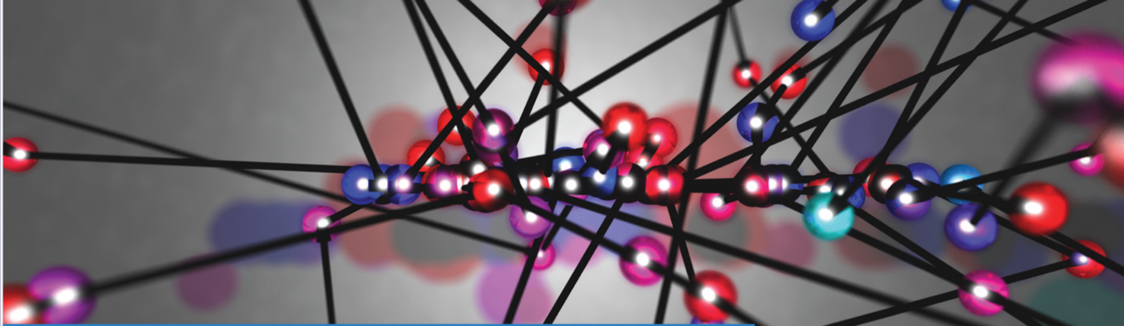

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Fundamentals of Charged Particle Transport in Gases and Condensed Matter

Robert Robson
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Fundamentals of Charged Particle Transport in Gases and Condensed Matter

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Fundamentals of Charged Particle Transport in Gases and Condensed Matter

By
Robert E. Robson, Ronald D. White and
Malte Hildebrandt

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Cover Image: Simulations of energy deposition of positrons in liquid water, often used in modeling as a surrogate for human tissue. Points of higher (lower) energy deposition are indicated by blue (red) spheres, while trajectories of positrons between collisions, represented by black lines, are biased towards the direction of an applied electric field. Eventually the positrons slow down sufficiently to annihilate with the electrons of the medium, producing two back-to-back gamma rays, as in PET (positron emission tomography) investigations. (Courtesy of Wade Tattersall)

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Preface

The foundations of modern transport theory were laid 150 years ago in a seminal paper presented to the Royal Society of London by J. Clerk Maxwell. He formulated the equations of change for the physical properties of a gas, represented as moments or averages over a velocity distribution function and paid particular attention to the influence of collisions. Six years later, Ludwig Boltzmann, undoubtedly influenced by Maxwell's results, presented a kinetic equation to the German Physical Society in Berlin, whose solution furnished the required distribution function. In spite of early criticism and subsequent intense scrutiny, Boltzmann's equation has withstood the test of time and has gone on to become a mainstay in the field of non-equilibrium statistical mechanics, in general, and charged particle transport, in particular, the subject of this book. The key to the success and longevity of Boltzmann's equation is not only its ability to furnish accurate theoretical values of experimentally measured quantities, but also its remarkable flexibility and adaptability to systems and physics that Boltzmann could not possibly have foreseen. Thus, there are generalizations of the kinetic equation to condensed matter, as discussed in this book, and to quantum and relativistic systems, discussed elsewhere. In addition, there are many adaptations and applications of Boltzmann's equation to traditional and contemporary areas of basic physics research and technology. To take just one example of cutting edge science: laser acceleration of particles to very high energies over distances several orders of magnitude smaller than conventional accelerators has been modelled through methods which are similar, at least in principle, to the ideas of Boltzmann and Maxwell. It would take several volumes to do justice to all of the fields on which the Boltzmann equation has had an impact and any single exposition, like the present, is necessarily circumscribed. Nevertheless, the scope of this book is broad and, moreover, the treatment is unique in that we provide a unified approach to the transport theory of particles of various types (electrons, ions, atoms, positrons, and muons) in various media (gases, soft-condensed matter, and amorphous materials). The applications are many and diverse, ranging from traditional drift tube experiments, positron emission tomography, and muon-catalyzed fusion, through to recent developments in materials physics.

