have unknown parameters that are each estimated by a normal distribution with pdf $\frac{e^{\frac{-z^2}{2}}}{\sqrt{2\pi}}$. This is a direct result of the independence across data (conditioned on the underlying and sufficient parameters) and independence across parameters MLE assumptions. This implies for a large enough data sample, that data (given the parameters) is asymptotically normally distributed. Because (by Bayes rule) the probability of parameters given data is proportional to the probability of data given parameters, then the parameters (for the data being analyzed) are also normally distributed. Given the independence across parameters, then each parameter is normally distributed.

Therefore, for a single parameter: $LR = \frac{\frac{e^{\frac{-z^2}{2}}}{\sqrt{2\pi}}}{\frac{e^{\frac{-z^2}{2}}}{\sqrt{2\pi}}|with\ MLE\ applied} = \frac{\frac{e^{\frac{-(z-\bar{z})^2}{2}}e^{\frac{-(\bar{z}-\mu_0)^2}{2}}}{\sqrt{2\pi}}|for\ the\ null}{\frac{e^{\frac{-(z-\bar{z})^2}{2}}e^{\frac{-(\bar{z}-\mu)^2}{2}}}{\sqrt{2\pi}}|with\ MLE\ applied}$

Now, notice that $\frac{e^{\frac{-(z-\bar{z})^2}{2}}e^{\frac{-(\bar{z}-\mu)^2}{2}}}{\sqrt{2\pi}}$ is maximized when $\bar{z} = \mu$. Therefore, $\frac{e^{\frac{-(z-\bar{z})^2}{2}}e^{\frac{-(\bar{z}-\mu)^2}{2}}}{\sqrt{2\pi}}|with\ MLE\ applied = \frac{e^{\frac{-(z-\bar{z})^2}{2}}}{\sqrt{2\pi}}$, which implies that $LR = \frac{\frac{e^{\frac{-(z-\bar{z})^2}{2}}e^{\frac{-(\bar{z}-\mu_0)^2}{2}}}{\sqrt{2\pi}}|for\ the\ null}{\frac{e^{\frac{-(z-\bar{z})^2}{2}}}{\sqrt{2\pi}}|with\ MLE\ applied} = e^{\frac{-(\bar{z}-\mu_0)^2}{2}} = e^{\frac{-z^2}{2}}$ since the null assumes $\mu_0 = 0$ and $\bar{z} = z$.

Also notice that the log of the likelihood ratio $\ln(LR) = ln\left(e^{\frac{-z^2}{2}}\right) = \frac{-z^2}{2}$.

Therefore $-2\ln(LR) = -2\frac{-z^2}{2} = z^2$ for one parameter. It is trivial to see that for n parameters $-2\ln(LR) = \sum_{i=1}^{n} z_i^2 \sim \chi^2_{(n)}$ i.e. $-2\ln(LR)$ is distributed as chi-squared distribution with n degrees of freedom.

Two model specifications in which model 1 has k more parameters than model 2 imply:

$$(-2\ln(LR_{model\ 1})) - (-2\ln(LR_{model\ 2})) = \sum_{i=1}^{n+k} z_i^2 - \sum_{i=1}^{n} z_i^2 = \sum_{i=1}^{k} z_i^2 =\sim \chi^2_{(k)}$$

The likelihood ratio test is defined as: $LR_{test} = (-2\ln(LR_{model\ 1})) - (-2\ln(LR_{model\ 2})) = -2\ln\left(\frac{LR_{model\ 1}}{LR_{model\ 2}}\right)$

Therefore, if the calculation $LR_{test} = -2\ln\left(\frac{LR_{model\ 1}}{LR_{model\ 2}}\right)$, applied to the empirical data, is associated with a low p-value in the $\chi^2_{(k)}$ distribution then the null assumption does not hold and the variations brought by the k parameters are not explained by random variation alone. As such, they are statistically significant and therefore model 1 is considered a better specification than model 2.

This approach ultimately results in the statistically significant specification:

$$P[y = 1|R, r] = expit(\beta_R R + \beta_r r + \beta_{r2} r^2 + \beta_0)$$

Further details in relation to the application of this approach to the actual data are discussed in the analysis section.

It should be noted that there exists a risk of endogeneity and confounding in the MV regression. Specifically, the relevant information level (explanatory) variable and implied decision market's correctness (response) variable have been defined in the previous sub-sections using market prices from the same adjacent days. As such, the explanatory variable risks being affected by the response variable (endogenous problem) and there may also be some underlying hidden factor causing the variation observed in both variables (confounding problem). However, endogenous and confounding variables able to be controlled for by using instrumental variable (IV) and propensity scoring analysis (PSA) techniques (Zohoori and Savitz, 1997; Caliendo and Kopeinig, 2008; Crown, 2014) as detailed in the following.

6.3.5.2 Instrumental variable analysis method

IV is typically used when an explanatory variable in regression is endogenous (Winship and Morgan, 1999; Bascle, 2008). The instrumental variable is assumed to only influence the response via the (mediating) treatment with no direct response effect on the IV (Pearl, 1995). Identification of a logically consistent IV is key (Heckman and Pinto, 2015).

An arbitrage opportunity in the market for Presidential Candidates is an appropriate IV for the analysis of this chapter. Whilst this idea is suggested in the literature review of this chapter, it is worthwhile reemphasizing it here. Prediction markets are widely considered to be arbitrage free (Buckley and O'Brien, 2015; Hanson and Oprea, 2009). However, arbitrage opportunities have been observed in real-world markets, e.g., in the Iowa Electronic Market (Schmitz, 2011). Real-world markets are posited to always possess arbitrage opportunities (Herschberg, 2012) because arbitrage opportunities indicate exploitable rich private information (Chen and Pennock, 2010) and hence cause informed trading (Treynor, 1987); which, in turn, given the right conditions (Grainger et al., 2015) leads to the best possible prices. Therefore, arbitrage opportunities are logically a good instrumental variable for this study.

The key source of endogeneity is best depicted in the acyclic causal chain as follows. Arbitrage opportunities lead to traders profiting via informed trading (relevant information level increase) that then leads to increased decision market accuracy. Reverse causality of decision market accuracy causing arbitrage opportunities is not logical and hence the acyclic causality is important when applying this IV. As such, the current day's arbitrage opportunity in the market for Democrat Presidential Candidates requires to be quantified. A logical approach to do so is as follows. Given the close of previous day price is $P[C_{t-1}]$, an arbitrage opportunity exists at the beginning of the current day if $P[C_{t-1}] \neq 1$. Specifically, the opportunity exists because C_{t-1} represents all candidate stocks and therefore theoretically $P[C_{t-1}] = 1$. However, in real-world markets arbitrage opportunities do exist for a period of time until traders take advantage of them to ultimately remove them. The arbitrage opportunity is $1 - P[C_{t-1}]$ at the start of the current day. At the close of the current day the arbitrage opportunity is $1 - P[C_t]$. Notice that if traders have acted to reduce the arbitrage opportunity throughout the day that $(1 - P[C_t])^2 < (1 - P[C_{t-1}])^2$. Rearranging this suggest that when $1 - \frac{(1-P[C_t])^2}{(1-P[C_{t-1}])^2} > 0$ then the arbitrage opportunity has reduced throughout the day. The term $1 - \frac{(1-P[C_t])^2}{(1-P[C_{t-1}])^2}$ may also be considered a measure of the size of arbitrage opportunity that traders could feasibly secure. Therefore, this measure of arbitrage will serve as the instrumental variable and denoted as $A_{IV} = 1 - \frac{(1-P[C_t])^2}{(1-P[C_{t-1}])^2}$ in the following investigation.

Two stage least squares (2SLS) approach is typically used in addressing the endogeneity problems for linear regressions, but in a binary logit regression context the control function method is typically applied (Lewbel et al., 2012). The control function is similar to the 2SLS in that in the first stage a linear regression of the potentially endogenous variable on the instrument variables (IVs) (and/or other exogenous variables is performed), which is used to generate residuals that are orthogonal to the IVs. In the second stage the residuals are the only additional term to the original probit[42] regression. If coefficient of the residuals is not statistically significant then this is equivalent to not rejecting a null hypothesis stating that there is no endogenous problem (Clarke and Windmeijer, 2012) i.e. there is no endogenous problem is retained.

The details of this are discussed in the analysis section.

The use of IVs do not control for a confounding variable problem (Wunsch et al., 2006). As such propensity score analysis on a fine strata is utilized as discussed in the following section.

6.3.5.3 Propensity scoring analysis method

Propensity scores analysis (PSA) is used to control for confounding variables and can be assigned to single, multiple, discrete and continuous treatments (Egger and Von Ehrlich, 2013). These generalized propensity scores (Kluve et al., 2012), applied to a continuous treatment, may simply discretize the treatment continuum (Bia and Mattei, 2008). For example, the maximum likelihood estimation technique have been used to determine unknown parameters for continuous treatment propensity scores (Hirano and Imbens, 2004); the approach used here is consistent with these ideas.

The propensity score effectively partitions or stratifies empirical data such that all observed covariates appear random in each strata or partition (Joffe and Rosenbaum, 1999; Austin, 2011). Therefore, a regression can be performed such that, in any strata, the response y (correctness of implied decision market) is not a function of other observed covariates X and only a function of relevant information level $\boldsymbol{\hat{r}}$**.** That is, $P[y|\hat{r}] = expit(\beta_{\hat{r}}\hat{r} + \beta_0)$ whereby $\hat{r} = \begin{cases} 1\ if\ r > \tau \\ 0\ if\ r \leq \tau \end{cases}$ and τ is an unknown parameter the MLE whose value is calculated in the

[42] A probit regression will be used given that STATA ivprobit will be used as an IV control function controlling for endogeneity.

same way as $\beta_{\hat{r}}$ and β_0 . Simply put, $\beta_{\hat{r}}$, β_0 and τ maximize the likelihood of observing the empirical data.

The propensity score $\pi(X) = P[\hat{r} = 1|X]$ is arrived at via a binary logit regression. Notice that this deliberately simple specification allows two trivial but important relationships to hold.

Firstly, given $\hat{r} \in \{0,1\}$,

$$\begin{aligned}
P[\hat{r} = 1|\pi(X)] &= E_{[\hat{r}|\pi(X)]}[\hat{r}|\pi(X)] = E[\hat{r}|\pi(X)] \\
&= E_{X|\pi(X)}\left[E_{[\hat{r}|\pi(X)|(X|\pi(X))]}[\hat{r}|\pi(X)|(X|\pi(X))]\right] \\
&= E_{X|\pi(X)}\left[E_{[\hat{r}|X|(X|\pi(X))]}[\hat{r}|X|(X|\pi(X))]\right] = E_{[\hat{r}|X]}[\hat{r}|X] \\
&= P[\hat{r} = 1|X]
\end{aligned}$$

That is, $P[\hat{r} = 1|\pi(X)] = P[\hat{r} = 1|X]$ simply means that the probability of the treatment conditioned on the *scalar* propensity score of covariates is equal to the probability of the treatment conditioned on the *vector* of covariates; where conditioning on a scalar greatly simplifies analysis.

Secondly, $\pi(X)$ effectively partitions the empirical data such that the relevant information level explanatory variable is independent of other observed covariates i.e. the strata of empirical observations conditioned on the propensity score is effectively randomized as required and said to have balanced the empirical data (Rosenbaum and Rubin, 1983b).

In this chapter, given the other observed covariates the propensity score is generated via an initial binary logit regression

$$\begin{aligned}
&P[\hat{r} = 1|R, DJIA, DJIAprev, A, S, Re, D] \\
&\qquad = expit(\beta_R R + \beta_{DJIA} DJIA + \beta_{DJIAprev} DJIAprev + \beta_A A + \beta_S\, S + \beta_{Re}\, Re \\
&\qquad + \beta_D D + \beta_0)
\end{aligned}$$

to ultimately arrive at the statistically significant form:

$$P[\hat{r} = 1|DJIAprev,] = expit(\beta_{DJIAprev} DJIAprev + \beta_0)$$

However, this form does not in and of itself control for unobserved confounding variables. As such the following approach controls for these.

Propensity scores to define fine strata within which to perform a regression reduces the likelihood of an unknown confounding variable problem (Winship and Morgan, 1999; Rosenbaum and Rubin, 1983b).

Consider the usual approach to stratified propensity score analysis (Joffe and Rosenbaum, 1999; Austin, 2011) applied to this chapter. There exist known covariates C, the relevant information level $r = \hat{r}$ explanatory variable of interest and the response y (a dichotomous variable recording that the implied decision market was correct '1' or not '0'). A propensity score strata is established by collecting all observations $\{(r, y) \in \{0,1\} \times \{0,1\}: P[r|C] \in [a, a + \Delta a]\}$. For small enough Δa known covariates are independent of r and effectively appear randomized in the strata $[a, a + \Delta a]$. As such the regression $P[y|r]$ may be used to test the statistical significance of relevant information level r. However, unobserved covariates not incorporated into C may cause a confounding problem (Joffe and Rosenbaum, 1999).

The problem of unknown confounders has been typically dealt with by using sensitivity analysis to determine what is required for an unknown confounder to change the conclusion of an analysis (Imbens, 2003; Rosenbaum, 2002; Rosenbaum and Rubin, 1983a). A significant limitation of the sensitivity approach is the misspecification of the functional form of the unknown confounder, computational intractability caused by the need to calculate a sufficient sample of the sensitivity region, and the possibility of type 1 and 2 errors (Clarke, 2006). The problem of unknown confounders may also be controlled for by creating sufficiently fine strata to effectively eliminate the variation of the unknown confounder in the strata sample (McNamee, 2005; Fitzmaurice, 2006). However, there is a trade-off between a large enough strata sample size to test significance and a small enough one to "reduce residual confounding" (Williamson and Forbes, 2014). A mathematically rigorous investigation of controlling for unknown confounders by using fine enough or calibrated strata has provided a theoretically rigorous basis for this approach (Lin et al., 1998). But, a simple mathematical argument applied to this study is as follows:

$P[y|r] = P[y|r, u]P[u|r] + P[y|r, u']P[u'|r]$; u is an unknown confounder and u' represents its complement.

Then for a small enough propensity score strata the confounder does not have full support i.e. either $P[u'|r] = 0$ or $P[u|r] = 0$. Without loss of generality, if $P[u'|r] = 0$ then $P[y|r] =$

$P[y|r,u]$ which implies r is independent of u. Hence, $P[y|r]$ is not confounded by unknown random variable u as required.

6.4 Analysis of the IEM 2008 Presidential Election Data

Analysis is performed on IEM stock price data for 3 consecutive months (May, June, and July 2008) in the 2008 Presidential Election. Specifically, data include prediction market stock prices at close of IEM market day and associated with Democrat party and candidates (Obama, Clinton, Edwards, and other candidates). The relevant information level proxy r was constructed using closing prices of adjacent days data. Those days with r in the allowed closed interval $[0,1]$ were kept for analysis. In total 144 observations are used in the analysis.

The significance of relevant information level to the probability of correct implied decisions within Iowa Electronic Market's (IEM's) is of primary interest. As discussed in the previous section, on each day, a relevant information level proxy r and correctness proxy y for each correct implied decision is recorded. Simply put, a regression of y versus r is performed in this section to discern the significance of relevant information level. Other covariates are initially incorporated into the regression to discern their effect; given that this real-world empirical setting, in contrast to the experimental setting, is complex. That is, in an experiment, a control and treatment pair of markets being identical in every way with the possible exception of relevant information level may behave the same or differently. If multiple control and treatment pairs (differing only in the value of the relevant information level) are observed with random variation across all covariates and a statistical difference is detected across these comparisons, then the only logical reason for this difference is that relevant information level is a statistically significant effect. In contrast, the real-world empirical setting of this chapter does not guarantee randomization across all covariates with respect to relevant information level.

To properly determine the significance of relevant information level on the probability of correct decisions in this IEM conditional (implied decisions) prediction market, three methods are applied to the empirical data. Firstly, the multivariate analysis (MV) method is applied on all variables that are identified in the literature review as pertinent. The MV identifies statistically significant explanatory variables that best explain the variation of the dependent (correct implied decision) variable. The potential for endogeneity problems are not directly addressed by MV and entirely possible given the construction of implied decision and relevant information level proxies; both being functions of close prices of stocks on the current and previous day.

To control for the potential problem of endogeneity, an instrumental variable control function (IV) approach is used. The close prices of previous and current days will serve as Instrumental Variable (IVs). Relevant information level is considered the only mediating variable for the IVs on the response, and the response does not affect (or feedback to) the IVs. This causal chain reduces the chance of an endogeneity problem, however, there remains the possibility that the treatment (relevant information level) and response (correct implied decision) is confounded by another variable i.e. they are both correlated with another covariate and therefore the ‘causal’ significance of relevant information level on the probability of correct implied decision is put into question. In short, the variations of correct implied decision and relevant information level could instead be caused by another variable that is correlated with both.

To control for the potential problem of relevant information level correlated with other variables and thereby muddying the significance of relevant information level, the fine stratification propensity scoring analysis (PSA) method is applied. Put simply, PSA collects (or partitions) observations that have the same probability of being treated given the other known covariates. Relevant information level is considered the treatment possible causing the variations in the (correct implied decision) response. Importantly, in this PSA derived collection of observations, the variations in the treatment across observations does not depend on the variation of the other known covariates. A sufficiently fine PSA strata then minimizes the probability of confounding caused by variations in unknown covariates. Ultimately, there is a low likelihood of confounding by known and unknown covariates. Therefore, in this setup, if relevant information level is found to be statistically significant, then being disentangled from other known and unknown variables, it can be truly said to be significant. An important qualification is that it can only be said to be significant within the PSA derived fine strata of observations. However, this is sufficient to reject a null hypothesis “relevant information is not significant for ‘any’ collection of observations”.

The combination of MV, IV and PSA emphasizes different insights and provides a compelling case for the statistical significance of relevant information level. The following subsections provide the details of the applied methods.

6.4.1 Multivariate analysis method

The multivariate analysis (MV) method is applied on all variables that literature has suggested as pertinent to the 2008 Presidential election setting; including age, sex, race, campaign receipts and disbursements of candidates, and the state of the economy (represented in the DJIA and previous day's DJIA). The first and second order relevant information level proxy information is included; with the second order included to cater for possible higher order real-world effects. Table 6.1 is the best MV specification arrived at by utilizing the likelihood ratio test along with the forward and backward stepwise regression as described in the previous section. Appendix D: Chapter 6 Appendix 4 provides an annotated STATA code that justifies Table 6.1 and Appendix D: Chapter 6 Appendix 3 justifies that the normality assumption holds and not continuity correction factor is required.

Table 6.1
Binary logistic regression of the implied decision market being consistent with the real-world outcome (Obama) vs. relevant information level ($\boldsymbol{r}$ and higher order $\mathbf{r^2}$) and race.

Variable	B coefficient	SE standard error	exp(B)
constant	1.286***	0.520	
r (relevant information level proxy)	-5.316**	2.129	0.007
r^2 (second order effect of r)	5.729***	2.118	361.044
race (Afro-American candidate)	-3.088***	0.984	0.114
race x r (interaction between relevant information level and information on race)	1.570	0.469	0.114
df (degrees of freedom)		4	
$\chi 2$		34.88	
p-value		0.000	
number of observations		144	
proportion of observations with dependent equal to '1'		59.7%	

Note: Other covariates considered are Economic (DJIA current and previous day) and Candidate attributes (age, sex, campaign receipts, campaign disbursements).
race is coded as '1' representing that the candidate is Afro-American and '0' representing that the candidate is not.
*p<0.10 **p<0.05 *** p<0.01

Table 6.1 depicts that race and relevant information level (first and second order) are significance to at least at a 5% level. The race-relevant information level interaction term is included because the likelihood ratio test shows a better specification with it than without.

Table 6.1 implies the function

$logit(P[Y = 1|relevant\ information\ level = r, race = R]) = 5.7r^2 - 5.3r + 1.6rR - 3.1R + 1.5$ which is visually depicted in Figure 6.1.

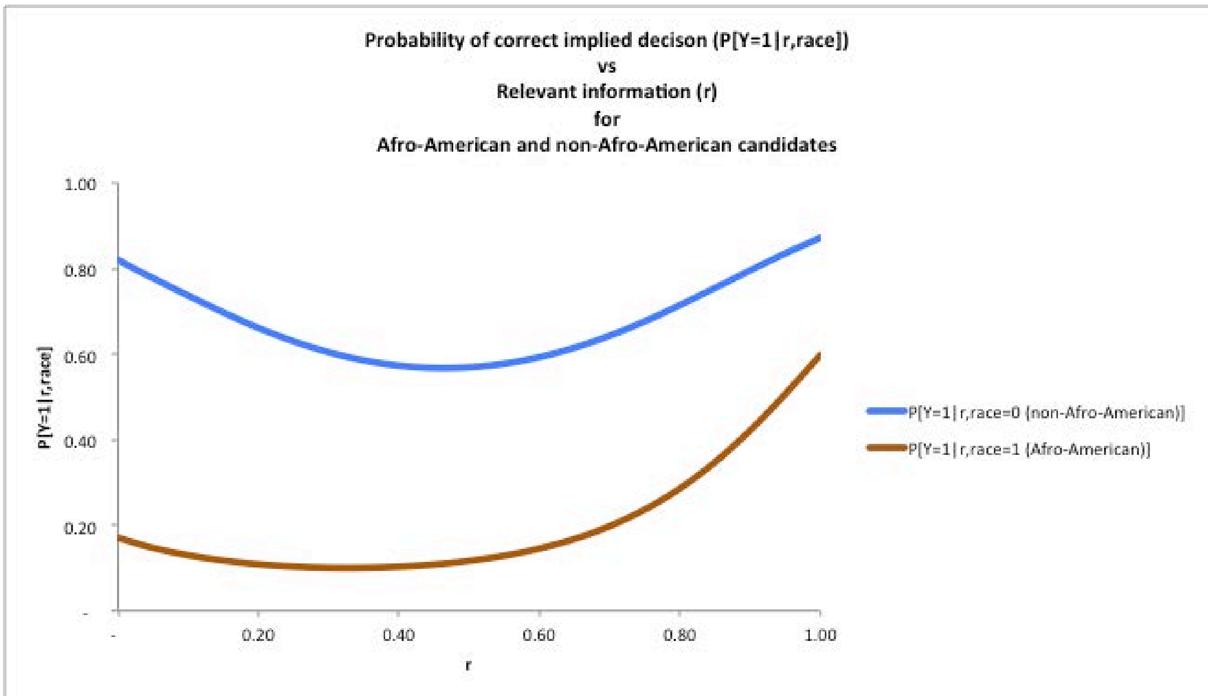

Figure 6.1 Binary logistic regression of the implied decision market being consistent with the real-world outcome (Obama) vs. relevant information level (r and higher order r^2) and race.

Figure 6.1 shows that the probability of the implied decision market is greatly reduced when comparing Afro-American stocks to non-Afro-American stocks. However, as relevant information level increases and a greater proportion of traders share information then the performance of the implied decision market on Afro-American stocks and non-Afro-American stocks converge. At a relevant information level of zero, no information is shared between Afro-American stocks and non-Afro-American stocks. As information is shared (as relevant information level increases above zero), the Afro-American stock information and non-Afro-American stock information is shared across stocks with the former initially pulling down the performance of the latter and the latter pulling up the performance of the former.

Further increases in relevant information level then lead to improved implied decision market performance in both stocks. In short, there is an initial race bias that eventually begins to correct when more information is shared.

Of central interest is the effect of relevant information on correct decisions holding all else constant. This may be investigated by using the odds ratio. Specifically, the odds ratio defined as the odds of a correct decision made on the Obama (Afro-American) stock with respect to the odds of a correct decision made on a non-Afro-American stock at a particular relevant information level, provides a means to isolate the effect of relevant information level on decision market performance. Additionally, it is a useful means to remove the high p-value race-relevant information level interaction and the higher order relevant information level term. Formally, since $logit(P[Y = 1|relevant\ information\ level = r, race = R]) = 5.7r^2 - 5.3r + 1.6r \times R - 3.1 \times R + 1.5$ and the odds ratio is defined here as

$$OddsRatio(r) = OR(r) = \frac{\left(\frac{P[Y = 1|r, R = 1]}{(1-P[Y = 1|r, R = 1])}\right)}{\left(\frac{P[Y = 1|r, R = 0]}{(1-P[Y = 1|r, R = 0])}\right)} \text{ then,}$$

$$log\big(OddsRatio(r)\big) = (5.7r^2 - 5.3r + 1.6r \times 1 - 3.1 \times 1 + 1.5) - (5.7r^2 - 5.3r + 1.6r \times 0 - 3.1 \times 0 + 1.5) = 1.6r - 3.1$$

This is relationship is best depicted in Figure 6.2.