One of the problems in writing a book such as this has been to overcome the perception that transport theory, beyond the simplistic mean free path arguments of some undergraduate books and courses, is somehow excessively difficult. On the one hand, it is true that a rigorous solution of the Boltzmann kinetic equation in phase space requires sophisticated mathematics and numerical procedures, and even the senior author of a well-known, formidable treatise on kinetic theory is reputed to have compared the exercise to "chewing glass." On the other hand, the original approach of Maxwell, using moment or "fluid" equations in configuration space, provides a complementary, semi-quantitative picture from which it is possible to obtain physical understanding while maintaining rigour. Both methods are employed in this book to provide a comprehensive treatment of charged particle transport phenomena.

The material has formed the basis of lecture courses given over the past 10 years in Australia and the United States at the senior undergraduate and graduate student level.

We thank Professor Michael Morrison of the University of Oklahoma; Professor Zoran Petrovic of the Institute of Physics, Belgrade; Professor Toshiaki Makabe of Keio University; and Dr. Bernhard Schmidt, originally at the University of Heidelberg and nowadays at DESY, Hamburg, for stimulating discussions and encouragement over many years. The dedication and contributions of the past and current staff, postdoctoral researchers, and post-graduate students at James Cook University cannot be understated. Particular thanks go to Kevin Ness, Bo Li, Sasa Dujko, Daniel Cocks, Gregory Boyle, Bronson Philippa, Wade Tattersall, Peter Stokes, Madalyn Casey, and Nathan Garland. The support of the Alexander von Humboldt Foundation, the Paul Scherrer Institut, the Australian Research Council, and James Cook and Griffith Universities is gratefully acknowledged.

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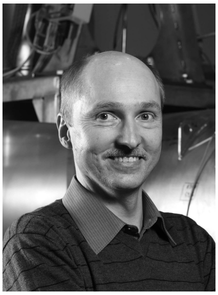
About the Authors

Robert Robson obtained his PhD in theoretical physics in 1972 at the Australian National University, Canberra. After a postdoctoral fellowship at the University of Alberta, Canada, he went on to lecture and research in physics and meteorology in Australia, the United States, Japan, and Europe. He was an Alexander von Humboldt Fellow at the University of Düsseldorf, Germany and held the Hitachi Chair of Electrical Engineering at Keio University, Japan. His most recent research has been on soft-condensed matter and amorphous semiconductors at James Cook University, Australia, and modelling relativistic electron beams in plasma-based accelerators at Deutsches Elektronen-Synchrotron (DESY), Hamburg. He is a distinguished member of the Australian Association of von Humboldt Fellows, and a Fellow of the American Physical Society and the Royal Meteorological Society.

Ronald White obtained his PhD in theoretical physics in 1997 with a study of electron transport in gases relevant to plasma processing studies at James Cook University. After research appointments in Australia, Japan, and the United States, he returned to James Cook University where he took up a lectureship in 2002 in the Mathematics Department. He was promoted to full Professor in 2015 and is currently the Head of Physical Sciences at James Cook University, which encompasses the Mathematics, Physics, and Chemistry Departments. His research interests are focused on the non-equilibrium transport of charged particles in gases, soft-condensed matter, liquids, and organic matter.



Malte Hildebrandt studied physics at the University of Heidelberg, Germany, and worked for his diploma thesis on electron swarm experiments. In 1999, he completed his PhD on the development of particle detectors for high energy physics experiments. He went on to a postdoctoral position at the University of Zürich, Switzerland, and moved later to the Paul Scherrer Institut, Switzerland. Since 2009, he has been Head of the Detector Group of the Laboratory for Particle Physics at the Paul Scherrer Institut. His work focuses on particle detectors, in particular, gaseous detectors, for charged particles and neutrons.

Glossary of Symbols and Acronyms

Symbol	**Meaning**
$\mathbf{a}$	external force per unit mass
α	scaling factor for velocity $\sqrt{\frac{m}{k_B T}}$
b	impact parameter
χ	scattering angle in centre of mass
D	diffusion tensor (starred quantities are "flux" while non-starred are "bulk")
ϵ	energy
ε	spatially uniform energy
E_e or E_eff	equivalent of effective electric field
$f(\mathbf{v})$, $f_0(\mathbf{v}_0)$	particle and neutral velocity distribution functions
$\phi(\tau)$	relaxation function for de-trapping
$\phi_m^{(\nu,l)}(\mathbf{v})$	Burnett function
$\mathbf{g}$, $\mathbf{G}$	relative and centre-of-mass velocities
γ	gradient energy parameter
Γ	particle flux
I, U	electric current and applied voltage
J, $J^\dagger$	collision operator and its adjoint
$\mathbf{J}_q$	heat flux vector
K, $\mathcal{K}$	mobility and reduced mobility coefficients
K_j	spectral wave number
λ_D	Debye length
m, m_0	particle and neutral molecular masses
n, n_0	particle and neutral number densities
N	total particle number
ν_m, ν_e	momentum and energy-transfer collision frequencies
$\vec{\nu}_i$, $\overleftarrow{\nu}_i$	inelastic and superelastic collision frequencies
ν_I	ionization collision frequency
ν_*	reactive loss collision frequency
$\widetilde{\nu}_m$	structure-modified collision frequency
ω, Ω_L	angular frequency of applied electric field, gyrofrequency of magnetic field
Ω	inelastic collision transfer term
P	pressure tensor
$\sigma(g,\chi)$	differential cross section

Symbol	**Meaning**
$\sigma^{(l)}, \sigma_m$	lth partial and momentum-transfer cross sections
$S(K, \Omega)$	structure function
T, T_0, T_b	particle, neutral, and basis temperatures
$\mathbf{v}, \mathbf{v}_0$	velocities of particles and neutrals
$\langle\rangle, \langle\rangle_0$	averages over particle and neutral velocities
$\langle\mathbf{v}\rangle$	average particle velocity
$\mathbf{v}_d$, $\mathbf{v}_d^*$	bulk and flux drift velocities
$\hat{\mathbf{v}}$	unit vector in direction of $\mathbf{v}$
$\langle\mathbf{vv}\rangle$	second rank tensor with components $\left\langle v_i v_j \right\rangle$
$V(r)$	interaction potential
$w(\alpha, v)$	Maxwellian distribution function
$Y_m^{(l)}(\hat{\mathbf{v}})$	spherical harmonic
Z	plasma dispersion function

BGK	Bhatnagar–Gross–Krook
μCF	muon-catalyzed fusion
PET	positron emission tomography
MTT	momentum transfer theory
GER	generalized Einstein relation

CHAPTER 1

Introduction

1.1 Boltzmann's Equation

1.1.1 A little history

In 1872, Ludwig Boltzmann proposed a kinetic equation of the form

$$\left(\frac{\partial}{\partial t} + L\right) f = \left(\frac{\partial f}{\partial t}\right)_{\text{col}} \tag{1.1}$$

for the velocity distribution function f of a low density gas, where L is a linear "streaming" operator in phase space, and $\left(\frac{\partial f}{\partial t}\right)_{\text{col}}$ accounts for binary, elastic collisions between the constituent atoms [1]. The expression for the latter was formulated on the basis of an Ansatz (or hypothesis), which effectively introduces an arrow of time into the evolution of the system, leading to the H-theorem and establishing a connection with the second law of thermodynamics. Although Boltzmann suffered criticism from his contemporaries, and the Ansatz has been the subject of considerable critical scrutiny since then, no satisfactory alternative has emerged, and the Boltzmann equation, modified by Wang Chang et al. to include inelastic collisions [2,3] remains to this day the preferred means of investigating gases in a non-equilibrium state.

Boltzmann's equation and the distribution function f play the same role in *kinetic theory* as do Schrödinger's equation and the wave function ψ in quantum mechanics. Once f is obtained from solution of Equation 1.1 all quantities of physical interest can be obtained as appropriate velocity "moments," similar to expectation values formed with $|\psi|^2$ in quantum physics (see Appendix A).