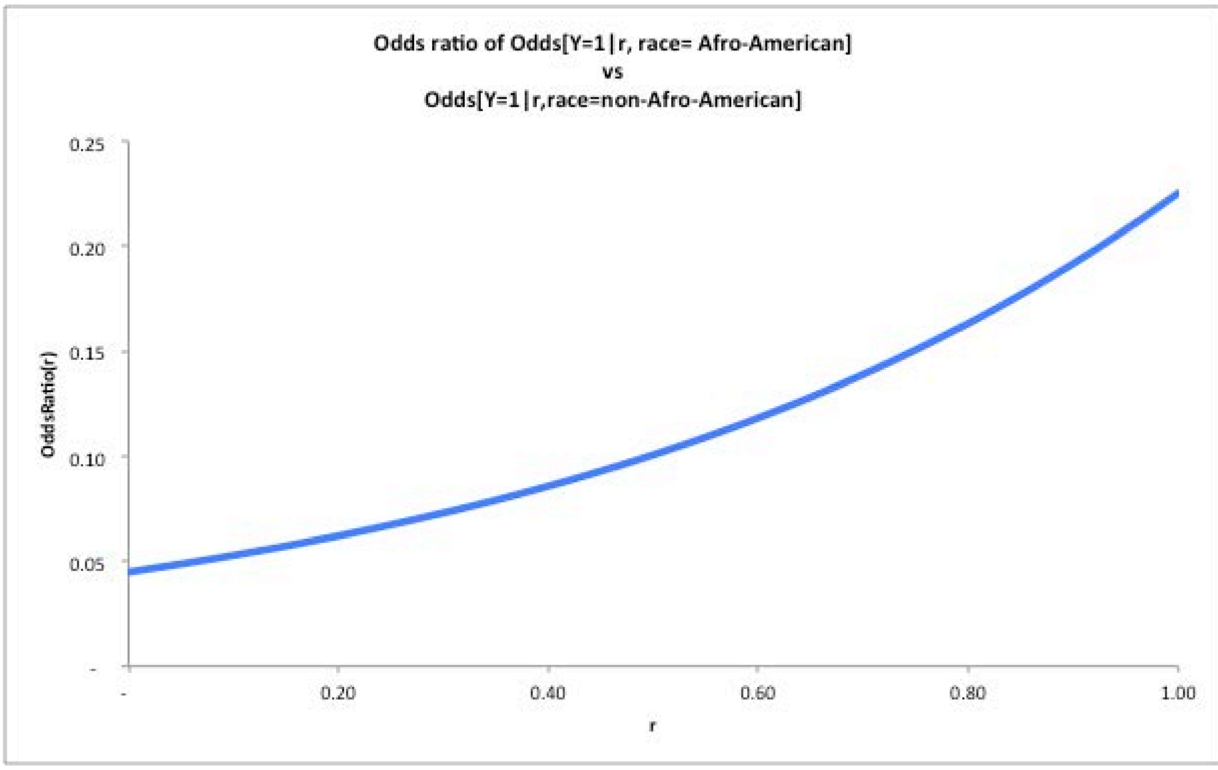

Figure 6.2 Odds ratio of correct implied decisions vs. relevant information level (*r*).

In short, graph 2 demonstrates that the odds ratio increases with relevant information level.

However, the possibility of endogeneity may exist, and thus, the use of an instrumental variable approach is justified to control for endogeneity.

6.4.2 Instrumental variable (IV) analysis using the control function approach

In the following, Table 6.2 depicts a probit regression that shows relevant information level to be statistically significant. Table 6.3 then uses the IV control function approach to reject the possibility of endogeneity; more strictly put, retain no endogeneity.

A probit regression of the response (day’s implied decision market selects Obama) onto the explanatory variables including relevant information level and race is depicted in Table 6.2.

Table 6.2

Probit model of the implied decision market being consistent with the real-world outcome (Obama) vs. relevant information level ($\mathbf{r}$ and higher order $\mathbf{r^2}$) and race.

Variable	B coefficient	SE standard error	exp(B)
constant	0.793***	0.273	
r (relevant information level proxy)	-3.028***	1.239	0.048
r^2 (second order effect of r)	3.529***	1.239	34.090
race (Afro-American candidate)	-1.310***	0.269	0.270
df (degrees of freedom)		3	
χ2		33.59	
p-value		0.000	
number of observations		144	
proportion of observations with dependent equal to '1'		59.7%	

Note: Other covariates considered are Economic (DJIA current and previous day) and Candidate attributes (age, sex, campaign receipts, campaign disbursements).
race is coded as '1' representing that the candidate is Afro-American and '0' representing that the candidate is not.
*p<0.10 **p<0.05 *** p<0.01

For the IV control function approach, relevant information level is linearly regressed onto the instrumental variable subspace; i.e. $A_{IV} = 1 - \frac{(1-P[C_t])^2}{(1-P[C_{t-1}])^2}$ and also $A^2{}_{IV}$ since at least two IVs are required to project possible endogenous variables r and r^2 onto. This regression is then used to calculate residuals as proxy for the true errors. A probit regression of the response onto the relevant information level (r and r^2), race and residuals is used to determine the presence of endogeneity.

Table 6.3 is the resulting control function analysis; with STATA's ivprobit utilized. Table 6.3 depicts that there is no endogeneity problem. Appendix D: Chapter 6 Appendix 5 provides the specific details and an annotated STATA code for Table 6.3.

Table 6.3

IV control function approach depicting no endogeneity problem in probit model (the implied decision market being consistent with the real-world outcome (Obama) vs. relevant information level ($\boldsymbol{r}$ *and higher order* $\mathbf{r^2}$*) and race).*

Variable	B coefficient	SE standard error	exp(B)
constant	0.809***	0.281	
r (relevant information level proxy)	-3.171**	1.354	0.048
r^2 (second order effect of r)	3.691***	1.382	34.090
race (Afro-American candidate)	-1.323***	0.271	0.270
df (degrees of freedom)		3	
χ2		28.60	
p-value		0.000	
number of observations		144	
proportion of observations with dependent equal to '1'		59.7%	

Note: Other covariates considered are Economic (DJIA current and previous day) and Candidate attributes (age, sex, campaign receipts, campaign disbursements).
race is coded as '1' representing that the candidate is Afro-American and '0' representing that the candidate is not.
*p<0.10 **p<0.05 *** p<0.01

Wald test of exogeneity
χ2 = 1.05
df (degrees of freedom) = 2
p-value = 0.592
Therefore, fails to reject the null: no endogeneity problem.

Table 6.3 shows that the IV residuals are not statistically significant and hence no endogeneity problem exists in the model specification.

Whereas Table 6.2 depicts a probit regression, Table 6.3 uses the IV control function to reject the possibility of endogeneity. **Therefore, relevant information level is statistically significant when controlling for possible endogeneity.**

The IV control function approach adjusts for endogeneity but it is still possible that the co-variation observed between the two variables of interest (relevant information level and correct implied decision) may be the result of a confounding variable causing the variation of both r and $P[Y = 1|r, race]$. The use of fine strata propensity score analysis can reduce the likelihood of a confounding variable problem as previously discussed. The application of which is as follows.

6.4.3 Propensity scoring analysis method

As discussed previously, to control for possible confounding variables, generalized propensity score analysis (PSA) to balance the continuous treatment (relevant information level) is justifiably employed to provide a means to partition the data, such that other observed variables are effectively random within each partition with respect to the treatment. This is achieved as follows:

Let $r_b = \begin{cases} 1 \; if \; r \leq \tau \\ 0 \; if \; r > \tau \end{cases}$ such that parameters θ and τ satisfy $L(\theta, \tau; \text{data}) = \max_{\delta,t} L(\delta, t; \text{data})$; where $L(.)$ is the likelihood of observing the data and a function ($L(\theta, \tau; \text{data}) = f(\theta, \tau)$) of parameters θ and τ maximize this likelihood. That is, the unknown threshold parameter τ is treated in the same way as other unknown parameters θ in the maximum likelihood estimation. Notice that $\tau = \left\{t: \max_{\theta,t}(\sum ln(P[y|r, C; \theta, t]))\right\}$ when other parameters are (log-likelihood MLE) maximized at θ, and since t implies the value $r_b = \begin{cases} 1 \; if \; r \leq \text{t} \\ 0 \; if \; r > \text{t} \end{cases}$ it is sufficient to write $\tau = \left\{t: \max_{\theta,t}(\sum ln(P[y|r_b(t), C; \theta, t]))\right\}$. For large N and for a given value of t, then the number of terms $P[y|r_b(t), C; \theta, t]$ having the same covariate vector C is $P[C|r_b(t)]N$. Therefore, $\tau = \left\{t: \max_{\theta,t}(\sum P[C|r_b(t)]N ln(P[y|r_b(t), C; \theta, t]))\right\}$ may be written. The Taylor series approximation of the natural logarithm is useful to estimate τ. The useful approximation is trivially arrived at. That is, $\ln(1-x) = -x + x^2 + O(x^3)$ and let $P[y|r_b(t), C; \theta, t] = 1 - x$ implies $ln(P[y|r_b(t), C; \theta, t]) = -P[y|r_b(t), C; \theta, t] + (P[y|r_b(t), C; \theta, t])^2 + O((P[y|r_b(t), C; \theta, t])^3) \approx -P[y|r_b(t), C; \theta, t]$. Therefore, strictly $\tau \approx \left\{t: \max_{\theta,t}(\sum P[C|r_b(t)]N(-P[y|r_b(t), C; \theta, t]))\right\}$, but given the stochastic nature of observations and the fact that τ is simply an estimate, to simplify notation an equality will be denoted i.e. $\tau = \left\{t: \max_{\theta,t}(\sum P[C|r_b(t)]N(-P[y|r_b(t), C; \theta, t]))\right\}$.

This expression then simplifies further to $\tau = \left\{t: \max_{\theta,t}(-\sum NP[y|r_b(t); \theta, t])\right\}$ which is the same as $\tau = \left\{t: \max_{\theta,t}(-\sum P[y|r_b(t); \theta, t])\right\}$. Using again the Taylor approximation allows one to rewrite the last equation as $\tau = \left\{t: \max_{\theta,t}(\sum ln(P[y|r_b(t); \theta, t]))\right\}$. This is the familiar MLE log-likelihood maximization to specify $P[y|r_b(t); \theta, t]$ with the most statistically significant values for parameters θ and t that most likely explain the data observed.

Importantly, notice that the regression is simply y onto $r_b(t)$; with other covariates C absent. For each given value of t, binary logit regression on $P[y|r_b(t); \theta, t]$ calculates θ.

Finally, given the calculated θ, the likelihood ratio test can be used to measure the statistical significance of including versus excluding $r_b(t)$ in the model specification at each value of t. This allows comparison across different datasets having different $r_b(t)$. The most (likelihood ratio based) statistically significant specification is the one that most requires $r_b(t)$ in the model to explain the data. That is, at that t including $r_b(t)$ gives the greatest improvement to the model's likelihood of observing the associated data. It can be said that at this $t = \tau$ the likelihood is maximized. A more formal proof for this is as follows:

Given θ, consider again $\tau = \left\{t: \max_t(\sum ln(P[y|r_b(t)]))\right\}$. This is the same as $\tau = \left\{t: \max_t(\sum ln(P[y|r_b(t)]) - K)\right\}; K\ is\ some\ constant$. Let $K = \sum ln(P[y])$ i.e. the model for which the null hypothesis that $r_b(t)$ is not significant holds. Then $\tau = \left\{t: \max_t(\sum ln(P[y|r_b(t)]))\right\}$ is the same as $\tau = \left\{t: \max_t(\sum ln(P[y|r_b(t)]) - \sum ln(P[y]))\right\}$. This final equation is the same as $\tau = \left\{t: \max_t\left(-2(\sum ln(P[y]) - \sum ln(P[y|r_b(t)]))\right)\right\}$. But, $-2(\sum ln(P[y]) - \sum ln(P[y|r_b(t)]))$ is simply $-2ln(LR) \sim \chi^2$; where LR denotes the likelihood ratio. Therefore, given θ, $t = \tau$ maximizes the value of χ^2. Moreover, with the same number of parameters, and therefore the same degrees of freedom, the likelihood ratio (LR) test with the lowest p-value across all t is at $t = \tau$.

The STATA code in Appendix D: Chapter 6 Appendix 6 performs this maximization and finds that the LR test has its lowest p-value at $\mathrm{t} = 0.75$. Therefore, $\tau = 0.75$ is a pragmatic estimate.

Table 6.3 is the result of setting the threshold to $\tau = 0.75$ and constructing the best specification of the propensity score $P[r_b = 1|DJIAprev]$. The best specification is obtained using likelihood ratio test along with the forward and backward stepwise regression. Appendix 6 provides an annotated STATA code that justifies Table 6.4. The normality assumption holds and no continuity correction factor is required.

Table 6.4

Binary logistic regression of the $\boldsymbol{r_b}$ (propensity score balanced relevant information level) vs. previous day's DJIA (DJIAprev).

Variable	**B coefficient**	**SE standard error**	**exp(B)**
constant	-6.937**	3.426	
DJIAprev (previous day's DJIA)	0.001**	0.000	0.000
df (degrees of freedom)		1	
$\chi 2$		4.95	
p-value		0.026	
number of observations		144	
proportion of observations with dependent equal to '1'		61.5%	

Note: Other covariates considered are Economic (DJIA current) and Candidate attributes (age, sex, campaign receipts, campaign disbursements).
r_b is coded as a dummy based on a cumulative distribution of r partitioned by a threshold τ that is a parameter treated like other parameter of the maximum likelihood estimation associated with the correct implied decision observations. Specifically, 'r_b =1' denotes $r < \tau$ and '0' otherwise.
*p<0.10 **p<0.05 *** p<0.01

This propensity score is used to identify a stratum of data. Subsets of this stratum are iteratively taken and logit regressions performed on each. The balance between a fine stratum and statistical significance is found. Importantly, the propensity score is used to partition the data (where partition k contains N_k of the total M terms) in such a way that the balancing score $r_b(\tau)$ concurrently balances the entire data and the partition.

More formally, it is trivial to see that if $r_b(\tau_k)$ is the balancing score derived for partition k and $r_b(\tau)$ is the balancing score for the entire data set, then it is sufficient that $\tau_k = \tau$ to ensure logical consistency and use of the same balancing score at the aggregate and partitioned level. A further implication is that, in partition k, $\tau = \left\{t: \max_{\theta,t}(-\sum P[y|r_b(t);\theta,t])\right\}$ and $\tau = \left\{t: \max_{\theta,t}(\chi(t)^2;\theta,t)\right\}$ is guaranteed to hold for the fine strata analysis that is used to control for unobserved covariates.

The fine strata PSA approach removes full support of unknown confounders as previously discussed. Table 6.5 is the regression performed on the data of a fine stratum. Appendix D: Chapter 6 Appendix 7 provides the STATA code details.

Table 6.5

Fine strata PSA binary logistic regression of the implied decision market being consistent with the real-world outcome (Obama) vs. $\boldsymbol{r_b = 1 - rbalanced}$ *(propensity score rbalanced is the relevant information level in the usual sense).*

Variable	B coefficient	SE standard error	exp(B)
constant	1.099**	0.516	
r_b (propensity score balanced relevant information level)	-1.224**	0.626	0.294
df (degrees of freedom)		1	
$\chi 2$		4.12	
p-value		0.042	
number of observations		52	
proportion of observations with dependent equal to '1'		58%	

Note: r_b is coded as a dummy based on a cumulative distribution of r partitioned by a threshold τ that is a parameter treated like other parameter of the maximum likelihood estimation associated with the correct implied decision observations. Specifically, 'r_b =1' denotes $r < \tau$ and '0' otherwise; which means that relevant information level rbalanced $= 1 - r_b$.. Notice other covariates were sufficiently randomised by using the balancing score. Fine stratification reduces the likelihood of unknown confounder effects.
*p<0.10 **p<0.05 *** p<0.01

In the empirical data r is an observed random variable and therefore so is r_b. The expected value of r_b is simply $E[r_b] = 1 \times \Pr[r \le \tau] + 0 \times \Pr[r > \tau] = \Pr[r \le \tau]$ within some fine strata near the balancing propensity score $E[r_b|other\ covariates]$. Since r is continuous then so is $\Pr[r \le \tau] = E[r_b]$. The relationship $\Pr[r \le \tau] = E[r_b]$ is important in that it connects the continuous measure of relevant information level r with the discrete measure r_b. A simple reductio ad absurdum proof best depicts the merit of this relationship as follows: given r_b in $E[Y|r_b]$ is statistically significant assume that r is not statistically significant. Notice that r_b is simply a function of r; i.e. $r_b = 1[r \le \tau] = \mathrm{f}(\mathrm{r})$. Hence, $E[Y|r_b] = E[Y|\mathrm{f}(\mathrm{r})]$ and since r is not significant then the contradiction $E[Y|r_b] = E[Y]$ occurs i.e. r_b is not statistically significant. Therefore, the assumption that r is not statistically significant is false. In short, if r_b in $E[Y|r_b]$ is statistically significant then so must be r in $E[Y|r]$.

Also noteworthy is the following:

$$E_Y[Y|r_b] = E_{E[r_b]|r_b}\left[E_{Y|E[r_b]|r_b}[Y|r_b, E[r_b]|r_b]\right] = E_{r_b}\left[E_{Y|r_b}[Y|r_b, E[r_b]]\right] = E_Y\left[Y|E[r_b]\right]$$

Since Y is an indicator function, the last term of the last equation is:

$Pr\left[Y = 1|E[r_b]\right] = expit(\beta E[r_b] + \alpha)$; a logistic regression continuous in variable $E[r_b]$. Define $E[r_{balanced}] = 1 - E[r_b]$. Then for increasing r, $\Pr[r \le \tau]$ decreases, and since

$\Pr[r \leq \tau] = E[r_b]$, $E[r_{balanced}]$ decreases. In short, $E[r_{balanced}]$ is consistent with dichotomous r_b, and a suitable positively correlated continuous proxy for r so that $Pr\big[Y = 1|E[r_{balanced}]\big]$ may be graphed as depicted in Figure 6.3.

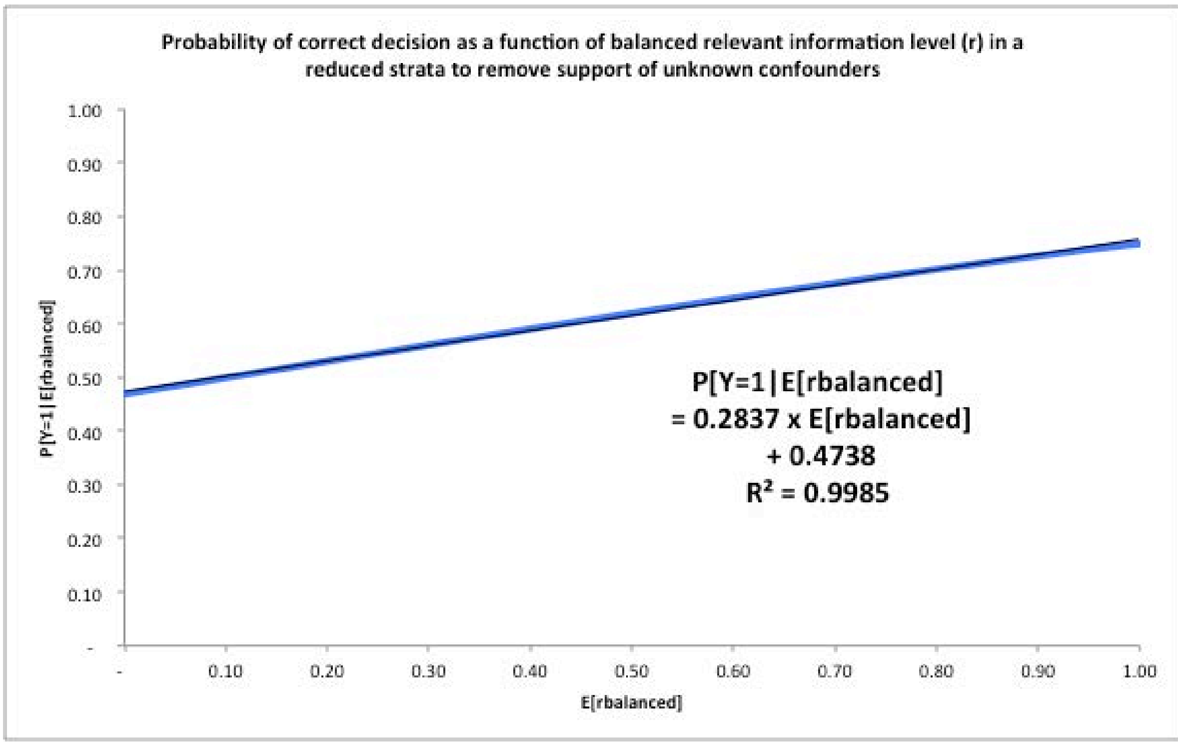

Figure 6.3 Binary logistic regression of the implied decision market being consistent with the real-world outcome (Obama) vs. fine strata PSA proxy for relevant information level.

Figure 6.3 depicts linearity consistent with theory and revealed in computer simulations of previous chapters. Importantly, it has removed all other variables that complicate the empirical setting and finds the probability of a correct implied decision increases with increasing relevant information level.

6.5 Discussion

The previous two chapters on computer simulations and games with humans in prediction and decision markets found relevant information level plays a statistically significant role in convergence to the DCE. The theoretical model developed in the simple prediction and decision market model chapter inspired the investigation into relevant information level. Relevant information level is simply the proportion of traders placing informed bids in a prediction or decision market. Theory suggests that relevant information level is an important ingredient for prediction and decision markets to attain the best possible prediction and decision (Grainger et al., 2015). Specifically, that the best possible prediction (being that elicited when all traders are fully informed) is achieved if and only if all traders bid in an informed way, and the probability of the best possible decision increases linearly with relevant information level in a decision market. The computer simulation chapter revealed that relevant information level is statistically significant in prediction and decision market simulations inhabited by idealized algorithmic traders. The prediction market games with human participants chapter revealed that relevant information level was statistically significant in prediction markets having human traders. However, both computer simulations and games with human participants are in laboratory settings. This chapter extends the investigation of relevant information level into the real-world. Ultimately, data from a real-world political election market is analyzed to reveal that relevant information level is statistically significant in real-world prediction (and their implied decision) markets.

The real-world political election market data are sourced from the Iowa Electronic prediction market (IEM) 2008 U.S. Presidential Elections. The IEM is considered a well-functioning (most of the time) political prediction market that regularly outperforms polls (Berg et al., 2008b). For example, whereas, polls showed that McCain was more likely to win over Obama (Abramowitz, 2008), the IEM prediction market attributed to Obama a high probability of success (Arnesen and Bergfjord, 2014). Because political parties and their candidates imply conditional probabilities (e.g. the party's chance increases when the chance of a candidate leading that party is elected), the prediction market contains an implied decision market. The analysis of the implied decision market is important for two reasons. Firstly, the correctness of a daily decision is easily considered right or wrong (e.g. Clinton will be elected (implied decision market) decision is wrong given that Obama was elected), whereas a prediction market probability does not succumb to such a categorization e.g. is a 70% (prediction market) chance of Clinton being elected on a particular day right or wrong? Secondly, the ultimate aim of this thesis is to facilitate the construction of *high quality* decision markets in not for profit organizations. Prediction markets are used in well-known firms (Thompson, 2012) but are certainly not a typical decision support tool. The lack of uptake is speculated in this thesis as arising from the lack of a quality signal of what a good prediction market is. In short, there is arguably a market failure caused by the inability to separate peach from lemon (Akerlof, 1970) prediction and decision markets. It is advocated throughout this thesis that high relevant information prediction and decision markets are the guarantee and quality signal of well-functioning prediction and decision markets.

The literature review on the political economy with an emphasis on prediction markets guided the methodology to analyze the associated empirical data. The insights garnered identified key determinants. These determinants formed the explanatory variables upon which to regress the response (i.e. correctness of the implied daily decision market). In short, the regressions were refined to relevant information level and race. The statistical significance of race was in agreement with literature. That is, the race of candidates (Afro-American vs. non-Afro-American) was an important factor for the 2008 Presidential election (Finn and Glaser, 2010) which in turn affected voter turnout (Greenwald et al., 2009) that has traditionally differed between demographic groups (Harder and Krosnick, 2008).

Other variables were considered and analyzed given they were considered in literature as determinants for political election outcomes. For example, the comparison of two political stock markets run during the Berlin 1999 state elections suggested “media coverage” as a determinant of the prediction differences. “Media coverage” was incorporated into this analysis under the broader proxy for marketing expenditure i.e. campaign receipts and disbursements (Hansen et al., 2004). Sex of the candidate was a salient issue in the 2008 presidential elections when Biden was selected instead of Clinton as Obama’s running mate (Saldin, 2008); as such it is properly included here as a dummy variable (Garavaglia and Sharma, 1998). Quantitative studies show that the economic performance of candidates matters (Lewis-Beck et al., 2008), with voter reacting to punish poor economic performance (Lewis-Beck and Stegmaier, 2000); as such the DJIA index for current and previous day is included in this analysis as the proxy associated with economic performance of the candidate. Proxies used for lobbying and marketing abilities (considered important e.g. Obama used social media extremely well (Wattal et al., 2010)) were the receipt and disbursement data by candidate.