The centenary of Boltzmann's work was marked by a special publication [4] of both a biographical and scientific nature, which illustrated the extent of the influence that this remarkable equation has had on many areas of physics, involving both gases and condensed matter. Indeed, Boltzmann's contributions to the wider field of statistical mechanics are profound and are remembered in a special way (see Figure 1.1).

1.1.2 From the "golden" era of gas discharges to modern times

The emergence of Boltzmann's equation in the latter part of the nineteenth century coincided with an era of great interest in electrical discharges

Figure 1.1 The equation $S = k \log W$ linking entropy S with the number of microstates W of a system appears on Boltzmann's memorial headstone in Vienna.

in gases, though mutual recognition took some time. These investigations were motivated by the earlier observation of striations (alternating light and dark bands in the discharge) by Abria [5] (and more recently [6]), and culminated in the seminal drift tube experiments around the turn of the century and in the early 1900s. For example, Kaufmann and Thomson independently determined the elementary charge-to-mass ratio, e/m, which in turn led to Thomson's discovery of the electron, while the seminal experiment of Franck and Hertz confirmed Bohr's predictions of the quantized nature of atoms. As a result, there has been tremendous progress in science and technology, and it is not surprising that in the first three decades of the twentieth century, the field produced more than its fair share of Nobel laureates. Historical surveys of the "golden era" of drift tube experiments have been given by a number of authors, including Brown [7], Müller [8], Loeb [9], and Huxley and Crompton [10].

Investigations of gaseous discharges also spawned the field of plasma physics, with applications ranging from hot, fusion plasmas ($T \sim 10^6 K$ or more), with the promise of virtually limitless clean energy, to low temperature ($T \sim 10^4 K$) plasmas, of such importance in the microchip fabrication industry [11–13] and finally through to low density, low energy "swarms" of electrons and ions in gases [14], with applications in such diverse areas as fundamental atomic and molecular physics [15] and gaseous radiation detectors [16]. In the course of time, Equation 1.1 has come to be regarded as *de rigueur* for analyzing experiments involving charged particles in gases and condensed matter [17], along with applications of both a technological and scientific nature.

1.1.3 Transport processes: Traditional and modern descriptions

In general, non-equilibrium systems are characterized by non-uniformity and gradients in properties which result in an irreversible flow or "flux" of these properties in such a direction as to restore uniformity and equilibrium. Such *transport processes* are *traditionally* represented by well-known empirical linear flux-gradient relations, such as Fourier's law of heat conduction, and Fick's law of diffusion of matter, in which the constants of proportionality define *transport coefficients*, namely, the thermal conductivity and diffusion coefficient tensor, respectively. These coefficients can be calculated theoretically from approximate solution of the Boltzmann's equation, through linearizing in temperature and density gradient, respectively. However, one should be cautious in applying these traditional ideas to interpret drift tube experiments, for two reasons:

- Experiments are traditionally analyzed using the *diffusion equation*, which represents overall particle balance in the bulk of the system, and the coefficients in the diffusion equation differ from those defined by Fick's law when particles are created or lost, for example, by ionization and attachment, respectively. In these circumstances, experiments do not measure the traditional transport coefficients.
- Flux-gradient relations and the diffusion equation are valid only for systems which have attained a state called the *hydrodynamic regime*. Some systems never get to that state and are intrinsically non-hydrodynamic, for example, the steady state Townsend and Franck-Hertz experiments. Neither Fick's law nor the diffusion equation are physically tenable in these cases, and neither is description in terms of transport coefficients (however defined) possible. Measurable properties can be calculated theoretically only by solving Boltzmann's equation without approximation.

1.1.4 Theme of this book

In essence, Boltzmann's equation takes us from the laws of physics governing behaviour on the microscopic (atomic) scale, collisions in particular, to the level of macroscopically measurable quantities. The microscopic–macroscopic connection is the theme of our discussion, and explaining just *how* the connection is made provides the substance of this book. Put succinctly, the program is to solve Equation 1.1 for f, and then form velocity averages to find the macroscopic quantities of interest, for example, electric currents, or total particle number, which are measured in experiment.

1.2 Solving Boltzmann's Equation

1.2.1 The path to solution

- *Chapman–Enskog method:* The Chapman–Enskog method [18] is a perturbative procedure which was developed about 100 years ago to solve Boltzmann's equation for systems close to equilibrium. It was applied to gaseous ions in the 1950s by Kihara [19] and Mason and Schamp [20] but, by virtue of the limitations of the procedure, results could be obtained for only the weak field regime. Given that the systems of interest are often driven far from equilibrium by strong fields, this procedure is inadequate for most purposes.
- *Light particles, Lorentz approximation:* It was recognized early on that $\left(\frac{\partial f}{\partial t}\right)_{\text{col}}$ could be approximated in differential form for electrons undergoing elastic collisions in gases [18,21]. This simplification, together with an assumption of near-isotropy of f in velocity space, originally attributed to Lorentz [22], enables Boltzmann's equation to be solved, sometimes analytically, without any restriction on the magnitude of the field. These ideas underpin the field of gaseous electronics [23], which has maintained a distinct identity over many decades.
- *Light particles in liquids and soft matter:* Cohen and Lekner [24] modified $\left(\frac{\partial f}{\partial t}\right)_{\text{col}}$ to account for coherent scattering of electrons in liquids and, as for gaseous media, f was also assumed to be nearly isotropic in velocity space. Nevertheless, Cohen and Lekner's results have become well established in the literature and provide the basis for more sophisticated transport analysis of both electrons and positrons in liquids and soft-condensed matter.
- *Light particles, inelastic processes:* In many cases of interest, electrons also undergo inelastic collisions with the molecules of the medium, and consequently $\left(\frac{\partial f}{\partial t}\right)_{\text{col}}$ no longer assumes a simplified

differential form. The Lorentz approximation is also questionable if inelastic processes are significant and, all in all, solution of Boltzmann's equation becomes more difficult. In fact, the degree of difficulty is on a par with heavier ions, for which there is significant anisotropy in velocity space even if inelastic processes are absent. This points towards the need for a general procedure for solving Boltzmann's equation for particles of all masses and types.

- *Wannier's theory:* In the 1950s, Wannier [25] solved Boltzmann's equation for dilute ions in gases in the strong field regime, though specifically for special models of interaction. He also formulated a relationship between the mean ion energy and average velocity, and sowed the seeds of an idea for a semi-quantitative alternative to rigorous numerical solution of Boltzmann's equation, which is nowadays called "momentum-transfer theory."
- *The Viehland–Mason solution for ions:* Around the time of the Boltzmann centenary in 1972, computing power had reached a level where rigorous numerical solution of the Boltzmann equation for ions had become possible for realistic forms of interaction, and without resorting to any perturbation method. In a series of papers commencing in 1975, Viehland, Mason, and collaborators developed a general method of solution of Boltzmann's equation for dilute ions in gases in electric fields of arbitrary strength [26–28]. The modern era of charged particle kinetic theory can be traced from this time.
- *Towards a unified kinetic theory:* Lin et al. [29] combined the essentials of the Viehland–Mason approach with Kumar's tensor formalism, adapted from atomic and nuclear physics [30], to develop a rigorous solution of Boltzmann's equation, modified to include inelastic collisions for light particles, avoiding the traditional *a priori* assumption of near-isotropy of f in velocity space. The method has been refined over the years, and nowadays provides the basis of a comprehensive kinetic theory of charged particles, ions, electrons, positrons, muons, and so on, in both gases and condensed matter. The reader can find a number of reviews and books detailing developments from the immediate post-Viehland–Mason era to more modern times [31–35].
- *Charge carriers in semiconductors:* The kinetic theory of free charge carriers (electrons and holes) scattered by phonons (lattice vibrations) in crystalline semiconductors was developed in parallel to gases [17]. It is sometimes remarked that there exists a one-to-one correspondence with scattering of charged particles from molecules and atoms in gases, even though the collision term $\left(\frac{\partial f}{\partial t}\right)_{\text{col}}$ in the kinetic equation (still referred to as

"Boltzmann's equation") is different. The role of transport theory in understanding experiments related to the development of solid-state devices including the transistor has a long history [36]. On the other hand, charge carriers are said to exhibit anomalous behaviour in disordered, non-crystalline amorphous media, such as organic semiconductors, due to trapping effects. These materials are being intensely investigated [37] and it appears that yet another technological revolution is underway [38,39]. The kinetic theory associated with these processes is, however, a "work in progress," with only simple forms of $\left(\frac{\partial f}{\partial t}\right)_{\text{col}}$ having so far been employed [40,41].