The dichotomous response variable (correct or incorrect implied decision market) justified the use of a binary logit model and analysis. Significance testing of variables to arrive at the best model specification was the objective. Whilst significance levels were used to demarcate significant from not significant, criticisms on the use of the arbitrary p value of 0.05 was taken on board; as such the best (statistical) practice suggests to report the p-value for the reader to determine significance or not (Murtaugh, 2014). Model specification employed backwards and forwards stepwize regression with the log of the likelihood ratio being proportional to a chi-squared distribution allowing the comparison of a model with and without a variable (Buse, 1982). Given that the likelihood ratio is directly related to other test (e.g. Wald and LaGrange) (Siniksaran, 2005) it was considered a sufficiently powerful means to identify the best regression model specification.

Post hoc tests to ensure normality assumption held given analysis of the type employed in this chapter has been found to be inconsistent under non-normality (Bera et al., 1984) and the related and equally important check that a continuity correction factor was not required (Guillen, 2014) was undertaken.

A multivariate binomial response model was constructed. Explanatory variables suggested by the literature review were used. The best model specification depicted in Table 6.1 was one in which relevant information level and race variables were the only explanatory variables. Figure 6.1 conveyed that racial bias distorts the correctness of the decision market. Specifically, the probability of the correct implied decision was worse for Afro-American stocks when compared to non-Afro-American stocks. However, as proportionally more information is shared (i.e. relevant information level increases), the relatively more correct pool of non-Afro-American stock information improves the Afro-American stock information. However, the relatively less correct Afro-American stock information introduces misinformation into the pool of non-Afro-American stock information. Graphically, the top curve (non-Afro-American stock) is pulled down and the bottom curve (Afro-American stock) is pulled up. As relevant information increases further still both stocks eventually increases the likelihood of being correct with increases in relevant information level.

The unexpected sign, considered an opportunity to reveals and deeper interesting dynamic (Kennedy, 2005), in this case the negative gradient at low relevant information levels can be explained. There is a commonly held racial bias; as confirmed in literature (Finn and Glaser, 2010). This common information can erroneously bias market prices (Treynor, 1987). In this setting, possessing private information in a market where private information is not readily shared can make an individual worse off as the effects of common information dominate (Galanis, 2010). In one experimental setting with low information sharing, a market group with low levels of information outperformed groups with higher levels of information (Camerer et al., 1989). Another explanation found in prospect theory suggests that in contexts with high information uncertainty or lacking of information, traders are motivated to 'prospect' (seek out) arbitrage opportunities which in turn leads to corrections in market prices (Kahneman and Tversky, 1979). This may also explain the higher than expected probability of correct decision when relevant information level is very low.

It is interesting to note that this common information (racial) bias, which interacts with relevant information level, was not seen in the computer simulations or the games with human participants. As previously mentioned both the computer simulations and games with human participants provided a setting in which no information was brought in by the trader and the trading screen was the source of all information. This likely explains the absence of the higher order common information bias not seen in those laboratory settings.

The odds ratio depicted in Figure 6.2 provided a means to view the MV as a function on relevant information only i.e. the algebra resulted in the race term being effectively removed from the graph. In so doing, the higher order effect was removed and the odds ratio increased with increasing relevant information level. Prima facie it appears that increases in relevant information lead to increases in the odds ratio of correct implied decisions. However, there is a possible endogenous problem given that the odds ratio may itself inform trades and increase proportion of informed trades (relevant information level). This possibility justified the use of an instrumental variable that controlled for this potential endogeneity.

The IV was logically derived and used to control for endogeneity by using the control function IV approach that was considered in literature as applicable to the dichotomous response setting. The probit regression of Table 6.2 found relevant information level as statistically significant. Table 6.3 utilized the IV control function approach to determine that relevant information level does not likely suffer from endogeneity. However, whilst an endogeneity does not likely exist, the confounding problem (in which some other variable may be causing the variations in both response and explanatory variables) is possibly present. IVs have been considered in econometric literature as a means to control for potential confounding, however, propensity score analysis is considered a relatively better approach to control for confounding (Klungel et al., 2004). Propensity score analysis (PSA) is a way of stratifying the data so that within a strata other covariates (being potentially known confounders) relative to the treatment variable are effectively randomized (Rosenbaum and Rubin, 1983b).

Whilst PSA effectively controls for known confounders, the problem of unknown confounders causing selection bias is not immediately resolved (Imbens and Rubin, 2015).

Unknown confounding has been dealt with by using sensitivity analysis to determine what is required for an unknown confounder to change the conclusion of an analysis (Imbens, 2003; Rosenbaum, 2002; Rosenbaum and Rubin, 1983a). However, a significant limitation of sensitivity analysis is the misspecification of the functional form of the unknown confounder, computational intractability caused by the need to calculate a sufficient sample of the sensitivity region, and the possibility of type 1 and 2 errors (Clarke, 2006). In contrast, the problem of unknown confounders can also be controlled for by creating sufficiently fine strata to effectively eliminate the variation of the unknown confounder in the strata sample (McNamee, 2005; Fitzmaurice, 2006).

Given the sufficiently large data set, fine stratification was applicable to this analysis (Luiz and Cabral, 2010). That is, the use of sensitivity analysis to ascertain what unknown effect was required to invalidate statistical significance (Lin et al., 1998; Rosenbaum and Rubin, 1983a) was not of central interest, rather the use of the fine strata approach (Yang et al., 2015) arguably inherent in sensitivity analysis literature (Rosenbaum and Rubin, 1983a), was used to construct a randomized strata of measured covariates whilst reducing the likelihood of confounding variation from unmeasured and potentially confounding variables. Table 6.4 and Figure 6.3 are the result of the applications of the fine strata PSA approach. It shows a strong linear relationship in which the probability of correct implied decision increases with increasing relevant information level. This is compelling evidence validating the theoretical model; particularly in light of the theory and computer simulations in Chapter 4 having attained the same linear relationship.

6.6 Conclusion & Future Research

Three statistical methods (MV, IV and PSA) are used to determine the relationship between the probability of implied correct decisions and the relevant information level in empirical data sourced from a real-world prediction market. The three methods emphasize different aspects and combined provide compelling evidence that relevant information level plays a statistically significant role in real-world prediction markets. Noteworthy is the linear relationship observed in the PSA providing a compelling case for a valid theoretical model considered in previous chapters.

One limitation of this study is that personality of candidates has not been considered. Personality is difficult to quantify, but a longitudinal psychological study has found it to be correlated with voter preference (Jost et al., 2009). However, it has been excluded here because of the lack of time, data, and an easily identifiable and credible proxy for it. Future studies beyond the scope of this thesis may investigate the candidate personality covariates further.

In conclusion, MV, IV and PSA on real-world (IEM 2008 Presidential Election observations) are consistent with theoretical expectations. Importantly, the analysis is strong evidence that relevant information level plays a statistically significant role in real-world prediction markets. Furthermore, increasing relevant information level in real-world prediction markets leads to an increase in the probability of the implied decisions being correct. In short, this chapter reveals that, in the real-world, high relevant information levels imply that prediction and their inherent implied decision market incorporates information to achieve best possible decisions.

The policy implications of the analysis of this chapter and the theoretical, computer simulation and games with humans' studies of previous chapters will be explored in the next and final chapter. Specifically, policy implications as they relate to constructing high quality real-world prediction and decision markets will be recommended.

Chapter. 7 Policy Implications for Project Prioritization

“Market efficiency is a premise, not a conclusion.”
Jack Treynor

Key Message of Chapter:

- **The combination of the theoretical model and the three investigations of previous chapters validate the significance of relevant information level as a quality signal for prediction and decision markets.**
- **A hypothetical example conveys the way to construct decision markets, of the type proposed in this thesis, to solve the problem of selection and prioritization of the best possible portfolio of projects for a not-for-profit firm.**
- **Policies for contemporary real-world prediction markets are advocated which improve their relevant information level and therefore their performance.**

This final chapter connects to research questions 2 and 3 by explaining, via an example, how to build high quality decision markets to select, with high probability, the best possible portfolio of project problems in a not-for-profit setting. Specifically, it conveys three important policy implications. It uses an example [43], being a hypothetical Indigenous (Australian) Not-For-Profit firm; a means to best convey the abstract ideas covered (Hasinoff, 2011).

Ultimately, this thesis investigation finds that high relevant information level guarantees high quality decision markets. Notably, relevant information level was identified via the new theoretical model of this thesis and then three logically sequenced investigations tested and validated the statistically significant effect of relevant information level on prediction and decision markets applied to portfolio of project selection problems in not for profit settings. In short, high relevant information level is a quality signal to guarantee a high-quality decision support tool. The novel contribution of this thesis is to introduce relevant information level as a means to measure how well market information is efficiently aggregated to make the best possible predictions and decisions. It is hoped that this may stimulate greater utilization of high quality prediction and decision markets in firms.

The selection of the best possible portfolio of projects for the firm to invest in, i.e., corporate strategy (Porter, 1989) is a combinatorially hard problem that is addressed in a computationally tractable way by decision markets. Relevant information level is crucial to

[43] The example is informed by the author’s experience of working in portfolio project management roles in Indigenous Not-For-Profits and his quest for a suitable and high quality decision support tool.

the proper application of prediction and decision markets to project prioritization problems and importantly signals the quality and robustness of the prediction and decision market as a decision support tool.

Finally, the ultimate goal of this thesis being to research the effective application of prediction markets to project prioritization decisions in not-for-profit firms is attained through measuring and publicizing relevant information level of running prediction and decision markets, and also by introducing trading rules that incentivize high relevant information level trading. In short, high relevant information level prediction markets underpinning decision markets can guarantee a highly probable solution to the (combinatorial NP hard) selection of the best possible portfolio of projects problem in not-for-profit firms.

7.1 Introduction

The overall objective of this thesis is to design a decision market and policies consistent with maximizing the probability of selecting the best possible project portfolios in a not-for-profit setting. The previous chapters on theoretical, experimental and real-world analysis prima facie suggest that policy recommendations that ensure high relevant information levels can improve prediction and decision markets to achieve this objective.

This chapter will address the policy implications in detail under three main topics. Firstly, high relevant information level guarantees improved prediction and decision markets. Secondly, an implied decision market within prediction markets is an appropriate decision support tool for firms. Thirdly, an example application of prediction markets to an Indigenous not-for-profit firm to ascertain the best possible portfolio of projects will educe, in a concrete and relatable way, the big ideas contributed by the entire thesis.

A review of related literature is undertaken in section 7.2. Section 7.3 will highlight the significance of relevant information level to improvements in prediction and decision markets. Section 7.4 will emphasize the important relationship between prediction and decision markets. Section 7.5 will consider a hypothetical not-for-profit firm faced with a project prioritization problem. The application of prediction and decision markets to this example will draw out the corporate strategy view of projects selection, the combinatorial NP hard problem it entails, the link between the decision market and utility framework, and the use of short decision market games as decision support tools. Finally, section 7.6 will conclude with a statement outlining the contribution of this thesis and proposed postdoctoral investigations.

7.2 Decision-Making Relating to Project Prioritization in an Indigenous Not-For-Profit Setting

Poor project prioritization to select the portfolio of projects to implement in not-for-profit settings is considered a key challenge for the 21st Century (Lacerda et al., 2016; Silvius and Schipper, 2014). Specifically, significant scarce resources on a global level are wasted through poor project portfolio management (Lacerda et al., 2016; Sánchez, 2015).

Projects have been typically ranked by attempting to aggregate their multiple attributes (Butler et al., 2001). To this end an attempt to leverage utility theory (Bernoulli, 1954) to prioritize projects has been attempted (Li et al., 2003). Calculus and set theory underlies the modern conceptualization of utility theory (Leonard, 1995). However, social utility theory, applicable to project prioritization for the good of society, suffers from logical inconsistencies (Fishburn, 1968). There have been many variants of utility theory that attempted to circumvent these logical problems (Schoemaker, 1982). Throughout these attempts, it was an assumption that firms maximize profits and individuals rationally maximize utility (Hammond, 1997; McCubbins et al., 2012). Empirical evidence, however, suggests that firms may not always profit maximize (Crossan, 2005) and utility maximization behavior by individuals (particularly in uncertain contexts) is not always observed (Van Den Bergh et al., 2000). In short, real-world evidence suggests that individual, social, and firm decisions are not the ideal result of individual utility, social welfare and profit maximization functions respectively (Kahneman, 2003; Van Den Bergh et al., 2000; Conitzer, 2009).

Pareto optimality became an alternative to the utility approach (Maskin, 2008). The Paretian concept underpinned the measurement of the success of social decisions in terms of GDP and then beyond GDP to include corrected (non-market-values) GDP (Fleurbaey, 2009). The Sen-ian capability concept also informed social decision-making by highlighting the barriers to informed decision making (Britz et al., 2013) and ultimately birthed the Capability Index (Fleurbaey, 2009). Despite these innovations, variants of cost benefit analysis (e.g., social return on investment) remain the dominant measures of success of social investments (Pathak and Dattani, 2014). For example, conservation work has typically applied cost benefit analysis subject to a budget constraint (Pannell and Gibson, 2016).

Corporations are considered a key agent for effective resource allocations and project prioritization in today’s world (Vishnevskiy et al., 2015) and good corporate governance is considered as necessary to good project prioritization and decision-making (Shleifer and Vishny, 1997). Corporate governance frames the corporate strategy that decides what projects a firm should be in or out of (Porter, 1989) and what market should be entered or exited (Brandenburger and Nalebuff, 1995). Aligning the principal and agent is key to good corporate governance (Crossan, 2005); a problem that has been considered within a utility framework (Ross, 1973). It has also been argued that there is no alignment problem (Fama, 1980), however, given that utility maximization is not the same as profit maximization the problem most likely exists (Olsen, 1973).

Small to Medium Enterprises (SMEs) contravene the Modigliani Miller separation theorems that are usually satisfied in larger firms (Miller, 1988) in that capital structure in SMEs most certainly impact operations decision making (Romano et al., 2001). In the Indigenous (Australian) setting, SMEs are typically not-for-profit in nature and employ adaptive management practices, i.e., a virtuous cycle of decisions affecting corporate knowledge that, in turn, affects future decisions as management adapts to the new knowledge (Berkes et al., 2000). These not-for-profit SMEs are structured as either public-private-partnership, social enterprise, or impact investments, but all pursue multiple objectives (Lehner and Nicholls, 2014) including those which also retain, as one of many objectives, the profit motive (Pathak and Dattani, 2014).

In such cognitively difficult contexts rules of thumb are used by human managers (Loomes, 1998). The use of decision support tools by managers in the form of ICT-based systems to predict and decide in these difficult contexts has come to be considered useful (Heiko et al., 2015). Typically multiple forecasting tools are used in order to triangulate the best possible prediction or decision; with scenario and sensitivity analyses also applied where possible (Clemen, 1989). The overlap of economics and computer science has led to sophisticated and user friendly decision support tools (Heiko et al., 2015; Bonney et al., 1999). However, potentially useful decision support tools (e.g., prediction and decision markets) can face uptake challenges that are possibly due to the absent quality signals guaranteeing their effectiveness (Scott and Scott, 2016).

7.3 Relevant Information and Policy Implications

In engineering sciences, information contained in a message with meaning, compared to a message that has “pure nonsense”, can be equal in Shannon’s sense of information (Weaver, 1949). This contrasts with the concept of ‘relevant information’ introduced by this thesis. Whilst Shannon’s information is a measure of surprising syntax, relevant information is a measure of shared semantics. However, both provide insights into the dynamics of information.

Relevant information level is simply the proportion of traders sharing their private information in a market; the value of which implies the dynamics expected in the prediction and decision market. Of fundamental importance is establishing prediction and decision markets with full relevant information, i.e., all traders share their private information. In such a market, where learning from prices takes place in a typical way, the prediction and decision market will provide the best possible prediction and decision respectively (Grainger et al., 2015).

Two policy implications result from the theoretical, laboratory and empirical investigations of previous chapters. Specifically, to improve the performance of prediction and decision markets the use relevant information level as a measure of market performance and the introduction of trading rules that ensure high relevant information levels are recommended. These policy implications will be discussed in detail in the following sections.

7.3.1 Relevant information level to measure prediction and decision market performance

The idea of *quantifying market efficiency* as undertaken in the real-world analysis chapter is an original contribution of this thesis. It demonstrates that the increase in relevant information level (i.e., the proportion of market traders sharing their private information) in prediction markets with implied decision markets increases the probability of a correct implied (decision market) decision. Since reliable information related parameters in prediction markets improves trader performance (Jumadinova and Dasgupta, 2011), publicizing relevant information level of the market would logically improve predictions and decisions. This is a strategy reminiscent of Akerlof’s quality signals to improve markets (Akerlof, 1970). In this sense, low relevant information levels would signal to traders’ markets with high arbitrage opportunities and incentivize them to trade in an informed way so as to profit. Their informed trading would, in turn, increase the relevant information level in the market. This technique of publicizing the quality of a prediction and decision market by announcing the relevant information level is no different to institutionalized corporate monitors that publicize metrics to reveal good or bad corporate governance in a stock market firm (Clark and Hebb, 2004). Both act to reveal market information asymmetry imperfections so as to ultimately improve the trading context.

7.3.2 Introduction of trading rules to ensure high relevant information levels in prediction and decision markets

In addition to publicizing the relevant information level of prediction and decision markets, more aggressive changes may be implemented to improve predictions and decisions. For example, a trader bidding the previous round market price provides no new information to the market. Hence to avoid this pathological trade (that acts to reduce the relevant information level), the trading platform can be programmed to prevent execution of such a trade. An additional measure may be to provide traders with a selection of bids that they may choose from; which are all relevant information level bids. However, new trading rules incentivize different behaviors and the ramifications of this on the prediction and decision market may be adverse. The theoretical model, computer simulations, games with humans, and real-world analysis did not constrain bids. To analyze the ramifications of such a trading rule change, a first step is a more general theoretical model. Appendix E: Chapter 7 Appendix 1 provides a new theoretical model that generalizes the price formation mechanism; and at least allows for this trading rule change. Relevant information level is still found to be significant and convergence to the best possible prediction still occurs. Despite this theoretical confirmation, laboratory and real-world validation of this should still be performed.

7.4 The Important Prediction and Decision Market Link

There is an important link between decision and prediction markets. In short, prediction markets should imply a decision market in order to be a useful decision support tool to a firm. This subtlety requires a deeper explanation as follows.

7.4.1 Prediction markets should imply a decision market

Consider a firm pondering whether to implement project A (or not to implement project A), in order to achieve desired outcome O. If it constructs a prediction market in desired outcome O and project A, then the prediction market will elicit probabilities $P[O]$, $P[A]$, and $P[A'] =$

$1 - P[A]$. It may be the case that $P[A] > P[A']$ and the firm then acts to implement project A. However, this can be the wrong action for the firm to take. Since $P[O] = P[O|A]P[A] + P[O|A']P[A']$ it is entirely possible that $P[O|A'] > P[O|A]$ and hence the firm should not have implemented project A. Therefore, it is important that conditional probabilities $P[O|A]$ and $P[O|A']$ are implied by the prediction market, either explicitly or implicitly and hence an implied decision market is required to exist in a corporate (firm) prediction market.

7.4.2 Decision markets instead of prediction markets as decision support tools

Ultimately a firm is interested in choosing the project(s) that maximizes the probability of the desired outcome. To the best of the author's knowledge, all decision support tools of the information market type are prediction markets. This thesis advocates a departure from prediction markets that predict a future state of the world towards conditional probability information found in decision markets that unambiguously select a project investment. Specifically, probabilities $P[O]$, $P[A]$, and $P[A']$ are ambiguous pieces of information. For example, if $P[A] = 0.65$ should the firm invest in project A? In contrast, $P[O|A'] > P[O|A]$ unambiguously signals that project A should not be implemented[44]. The difficulty of determining whether a prediction market prediction is good (Damnjanovic et al., 2012) then becomes immaterial. Decision markets are both more applicable to the firm (Hanson, 1999) and unambiguous as a decision support tool.

7.5 Example Application of Prediction Markets to a Project Prioritization Problem in a Not-For-Profit Firm

The application of a (100% relevant information level) prediction market (with an implied decision market) to an Indigenous (Australian) not-for-profit firm's project prioritization problem is considered. This example is used to demonstrate how prediction markets of the type studied in this thesis can aid a firm's corporate strategy, i.e., what projects are in or out of the firm's project portfolio. The following sections also provide step-by-step detail on how the prioritization using prediction markets is applied, and why it works.

7.5.1 Indigenous firm setting

Adaptive management has been applied in typical Small to Medium Enterprise (SME) Indigenous Australian firms (Berkes et al., 2000) whereby the impact of decisions builds

[44] An important qualification to this is that the robustness and sensitivity of this inequality to information uncertainty should also be considered.

knowledge which in, turn, affects future decisions. These Indigenous SMEs are typically not-for-profit and may operate as public-private-partnerships, social enterprise, or impact investments (Lehner and Nicholls, 2014). Typically, the objective is not just to maximize dollar profit, rather, it is to achieve social and financial returns which may include land, health, education, social, economic and employment outcomes (Pathak and Dattani, 2014).

To contend with the difficult multi-objective (Indigenous) setting, the Millennium Ecosystem Assessment initiated by the United Nations in 2000 emphasized a decision making approach underpinned by monetary quantification (Gewin, 2002) which includes Indigenous assets, e.g., land and sea, fauna and flora. However, there are other ways to make decisions, including utilizing probabilities (Von Neumann and Morgenstern, 2007), scoring rules (Gneiting and Raftery, 2007) and other non-monetary metrics (Chiapello, 2015). Moreover, monetary quantification underpinning and informing Indigenous decision making has been considered inappropriate and inept to deal with the complex Indigenous Community setting (Marre et al., 2016). The decision market of this thesis provides a means for an Indigenous not-for-profit SME to make good decisions in an Indigenous Community context using a decision market that does not require the monetization of Indigenous assets.

Typically such settings, particularly related to conservation activities, have employed a benefit cost approach subject to budget constraints (Pannell and Gibson, 2016). That is, the best portfolio of projects selected is ranked according to a benefit cost ratio with the top tier implemented until the given budget is exhausted. However, Porter's corporate strategy concepts whereby the best practice for firms is considering what projects a firm should place into or out of its project portfolio (Porter, 1989) is not adequately addressed by using benefit cost ratios. Projects in a portfolio interact to ultimately create a portfolio that is more or less than the sum of its (project) parts. For n potential projects the firm may therefore need to consider 2^n possible project portfolios. Complicating this portfolio of project selection problem further is the finding that SME capital structure affects decision making (Romano et al., 2001); a situation that is anathema to the idealized Modigliani Miller separation theorem (Miller, 1988).

In the following sections the theory and application of a decision market to select the best possible portfolio of projects for a given budget for this example is presented.

7.5.2 Corporate strategy view of project prioritization and application of decision market

The corporate strategy view of project prioritization is simply that which attempts to identify the best possible portfolio of projects that increases the chance of achieving the desired outcome for a given budget (Mintzberg, 1987). To this end, a decision market can establish a well-ordered list of projects (best to worst) such that successively adding the next best project from the list improves the project portfolio. This is easily achieved as follows:

Consider n projects from which the firm will select a subset of projects to invest in. Let P_i and P_{i+1} denote projects i and $i+1$ and let π denote the remaining $n-2$ projects. Specifically, let $P_i = 1$ indicate that project i is undertaken and $P_i = 0$ indicate that it is not undertaken. To simplify notation define the complement $P'_i = 1 - P_i$.

The decision market is run and projects are selected implying that the values $P_k; k = 1\ to\ n$ are known.

Without loss of generality the value $P_{i+1} = 1$ of project $i+1$ is conditioned on information including the values of P_i and those in π.

Formally, the decision market determines that $\boldsymbol{P}[(\boldsymbol{O}|\boldsymbol{P_{i+1}})|\boldsymbol{\pi},\boldsymbol{P_i}] > \boldsymbol{P}[(\boldsymbol{O}|\boldsymbol{P'_{i+1}})|\boldsymbol{\pi},\boldsymbol{P_i}]$ where O represents the desired outcome, such as profit, may be rewritten as $P[O|P_{i+1},\pi,P_i] > P[O|P'_{i+1},\pi,P_i]$.

Notice that $P[OP_{i+1}|\pi,P_i] + P[OP'_{i+1}|\pi,P_i] = P[O|\pi,P_i]$ may be rewritten as $P[O|P_{i+1},\pi,P_i]P[P_{i+1}|\pi,P_i] + P[O|P'_{i+1},\pi,P_i]P[P'_{i+1}|\pi,P_i] = P[O|\pi,P_i]$.

But since $P[O|P_{i+1},\pi,P_i] > P[O|P'_{i+1},\pi,P_i]$ then $P[O|P_{i+1},\pi,P_i]P[P_{i+1}|\pi,P_i] + P[O|P_{i+1},\pi,P_i]P[P'_{i+1}|\pi,P_i] > P[O|\pi,P_i]$. Simplifying further, $P[O|P_{i+1},\pi,P_i]\big[P[P_{i+1}|\pi,P_i] + P[P'_{i+1}|\pi,P_i]\big] > P[O|\pi,P_i]$ implies $\boldsymbol{P}[\boldsymbol{O}|\boldsymbol{P_{i+1}},\boldsymbol{\pi},\boldsymbol{P_i}] > \boldsymbol{P}[\boldsymbol{O}|\boldsymbol{\pi},\boldsymbol{P_i}]$.

Now $P[O|P_{i+1},\pi,P_i] > P[O|\pi,P_i]$ may be rewritten as $\boldsymbol{P}[(\boldsymbol{O}|\boldsymbol{P_{i+1}})|\boldsymbol{\pi},\boldsymbol{P_i}] > \boldsymbol{P}[(\boldsymbol{O}|\boldsymbol{P_i})|\boldsymbol{\pi},\boldsymbol{P_i}]$.

In short, the decision market selects each project P_{i+1} when $P[(O|P_{i+1})|\pi,P_i] > P[(O|P'_{i+1})|\pi,P_i]$ and establishes a well-ordered list of project priorities whereby project $i+$

1 is listed as a higher priority than project i when $P[(O|P_{i+1})|\pi, P_i] > P[(O|P_i)|\pi, P_i]$. Notice also that as the portfolio of projects adds the next best project, the likelihood of achieving the desired outcome O increases; since $P[O|P_{i+1}, \pi, P_i] > P[O|\pi, P_i]$.

Therefore, the best possible portfolio of projects may be chosen using the list of project priorities generated by the decision market; with the number of projects selected constrained only by the firm's budget.

This is computationally feasible as is described in the following section.

7.5.3 Best possible projects portfolio as an NP hard problem solved in a computationally tractable way by prediction and decision markets

The selection of a subset of n projects to form the best possible portfolio is a combinatorially NP-hard problem whereby each project i is either in or out, i.e., $P_i = 1\ or\ 0$ respectively. Therefore, there are 2^n possible portfolio combinations (including the do-nothing portfolio) that require consideration by the firm's management. This is by no means a trivial undertaking but is made simple by a decision market which selects project P_{i+1} when $P[(O|P_i)|other\ projects + other\ information] >$ $P[(O|P'_i)|other\ projects + other\ information]$ for all $1 = 1\ to\ n$. That is n decision markets are required. Since each decision market comprises four prediction market stocks ($P[OP_i|information]$, $P[P_i|information]$, $P[OP'_i|information]$, and $P[P'_i|information]$), then there are $4n$ prediction markets run. Therefore, although the complexity of the project prioritization problem increases as 2^n the prediction market calculations increase in proportion to $4n$. Hence the combinatorial problem is made computationally tractable.

7.5.4 The prediction and decision market link to utility and money

The Arrow Impossibility Theorem demonstrates how four deterministic assumptions to satisfy a social preference generate a logical contradiction (Arrow, 1974). In contrast, the decision market elicits individual private information to determine the best possible portfolio of projects to achieve a desired outcome. That is, the objective of the decision market is not to maximize social utility; rather, it is to maximize the probability of selecting projects to achieve a desired outcome determined by that group. The specific link of decision markets to social utility and money may be portrayed as follows.

Ultimately, the decision market identifies the best possible portfolio of projects Π to maximize the probability of achieving the outcome O, i.e., $P[O|\Pi]$ is the maximum probability across all possible project combinations. Notice that if outcome O is the maximization of firm profit then Π are the projects that will most likely achieve the maximum profit. In contrast, if outcome O is maximizing the social satisfaction in an Indigenous Community then Π are the projects that will most likely achieve this maximization of a proxy for social utility or welfare of that Indigenous Community.

In short, the decision market may be considered a generalization of economic maximization problems given it encompasses both profit and not-for-profit settings in the way described. Specifically, there is a simple link to the (economic) present value. Consider a not-for-profit decision market with n projects indexed by P_i. Let project k be a risk-free bond of value V_k, and $p_k = P[P_k|information]$ be the probability that the decision market assigns to investing in that bond, then the expected value of the bond is $V_k p_k$.

In a rational, risk-neutral and myopic decision market (decision support tool) game, the proportion of game money invested in P_i by a fully informed trader in the final round of the game is α_i. Each P_i has an expected value for that trader of $V_i p_i$; whereby p_i is revealed by the decision market, and V_i is inherent. The value of V_i and therefore the present value of project P_i can be calculated in the following way.

The rational, risk-neutral, myopic trader playing this decision market game maximizes total wealth $T = \sum_i \alpha_i \varepsilon V_i p_i + \lambda(1 - \sum_i \alpha_i)$ where ε is the implied exchange rate of game dollars per real-world dollar. This implies that $\frac{\partial T}{\partial \alpha_j} = \varepsilon V_j p_j - \lambda = 0$ and $\frac{\partial T}{\partial \alpha_k} = \varepsilon V_k p_k - \lambda = 0$. Therefore, $V_j = \frac{V_k p_k}{p_j}$ is the present value of P_j may be known; given that V_k is the specified value of the bond and p_j and p_k are revealed by the decision market. Since trades are undertaken in a market with the highest relevant information level then this may also be considered the best possible estimate of the present value.

7.5.5 Short run decision market game as a decision support tool

The prediction and decision market games explored in this study may find utility in a real-world context; however, such applications are beyond the scope of this thesis but could be pursued in postdoctoral studies. These prediction and decision market games differ from prediction market software currently in the marketplace; mainly because of the emphasis placed on high relevant information level prediction and decision markets in this study. Relevant information level is simply, but importantly, a measure of prediction and decision market efficiency.

The decision market games proposed in the theoretical chapter and trialled in the game with human participants chapter have features worth highlighting.

In real-world prediction markets, stocks are paid based on a real-world event being realized. The issue of accurately specifying the outcomes has been previously called into question whereby the contract wording of the payout event and the real-world event that occurred led to ambiguity over whether to pay a stock or not, e.g., Tradesport's required tightening of the payout terms for the Yasser Arafat departure from the Palestinian state by the end of 2005 that was complicated by his hospitalization in Paris (Wolfers and Zitzewitz, 2006). Events that may be in the distant future also complicate a prediction market, requiring the market to run the full span of the long time horizon event, e.g., a prediction market for climate outcomes (Hsu, 2011).

The features of the decision market game proposed in this thesis solve both the ambiguity of payment and long running market problem. Specifically, the decision market game payouts are specified on game states and not real-world states. That is, whilst private information about the real-world is aggregated (at a high relevant information level where a best possible decision is made), the game simply specifies an end of game date at which the best possible decision is the one with the highest conditional probability, e.g., project A is chosen if the decision market game reveals that $P[O|A] > P[O|A']$ at 100 days from today. Notice two things. Firstly, that an arbitrary[45] end of game date is possible with values of $P[O|A]$ and $P[O|A']$ provided by the game. Secondly, that the payout is based on the game state $P[O|A] > P[O|A']$ and not a real-world event. Therefore, short games for long real-world settings are possible to run, and the problem of ambiguity of payout is avoided.

7.5.6 Application of prediction markets to project prioritization decisions in a not for profit Indigenous Australian firm

Consider a hypothetical Indigenous Australian SME called Gelam-B with a \$2m project budget and choosing amongst five possible projects each having a budget of \$600k for the next two years, i.e., only three projects can be funded. Gelam-B has considered contemporary

[45] Provided that it is prior to the actual real-world outcome being revealed.

decision support tools including discounted cash flow, non-market valuations, scoring rules and other approaches, but has instead decided to implement an internal prediction and decision market such as the one in the games with humans chapter. Gelam-B's decision market is called Sub-eh and will be used to find the best possible projects for Gelam-B's project portfolio to maximize the chance of achieving the desired outcome, i.e., having healthy Families, Country, and Economy.

7.5.6.1 Gelam-B's possible projects

The projects in this example are Junior Ranger project, Indigenous Protected Area (IPA) project, Back to Country project, Primary Health Care project, and City to Country Employment project. The Junior Ranger project involves Elders of the Indigenous Community taking kids on Country to teach culture and language in order to care for Country and re-establish respect for Elders so as to combat the broken social fabric that has plagued the Community. The IPA project involves ensuring Native Title is not extinguished and working with Government and scientists to Care for Country. The Back to Country project involves getting people back on Country and obtaining Native Title and Land Rights by demonstrating the Community's connection to Country. The Primary Health Care project involves working with the local health clinic doctors and preventing early mortality and reducing mental health problems that have been on the rise. The City to Country Employment project involves partnerships with city-based corporations whereby Community people are positioned for 6 months in a city job and then return to Community to consider study, work, and local entrepreneurial opportunities.

7.5.6.2 Sub-eh decision market game is run for 60 days to prioritize projects

Gelam-B's staff and other interested Community participants play the prediction and decision market game (called Sub-eh). At the start of the game a Community meeting explains the objective of the game, i.e., to assist with choosing the combination of projects that most likely improve Community people's wellbeing, their Country and their opportunities for good jobs. The game is played for 60 days and informal discussions are stimulated by it, which in turn informs trading. Every day the relevant information level is measured (in a manner similar to the real-world analysis chapter) and publicized. Relevant information was found to be very high and the maximum in the last 4 days of trading. On the 60th day the game is halted and the winner of the game awarded the game prize, i.e., a holiday to Canada.

The games are ranked from the game with the highest conditional probability to the game with the lowest conditional probability. The ranking from highest to lowest is hypothetically as follows: Back to Country project, City to Country Employment project, Primary Health Care project, IPA project, and Junior Ranger project. Since only 3 projects can be selected, Gelam-B implements a project portfolio containing Back to Country project, City to Country Employment project, and Primary Health Care project.

In summary, by implementing the prediction and decision markets game with staff, intended beneficiaries and other stakeholders, in this imagined scenario Gelam-B is confident that the best possible portfolio of projects has been chosen; given that the daily relevant information level was very high and the maximum in the last 4 days of trading. In this hypothetical scenario, it has successfully applied a well-functioning prediction and decision market to a project prioritization problem in a not-for-profit setting.

7.6 Conclusion & Future Research

The strategy of the thesis was to construct a theoretical model that revealed the important facets in an idealized setting. High relevant information level was revealed as an important theoretical element for well-functioning prediction and decision markets. This revelation by theory was then tested in a computer simulation that provided the advantage of numerous prediction and decision market scenarios to verify, at a very high level, that relevant information level played a statistically significant role in ensuring the convergence to DCE. The need to extend the investigation beyond algorithmic traders resulted in prediction market games with human participants. Even in a market inhabited by human traders relevant information level was found to play a statistically significant role in ensuring convergence to the DCE. However, both computer simulation and games with humans were undertaken in laboratory conditions. Therefore, data from a real-world prediction market were analyzed to ultimately reveal that relevant information level was statistically significant in the real-world.

In short, the principal contribution of this thesis is the *provision of a means to measure the quality of prediction and decision markets as decision support tools for firms*. If taken up, this thesis' contribution has the potential to generate positive change in the real-world; as follows. *The theoretical model chapter provides the logical foundations of why high relevant information level guarantees high quality prediction and decision markets. The real-world analysis chapter shows in detail how to measure relevant information in real-world* *prediction and decision markets. The games with humans' chapter demonstrate how to build a short-lived decision market game as potential decision support tool for a not-for-profit setting even with long-term projects. The computer simulations chapter establishes how decision markets may be built from prediction markets to attain the best possible predictions and decisions. Importantly, all three investigations revealed relevant information level was a statistically significant effect for improving portfolio of project selection in their respective not for profit setting. Finally, the example of this chapter provides a 'how-to' guide for applying prediction and decision markets to prioritize projects in real-world not-for-profit firms.*

In conclusion, the theoretical model of this thesis was kept deliberately simple and used only to reveal a useful feature (relevant information level) to be validated in simulations, games and real-world data. Postdoctoral research building upon the work of this thesis is proposed as follows. Firstly, a deeper theoretical investigation may be advanced to reveal other interesting features in a more general theoretical setting. For example, Appendix E: Chapter 7 Appendix 1 exhibits the beginnings of a more general theoretical model that still shows relevant information level as important but brings with it the advantage of a theoretical study of strategic trading and a more general price formation mechanism. Insights revealed by such a model would then undergo validation in laboratory and real-world analysis. Secondly, in parallel to this theoretical work, building a high relevant information level prediction and decision market game in a real-world not-for-profit firm would add to this research and provide an experimental setting to explicitly evaluate the specific prediction and decision market game design used in the games with human participants' chapter of this thesis. It would also permit initial assessment of the potential impact of the findings set out in this thesis[46]. Finally, post-doctoral collaborations with other prediction and decision market builders (e.g. contributing to the continuous improvement of the Iowa Electronic Market) are envisaged. Specifically, publicizing relevant information level, as a measure of current real-world prediction and decision market quality, is an immediately possible contribution. A longer-term aspiration is transforming real-world public prediction markets and internal corporate prediction markets into decision markets that identify the best possible portfolio of projects to ultimately improve the mobilization of scarce global economic resources.

[46] At the time of writing, a post-doctoral opportunity in the form of joint venture between the University and an Indigenous not for profit health organization to build a project selection decision market of the type proposed in this thesis exists.

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Appendices

Appendix A: Chapter 3 Appendices

Chapter 3 - Appendix 1 Axioms, theorems and proofs for the theoretical model

In this appendix the mathematical formalism on which this chapter is based is provided. Importantly, detailed proofs build towards the new simple decision market and the implication that *relevant information is an important ingredient and signals the quality of prediction and decision markets*.

Definition (direct communication equilibrium [DCE]):

The market equilibrium reached when *all* agents have bids conditioned on full information x (i.e. all agents bid $E[f(s)|x]$) shall be called the *direct communication equilibrium (DCE).* Notice that since the market price is the average of all bids then it is $E[f(s)|x]$.

Axiom 1 and the private information stage property

Axiom 1 is motivated by the notion that a trader will ideally learn something from the first-round market price. Specifically, axiom 1 introduces into the model the idea that, irrespective of the participating traders, no first-round market price is formed where an agent learns *not a single piece* of new information. That is, in this new model any m of the n traders (where $m \leq n$) may be selected to form a first-round market price that causes all of the m traders to learn something new.

For example, consider these m traders. They form a first-round market price p. Arbitrarily chosen trader i always knows its own private information bit x_i. Now, if trader i learns no new piece of information from market price p then it must believe that there are two ways to form this market price. Trader i may consider that the market price was either formed by calculating $p = \frac{\sum b_i}{m} = \frac{\sum E[f(s)|x_i]}{m} = \frac{E[f(s)|x_1]+\cdots+E[f(s)|x_i]+\cdots+E[f(s)|x_j]+\cdots+E[f(s)|x_m]}{m}$, or formed by calculating $p = \frac{\sum b_i}{m} = \frac{\sum E[f(s)|x_i]}{m} =$ $\frac{E[f(s)|x'_1]+\cdots+E[f(s)|x_i]+\cdots+E[f(s)|x'_j]+\cdots+E[f(s)|x'_m]}{m}$; where x'_j denotes the opposite bit value of x_j and all bit values except for x_i are opposite bit values (e.g. for $k \neq i$, if $x_k = 1$ in the first equation then $x'_k = 0$ in the second equation).

Importantly, notice that agent i does not know *with certainty* the information bit x_j of any other agent j. That is, agent i can only infer that market price p resulted from either

(i) A market in which the bit value was x_j where $p = \frac{E[f(s)|x_1]+\cdots+E[f(s)|x_i]+\cdots+E[f(s)|\boldsymbol{x_j}]+\cdots+E[f(s)|x_m]}{m}$, or

(ii) A market in which the bit value was x'_j where $p = \frac{E[f(s)|x'_1]+\cdots+E[f(s)|x_i]+\cdots+E[f(s)|x'_j]+\cdots+E[f(s)|x'_m]}{m}$.

Combine (i) and (ii) to form

$$\frac{E[f(s)|x_1]+\cdots+E[f(s)|x_i]+\cdots+E[f(s)|\boldsymbol{x_j}]+\cdots+E[f(s)|x_m]}{m} = \frac{E[f(s)|x'_1]+\cdots+E[f(s)|x_i]+\cdots+E[f(s)|x'_j]+\cdots+E[f(s)|x'_m]}{m}.$$

Simplify this further to form the equation

$$E[f(s)|x_1] + \cdots + E[f(s)|\boldsymbol{x_j}] + \cdots + E[f(s)|x_m]$$
$$= E[f(s)|x'_1] + \cdots + E[f(s)|x'_j] + \cdots + E[f(s)|x'_m]$$

Or more concisely write:

$$\sum_{k\neq i}^{m-1} E[f(s)|x_k] = \sum_{k\neq i}^{m-1} E[f(s)|x'_k]$$

Notice that this equation signifies that agent i only knows its private information bit x_i and cannot infer with certainty some other agent's information bit from the first-round market price. This would contravene axiom 1.

Axiom 1 requires that arbitrary agent i learns something from the first-round market price when trading with any group of traders. So it must be the case that the following property holds:

Private information stage property: For any agent i, $\sum_{k\neq i}^{m-1} E[f(s)|x_k] \neq \sum_{k\neq i}^{m-1} E[f(s)|x'_k]$ for all $m \leq n$, when at the private information stage.

Axiom 2 and the full information stage property

Axiom 2 is motivated by the notion that when all traders learn (know with certainty) all private information bit values at the full information stage, the market price cannot cause any trader to unlearn this full information $x = (x_j, x_{-j})$. This implies that, at the full information stage, agent i (where $i \neq j$) cannot observe a market price $p = E\left[f(s)\middle|x_{-j}\right]$. If they did observe this, they would not know the private information bit value of agent j with certainty; since $p = E\left[f(s)\middle|x_{-j}\right] = P\left[x_j\middle|x_{-j}\right] \times E\left[f(s)\middle|x_j, x_{-j}\right] + P\left[x'_j\middle|x_{-j}\right] \times E\left[f(s)\middle|x'_j, x_{-j}\right]$ where $P\left[x_j\middle|x_{-j}\right] \in (0,1)$. In effect observing market price $p = E\left[f(s)\middle|x_{-j}\right]$ causes agent i to *unlearn* x_j when at the full information state (since x_j is not known with certainty since $P\left[x_j\middle|x_{-j}\right] \in (0,1)$). This would contravene axiom 2.

It is trivial to show:

$$p = E\left[f(s)\middle|x_j, x_{-j}\right] = E\left[f(s)\middle|x'_j, x_{-j}\right] \Longrightarrow p = E\left[f(s)\middle|x_{-j}\right]$$

and

$$p = E\left[f(s)\middle|x_j, x_{-j}\right] \neq E\left[f(s)\middle|x'_j, x_{-j}\right] \Longrightarrow p = E\left[f(s)\middle|x_j, x_{-j}\right] \neq E\left[f(s)\middle|x_{-j}\right]$$

where $P\left[x_j, x_{-j}\right], P\left[x'_j, x_{-j}\right], P\left[x_{-j}\right] \in (0,1)$.

Thus, for the purposes of the following proofs, it is sufficient to require that the following property holds:

Full information stage property: Market prices at this stage must be such that $p = E\left[f(s)\middle|x_j, x_{-j}\right] \neq E[f(s)|x'_j, x_{-j}]$ in order that the information bit value of agent j is certainly known to some other agent i.

Definition (proper market price):

If a market price possesses both the "*private information stage property*" and the "*full information stage property*" it shall be called a **proper market price**.

Definition (proper information market):

Chen's prediction market model modified to require a proper market price, shall be called a *proper information market*.

Definition (relevant information):

The following lemmas provide a formal definition of relevant information.

Lemma 1.1 (relevant information): If agent i bids $b_i = E[f(s)|x_i] = E[f(s)|x'_i]$ this is the same as if they bid $b_i = E[f(s)]$ and it is said that "x_i is not relevant information and agent i does not express a relevant information in their bid". Otherwise it is said "x_i is relevant information and agent i expresses relevant information in their bid".

Proof:

Given $E[f(s)|x_i] = E[f(s)|x'_i]$

This means $P[f|x_i] = P[f|x'_i]$ where $f\ is\ "f(s) = 1"$.

By Bayes' theorem $\frac{P[x_i|f]P[f]}{P[x_i]} = \frac{P[x\prime_i|f]P[f]}{P[x\prime_i]}$.

And simplifying $P[x_i f]P[x'_i] = P[x'_i f]\ P[x_i]$.

Simplifying further to obtain $P[x_i f] = P[f]\ P[x_i]$,

Rearrange to obtain $E[f(s)|x_i] = E[f(s)]$ as required. ■

Lemma 1.2 (relevant information in general): Agent i bidding such that $b_i = E[f(s)|x_j, Y] = E[f(s)|x'_j, Y]$ is the same as if they bid $b_i = E[f(s)|Y]$ and it is said that "x_j is not relevant information and agent i does not express this relevant information in their bid". Otherwise it is said "x_j is relevant information and agent i expresses this relevant information in their bid".

Proof:

Given $b_i = E[f(s)|x_j, Y] = E[f(s)|x'_j, Y]$

This means $P[f|x_j, Y] = P[f|x'_j, Y]$ where $f\ is\ "f(s) = 1"$

By Bayes' theorem $\frac{P[x_j,Y|f]P[f]}{P[x_j,Y]} = \frac{P[x\prime_j,Y|f]P[f]}{P[x\prime_j,Y]}]$

And simplifying $P\left[x_j fY\right]P[x'_j Y] = P[x'_j fY]\ P[x_j Y]$

Simplifying further to obtain $P\left[x_j fY\right]P[Y] = P[fY]\ P[x_j Y]$

Rearrange to obtain $b_i = E[f(s)|x_j Y] = E[f(s)|Y]$ as required.

■

Main information market theorems

The main theorems for the new prediction market model are now presented and ultimately show that a simple decision market may be established using them.

Theorem 1 (Relevant information as sufficient for DCE convergence):

"All agents express relevant information" is a sufficient condition for convergence to the direct communication equilibrium (DCE) in a proper information market.

Proof:

Given "all agents express relevant information" in a proper information market.

At the end of the first round the market price will be revealed and is $p = \frac{E[f(s)|x_1]+\cdots+E[f(s)|x_n]}{n}$. Any agent i may consider all possible information vectors that attained p and form a set which contains them $X = \left\{a \in B^n | p = \frac{E[f(s)|a_1]+\cdots+E[f(s)|a_n]}{n}\right\}\ where\ B = \{0,1\}$. Agent i may reason that X is not empty as it at the very least contains the actual information vector x which was responsible for p. Agent i may also reason that X is a singleton set containing only x via the following argument:

Agent i **assumes that some other information vector $\boldsymbol{y}$ leads to $\boldsymbol{p}$** where $y \in X$ and $y \neq x$. They may write $p = \frac{E[f(s)|x_1]+\cdots+E[f(s)|x_n]}{n} = \frac{E[f(s)|y_1]+\cdots+E[f(s)|y_n]}{n}$. This may be simplified to $E[f(s)|x_1] + \cdots + E[f(s)|x_n] = E[f(s)|y_1] + \cdots + E[f(s)|y_n]$. Since $x_j, y_j \in \{0,1\}$ "replace y_j in the last equation with x_j if $y_j = x_j$" or "replace y_j in the last equation with x'_j if $y_j \neq x_j$" where x'_j denotes the opposite bit value of x_j. For example: $E[f(s)|x_1] + E[f(s)|x_2] + E[f(s)|x_3] + \cdots + E[f(s)|x_n] = E[f(s)|x_1] + E[f(s)|x'_2] + E[f(s)|x_3] + \cdots + E[f(s)|x'_n]$. Simplifying further and eliminate from both sides of the equation identical terms e.g. $E[f(s)|x_1]$ is eliminated from both sides but $E[f(s)|x_2]$ and $E[f(s)|x'_2]$ remain given the relevant information assumption. Hence after simplification there is at least one term on both sides that are different and in general there are *l* terms that remain on both sides since $y \neq x$. That is, either $E[f(s)|x_k] = E[f(s)|x'_k]$ or $\sum^l E[f(s)|x_k] = \sum^l E[f(s)|x'_k]$ where $k \neq i$. However since "all agents express relevant information" this means $E[f(s)|x_k] = E[f(s)|x'_k]$ is not allowed by Lemma 1.1. Furthermore since the proper market price private information stage property holds

$\sum^{m-1} E[f(s)|x_k] \neq \sum^{m-1} E[f(s)|x'_k]$ for all $m \leq n$. Which implies the contradiction $\sum^{l} \; E[f(s)|x_k] \neq \sum^{l} \; E[f(s)|x'_k]$. **Therefore it must be the case that "there does not exist some other information vector y leading to p".**

Thus, agent i finds only information vector x leads to the first-round price p. Hence in round 2 all agents submit the bid $E[f(s)|x]$ which is exactly the direct communication equilibrium.

■

The 'necessary' condition proof shall now be considered.

Theorem 2 (Relevant information as necessary for DCE convergence):

"All agents express relevant information" is a necessary condition for convergence to the direct communication equilibrium (DCE) in a proper information market.