1.2.2 A complementary approach: Fluid modelling

After solving the Boltzmann equation as described above, quantities of physical interest are formed by taking appropriate velocity averages of f. An alternative approximate, more computationally economical and physically appealing alternative is to find the averages directly by solving approximate moment or fluid equations in configuration space. These equations can be formed either by taking velocity moments of Boltzmann's equation, or from first principles, as Maxwell [42] did 6 years before Boltzmann formulated his kinetic equation. In fact, the roles can be completely reversed, as we show in this book, and Boltzmann's equation can be obtained (and later solved) using the moment equation method.

Maxwell paid particular attention to the collision terms in the moment equations and showed that they could be evaluated exactly for a particular model, in which the interaction varied inversely as the fifth power of the distance. The Maxwell model, which corresponds to a point-charge, induced dipole interaction, is particularly suitable as a first approximation when discussing charged particles in gases. It provides the basis for "momentum transfer" theory [33], which has proved particularly successful in semi-quantitative fluid modelling of charged particle transport phenomena [43].

1.3 Experiment and Simulation

1.3.1 An idealized apparatus

Although this book focuses on theory, we touch briefly on experiments [10,15,34,36,44,45] though it is not possible to discuss technical details. We instead focus on principles of operation, following the style of Kumar [14], using as an example the idealized experimental arrangement shown in Figure 1.2.

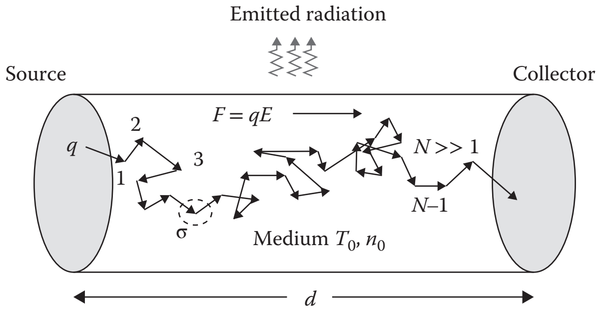


Figure 1.2 A schematic representation of an experiment in which particles of charge q are emitted by the source electrode and travel through a medium of known properties to a collecting electrode a distance d away under the influence of an electric field. Collisions are represented by the vertices of the trajectory and are characterized by appropriate scattering cross sections σ.

Particles of charge q emitted by a source electrode are forced by a uniform electric field E to move a distance d through a chamber containing a medium of known properties (gas or condensed matter) to a collecting electrode. Particle number density n is assumed sufficiently small so that mutual interactions are negligible in comparison with interactions of particles with the constituents of the medium. Such collisions are assumed to be local, that is, to take place in a region small compared with any macroscopic dimension, effectively at a point, and are represented by the vertices in the particle trajectory shown in the figure.

The source may operate in a pulsed or continuous mode. In some experiments, particles incident on the collecting electrode form the current measured in an external circuit. In the Franck–Hertz experiment [46], there is a modulating grid in front of the collecting electrode. In the Cavalleri experiment [10], it is the total number of particles within the chamber that is determined as a function of time. In yet other experiments, the radiation emitted by atoms and molecules returning to a lower energy level after excitation in a collision may be used as a diagnostic tool, as in the photon flux technique [47].