Proof:

Given the market has attained the direct communication equilibrium in a proper information market, all agents know all private information in the market $x = (x_j, x_{-j})$ and the market is said to be at the full information stage.

Assume that there exists at least one piece of information that is not relevant.

This assumption at the very least implies the case where a single bit of information x_j is not relevant.

Any arbitrarily chosen agent i will bid such that

$b_i = E\left[f(s)\middle|x_j, x_{-j}\right] = E[f(s)|x'_j, x_{-j}] = E[f(s)|x_{-j}]$ by lemma 1.2.

This means that the market price formed will be

$p = \frac{\sum^{n} E[f(s)|x_{-j}]}{n} = E[f(s)|x_{-j}]$

But since, $E\left[f(s)\middle|x_j, x_{-j}\right] = E[f(s)|x'_j, x_{-j}] = E[f(s)|x_{-j}]$,

then $p = E\left[f(s)\middle|x_j, x_{-j}\right] = E[f(s)|x'_j, x_{-j}]$.

This contradicts the proper market price full information stage property whereby market prices at this stage must be such that $p = E\left[f(s)\middle|x_j, x_{-j}\right] \neq E[f(s)|x'_j, x_{-j}]$ in order that the information bit value of agent j is certainly known to some other agent i.

In short, the assumption implies a market price that causes 'unlearning' of x_j at the full information stage in contravention to axiom 2.

Therefore it must be the case that ***"All agents express relevant information".***

■

The next logical step is now taken to construct a proper information (prediction) market with *multiple contracts*. Specifically, the convergence of this multi-stock model towards the direct communication equilibrium is of interest. Theorem 3 is a formal statement of such a model.

Definition (Proper information market with r stocks):
Suppose r stocks are traded in a proper information market. The arbitrarily chosen k^{th} stock is traded in a proper information market and represented by $F^{(k)}$. $F^{(k)}$ pays \$1 when the function $f^{(k)}(s) = 1$ and pays \$0 otherwise; where $B = \{0,1\}$ and $s \in B^N$ is the usual representation of the state of the world with commonly know probability distribution $P(s)$. Assume that arbitrarily chosen agent i, in the first-round bid, expresses its 1 bit of relevant information $x_i^{(k)}$ pertaining to the value of $f^{(k)}(s)$. Uncertainty of agents' information is commonly known in probability distribution $Q(x^{(k)}|s)$ where $x^{(k)} \in B^n$ and n is the number of agents trading in the market. The collection of all r stocks traded in this way shall be called a *proper information market with r stocks*.

Theorem 3 (Proper information market equilibrium with r stocks):
A proper information market with r stocks converges to a market equilibrium in which each stock attains its DCE when all agents express relevant information.

Proof:

Given a proper information market with r stocks denote $x_i^{(k)}$ as private information of arbitrarily chosen agent i for the arbitrarily chosen k^{th} stock. ***"All agents express relevant***

information" is a necessary and sufficient condition for convergence to the DCE of a stock by previous theorems 1 and 2. Therefore, the price of contract $F^{(k)}$ converges to DCE $E[f^{(k)}(s)|x^{(k)}]$. Since k is arbitrary, all r stocks converge to their respective direct communication equilibrium and hence the entire market reaches equilibrium.

■

Simple Decision Market Model

Proper information (prediction) markets with *r* stocks are used to build a simple decision market model. In order to do so, a variation to the payout structure of the contracts is considered.

Definition (t-contract): A stock contract $F^{(k)}$ that pays \$t when $f^{(k)}(s) = 1$ and pays \$0 when $f^{(k)}(s) = 0$ we shall call a t-contract.

Lemma 4.1 (convergence of t-contracts): A t-contract converges to the DCE $E[f^{(k)}(s)|x^{(k)}]t$ in a proper information market where "all agents express relevant information".

Proof: Consider a t-contract $F^{(k)}$. Then it pays \$t when $f^{(k)}(s) = 1$ and pays \$0 when $f^{(k)}(s) = 0$. The currency (\$) could easily be exchanged for some other currency (φ) where \$t = $\varphi 1$. Hence, consider $F^{(k)}$ as paying $\varphi 1$ when $f^{(k)}(s) = 1$ and paying $\varphi 0$ when $f^{(k)}(s) = 0$. This is the usual 1-contract that has been considered in the previous theorems. Given "all agents express relevant information", it converges to the DCE $\varphi E[f^{(k)}(s)|x^{(k)}]$ by previous theorems. Since \$t = $\varphi 1$, the t contract converges to the DCE $\$E[f^{(k)}(s)|x^{(k)}]t$

■

Definition (Derivative): Given two contracts $F^{(k)}$ and $F^{(J)}$. It is said that $F^{(k)}$ is a derivative of $F^{(J)}$, if payment of $F^{(J)}$ causes payment of $F^{(k)}$. Formally, this shall require $\Pr[f^{(k)}(s) = 1|x_i^{(k)}] = \Pr[f^{(J)}(s) = 1|x_i^{(J)}]$ in the first round.

Theorem 4 (Derivative attains DCE): If $F^{(k)}$ is a derivative of $F^{(J)}$, with the former being a w-contract and the latter a 1-contract, then $F^{(k)}$ converges to the DCE $E[f^{(J)}(s)|x^{(J)}] \times w$ in a proper information market "where all agents express relevant information".

Proof: Consider the w-contract $F^{(k)}$. Agent i bids

$b_i^{(k)}(\mathrm{w}-\mathrm{contract}) = Pr\left[f^{(k)}(s) = 1|x_i^{(k)}\right] \times w$ in the first round.

But given $F^{(k)}$ is a derivative of $F^{(J)}$ then it must be the case that

$\Pr[f^{(k)}(s) = 1|x_i^{(k)}] = \Pr[f^{(J)}(s) = 1|x_i^{(J)}]$.

which means

$$b_i^{(k)}(\mathrm{w}-\mathrm{contract}) = Pr\left[f^{(J)}(s) = 1|x_i^{(J)}\right] \times w$$

which is equivalent in form to a first round bid on w-contract in $F^{(J)}$.

Since "all agents express relevant information" and by lemma 6.1, $F^{(k)}$ converges to the DCE $E[f^{(J)}(s)|x^{(J)}] \times w$.

■

Definition (Probability Derivative): If $F^{(k)}$ is a derivative of $F^{(J)}$ in a proper information market where all agents express relevant information and the DCE of $F^{(k)}$, denoted by $price(F^{(k)})$, equals the probability $\Pr[f^{(k)}(s) = 1|x^{(k)}]$, it is said that $F^{(k)}$ is a probability derivative.

Definition (Probability of payment $\phi_t(F^{(k)})$): The probability as at round t, that $F^{(k)}$ pays \$1 at some future point in time, is represented by $\phi_t(F^{(k)})$. Notice that if $F^{(k)}$is either a 1-contract or a probability derivative that $\phi_t\left(F^{(k)}\right) = \mathrm{price}(F^{(k)})$ at the DCE; that is the probability of the contract paying \$1 is directly reflected in the price of the contract.

Theorem 5 (Probability derivative w-contract requirement): If $F^{(k)}$ is a derivative of $F^{(J)}$, with the former being a w-contract and the latter a 1-contract, then $w = \frac{\phi_t(F^{(k)})}{\phi_t(F^{(J)})}$ ensures $F^{(k)}$ is a probability derivative in a proper information market where all agents express relevant information.

Proof: Consider the w-contract $F^{(k)}$.

In the first round ($t = 1$) Agent i observes $\phi_t(F^{(k)})= Pr\left[\mathrm{f}^{(k)}(\mathrm{s}) = 1|\mathrm{x}_i^{(k)}\right]$ and

$\phi_t(F^{(J)})= Pr\left[\mathrm{f}^{(J)}(\mathrm{s}) = 1|\mathrm{x}_i^{(J)}\right]$

Therefore they believe that $w = \frac{\phi_t(F^{(k)})}{\phi_t(F^{(J)})} = \frac{Pr\left[\mathrm{f}^{(k)}(\mathrm{s})=1|\mathrm{x}_i^{(k)}\right]}{Pr\left[\mathrm{f}^{(J)}(\mathrm{s})=1|\mathrm{x}_i^{(J)}\right]}$.

Because w-contract $F^{(k)}$ is also a derivative of $F^{(J)}$, in the first round Agent i bids

$$\begin{aligned}
\mathrm{b}_i^{(k)}(\mathrm{w}-\mathrm{contract}) &= Pr\left[\mathrm{f}^{(J)}(\mathrm{s}) = 1|\mathrm{x}_i^{(J)}\right] \times \$\mathrm{w} \\
&= Pr\left[\mathrm{f}^{(J)}(\mathrm{s}) = 1|\mathrm{x}_i^{(J)}\right] \times \frac{Pr\left[\mathrm{f}^{(k)}(\mathrm{s})=1|\mathrm{x}_i^{(k)}\right]}{Pr\left[\mathrm{f}^{(J)}(\mathrm{s})=1|\mathrm{x}_i^{(J)}\right]} \\
&= Pr\left[\mathrm{f}^{(k)}(\mathrm{s}) = 1|\mathrm{x}_i^{(k)}\right]
\end{aligned}$$

This is the usual 1-contract considered previously. Because all agents express relevant information, the DCE is attained in the second round. Therefore, $price\left(F^{(k)}\right)$ equals the probability $Pr\left[\mathrm{f}^{(k)}(\mathrm{s}) = 1|\mathrm{x}^{(k)}\right]$ as required.

■

Comment 5.1: Notice at DCE $\phi_t(F^{(k)})= Pr\left[\mathrm{f}^{(k)}(s) = 1|\mathrm{x}^{(k)}\right] = price(F^{(k)})$ and $\phi_t(F^{(J)})= Pr\left[\mathrm{f}^{(J)}(\mathrm{s}) = 1|\mathrm{x}^{(J)}\right] = price(F^{(J)})$ which implies $\mathrm{w} = \frac{\mathrm{price}(\mathrm{F}^{(k)})}{\mathrm{price}(\mathrm{F}^{(J)})}$.

Definition (Decision market contract pairs): It is said "$F^{(k)}$ on $F^{(J)}$ is the decision market pair for O and P", if $F^{(k)}$ represents the event "O and P occurs", $F^{(J)}$ represents the event "P occurs", and $F^{(k)}$ is a probability derivative of the 1-contract $F^{(J)}$

Theorem 6 (Decision market contract payout structure): Assume a proper information market where all agents express relevant information, and all market prices have attained their respective DCE. If $F^{(k)}$ on $F^{(J)}$ is the decision market pair for O and P, then the quotient of the market price of $F^{(k)}$ and the market price $F^{(J)}$ is equal to "the probability of O conditional on P and full information (denoted simply as $Pr[O|P]$)[47]", and $F^{(k)}$ is a w-contract, where $w = \frac{price(F^{(k)})}{price(F^{(J)})} = Pr[O|P]$.

Proof: by assumptions, at the DCE

$Pr[f^{(k)}(s) = 1|x^{(k)}] = Pr[OP]$ and $Pr[f^{(J)}(s) = 1|x^{(J)}] = Pr[P]$

Since $F^{(J)}$ is a 1-contract, $Pr[f^{(J)}(s) = 1|x^{(J)}] = price(F^{(J)})$.

Since $F^{(k)}$ is a probability derivative, $Pr[f^{(k)}(s) = 1|x^{(k)}] = price(F^{(k)})$

Therefore, $Pr[O|P] = \frac{Pr[OP]}{Pr[P]} = \frac{Pr[f^{(k)}(s)=1|x^{(k)})]}{Pr[f^{(J)}(s)=1|x^{(J)}]} = \frac{price(F^{(k)})}{price(F^{(J)})}$

Furthermore, $w = \frac{price(F^{(k)})}{price(F^{(J)})}$ by comment 5.1

Therefore, $w = \frac{price(F^{(k)})}{price(F^{(J)})} = Pr[O|P]$, as required.

■

Theorem 7 (Decision market selection rule): Given a proper information market of 2M contracts where all agents express relevant information. Specifically, let y index the contract, namely y = 1, to …, M and for each y let there be contracts $F^{(ky)}$ and $F^{(Jy)}$. Let $F^{(ky)}$ on $F^{(Jy)}$ be the decision market pair for O and P_y, where $F^{(ky)}$ is a w_y-contract. Also let there exist a selection rule that ensures only contracts $F^{(Ky^*)}$ and $F^{(Jy^*)}$ are paid if they satisfy the condition $y^* = argmax_y(w_y)$, then this market setup is guaranteed to select the best[48] P_y . This shall be called a decision market for P given O.[49]

Proof: by theorem 6, $w_y = Pr[O|P_y]$

By substitution of this into the given selection rule

$$y^* = argmax_y(Pr[O|P_y])$$

[47] Strictly the conditional probability should be denoted $Pr[O|P, x^{(k)}, x^{(J)}]$ but for brevity and without loss of meaning, $Pr[O|P]$, $Pr[O|P]$ and $Pr[O|P]$ is denoted.

[48] Best P_y is that P_y that when selected maximizes the likelihood of O.

[49] This decision market is applicable to selecting the "Project" (from amongst alternative projects) that provides the greatest likelihood of achieving a desired "Outcome".

The best decision theoretic P_y satisfies this equation (Othman and Sandholm, 2010). Therefore, the best P_y is selected.

■

Appendix B: Chapter 4 Appendices

Chapter 4 - Appendix 1: Proof of Corollary of Expected Linearity in Decision Markets

Corollary

"The probability $\boldsymbol{\rho}$ that a decision market selects the best possible decision (i.e. the decision based on fully informed prediction markets at their DCE) increases linearly in relevant information level $\boldsymbol{r'}$ from $\boldsymbol{\rho = P[A]}$ at $\boldsymbol{r' = 0}$ to $\boldsymbol{\rho = 1}$ at $\boldsymbol{r' = 1}$. In mathematical notation: $\boldsymbol{\rho(r') = (1 - P[A])r'} + \mathbf{P[A]}$"

Proof:

Consider the following indicator functions

$$S_t(r_t, x) = \begin{cases} 1 \ if \ \frac{p_{O\&A}}{p_A} > \frac{p_{O\&A\prime}}{p_{A\prime}} \ in \ markets \ with \ r = r_t \\ \quad 0 \ \ otherwise \end{cases} ; \boldsymbol{given\ x}$$

$$S_c(r_c, x) = \begin{cases} 1 \ if \ \frac{p_{O\&A}}{p_A} > \frac{p_{O\&A\prime}}{p_{A\prime}} \ in \ markets \ with \ r = r_c = 1 ; \boldsymbol{given\ x} \\ 0 \ \ otherwise \end{cases}$$

Notice: they may be thought of as selections made in a treatment decision market and the control (DCE based) decision market respectively.

A value of '1' indicates the decision market selected A, whereas a value of '0' indicates the complement A' is selected.

Define another indicator function

$$J(r_t, r_c, x) = S_t(r_t, x)S_c(r_c, x) + \big(1 - S_t(r_t, x)\big)(1 - S_c(r_c, x))$$

Notice that $J(r_t, r_c, x)$ has a value of '1' when the markets select the same and '0' otherwise. Furthermore, $J(r_c, r_c, x) = 1$.

Also notice the trivial result: $\rho(r_t, r_c) = P['markets\ select\ the\ same'] = E_x[J(r_t, r_c, x)]$ denotes that the probability of same selection may be obtained by calculating expected value of J over all possible x. This holds because the expected value of the indicator function (e.g. J) of an event equals the probability of that event.

When the relevant information level $r_t = 0$ then $S_t(r_t = 0, x) = constant\ for\ all\ x$ since all prices in the market are not conditioned on x and therefore do not vary as x changes. For example, without loss of generality, say $S_t(r_t = 0, x) = 1$ then $E_x[J(r_t = 0, r_c, x)]$ is simply the probability that $S_c(r_c, x) = 1$ considered over all values of x in the control (DCE based) decision market. This equals the probability of A: $P[A]$.

Hence so far

$\rho(r_t = 1,1) = 1$

and

$\rho(r_t = 0,1) = P[A]$.

Of interest is considering other values in the open interval $r_t \in (0,1)$.

Define

$$S_t(r_t = r', x) = \begin{cases} S_t(r_t = 1, x)\ chosen\ r' \times 100\%\ of\ the\ time \\ S_t(r_t = 0, x)\ chosen\ (1 - r') \times 100\%\ of\ the\ time \end{cases}$$

It is simply a stochastic function that selects a market with full relevant information $r' \times 100\%$ of the time and selects a market with no relevant information otherwise. The expected relevant information level associated with $S_t(r_t = r', x)$ may be calculated as $r' \times 1 + (1 - r') \times 0 = r'$. That is, prior to $S_t(r_t = r', x)$ choosing full or zero relevant information level markets it can only be stated that the expected relevant information level of the market is r'.

Now, consider the following two scenarios:

A market with fixed r' relevant information

OR

A market chosen according to stochastic function $S_t(r_t = r', x)$

Require (1) and (2) to be identical in every way with the exception of relevant information level.

But, as shown above, the expected relevant information level for both is $\boldsymbol{r'}$.

Therefore, on average they are identical in every way and also have the same expected relevant information level $\boldsymbol{r'}$. That is, on average they are the same (**).

Define indicator functions for each of the respective scenarios as:

$$J(r', r_c, x) = S_t(r', x)S_c(r_c, x) + \left(1 - S_t(r', x)\right)(1 - S_c(r_c, x))$$

and

$$J(r', r_c, x) = \begin{cases} J(1, r_c, x) \; chosen \; r' \times 100\% \; of \; the \; time \\ J(0, r_c, x) \; chosen \; (1 - r') \times 100\% \; of \; the \; time \end{cases}$$

The expected value of the last equation can be calculated and will equal the expected value of the first equation; since by (**), on average they are the same.

That is:

$$\rho(r', r_c) = E_x[J(r', r_c, x)] = r' \times E_x[J(1, r_c, x)] + (1 - r') \times E_x[J(0, r_c, x)]$$

Simplifying further,

$$\rho(r', r_c) = r' \times \rho(r_t = 1,1) + (1 - r') \times \rho(r_t = 0,1)$$

$$\rho(r', r_c) = r' \times 1 + (1 - r') \times P[A]$$

$$\rho(r', r_c) = (1 - P[A])r' + P[A]$$

or more succinctly:

$$\boldsymbol{\rho(r') = (1 - P[A])r' + \mathbf{P}[\mathbf{A}]}$$

Therefore the probability of selecting the best decision increases linearly in relevant information $\boldsymbol{r'}$ from $\boldsymbol{\rho = P[A]}$ at $\boldsymbol{r' = 0}$ to $\boldsymbol{\rho = 1}$ at $\boldsymbol{r' = 1}$. ■

Chapter 4 - Appendix 2: Agents' Information Generation and Computationally Tractable Binomial Technique

Formally, using the binomial theorem technique, allows for any agents' information vector $x = (x_1, \ldots, x_n)$, the value $\mathrm{Q}(\mathrm{x}|\mathrm{s}) = \mathrm{k}Q(x_1|s) \ldots Q(x_\mathrm{n}|s)$ for $\mathrm{k} > 0$ (i.e. dependence across agents' information) to be constructed. The fundamental ideas of this technique is explored in the following simple example and then the exact construction of $\mathrm{Q}(\mathrm{x}|\mathrm{s})$ utilised in the simulation is presented.

Example of the binomial technique to generate agent information probability distribution
For the sake of pedagogic exposition of the binomial technique, consider a market with only two agents.

Assume agent information bits that are assigned and conditioned on the state of the world are $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$ which in turn are relevant information and not-relevant information respectively.

Formally,
$\boldsymbol{P[x_1|s] = p}$ and $\boldsymbol{P[x_1|s'] = r}$, $\boldsymbol{p \neq r}$, i.e., relevant information
$\boldsymbol{P[x_2|s] = q}$ and $\boldsymbol{P[x_2|s'] = q}$, i.e., not relevant information

Also allow dependence to exist across information $\boldsymbol{x_1}$ and $\boldsymbol{x_2}$.
Notice that $\boldsymbol{P[x_1x_2|s] + P[x'_1x_2|s] + P[x_1x'_2|s] + P[x'_1x'_2|s] = 1}$

Without loss of generality let $\boldsymbol{p, q > 0.5}$ and define $\boldsymbol{P[x_1x_2|s] = (1 + \varepsilon)P[x_1|s]P[x_2|s]}$, where $\boldsymbol{\varepsilon > 0}$. That is, dependence holds here. Notice for independence $\boldsymbol{\varepsilon = 0}$.

Rewrite this as $\boldsymbol{P[x_1x_2|s] = (1 + \varepsilon)pq}$ where $\boldsymbol{\varepsilon > 0}$.

Also define $\boldsymbol{P[x'_1x_2|s] = (1 - \delta)P[x'_1|s]P[x_2|s]}$, $\boldsymbol{P[x_1x'_2|s] = (1 - \delta)P[x_1|s]P[x'_2|s]}$, $\boldsymbol{P[x'_1x'_2|s] = (1 - \delta)P[x'_1|s]P[x'_2|s]}$ where $\boldsymbol{\delta > 0}$.

Or more simply $\boldsymbol{P[x'_1x_2|s] = (1 - \delta)p'q}$, $\boldsymbol{P[x_1x'_2|s] = (1 - \delta)pq'}$, $\boldsymbol{P[x'_1x'_2|s] = (1 - \delta)p'q'}$; where $\boldsymbol{p' = 1 - \mathbf{p}}$ and $\boldsymbol{q' = 1 - \mathbf{q}}$.

Notice that $(\mathbf{1}+\boldsymbol{\varepsilon})\boldsymbol{pq}$ is greater than $(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q}$, $(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq'}$, and $(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q'}$since $\boldsymbol{p},\boldsymbol{q}>\mathbf{0.5}$.

Also notice that $\mathbf{1}=(\boldsymbol{p}+\boldsymbol{p'})(\boldsymbol{q}+\boldsymbol{q'})=\boldsymbol{pq}+\boldsymbol{p'q}+\boldsymbol{pq'}+\boldsymbol{p'q'}$ and hence $(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq'}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q'}<(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq'}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q'}<(\mathbf{1}-\boldsymbol{\delta})$. Therefore there exists $\boldsymbol{\varepsilon}>\mathbf{0}$ in $(\mathbf{1}+\boldsymbol{\varepsilon})\boldsymbol{pq}$.

Substitute into $\boldsymbol{P}[\boldsymbol{x_1x_2}|\boldsymbol{s}]+\boldsymbol{P}[\boldsymbol{x'_1x_2}|\boldsymbol{s}]+\boldsymbol{P}[\boldsymbol{x_1x'_2}|\boldsymbol{s}]+\boldsymbol{P}[\boldsymbol{x'_1x'_2}|\boldsymbol{s}]=\mathbf{1}$ to obtain $(\mathbf{1}+\boldsymbol{\varepsilon})\boldsymbol{pq}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq'}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q'}=\mathbf{1}$ and rearrange this to obtain $(\mathbf{1}+\boldsymbol{\varepsilon})\boldsymbol{pq}-(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq}+[(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq'}+(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{p'q'}]=\mathbf{1}$ which simplifies to $(\mathbf{1}+\varepsilon)\boldsymbol{pq}-(\mathbf{1}-\boldsymbol{\delta})\boldsymbol{pq}+[(\mathbf{1}-\boldsymbol{\delta})]=\mathbf{1}$ and further simplifies to $\boldsymbol{\varepsilon}=\frac{\boldsymbol{\delta}(\mathbf{1}-\boldsymbol{pq})}{\boldsymbol{pq}}$.

That is, it is possible to choose $\boldsymbol{\delta}$ to find a suitable $\boldsymbol{\varepsilon}$.

In summary, a distribution $\boldsymbol{Q}(\boldsymbol{x}|\boldsymbol{s})$ is specified with a mix of relevant and not-relevant information which has dependence across information.

This technique may be generalized further.

The actual algorithm used to establish the (dependence across) agent information distribution in the simulations

Firstly introduce an additional requirement, i.e., if most agents for example received an information bit of 1 then the state of the world is more likely to be $s=1$. Thus, the actual algorithm used is a modification of the above example, but utilizes the same ideas.

$$P[x_1,\dots,x_n|s]+\cdots+P[other\ combination|s]+\cdots+P[x'_1,\dots,x'_\mathrm{n}|s]=1$$

let $P[x_1,\dots,x_n|s]=\delta P_1$; where $P_1=P[x_1|s]\dots P[x_\mathrm{n}|s]$

and $P[x_1,\dots,x_n|s]=\varepsilon P_0$; where $P_0=P[x'_1|s]\dots P[x'_\mathrm{n}|s]$

and $P[other\ combination|s]=\propto P_{other}$; where $P_{other}=\prod_{i=1}^{n}P[y_\mathrm{i}|s]$ and $y_\mathrm{i}=x_\mathrm{i}$ or $y_\mathrm{i}=x'_\mathrm{i}$ and $\exists j,k\ such\ that\ y_\mathrm{j}=x_\mathrm{j}\ and\ y_\mathrm{k}=x'_\mathrm{k}$

Let $\delta>1$ to ensure that given a state of the world of 1 there is a greater probability of receiving information bit values of 1, i.e., $P[x_1,\dots,x_n|s]>P[x_1,\dots,x_n]$ where for simplicity

$x_{\mathrm{i}} = 1$ and $x'_{\mathrm{i}} = 0$. Similarly, require $\varepsilon < 1$ to ensure that in a state of the world of 1 there is a relatively low probability of receiving information bit values of 0, i.e., $P[x'_1, \ldots, x'_n|s] < P[x'_1, \ldots, x'_n]$.

Rearrange the first equation
$(\delta - \propto)P_1 + \propto (addition\ of\ \boldsymbol{all}\ combinations) + (\varepsilon - \propto)P_0 = 1$; where
$(addition\ of\ \boldsymbol{all}\ combinations) = P_1 + \sum_x P_{otherx} + P_0$ and $\sum_x P_{otherx}$ includes all possible arrangements except P_1 and P_0.
There $(\delta - \propto)P_1 + \propto + (\varepsilon - \propto)P_0 = 1$.

Importantly, require: $P[x_1|s] > P[x'_1|s]$
Using $(\delta - \propto)P_1 + \propto (addition\ of\ all\ combinations) + (\varepsilon - \propto)P_0 = 1$
and collecting from it only the terms with x_1

$$P[x_1|s] = (\delta - \propto)P_1 + \propto (P[x_1|s])$$

similarly

$$P[x'_1|s] = \propto (P[x'_1|s]) + (\varepsilon - \propto)P_0$$

Hence,

$$(\delta - \propto)P_1 + \propto (P[x_1|s]) > \propto (P[x'_1|s]) + (\varepsilon - \propto)P_0$$

since $P[x_1|s] > P[x'_1|s]$
for simplicity ensure $(\delta - \propto)P_1 > (\varepsilon - \propto)P_0$
and ensure $P_1 > P_0$
Therefore it is possible to choose $\delta - \propto > \varepsilon - \propto$ or more succinctly $\delta > \varepsilon$

Now since

$$P[x_1|s] = (\delta - \propto)P_1 + \propto (P[x_1|s])$$

and

$$P[x'_1|s] = \propto (P[x'_1|s]) + (\varepsilon - \propto)P_0$$

Rearrange these equations to obtain

$$P[x_1|s](1 - \propto) = (\delta - \propto)P_1$$

and

$$P[x'_1|s](1 - \propto) = (\varepsilon - \propto)P_0$$

Choose $\propto > 1$ then the left hand side of the both of the above equations is negative. Hence $\delta < \propto$ and $\varepsilon < \propto$ for the right-hand-side to also be negative.

Dividing one equation by the other to obtains

$$\frac{P[x_1|s]}{P[x'_1|s]} = \frac{(\delta - \propto)P_1}{(\varepsilon - \propto)P_0}$$

Simplify the situation and insist that all agents receive information in a similar way, i.e., $P[x_{\mathrm{i}}|s] = u$ and $P[x'_{\mathrm{i}}|s] = d$ for constants e.g. $u = 0.8, d = 0.2$. This simply means the agent knows there is an 80% chance of receiving a bit value of 1 if the state of the world bit value is 1 and there is a 20% chance of receiving a bit value of 0 if the state of the world bit value is 1.

Thus $P_1 = u^n$ and $P_0 = d^n$

And rewrite $\frac{P[x_1|S]}{P[x'_1|S]} = \frac{(\delta - \propto)P_1}{(\varepsilon - \propto)P_0}$ as

$$\frac{u}{d} = \frac{(\delta - \propto)u^n}{(\varepsilon - \propto)d^n}$$

Rearrange this to obtain

$\delta = \frac{d^{n-1}}{u^{n-1}}(\varepsilon - \propto) + \propto$, and notice that $\delta < \propto$ if $\varepsilon < \propto$, as required.

Henceforth choose $\propto = 1 + k$ for some $k > 0$

And

$$\varepsilon = 1 - k$$

then

$$\delta = \frac{d^{n-1}}{u^{n-1}}(-2k) + (1 + k)$$

Ensure that $\delta > 1$ by choosing $\frac{d^{n-1}}{u^{n-1}}(2k) < k$

That is, $\frac{1}{2} > \frac{d^{n-1}}{u^{n-1}}$; satisfied by $u = 0.8$ and $d = 0.2$ for $n > 1$

Hence a quick way of generating the conditional probability for agents' information has been established.

Summary Steps

Given $P[x_{\mathrm{i}}|s] = u$ and $P[x'_{\mathrm{i}}|s] = d$

To find any $P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right]$ value consider the equation

$(\delta)P_1 + \propto (addition\ of\ other\ combinations) + (\varepsilon)P_0 = 1$

Now, let $num1$ and $num0$ denote the number of bit values in the collection $x_{\mathrm{i}}, \dots, x_{\mathrm{j}}$ that are 1 and 0 respectively.

For $1 < j \leq n$

Case (i): $\boldsymbol{x_i}, \dots, \boldsymbol{x_j}$ is a mixture of zeros and ones

This does not involve the end terms, i.e., $P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = \propto$ $(addition\ of\ terms\ with\ x_{\mathrm{i}}, \dots, x_j\ in\ them)$

$$P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = \propto P[x_{\mathrm{i}}|s] \dots P\left[x_j \middle| s\right](addition\ of\ permutation\ of\ the\ rest)$$

$$P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = \propto P[x_{\mathrm{i}}|s] \dots P\left[x_j \middle| s\right](1) = \propto u^{num1} d^{num0}$$

Case (ii): $\boldsymbol{x_i}, \dots, \boldsymbol{x_j}$ is all ones

This does not involve the right end term, i.e., $P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = (\delta)P_1 + \propto$ $(addition\ of\ \boldsymbol{other}\ terms\ with\ x_{\mathrm{i}}, \dots, x_j\ in\ them)$

$$P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = (\delta - \propto)P_1 + \propto (addition\ of\ \boldsymbol{all}\ terms\ with\ x_{\mathrm{i}}, \dots, x_j\ in\ them)$$

$$P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = (\delta - \propto)P_1 + \propto P[x_{\mathrm{i}}|s] \dots P\left[x_j \middle| s\right](addition\ of\ permutation\ of\ the\ rest)$$

$$P\left[x_{\mathrm{i}}, \dots, x_j \middle| s\right] = (\delta - \propto)P_1 + \propto P[x_{\mathrm{i}}|s] \dots P\left[x_j \middle| s\right](1) = (\delta - \propto)u^n + \propto u^{num1}$$

Case (iii): $\boldsymbol{x_i}, \dots, \boldsymbol{x_j}$ is all zeros

This does not involve the left end term, i.e., $P\left[x_{\mathrm{i}},\ldots,x_{j}\middle|s\right]=(\varepsilon)P_{0}+\propto$ $(addition\ of\ \boldsymbol{other}\ terms\ with\ x_{\mathrm{i}},\ldots,x_{j}\ in\ them)$

$$P\left[x_{\mathrm{i}},\ldots,x_{j}\middle|s\right]=(\varepsilon-\propto)P_{0}+\propto(addition\ of\ \boldsymbol{all}\ terms\ with\ x_{\mathrm{i}},\ldots,x_{j}\ in\ them)$$

$$P\left[x_{\mathrm{i}},\ldots,x_{j}\middle|s\right]=(\varepsilon-\propto)P_{0}+\propto P[x_{\mathrm{i}}|s]\ldots P\left[x_{j}\middle|s\right](addition\ of\ permutation\ of\ the\ rest)$$

$$P\left[x_{\mathrm{i}},\ldots,x_{j}\middle|s\right]=(\varepsilon-\propto)P_{0}+\propto P[x_{\mathrm{i}}|s]\ldots P\left[x_{j}\middle|s\right](1)=(\varepsilon-\propto)d^{n}+\propto d^{num0}$$

In a similar manner $P\left[x_{\mathrm{i}},\ldots,x_{j}\middle|s'\right]$ is established.

Chapter 4 - Appendix 3: Ensuring Proper Market Prices Hold in the Computer Simulations

In the first round (private information stage)

Suppose that two types of bids exist in the market. Those bids with relevant information shall be called peach bids and denoted b_{P0} and b_{P1} to represent bids conditioned on information 0 and 1 respectively (notice that $b_{P0} \neq b_{P1}$). Those bids with NOT-relevant information shall be called lemon bids and denoted b_{L0} and b_{L1} to represent bids conditioned on information 0 and 1 respectively (notice that $b_{L0} = b_{L1}$).

At the private information stage (in the first round) the price $p = \frac{\sum b_i}{n}$ may be written in the form $p = \frac{b_i + \alpha b_{P0} + \beta b_{P1} + \lambda b_{L0} + \tau b_{L1}}{n}$ where some agent i bids b_i and other agents' bids consist of $\alpha + \beta$ peach bids and $\lambda + \tau$ lemon bids. Suppose that in the simulation an odd number of peach bids occur (which implies that $\alpha \neq \beta$)

Now assume that there exists a price for some agent where it cannot learn any information from at least one other agent in the first round. Then for that agent, even if other agents' information bits all changed to the opposite bit values the market price is unchanged, i.e., $p = \frac{b_i + \alpha b_{P1} + \beta b_{P0} + \lambda b_{L1} + \tau b_{L0}}{n}$.

That is, $\frac{b_i + \alpha b_{P0} + \beta b_{P1} + \lambda b_{L0} + \tau b_{L1}}{n} = \frac{b_i + \alpha b_{P1} + \beta b_{P0} + \lambda b_{L1} + \tau b_{L0}}{n}$ which simplifies to $\alpha b_{P0} + \beta b_{P1} = \alpha b_{P1} + \beta b_{P0}$ and implies the contradiction $\alpha = \beta$ since peach bids require that $b_{P0} \neq b_{P1}$. Therefore, proper market prices must hold in the first round.

Computer simulated markets for which an odd number of peach bids exist will sufficiently ensure that proper market prices hold in the first round.

However, the first round proper market price condition breaks down only for “even peach bids and when $\alpha = \beta$”. This occurs only when the quantity of 1’s and 0’s are the same for peach bids; a low probability event for large enough groups of traders as is the case in the simulations. Thus, the computer simulation embraces generality and does not enforce an odd number of relevant information bits. In summary, a setup that ensures proper market prices are highly likely (i.e. an ‘almost everywhere’ type concept holds).

In the last round (full information stage)

The agent learns everything possible about all agents’ information from the first round price. Equilibrium is reached in the second round market price given subsequent rounds offer no further information to the agents.

Specifically, the agent learns the value of the relevant information x_R in the market, but cannot infer the information bit values of agents that do not express relevant information. As such, rounds subsequent to the first round will see identical agent bids $E[f(s)|x_R]$. If indeed $x_R = x$ then a full information market price is generated.

Now consider the simulation and the possibility that an agent does not attain full information but the full information (DCE) price is still reached.

That is, given $\boldsymbol{x_R} \neq \boldsymbol{x}$, in the simulations this would mean:

$$\mathrm{p} = \mathrm{E}[\mathrm{f(s)}|x] = \mathrm{P}[\mathrm{s} = 1|\mathrm{x}] = \frac{\mathrm{P}[\mathrm{x}|\mathrm{s} = 1]\mathrm{P}[\mathrm{s} = 1]}{\mathrm{P}[\mathrm{x}]}$$

$$p_R = \mathrm{E}[\mathrm{f(s)}|x_R] = \mathrm{P}[\mathrm{s}=1|x_R] = \frac{\mathrm{P}[x_R|\mathrm{s}=1]\mathrm{P}[\mathrm{s}=1]}{\mathrm{P}[x_R]}$$

where $x_R = (x_1, \dots, x_r)$ and $x = (x_1, \dots, x_r, x_{r+1}, \dots, x_n) = (x_R, x_{-R})$ and $\mathrm{P}[x_{-R}|x_R, \mathrm{s}=1] \neq \mathrm{P}[x_{-R}|x_R]$ since all bits in x_{-R} are relevant information bits given that all bits in x are.

Assume $\boldsymbol{p} = \boldsymbol{p_R}$

Then $\mathrm{p} = \mathrm{E}[\mathrm{f(s)}|x] = \mathrm{P}[\mathrm{s}=1|x_R, x_{-R}] = \frac{\mathrm{P}[x_R, x_{-R}|\mathrm{s}=1]\mathrm{P}[\mathrm{s}=1]}{\mathrm{P}[x_R, x_{-R}]} = \frac{\mathrm{P}[x_{-R}|x_R,\mathrm{s}=1]\mathrm{P}[x_R|\mathrm{s}=1]\mathrm{P}[\mathrm{s}=1]}{\mathrm{P}[x_{-R}|,x_R]\mathrm{P}[x_R]} = \frac{\mathrm{P}[x_{-R}|x_R,\mathrm{s}=1]}{\mathrm{P}[x_{-R}|,x_R]} p_R$

Which implies $\mathrm{P}[x_{-R}|x_R, \mathrm{s}=1] = \mathrm{P}[x_{-R}|, x_R]$; since $p = p_R$. But this contradicts with $\mathrm{P}[x_{-R}|x_R, \mathrm{s}=1] \neq \mathrm{P}[x_{-R}|, x_R]$ above.

Therefore in the simulation it can easily be seen that the DCE price is not the same as the price that is formed with less than full information. If it were, then this would violate the full information proper market price condition.

In summary, $\boldsymbol{p \neq p_R}$ for $\boldsymbol{x_R \neq x}$ as required. That is the full information proper market price condition holds in the simulation.

In summary, computer simulation tests of the hypotheses in markets that ensure proper market prices are run.

Chapter 4 - Appendix 4: The Key Piece of Code for the Computer Simulated Decision Market Function

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Appendix C: Chapter 5 Appendices

Chapter 5 - Appendix 1: Algorithmic Traders

The key features of the algorithmic traders populating the prediction market games with human participants are described in the following. The important feature is the simple specification of the algorithmic traders that align with the proper market prices condition.

Agent learning (from market price)

In the first round, algorithmic trading agents are required to submit bids conditioned on that agent's private information bit and also commonly known prior probability; which are available to traders in the game. In the second round, algorithmic trading agents are required to submit bids conditioned on that agent's private information bit and also the information attained via the previous round prediction market stock price. The prediction market stock price is simply a publicized probability estimate of the event underlying the stock, and may be used in conjunction with what the trader already knows to update their belief. Considering both the market and their own information, a trader may be inclined towards or away from the market information; given the uncertainty of information context of the game. In this game algorithmic traders consider the market price implied probability as converging towards the best available prior probability as the game is played. They update an estimate of the prior with their private information to ultimately arrive at their bid. The specific details of this Bayesian updating is as follows:

Agent i will reason as follows in the first round:

- $\mathrm{P}(\mathrm{s}=1)$ is the commonly known prior probability that the stock will pay \$1. We may think of this as information sourced from the world.
- In the first round algorithmic trader i bids in a Bayesian way $\mathrm{b_i} = \mathrm{E}[\mathrm{f(s)}|\mathrm{x_i}] = \mathrm{P}[\mathrm{s}=1|\mathrm{x_i}] = \frac{\mathrm{P}[\mathrm{x_i}|\mathrm{s}=1]\mathrm{P}[\mathrm{s}=1]}{\mathrm{P}[\mathrm{x_i}]}$.

Agent i will reason as follows in the second round:

- At the end of the first round, market price p is formed and reflects the current round's market belief of the probability of the outcome.

- The first round price p contains important information to update the prior probability $P[s = 1]$. It may me utilized in conjunction with the agent's private information bit to estimate a prior probability to arrive at a second round bid.
- Keeping things simple, the prior probability estimate may be considered the weighted average of the agent's private belief x_i and the public belief p. The assumption is that the agent will consider the true prior is somewhere between what it knows (privately) and what the market price has estimated. That is, $P[s = 1] = w_x x_i + w_p p$, where $w_x + w_p = 1$.
- After multiple rounds p changes and becomes a better estimate than the agent's private information, as such, in the limit, the agent will use the prior probability estimate $P[s = 1] = \lim_{w_p \to 1} w_x x_i + w_p p = p$.
- Therefore the agent's bid in rounds after round 1 is simply $b_i = E[f(s)|x_i] = \frac{P[x_i|s=1](w_x x_i + w_p p)}{P[x_i|s=1](w_x x_i + w_p p) + P[x_i|s=0](1-(w_x x_i + w_p p))}$
- Notice in the limit that $b_i = E[f(s)|x_i] = \frac{P[x_i|s=1](w_x x_i + w_p p)}{P[x_i|s=1](w_x x_i + w_p p) + P[x_i|s=0](1-(w_x x_i + w_p p))} \rightarrow \frac{P[x_i|s=1]p}{P[x_i|s=1]p + P[x_i|s=0](1-p)}$ as $w_p \rightarrow 1$

Ensuring proper market prices hold

The prediction market of this game is required to be one in which proper market prices exist, i.e., at the private and full information stage certain conditions must hold.

In short, the assumed conditions amount to the following statements:

- It is very unlikely to find a market in which the first round market price does not reveal information to an agent about other agents,
- It is very unlikely that when all agents possess full information about the market that a subsequent market price causes an agent to change their information.

Suppose that two types of bids exist in the prediction market. Those bids with relevant information shall be called peach bids and denoted b_{P0} and b_{P1} to represent bids conditioned on information 0 and 1 respectively (notice that $b_{P0} \neq b_{P1}$ given the relevant information requirement). Those bids with NOT-relevant information shall be called lemon bids and denoted b_{L0} and b_{L1} to represent bids conditioned on information 0 and 1 respectively (notice that $b_{L0} = b_{L1}$ given the NOT-relevant information requirement).

At the private information stage (in the first round) the market price $p = \frac{\sum b_i}{n}$ and utilizing notions in the previous paragraph this may be rewritten as $p = \frac{b_i + \alpha b_{P0} + \beta b_{P1} + \lambda b_{L0} + \tau b_{L1}}{n}$; where some agent i bids b_i and other agents' bids consist of $\alpha + \beta$ peach bids and $\lambda + \tau$ lemon bids. Suppose that there are an odd number of peach bids (which implies that $\alpha \neq \beta$)

Now assume that there exists a price for some agent where it cannot learn any information about other agents in the first round. Then for that agent, if other agents' information bits changed to the opposite bit values the price would remain unchanged (otherwise they would be able to infer information of other traders). Hence, $p = \frac{b_i + \alpha b_{P1} + \beta b_{P0} + \lambda b_{L1} + \tau b_{L0}}{n}$ is possible.

Thus, $\frac{b_i + \alpha b_{P0} + \beta b_{P1} + \lambda b_{L0} + \tau b_{L1}}{n} = \frac{b_i + \alpha b_{P1} + \beta b_{P0} + \lambda b_{L1} + \tau b_{L0}}{n}$ which simplifies to $\alpha b_{P0} + \beta b_{P1} = \alpha b_{P1} + \beta b_{P0}$ and since peach bids require that $b_{P0} \neq b_{P1}$ this implies the contradiction $\alpha = \beta$.

Therefore, in theory, an odd number of peach bids ensure learning occurs in the first round. Learning is not theoretically guaranteed for an even number of peach bids, however, this theoretical reasoning simply estimates the stochastic real-world and as such it may be said that learning is probable in this stochastic game, i.e., proper market prices probably hold in the first round of the game.

At the full information stage the price $p = \frac{\sum b_i}{n} = \frac{\sum \frac{P[x_i|s=1](w_x x_i + w_p p)}{P[x_i|s=1](w_x x_i + w_p p) + P[x_i|s=0](1-(w_x x_i + w_p p))}}{n}$ is formed. Assume that the proper market price does not hold at the full information stage. That is, there exists some agent i who may change its information to x'_i with the same full information market price being attained; Formally:

$$\frac{P[x'_i|s=1](w_x x'_i + w_p p)}{P[x'_i|s=1](w_x x'_i + w_p p) + P[x'_i|s=0](1-(w_x x'_i + w_p p))} = \frac{P[x_i|s=1](w_x x_i + w_p p)}{P[x_i|s=1](w_x x_i + w_p p) + P[x_i|s=0](1-(w_x x_i + w_p p))}$$

Without loss of generality let $x_i = 1$, then simplifying the equation obtains:

$$\frac{(1-P[x_i|s=1])(w_p p)}{(1-P[x_i|s=1])(w_p p)+(1-P[x_i|s=0])(1-w_p p)} = \frac{P[x_i|s=1](w_x+w_p p)}{P[x_i|s=1](w_x+w_p p)+P[x_i|s=0](1-(w_x+w_p p))}$$

$$\frac{1}{1+\frac{(1-P[x_i|s=0])(1-w_p p)}{(1-P[x_i|s=1])(w_p p)}} = \frac{1}{1+\frac{P[x_i|s=0](1-(w_x+w_p p))}{P[x_i|s=1](w_x+w_p p)}}$$

$$\frac{(1-P[x_i|s=0])(1-w_p p)}{(1-P[x_i|s=1])(w_p p)} = \frac{P[x_i|s=0]\left(1-(w_x+w_p p)\right)}{P[x_i|s=1](w_x+w_p p)} \ (*)$$

Without loss of generality assume that $P[x_i = 1|s = 1] > 0.5$ and $P[x_i = 1|s = 0] < 0.5$. This would mean that the left-hand-side of $(*)$ is of greater value than the right hand side. That is, $\frac{(1-P[x_i|s=0])(1-w_p p)}{(1-P[x_i|s=1])(w_p p)} > \frac{P[x_i|s=0](1-(w_x+w_p p))}{P[x_i|s=1](w_x+w_p p)}$ contradicts with $(*)$.

Therefore, the full information stage proper market price property holds.

In summary, analysis of these algorithmic traders in a stochastic prediction market setting suggest proper market prices likely hold in the prediction market web game of this study.

Chapter 5 - Appendix 2: Post Hoc Tests for Normality and an Unrequired Continuity Correction Factor

Post hoc tests to ensure that the distribution is normal and that a continuity correction factor is not required is as follows:

Testing normality assumption holds

Null hypothesis: normally distributed

Alternative hypothesis: not normally distributed

r	DFB (PM)	mean	z^2		Br	B0
					3.68738	-3.48025
0	0	0.029879	0.0308			
0.3	0	0.085174	0.093104			
0.5	0	0.162934	0.194648			
0.9	0	0.459686	0.850775			
0.95	1	0.50569	0.977496			
1	1	0.551598	0.812914			
0	0	0.029879	0.0308			
0.3	0	0.085174	0.093104			
0.5	1	0.162934	5.137466			
0.9	0	0.459686	0.850775			
0.95	0	0.50569	1.023022			
1	1	0.551598	0.812914			
0	0	0.029879	0.0308			
0.1	0	0.042635	0.044534			
0.3	0	0.085174	0.093104			
0.85	0	0.414358	0.707529			
0.95	1	0.50569	0.977496			
1	1	0.551598	0.812914			
0	0	0.029879	0.0308			
0.3	1	0.085174	10.74065			
0.5	0	0.162934	0.194648			
0.3	0	0.085174	0.093104			
0.5	0	0.162934	0.194648			
0.9	0	0.459686	0.850775			
0.95	0	0.50569	1.023022			
1	1	0.551598	0.812914			
0	0	0.029879	0.0308			
0.3	0	0.085174	0.093104			
0.5	0	0.162934	0.194648			
0.9	0	0.459686	0.850775			
		sum z^2	28.68409			
		p value	0.428679	retain null hypothesis of normality		

Therefore, since $p = 0.43$, the null hypothesis is retained. That is, a normal distribution across experiments holds.

Testing the continuity correction factor is not required

The continuity correction factor (CCF) is not required if $np > 5$ and $nq > 5$ in a binomial distribution. Refer the following analysis:

n	30
p	0.266667
q	0.733333
np	8
nq	22

since np and nq>5 then CCF is negligible and the normal and chi distributions are applicable to this binomial context
CCF: continuity correction factor

Therefore, since $np = 8 > 5\ and\ nq = 22 > 5$, an adjusting CCF is not required (Schader and Schmid, 1989) and the normal and chi-squared analyses may be suitably employed to analyze results.

Chapter 5 - Appendix 3: A Priori Experimental Theoretical Design

Considering time and budget an a priori theoretical design for the experiment was devised to maximize the power of test and minimize resources including human participant playing hours invested in the game. This is comprehensively described as follows

University resources (e.g. money and time) are required in this game play experiment. The initial game play estimate suggests 50 minutes per human participant with 110 participants in total. Given these investments are not insubstantial an attempt to design and justify an efficient experimental setup is made. Simply put, an experimental design that does not inherently provide the ability to test the hypothesis of interest needs to be avoided. Justifying a priori an efficient experimental design is in no way a trivial exercise and as such simplifying assumptions for the power of test calculations have been made. Because of these idealized assumptions a simple simulation of the experiment will also be undertaken to provide an additional design check. In short, this conservative analysis suggests that the proposed experimental design is likely fit for purpose.

Hypothesis of interest

Assume the experiment runs 40 games; 20 controls paired with 20 treatments. In the game, a control and its treatment are identical in every way with possible exception being the value of relevant information level r; whereby the control game has value $r = 1$ and the corresponding treatment game has value $r \in [0,1]$. When the control and the treatment are considered to behave the same a '1' is recorded, else a '0' is recorded; whereby the 'same' average order quantity in the final round is of interest. This dichotomy suggests the use of a binary logit response model. The maximum likelihood estimation results in a value for β_0 and β_r in the probability function:

$$P(control\ and\ treatment\ behave\ the\ same, r) = \frac{e^{\beta_0+\beta_r r}}{1+e^{\beta_0+\beta_r r}}.$$

The null hypothesis is $\boldsymbol{H_0}$: $\boldsymbol{\beta_r = 0}$ and the alternative $\boldsymbol{H_A}$: $\boldsymbol{\beta_r \neq 0}$.

Conservatively assume $P(r = 0) = 0.5$. From the theoretical model it is expected that $P(r = 1) = 1$, but conservatively assume that $P(r = 1) = 0.95$. It is yet to be experimentally determined in the study of this chapter that probability increases with increasing r; but these conservative assumptions still allow room to observe this possibility.

The assumption $P(r=0)=\frac{e^{\beta_0}}{1+e^{\beta_0}}=0.5$ implies $\beta_0=0$.

The assumption $P(r=1)=0.95$, implies $\beta_r=2.94$

If the null hypothesis is not true then a statistically significant difference may be observed between $P(r=0)$ and $P(r=0.5)$.

For the $n=20$ control-treatment results since the logit setup is inherently a binomial distribution the estimate for population mean p and variance $\frac{p(1-p)}{n}$ at a particular r is applicable. For example, a binomial distribution with mean $\mu=p=0.5$ and variance $\sigma^2=\frac{p(1-p)}{n}=0.0125\approx 0.1118^2$ exists for $r=0$. The additional assumption that a normal distribution with $\mu=0.5$ and $\sigma^2=0.0125\approx 0.1118^2$ for $r=0$, may be used to estimate the binomial distribution and simplifies the following calculations.

If it is assumed that the Null is true, then $\beta_r=0$ and since $\beta_0=0$, it must be the case that $\mu=0.5$ and $\sigma^2=0.0125\approx 0.1118^2$ in the standardised normal distribution $z=\frac{x-\mu}{\sigma}$; where x is the experimentally observed mean. If $z>1.645$, this would suggest that the null hypothesis could be rejected at a 5% significance level. Which would coincide with values of $x>z\sigma+\mu=1.645\times 0.1118+0.5=0.68$ under the null hypothesis assumptions.

If it is assumed that the null is not true then $\beta_r=2.94$ (i.e. it is not zero as was assumed in the null) and there would exist an approximate normal distribution with an estimated population mean $P(r=0.5)=0.813$ and a variance of $0.0076=0.087^2$. Now the observed experimental mean value of $x>0.68$ under this alternative assumption would correspond to $z>\frac{0.68-0.813}{0.087}=-1.53$. That is, the probability of observing this under the alternative assumption would be greater than 90%.

Notice that these considerations are simply the power of test concept whereby the region $x>0.68$ is rejected under the null at a 5% significance level and accepted as the alternative with at least 90% probability. The power of the test is simply the ability of the test to correctly reject the null. In short, the power of the test is 90%.

A ‘rule of thumb’ good power of test is generally considered greater than 80%. However the 90% estimate is only suggestive of a good experimental design (considering the many albeit conservative assumptions). As such, a simple simulation of the experiment to double-check this result is prudent and performed in Chapter 5 Appendix 4.

Chapter 5 - Appendix 4: A Priori Experimental Design Simulation

This simulation was performed as a double check to the power of test calculation in Chapter 5 Appendix 3.

Multiple simulations were run in excel with randomly produced data from the sigmoid shaped (logit) distribution in r. The question of interest is: does the data produced conform to the logit normality assumption and then also reveal the statistical significance of the relevant information level? Appendix Figure 1 and Appendix Figure 2 depict a typical simulation run. Data from each run was analyzed. Firstly a chi-squared test (see below Appendix Figure 1) finds that the normality assumption is not rejected. Thus the assumption of normality is satisfied.

random r	dce decision random linear	prediction	correct (count method)	z	chi sq (normality)		
0.25	1	0	0	1.02025962	1.040929693		
0.9	1	1	1	0.05613008	0.003150586		
0.15	0	0	1	-0.6273613	0.393582259		
0.5	1	1	1	0.33441199	0.111831378		
0.35	1	1	1	0.65303905	0.426460006		
0.75	1	1	1	0.10961071	0.012014507		
0.8	1	1	1	0.08769346	0.007690143		
0.65	1	1	1	0.17124761	0.029325744		
0.75	1	1	1	0.10961071	0.012014507		
0.35	0	1	0	-1.531302	2.344885772		
0.15	1	0	0	1.59397771	2.540764926		
0.85	1	1	1	0.07015868	0.004922241		
0.85	1	1	1	0.07015868	0.004922241		
0.25	0	0	1	-0.9801427	0.960679676		
0.9	1	1	1	0.05613008	0.003150586		
0.5	1	1	1	0.33441199	0.111831378		
0.8	1	1	1	0.08769346	0.007690143		
0.35	0	1	0	-1.531302	2.344885772		
0.45	1	1	1	0.41799165	0.174717022		
0.4	1	1	1	0.5224604	0.272964874		
num 1's	16		count (/20)		chi sq	df	p value
num 0's	4		16		10.80841345	18	0.90230912
total	20			post-hoc check: transform N(0,1)			retain
Bo	-2.270991596			0.05124389	mean is near 0		Null: normally distributed
Br	8.923509387			0.75239599	stdev is near 1		

Appendix Figure 1 Chi-squared test with the Null hypothesis being "normality exists".

Finally, if the null hypothesis $\beta_r = 0$ is assumed true the simulation reject this at the 5% significance level, as illustrated in the below Appendix Figure 2.

experiments	random r	dce decision random linear	z	chi sq (normality)	df	p value
1	0.25	1	3.112716439	9.689003628		
2	0.9	1	3.112716439	9.689003628		
3	0.15	0	-0.3212628	0.103209787		
4	0.5	1	3.112716439	9.689003628		
5	0.35	1	3.112716439	9.689003628		
6	0.75	1	3.112716439	9.689003628		
7	0.8	1	3.112716439	9.689003628		
8	0.65	1	3.112716439	9.689003628		
9	0.75	1	3.112716439	9.689003628		
10	0.35	0	-0.3212628	0.103209787		
11	0.15	1	3.112716439	9.689003628		
12	0.85	1	3.112716439	9.689003628		
13	0.85	1	3.112716439	9.689003628		
14	0.25	0	-0.3212628	0.103209787		
15	0.9	1	3.112716439	9.689003628		
16	0.5	1	3.112716439	9.689003628		
17	0.8	1	3.112716439	9.689003628		
18	0.35	0	-0.3212628	0.103209787		
19	0.45	1	3.112716439	9.689003628		
20	0.4	1	3.112716439	9.689003628		
	num 1's	16		chi sq	df	p value
	num 0's	4		155.4368972	18	6.49326E-24
	total	20				reject
	Bo	-2.270991596				Null: Br = 0
	Br	0				

Appendix Figure 2 Chi-squared test with the Null hypothesis being "$\beta_r = 0$".

In summary, these simple simulations find that if relevant information is significant then the proposed experimental design will likely detect this.

Chapter 5 - Appendix 5: 3 Pilots (Interface & Functionality Test, Decision Market Game Test, Prediction Market Game Test)

Pilot 1: Interface and functionality test of decision market webgame

- Modular programming with stubs and harness testing provided a low risk for coding errors.
- Five human traders played a decision market game and provided feedback on the game interface
  - Less information on the interface was preferred
  - The training video was useful
  - The sliders made the interface simple to use but it would be good if they automatically calculated costs of orders
- Feedback was incorporated into new webgame
  - Interface words were reduced
  - The training video was kept but shortened
  - The sliders were programmed to calculate costs of orders

Pilot 2: Decision market webgame test

- 10 decision market games were played and feedback provided by human traders
  - A concrete scenario was suggested instead of project A stocks in the original game. This was changed to an election game setting, but because of the risk of bias (due to a real-world election running at that time) the scenario was change to a "build a Dog Friendly Beach DFB stock" game, i.e., the DFB game scenario was concrete enough for humans to relate to it, and lowered the risk of humans bringing real-world information into the game.
  - The game was played amongst staff and within Dr Sizhong Sun's economics' class. The use of a powerpoint (learning scaffold) to "walk people through" the first round of the game was considered very useful. At this stage rudimentary analysis of results was only suggestive of the statistical significance of relevant information level. With such small samples, convergence issues to a solution were encountered when using statistical software; consistent with previous research findings that noted that convergence to a maximum likelihood estimation solution can be problematic in small samples (Allison, 2008).

Pilot 3: Prediction market webgame test

- Meetings with Dr Sun resulted in proposing just the prediction market part of the game be run; given the decision market game was built using prediction markets and relevant information level could be usefully tested at this building blocks level. The game was recoded into a prediction market game.
- The new prediction market game was first run amongst the PhD cohort and then at the college seminar. It functioned well and was considered ready for implementation

Chapter 5 - Appendix 6: Specific Details and Screenshots of the Actual Web-Based Prediction Market Game

The computer simulation work of the previous chapter is modified to allow human and computer agents trading in a web based prediction market game.

60 market games were played; wherein each of the 30 treatment games were paired to a control game such that they were identical in every way with the possible exception of the relevant information level r (where the control game had $r = 1$ and the treatment game had $r \in [0,1]$). The observation of interest is whether the treatment and control market converge to towards the same average order quantity by the end of the fifth round. The average order quantity is the average of traders' orders (bids) and the control market with all agents fully informed results in the best possible prediction (i.e. it is the equivalent to the direct communication equilibrium of the theoretical model). The statistic γ will record "1" or "0" will record whether a treatment market's equilibrium (the average order quantity) converges to the control (dce average order quantity) equilibrium. Specifically, $\gamma = 1$ will denote that the control and treatment average order quantities are within 5 unit of each other by the end of the final round.

Each market game will have 5 rounds. The researcher guides participants on the first round using a powerpoint presentation. The remaining four rounds are played with no assistance. A single market game will run for 20 minutes. The control and its treatment are run in the same tutorial class, with human traders assigned randomly to games. There are 5 tutorial classes ranging from business economics to statistics and environmental economics, and ranging from undergraduate to masters' candidate experience.

Each market will contain 20 agents in total of which 1 or 2 agents will be humans; the contract that pays, the uncertain information and the number of human traders being randomly assigned to each game. Each human trader plays only one market game to ensure no learning effects across market games.

Each market game will be assigned a relevant information level r corresponding to the proportion of the 20 agents assigned relevant information in the game. The associated relevant information bits were then randomly assigned to the market traders; which included human and computer traders. In short, an agent will either receive a relevant information bit or not, for the two stocks “DFB (Dog Friendly Beach) is built” stock and “DFB is not built” stock; if it receives a relevant information bit, the value of that bit will be 0 or 1. For example, an agent who receives relevant information bit of 1 for the “DFB (Dog Friendly Beach) is built” stock, has received private and uncertain information indicating that the DFB stock will be paid $1 at the end of round 5.

For each market round, an agent is expected to bid based on information implicitly contained in the game, i.e., a commonly known state of the world probability distribution, commonly know conditional probability distribution of information, their private bit of relevant information (if assigned one) and the previous round average market order quantity for all rounds after round 1. This information is sourced over multiple rounds of the game using the web page interface and sliders. Sliders provide a means for the human trader to determine cost of orders and tables and rules provide game relevant information for traders to make a best possible informed order quantity bid.

Implementation details

Given the participation of multiple human agents in each market game, a simple web-based interface that displays the key information was coded in php with records of trade stored as CSV files. Using php with no cookies or other complicating feature allowed the game to be played on any device with a web browser (e.g. phone, tablet, computer). Each trader was provided with a unique user name and password; with their identities kept anonymous per the H6263 ethics authorization. Traders were told that should they score the highest game money (being the sum of game bank balance remaining and the stocks they invested in that paid) by the end of the game across all web-games played that they would then be eligible for the opportunity of winning $200.

Key prediction market game screenshots

Human traders were presented with the following login screen and told they had a choice to play or not for an opportunity to win $200

INVESTMENT GAME LOGIN PAGE

Welcome to the Investment Game.

YOUR IMPLIED CONSENT TO TERMS OF PLAYING THE GAME.
Before you enter the game please read the following.
Terms of playing the game
If you agree to be involved in the study, you will be given play money to play a stockmarket type computer game for 2 rounds of the game.
Your aim is to make the most game money. You are to submit 4 quantity orders on each round of the game; there are 4 events that each need a quantity order.
Each round should take about 5 minutes of your time.

The person who makes the most game money across all games wins a $200 prize !!

Taking part in this study is completely voluntary and you can stop taking part at any time without explanation or prejudice.
Your responses and contact details will be strictly confidential. The data from the study will be used in research publications and reports. You will not be identified in any way in these publications.

If you have any questions about the study, please contact:
Mr Daniel Grainger :
Dr Sizhong Sun:
If you agree to the terms of playing the game and wish to proceed into the game please enter your login name and password and click the SUBMIT button.

Please be aware that pressing the SUBMIT button implies your consent to the Terms of playing the game.

LOGIN AND CONSENT

LOGIN NAME 1 - 1

The hypothetical scenario with training video was a remnant of pilots and made available to players – but the powerpoint guide used by the researcher for the first round bids made it redundant.

DISCLAIMER: The following game considers a hypothetical scenario.

INVESTMENT GAME

STEP 1: THE HYPOTHETICAL SCENARIO [DEFINITELY READ THIS !!]

Please CLICK on this HYPOTHETICAL SCENARIO

Information required to make an informed bid considering assets owned and private and public information is presented in the table.

STEP 2: LOOK AT THE INFORMATION IN THE BELOW TABLE

YOUR CURRENT POSITION			
Events	Number of units (stocks) you OWN	AVERAGE NUMBER of UNITS (STOCKS) bought by PLAYERS in the PREVIOUS ROUND	THE LATEST INFORMATION suggests that the PROBABILITY the EVENT OCCURS AT THE END OF THE GAME is
TPS builds the DFB	0	50	20%
TPS does NOT build the DFB	0	50	80%

YOUR REMAINING GAME MONEY = $250

TOTAL ASSETS
TPS builds the DFB STOCK: 0 units---- PAID $1 PER UNIT ONLY IF THE EVENT OCCURS AT THE END OF THE GAME
TPS does NOT build the DFB STOCK: 0 units---- PAID $1 PER UNIT ONLY IF THE EVENT OCCURS AT THE END OF THE GAME
Cash in hand: $250

Slider are used to order units of stock and also calculate the cost to assist human traders in making informed (ordering) decisions.

STEP 3: USE THE SLIDERS (below) TO BUY UNITS.

PLEASE CLICK HERE TO UNDERSTAND HOW THE NUMBER OF UNITS BOUGHT AFFECTS THE COST PER UNIT

The current time is: Tue:10:39:57, AND the market is open between 10AM (10:00:00) and 5PM (17:00:00) from Monday to Thusday.

NUMBER OF (TPS BUILDS DFB) UNITS YOU WOULD LIKE TO ORDER:
0 units x $0 per unit = $0 total cost for these units"
[Note: you are paid $1 for ALL "TPS builds the DFB" units you OWN only if that event occurs at the end of the game]

NUMBER OF (TPS DOES NOT BUILD DFB) UNITS YOU WOULD LIKE TO ORDER:
0 units x $0 per unit = $0 total cost for these units"
[Note: you are paid $1 for ALL "TPS does NOT build the DFB" units you OWN only if that event occurs at the end of the game]

Confirmation screen is a deliberate mechanism for the trader to pause and consider the trade they are committing to.

STEP 4: CONFIRM & AUTHORISE YOUR ORDERS WITH YOUR LOGIN NAME AND PASSWORD.

CAUTION: YOU CAN ONLY CLICK THE SUBMIT BUTTON ONCE PER DAY - SO YOU NEED TO BE SURE WHERE YOU PLACE ALL SLIDERS

LOGIN NAME [ ] - [ ]

PASSWORD [ ]

Submit

Chapter 5 - Appendix 7: Analysis of Results via a MLE Binary Response Logit Model

Independence across prediction market webgames is assumed.

$\gamma = 1$ denotes the convergence of the average quantity order of the treatment webgame to that of the control. Hence define: $[\gamma = 1] = F_{\gamma=1}(\beta; x) = \frac{e^{\beta x^T}}{1+e^{\beta x^T}}$, where $\beta = (\beta_0, \beta_r, \beta_h)$ and $x = (1, r, h)$. Since $\gamma = 1$ or $\gamma = 0$ then $F_{\gamma=0}(\beta; x) = 1 - F_{\gamma=1}(\beta; x) = \frac{1}{1+e^{\beta x^T}}$.

30 data points (of the form (r, h, γ)) and the maximum likelihood technique allow an estimate of $\beta = (\beta_0, \beta_r, \beta_h)$.

Since $\gamma = 1$ or $\gamma = 0$ for each market game (let k represent the number of games for which the result $\gamma = 1$ was observed) and independence across market games is assumed, then utilizing the maximum likelihood estimate technique maximizes:

$$L(\beta) = \prod_{j}^{k} F_{\gamma=1}(\beta; x_j) \prod_{i}^{40-k} F_{\gamma=1}(\beta; x_i)$$

to find the parameters $\beta = (\beta_0, \beta_r, \beta_h)$ given all observational data including experiment $d's$ data $x_d = (1, r_d, h_d)$.

Let $l = \ln(L(\beta))$ therefore $\frac{dl}{d\beta} = \frac{1}{L} \times \frac{dL}{d\beta} = 0$ implies that maximizing l is the same as maximizing L; assuming $L(\beta) \neq 0$. Furthermore a maximum for L does exist at β since $\frac{d^2 l}{d\beta^2} = \frac{1}{L} \times \frac{d^2 L}{d\beta^2} - \frac{1}{L^2} \times \frac{dL}{d\beta} = \frac{1}{L} \times \frac{d^2 L}{d\beta^2}$ and because l differentiated twice is a negative number at the stationary value β (without formally proving this it is enough to notice that l is the sum of natural logs of numbers less than 1 and hence is always negative with a negative value closest to zero (the maxima).

$\beta_0, \beta_r, \beta_h$ that maximise:

$$l(\beta) = \ln\big(L(\beta)\big) = \sum_{j}^{k} \beta_0 + \beta_r r_j + \beta_h h_j - \sum_{i}^{40} ln\big(1 + e^{\beta_0 + \beta_r r_i + \beta_h h_i}\big)$$

are identified at

$$\frac{\partial l}{\partial \beta_0} = k - \sum_{i}^{40} \frac{1}{1 + e^{\beta_0 + \beta_r r_i + \beta_h h_i}} = 0$$

$$\frac{\partial l}{\partial \beta_r} = \sum_{j}^{k} r_j - \sum_{i}^{40} \frac{r_i e^{\beta_0 + \beta_r r_i + \beta_h h_i}}{1 + e^{\beta_0 + \beta_r r_i + \beta_h h_i}} = 0$$

$$\frac{\partial l}{\partial \beta_h} = \sum_{j}^{k} r_j - \sum_{i}^{40} \frac{r_h e^{\beta_0 + \beta_r r_i + \beta_h h_i}}{1 + e^{\beta_0 + \beta_r r_i + \beta_h h_i}} = 0$$

Without an analytic solution to this maximization problem the **Newton-Raphson method** to estimate a numerical solution may be used as follows:

Let $f_0(\beta_0) = \frac{\partial l}{\partial \beta_0}, f_r(\beta_r) = \frac{\partial l}{\partial \beta_r}, f_h(\beta_h) = \frac{\partial l}{\partial \beta_h}$

These imply that $f'_0(\beta_0) = \frac{\partial f_0(\beta_0)}{\partial \beta_0}$

And the $(n+1)^{th}$ estimate for $\beta_{0,n+1} = \beta_{0,n} - \frac{f_0(\beta_{0,n})}{f'_0(\beta_{0,n})}$.

In a similar manner other parameters may be determined.

An initial *guess* for $(\beta_{01}, \beta_{r1}, \beta_{h1})$ and using $\beta_{0,n+1} = \beta_{0,n} - \frac{f_0(\beta_0)}{f'_0(\beta_0)}$ find the next estimate for β_0 is $\beta_{02} = \beta_{01} - \frac{f_0(\beta_{01})}{f'_0(\beta_{01})}$. Similarly use $(\beta_{01}, \beta_{r1}, \beta_{h1})$ to find β_{r2} and also $(\beta_{01}, \beta_{r1}, \beta_{h1})$ to find β_{h2}.

Now that a new estimate $(\beta_{02}, \beta_{r2}, \beta_{h2})$ is obtained the process is repeated.

After many iterations an estimate $\beta^\wedge = (\beta^\wedge{}_0, \beta^\wedge{}_r, \beta^\wedge{}_h)$ may be the global maxima; however this needs to be verified by randomly selecting a different initial guess and repeating.

If it is the global maxima the specific functional form for the model is determined:

$$\boldsymbol{F(x) = F^\wedge{}_{\gamma=1}(\beta^\wedge; x) = \frac{e^{\beta^\wedge x^T}}{1 + e^{\beta x^{\wedge T}}}}$$

To determine the significance of β^ the assumption of the binomial model approximating normality is made (which is tested by determining that a continuity correction factor is not required) and then the usual significance tests applied.

With this assumption notice that $L(\beta; under\ null\ hypothesis) = \prod\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2}\right] = \prod\left[e^{-\frac{1}{2}\left(\frac{\bar{x}-\mu_0}{\sigma}\right)^2}\right]\prod\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}\right]$

$$L(\beta; under\ alternative\ hypothesis) = \prod\left[e^{-\frac{1}{2}\left(\frac{\bar{x}-\mu}{\sigma}\right)^2}\right]\prod\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}\right]$$
$$= \prod\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\bar{x}}{\sigma}\right)^2}\right] since, under\ the\ alternative,$$

$maximisation\ of\ parameters\ implies\ highest\ likelihood$
$which\ also\ implies\ \mu = \bar{x}$

Therefore $-2 \times \ln\left(\frac{L(\beta; under\ null\ hypothesis)}{L(\beta; under\ alternative\ hypothesis)}\right) = -2 \times \ln\left(\prod\left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_0}{\sigma}\right)^2}\right]\right) = -2 \times \ln\left(e^{-\frac{1}{2}\Sigma\left(\frac{x-\mu_0}{\sigma}\right)^2}\prod\left[\frac{1}{\sigma\sqrt{2\pi}}\right]\right) =$
$\sum\left(\frac{x-\mu_0}{\sigma}\right)^2 \sim\chi^2\ with\ degrees\ of\ freedom\ equal\ to\ the\ number\ of\ parameters\ in\ \beta$
$less\ the\ number\ of\ fixed\ parameter\ (i.e. 1\ fixed\ parameter)$

To determine the best possible model, since $-2 \times \ln\left(\frac{L(\beta; under\ null\ hypothesis)}{L(\beta; under\ alternative\ hypothesis)}\right)$ and some other specification $-2 \times \ln\left(\frac{L(b; under\ null\ hypothesis)}{L(b; under\ alternative\ hypothesis)}\right)$ (with say 1 more variable than the previous specification) are both distributed as chi-squared distributions, then subtracting one distribution from the other arrives at a chi-squared distribution with 1 degree of freedom. If this “1 degree of freedom” distribution does not have a sufficiently low p value (e.g. $p < 0.05$) then the extra variable should be discarded.

Chapter 5 - Appendix 8: Rational, Risk-Neutral, and Myopic Incentive Compatible Game

The prediction market webgame is designed to be incentive compatible with a Rational, risk-neutral, and myopic trader. Mathematically, stated:

Consider trader i who possesses private information $x_i^{(k)}$ about stock k. Given the previous round price p (after the first round), trader i will bid $b_i = E[f^{(k)}(s)|x_i^{(k)}, p]$ if they are rational, risk neutral and myopic.

<u>The rules are as follows:</u>

Rule 1: each round each trader submits a quantity bid from 0 stocks to not greater than 100 stocks (using the sliders provided).

Rule 2: The quantity bid results in the trader seeing the cost of their order. The increased demand for the stock is assumed to increase the per unit price. Specifically, if the trader orders b stocks then the per unit price is $\frac{\mathrm{b}}{2}$ cents per stock; with a total cost for the order of $\frac{\mathrm{b}}{2} \times b$ cents.

Rule 3: At the end of the market game an event is revealed to have had occurred, the trader will receive 100 cents for each stock associated with that event; otherwise they receive 0 cents. For example, if at the end of the market game event A is revealed as occurring, then an agent who holds say 7 A stocks will receive 700 cents.

Rule 4: At the end of the market game, a total T is calculated; being the trader's game money remaining plus the money paid to each stock in accordance with rule 3. The trader with the greatest T is the winner and receives the opportunity to win $200.

Notice that rules 1 to 4 imply that the Total wealth (T) of a trader on each round is calculated as: $T = value\ of\ stock\ \times\ quantity\ of\ stocks\ ordered\ (i.e. "b") -$
$purchase\ price\ per\ stock\ \left(i.e. "\frac{\mathrm{b}}{2}"\right) \times\ quantity\ of\ stocks\ (i.e. "b") +$
$number\ of\ stocks\ held\ before\ order\ ("n_s") \times value\ of\ stock\ +$
$money\ in\ bank\ before\ order("B")$

Since a rational, risk neutral and myopic trader would perceive the $value\ of\ stock = E[f^{(k)}(s)|x_i^{(k)}, p]$, (or simply $E[f^{(k)}(s)|x_i^{(k)}]$ in the first round), then in mathematical notation:

$$T = E[f^{(k)}(s)|x_i^{(k)}, p] \times b - \frac{\mathrm{b}}{2} \times b + n_s \times E[f^{(k)}(s)|x_i^{(k)}, p] + B$$

Since the trader is rational, it attempts to maximize T. Therefore, the b that maximizes T may be found (by differentiating with respect to b and equating this to zero and noting that the second derivative is negative). As such:

$$\frac{dT}{db} = E\left[f^{(k)}(s)\middle|x_i^{(k)}, p\right] - b = 0$$

and $b_i = b = E[f^{(k)}(s)|x_i^{(k)}, p]$ is quantity bid ordered by the trader. Therefore the game rules incentivize, and are compatible with, a rational, risk neutral and myopic trader.

Chapter 5 - Appendix 9: Utility Based Proof that a Strategic Trader Will Likely Trade in The Same Way as a Rational, Risk-Neutral and Myopic Trader Given This Specific Prediction Market Web Game Setting

Consider a rational strategic trader who observes at some round in the market game the present value of their total wealth as T.

Specifically, they will calculate $T = n[V(x,\pi,b)] + bV(x,\pi,b) - \frac{b}{2}b + M$; where M is the remaining amount of game money, n is the number of stocks they currently own, $V(x,p,b)$ is the perceived present value of each stock (as they consider what they privately know x, the history of previous round market prices π, and the bid they are about to place b).

Remember from Chapter 5 Appendix 8 that the trading rule is such that a quantity order bid b requires the player to pay $\frac{b}{2}$ cents per stock ordered. That is, in this strategic trader setting, the trader pays $\frac{b}{2}b$ cents and receives b stocks that they value at $bV(x,\pi,b)$.

The value $V(x,\pi,b)$ they ascribe is a function of their bid b. This $V(x,\pi,b)$ can be broken into a rational, risk neutral, myopic value $E(x,\pi)$ and what is best described as strategic surplus value $S(x,\pi,b)$ which changes when the trader changes their quantity order b. $S(x,\pi,b)$ is the extra value that the stock has when it is used in a strategic bidding way, e.g., bidding so as to change the current or future price of it or another stock. Thus, $V(x,\pi,b) = E(x,\pi) + S(x,\pi,b)$. Notice that this is simply stating that the trader may increase their perceived present value of a stock $V(x,\pi,b)$ above its rational risk neutral myopic value $E(x,\pi)$ by changing their bid b, i.e., they will only participate in strategic trading for those bids that cause $S(x,p,b) \geq 0$; otherwise there is no value to putting cognitive effort into strategic trade.

Being rational, the trader will always trade to maximize the utility $U(T)$ that they experience from T. It is assumed that the trader's utility increases with increasing wealth T, i.e., non-satiation $\frac{dU(T)}{dT} > 0$ holds.

The only lever that the player has direct control over to influence future prices to maximize their utility is their bid b; utility as a function of bid is denoted $U(T(b))$. An optimal bid exists in the rational, risk neutral and myopic setting of Chapter 5 Appendix 8. If an optimal strategic bid does not exist then it is assumed the trader will trade in a rational, risk neutral and myopic way. However, if an optimal strategic bid does exist then it will maximize utility either within the open interval $(0,100)$ at $\frac{dU(T(b))}{db} = 0$ with the second derivative assumed as taking on a negative value (i.e. a maxima exists there), or the optimal bid will be at the end points of the allowable quantity orders, i.e., the optimal bid will be either 0 or 100.

First consider the open interval $b \in (0,100)$.
Now, $\frac{dU(T(b))}{db} = \frac{dU(T)}{dT} \times \frac{dT}{db} = 0$ and since $\frac{dU(T)}{dT} > 0$, this implies that $\frac{dT}{db} = 0$.

Since $T = n[V(x,\pi,b)] + bV(x,\pi,b) - \frac{b}{2}b + M$ differentiating obtains $\frac{dT}{db} = n[V'(x,\pi,b)] + V(x,\pi,b) + bV'(x,\pi,b) - b = 0$.

Rearranging this obtains $b = \frac{n[V\prime(x,\pi,b)]+V(x,\pi,b)}{1-V\prime(x,\pi,b)}$
But, since b only take on values in the open interval $(0,100)$ in this particular prediction market web game, then b cannot depend on n; else there is a possibility it could fall outside the interval. Therefore $V'(x,\pi,b) = 0$ must hold.

Since $V(x,\pi,b) = E(x,\pi) + S(x,\pi,b)$, the first derivative of this with respect to b is simply $V'(x,\pi,b) = S'(x,\pi,b)$. Therefore $S'(x,\pi,b) = 0$ must hold.

So $n[V'(x,\pi,b)] + V(x,\pi,b) + bV'(x,\pi,b) - b = 0$ becomes $V(x,\pi,b) - b = 0$.
This implies that the rational strategic trader bids: $b = V(x,\pi,b) = E(x,\pi) + S(x,\pi,b)$.

Now in this specific prediction market web game setting the bid b is a probability reported by the player of some event. This is consistent with the trader bidding on the complement of that event by submitting a bid $b_c = 1 - b$; the stocks of the complement event also being traded in the game.

Notice that the risk neutral myopic value of the complementary event $E_c(x, \pi)$ is simply $E_c(x, \pi) = 1 - E(x, \pi)$.

Using consistent reasoning across both stocks, the trader will bid $b_c = E_c(x, \pi) + S_c(x, p, \pi)$ on the complement event and partake in strategic play for bids that cause $S_c(x, \pi, b) \geq 0$.

Substituting into $b_c = E_c(x, \pi) + S_c(x, \pi, b)$ obtains $(1 - b) = \big(1 - E(x, \pi)\big) + S_c(x, \pi, b)$.

Adding equation $b = E(x, \pi) + S(x, \pi, b)$ and $(1 - b) = \big(1 - E(x, \pi)\big) + S_c(x, \pi, b)$ obtains $0 = S(x, \pi, b) + S_c(x, \pi, b)$. But since $S(x, \pi, b) \geq 0$ and $S_c(x, \pi, b) \geq 0$ it must be the case that $S(x, \pi, b) = S_c(x, \pi, b) = 0$.

That is, if the optimal bid is in the open interval $(\mathbf{0}, \mathbf{100})$ the trader never plays strategically and instead bids in a rational, risk neutral, and myopic way by placing a bid $\boldsymbol{b = V(x, \pi, b) = E(x, \pi)}$ for this specific prediction market web game.

However a strategic optimal bid could be either 0 or 100. That is either:
$T = n[V(x, \pi, 0)] + M$ is the maxima
or
$T = n[V(x, \pi, 100)] + 100V(x, \pi, 100) - \frac{100}{2}100 + M$ is the maxima.

Therefore, for this particular prediction market web game, a strategic bid in the open interval $(0,100)$ is the same as a rational, risk neutral and myopic bid, or, strategic bids of 0 or 100 (being not necessarily rational, risk neutral and myopic) are utility maximizing bids. In summary, a strategic trader will ‘likely’ trade in the same way as a rational, risk-neutral and myopic trader given this specific prediction market web game setting

Chapter 5 - Appendix 10: Prediction Market Web-Game Notes for the Five Tutorial Classes/Experiments

Experiment in tutorial class 1

Dr Hong Bo Liu's class

10am 28th july 2016

BX3024 + EC5207

4-006

8 students in attendance (30 students expected) – games 1 to 8 played, i.e., single human games with full control and treatment pairing

Player 7-1 highest human score (name hidden for confidentiality)

Comments: took about 20 mins to play and present

Students all agreed to play

Students told that they could possibly win $200 if highest score of all players

Experiment in tutorial class 2

Dr Hong Bo Liu's class

10am 29th july 2016

BX2022 + EC5206

27-004

games 9 to 12 (single human games i.e. 6 students) 13 to18 (2 player games i.e. 24 students) and also 25 (but 25 single player game needs exclusion given that it has no treatment game 26)

Player 14-2 highest human score (name hidden for confidentiality)

Comments: took about 30 mins to play and present

Students all agreed to play

Students told that they could possibly win $200 if highest score of all players

Experiment in tutorial class 3

Dr Michelle Esparon class

12.30pm 4th August 2016

EV2003

Engineering tutorial rooms (opposite DATSIP) games 25 to 36 (single human games i.e. 12 students) 19 to 24 (2 player games i.e. 24 students) and also 37 (but 37-2 double player game needs exclusion given that it has no treatment game or sufficient other players)

Player 21-2 highest human score (name hidden for confidentiality)

Comments: took about 30 mins to play and present

Students all agreed to play

Students told that they could possibly win $200 if highest score of all players

<u>Experiment in tutorial class 4</u>

Dr Michelle Esparon class

3.30pm 4th August 2016

EV2003

Engineering tutorial rooms (opposite DATSIP)

games 1 to 7 (single human games i.e. 13 students – need to exclude 7-1 as treatment 8-1 not played) 15 to 24 (2 player games i.e. 20 students)

7-1 does not have a treatment (8-1)

17-2 chose not to play but this is human.

Player 24-2 highest human score (name hidden for confidentiality)

Comments: took about 30 mins to play and present

Students all agreed to play with the exception of trader 17-2 but that is legitimate human behavior so they are included in the sample

Students told that they could possibly win $200 if highest score of all players

<u>Experiment in tutorial class 5</u>

Dr Michelle Esparon class

3.15pm 5th August 2016

EV2003

Engineering tutorial rooms (opposite DATSIP)

games 1 to 10 (single human games i.e. 10 students – may be a problem with attention as it is Friday end of day (however this is human behavior) – a possible understanding problem with 1 student, but will still include data given legitimate human behavior and the notion of bounded rationality

Player 8-1 highest human score (name hidden for confidentiality)

Comments: took about 15 mins to play and present, i.e., 5mins shorter than other games – researcher was possibly tired from a previous seminar presentation and possibly comparatively succinct in this presentation, but results will still be included given teacher of class said they noticed no difference in the researcher's presentation.
Students all agreed to play except for 10-1 so 9-1 has to be discarded given it is the control for the 10-1 treatment)
Students told that they could possibly win $200 if highest score of all players

Chapter 5- Appendix 11: Prediction Market Web-Game Analysis STATA Code

```
. do pmgameswithhumans

. use pmgameswithhumans.dta, clear

.
.  */ logit regress response (DFB) against  relevant information level r and the other possible covariate (hu
> m)
.  */ NB theory predicts that r is significant (so a statistically significant coefficient at the 5% level sh
> ould be detected)
.
.
. */ Now logit regress response (DFB) against the only treatment i.e. relevant information level r
. logit DFB r

Iteration 0:   log likelihood = -17.397455
Iteration 1:   log likelihood = -14.074976
Iteration 2:   log likelihood = -13.782413
Iteration 3:   log likelihood = -13.779285
Iteration 4:   log likelihood = -13.779284

Logistic regression                               Number of obs   =         30
                                                  LR chi2(1)      =       7.24
                                                  Prob > chi2     =     0.0071
Log likelihood = -13.779284                       Pseudo R2       =     0.2080

------------------------------------------------------------------------------
         DFB |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           r |    3.68738   1.666023     2.21   0.027      .422034    6.952726
       _cons |   -3.48025   1.370193    -2.54   0.011    -6.165778   -.7947214
------------------------------------------------------------------------------


.
. */ r is statistically significant
.
.
.
.
.
.
.
end of do-file

.
```

Appendix D: Chapter 6 Appendices

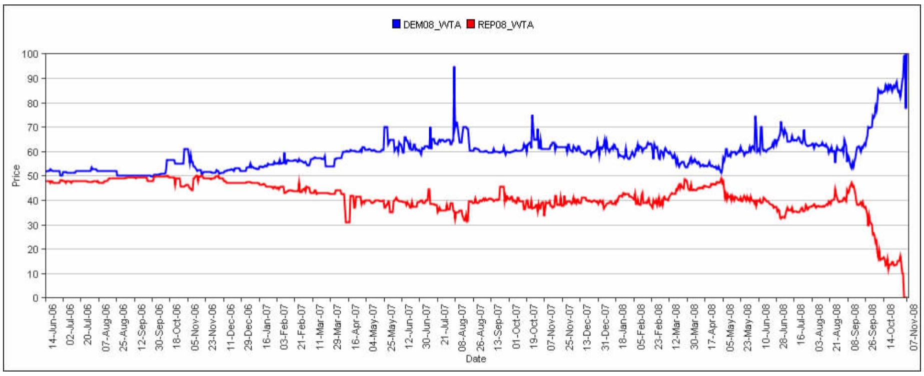
Chapter 6 - Appendix 1:2008 IEM Presidential Election Data

Raw data and asset descriptions source:

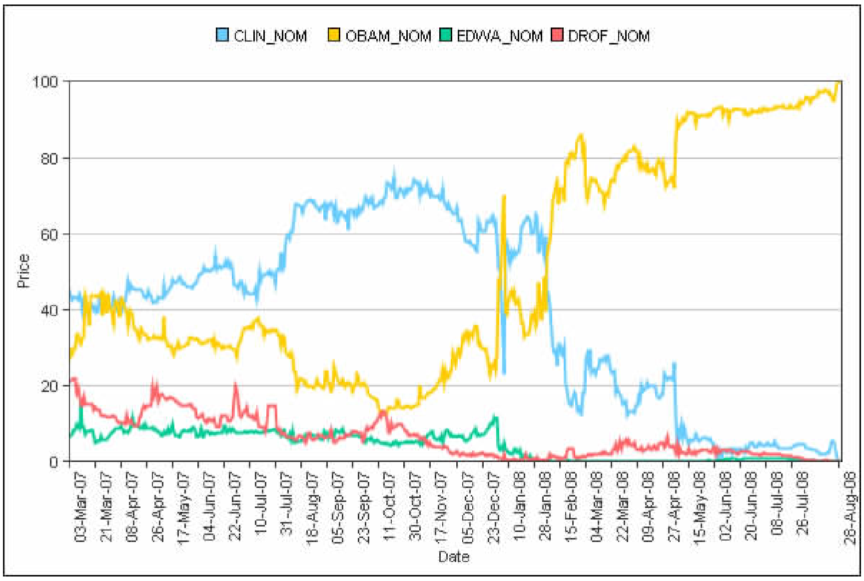
Iowa Electronic Markets. 2008. 2008 Presidential Election Prediction Markets. [ONLINE] Available at: https://tippie.uiowa.edu/iem/closed/pres08.html. [Accessed 18 November 2015].

1. Raw data

2008 Iowa Electronic Markets (IEM) raw data (daily close market price) for 2 Major parties (Democrats and Republicans) as below :



and also raw data (daily close market price) for 4 Democrat Candidates as below:



Period of analysis chosen was 3 months (May, June, July 2008); where liquidity of trade was high.

2. Asset Descriptions

<u>Market:</u>

Name:	PRES08_WTA
Description:	2008 US Presidential Election Winner-Takes-All Market
Open Date:	06/01/06 01:15 PM
Close Date:	11/07/08 04:05 PM

<u>Assets:</u>

Name	**Description**
DEM08_WTA	$1 if the Democratic Party nominee receives the majority of popular votes cast for the two major parties in the 2008 U.S. Presidential election, $0 otherwise
REP08_WTA	$1 if the Republican Party nominee receives the majority of popular votes cast for the two major parties in the 2008 U.S. Presidential election, $0 otherwise

<u>Market:</u>

Name:	DConv08
Description:	2008 Democratic Nomination Market
Open Date:	03/02/07 11:59 AM
Close Date:	11/07/08 05:37 PM

<u>Assets:</u>

Name	**Description**
CLIN_NOM08	$1.00 if Hillary Clinton wins the 2008 nomination; $0.00 otherwise
EDWA_NOM08	$1.00 if John Edwards wins the 2008 nomination; $0.00 otherwise
OBAM_NOM08	$1.00 if Barack Obama wins the 2008 nomination; $0.00 otherwise
DROF_NOM08	$1.00 if another candidate wins the 2008 Democratic nomination;

$0.00 otherwise

Market:

Name:	RConv08
Description:	2008 Republican Nomination Market
Open Date:	03/02/07 11:59 AM
Close Date:	11/07/08 05:37 PM

Assets:

Name	**Description**
GIUL_NOM08	$1.00 if Rudy Giuliani wins the 2008 nomination; $0.00 otherwise
HUCK_NOM08	$1.00 if Mike Huckabee wins the 2008 nomination; $0.00 otherwise
MCCA_NOM08	$1.00 if John McCain wins the 2008 nomination; $0.00 otherwise
ROMN_NOM08	$1.00 if Mitt Romney wins the 2008 nomination; $0.00 otherwise
THOMF_NOM8	$1.00 if former Senator Fred Thompson wins the 2008 Republican nomination; $0.00 otherwise
RROF_NOM08	$1.00 if another candidate wins the 2008 Republican nomination; $0.00 otherwise

Chapter 6 - Appendix 2: Raw Data Satisfying $r \in [0, 1]$

This raw data has been removed

Chapter 6 - Appendix 3: Tests for Normality and Continuity Correction Factor in MV

This raw data has been removed

Testing normal estimates are satisfactory holds

Testing whether a continuity correction factor (CCF) is required

The continuity correction factor is not required if $np > 5$ and $nq > 5$ in a binomial distribution. Refer the following analysis:

n	128
p	0.25
q	0.75
np	32
nq	96

since np and nq>5 then CCF is negligible and the normal and chi distributions are applicable to this binomial context
CCF: continuity correction factor

CCF is not required

Therefore, the normal and chi-squared analyses may be properly employed in this experiment.

Chapter 6 - Appendix 4: Multivariate Analysis STATA File

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Chapter 6 - Appendix 5: Instrumental Variable (IV) Analysis STATA File

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Chapter 6 - Appendix 6: Propensity Score Analysis STATA File

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Chapter 6 - Appendix 7: Fine Strata Propensity Score Analysis STATA File

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Appendix E: Chapter 7 Appendices

Chapter 7 - Appendix 1: A More General Prediction Market Model

Relevant information

We say agent i expresses relevant information in their bid if $b_{iP}(x_i) \neq b_{iP}(x'_i)$ and $b_{iF}(x_i x_{-i}) \neq b_{iF}(x'_i x_{-i})$ where $b_{iP}: B \rightarrow [0,1]$ is a function that maps private information to a bid price at the private information stage and $b_{iF}: B^n \rightarrow [0,1]$ is a function that maps full information across n agents to bid price at the full information stage.

Price function formation mechanism

We shall now specify how market price is established at the private information and full information stage.

Private information stage market price is denoted by $p_P(b_{1P}(x_1), \dots, b_{nP}(x_n))$ where $p_P: [0,1]^n \rightarrow [0,1]$ is a function that maps bids at the private information stage to a market price.

Full information stage market price is denoted by $p_F(b_{1F}(x), \dots, b_{nF}(x))$ where $p_P: [0,1]^n \rightarrow [0,1]$ is a function that maps bids at the full information stage to a market price.

Price function (private information stage) and separation of variables

The price function at the private information stage is $p_P(b_{1P}(x_1), \dots, b_{nP}(x_n))$. Since $x_k \in B$ then $b_{kP}(x_k)$ takes on two different values at most. Since there are n agents p_P takes on 2^n different values at most.

It is possible to find suitable functions $\rho_1, \dots, \rho_n$ such that $p_P\big(b_{1P}(x_1), \dots, b_{nP}(x_n)\big) = \rho_1\big(b_{1P}(x_1)\big) \dots \rho_n(b_{nP}(x_n))$ since $\rho_k\big(b_{kP}(x_k)\big)$ may take on 2 different values and hence the product $\rho_1\big(b_{1P}(x_1)\big) \dots \rho_n(b_{nP}(x_n))$ may take on 2^n different values at most. That is, the range of $\rho_1\big(b_{1P}(x_1)\big) \dots \rho_n(b_{nP}(x_n))$ has a sufficient cardinality for the possible prices p_P.

Proper market price revisited

To generalize our price formation mechanism beyond the Shapley Shubik market price requires no extra conditions and entails only the use of the separation of variables that we have discussed above.

Recall the proper market price properties:

For the private information stage we require:

$p(x_i, x_j, x^*) \neq p(x_i, x'_j, x^{**})$ where $x_j \neq x'_j$ and x_k in x^* implies x'_k in x^{**}

For the full information stage we require:

$p(x_i, x_j, x^*) \neq p(x_i, x'_j, x^{**})$ where $x_j \neq x'_j$ and x_k in x^* implies x_k in x^{**}

Theorem 1 (Relevant information as sufficient for dce convergence):

"All agents have relevant information" is a sufficient condition for convergence to the direct communication equilibrium in an information market having a proper market price.

Proof:

At the end of the first round the market price will be revealed and is p. Any agent i may consider all possible information vectors that attained p and form a set which contains them $X = \{a \in B^n | p_P(b_{1P}(a_1), \dots, b_{nP}(a_n)) = p\}$. Agent i may reason that X is not empty as it at the very least contains the actual information vector x which was responsible for p. Agent i may also reason that X is a singleton set containing only x via the following argument:

Agent i assumes that some other information vector $\boldsymbol{y}$ leads to $\boldsymbol{p}$ where $y \in X$ and $y \neq x$. They may write $p = p_P(b_{1P}(x_1), \dots, b_{nP}(x_n)) = p_P(b_{1P}(y_1), \dots, b_{nP}(y_n))$.

Using the separation of variables, we may rewrite the equation as:

$$\rho_1(b_{1P}(x_1)) \dots \rho_n(b_{nP}(x_n)) = \rho_1(b_{1P}(y_1)) \dots \rho_n(b_{nP}(y_n))$$

Since $x_j, y_j \in \{0,1\}$ we may "replace y_j in the last equation with x_j if $y_j = x_j$" or "replace y_j in the last equation with x'_j if $y_j \neq x_j$" where x'_j denotes the opposite bit value of x_j.

After this substitution is made, we may simplify the equation further and obtain

$$\rho_i(b_{iP}(x_i)) \prod_k^L \rho_k(b_{kP}(x_k)) = \rho_i(b_{iP}(x_i)) \prod_k^L \rho_k(b_{kP}(x'_k))$$

Notice that L must at least be 1 given $y \neq x$ and all agents have relevant information.

We may rewrite as:

$p_P(b_{iP}(x_i), b_{k1P}(x_{k1}) \dots, b_{kLP}(x_{kL})) = p_P(b_{iP}(x_i), b_{k1P}(x'_{k1}) \dots, b_{kLP}(x'_{kL}))$.

But this is of the form: $p(x_i, x_j, x^*) = p(x_i, x'_j, x^{**})$ where $x_j \neq x'_j$ and x_k in x^* implies x'_k in x^{**}. Hence we have arrived at a contradiction.

Therefore it must be the case that "there does not exist some other information vector leading to $\boldsymbol{p}$".

Therefore, agent i finds only information vector x leads to the first round price p. Hence in round 2 all agents know x and also each agent knows that all agents know x. Hence an equilibrium price is reached which is the same as if all agents directly communicated their information, i.e., the dce price is reached

■

Theorem 2 (Relevant information as necessary for dce convergence):

"All agents have relevant information" is a necessary condition for convergence to the direct communication equilibrium in a proper information market.

Proof:

Assume that the market has attained the direct communication equilibrium price.

If $x = (x_1, \dots, x_i, x_j, \dots, x_n)$ then the market price would be $p_F(b_{1F}(x), \dots, b_{nF}(x))$

If $x^\wedge = (x_1, \dots, x_i, x'_j, \dots, x_n)$ then the market price would be $p_F(b_{1F}(x^\wedge), \dots, b_{nF}(x^\wedge))$

Now assume that there exists at least one piece of information that is not relevant; say x_j is not relevant.

This means that x_j does not affect an agent's bid. Formally, we write the bid for agent k may be $b_{kF}\left(x_j x_{-j}\right) = b_{kF}(x'_j x_{-j})$. But notice that $x = x_j x_{-j}$ and $x^\wedge = x'_j x_{-j}$.

Hence we may write: $b_{kF}(x) = b_{kF}(x^\wedge)$.

Which upon substitution implies $p_F\left(b_{1F}(x), \dots, b_{nF}(x)\right) = p_F(b_{1F}(x^\wedge), \dots, b_{nF}(x^\wedge))$.

But this is of the form: $p\left(x_i, x_j, x^*\right) = p(x_i, x'_j, x^{**})$ where $x_j \neq x'_j$ and x_k in x^* implies x_k in x^{**}. Thus we arrive at a contradiction.

Therefore it must be the case that "All agents express relevant information". ■