For a *gaseous* medium, particles may be considered to collide with individual atoms and molecules and the various processes (elastic, inelastic, ionizing, reacting, etc.) are characterized by a corresponding binary scattering cross section σ. Collisions take place on a time scale small compared with any relevant macroscopic scale, and to all intents and purposes are instantaneous. In the *time-of-flight experiment*, an initial sharp pulse of particles injected at the source spreads at a constant rate about its centre-of-mass, which moves with constant velocity v_d through the medium, as shown in Figure 1.3. Although the pulse spreads in the course of time,

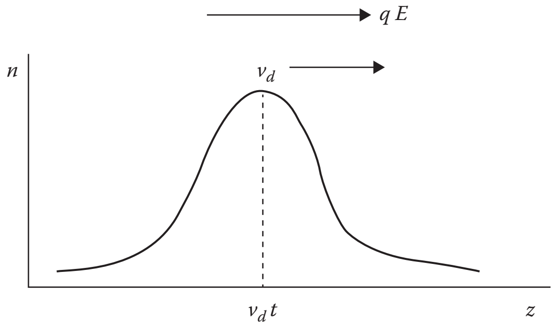


Figure 1.3 The number density n of charged particles as a function of distance z from the source at a time t after injection into the medium, initially as a sharp pulse. After a sufficient number of collisions, the pulse has spread out and its centre-of-mass travels with constant velocity v_d, determined by field, the scattering cross sections, and the properties of the medium. The width of the pulse increases with time t in proportion to the (longitudinal) diffusion coefficient.

it still retains its identity, and its two main properties (centre-of-mass velocity and width) are readily measurable. The same properties may be calculated from solution of Boltzmann's equation. Naturally, the theoretical values should be calculated to at least the same accuracy as the experimentally measured counterparts. Typically, the accuracy in swarm drift tube experiments is 0.1%–1.0% for the drift velocity v_d [10].

The picture is similar for charge carriers scattered from phonons in a *crystalline semiconductor,* and there too the time-of-flight experiment is the canonical experiment.

For a *soft matter* medium with short-range order, particles are scattered simultaneously (diffracted) by many constituent molecules. Nevertheless, the picture of local, instantaneous interactions at the vertices of Figure 1.2 prevails, and a pulse in a time-of-flight experiment in this medium generally maintains its distinct identity. There are cases, however (e.g., electrons in neon), where this is not the case [48], where electrons can get caught and released from "bubble" states.

For *amorphous materials,* such as organic semiconductors, this is generally not the case, where particles are trapped for finite times in localized states. The picture shown in Figure 1.2 still holds, but vertices now represent "collisions" (trapping/de-trappings) lasting finite times, rather than taking place instantaneously. Particles may be trapped for significant times over the entire length of the chamber; consequently, the particle density profile in a time-of-flight experiment is qualitatively quite different. In particular, there is no distinct travelling pulse in a time-of-flight experiment [37].

1.4 About this Book

In this book, we focus on non-relativistic, low density charged particles which interact predominantly with the background medium, and neglect mutual Coulomb interactions and self-consistent fields. The main aims are to:

- Formulate kinetic and fluid equations for charged particles in gases, soft-condensed matter, and amorphous materials, allowing for coherent scattering and/or trapping in localized states where necessary,
- Outline the basic techniques for solving the kinetic equation and for calculating transport properties,
- Understand the link between the microscopic processes and the macroscopic transport properties,
- Apply the theory to traditional and new areas of science, technology, and medical diagnostic techniques.

While rigour is a watchword, we use short arguments and simplified mathematics wherever possible to elucidate the physics.

The structure is as follows:

Part I: Fundamentals of kinetic theory, derivation of Boltzmann's related kinetic equations, as well as calculation of classical cross sections.

Part II: Simplified treatment of transport processes through a fluid equation analysis, in which Boltzmann's kinetic equation in phase space is replaced by a set of approximate "moment" equations in configuration space.

Part III: Procedures and techniques for solution of Boltzmann's equation.

Part IV: Applications include boundary effects and diffusion cooling, Franck–Hertz experiment, anomalous transport in amorphous semiconductors, calculation of positron range in positron emission tomography (PET), muon-catalyzed fusion, and gaseous radiation detectors.

Part V: Gives a series of appendices providing extra information, miscellaneous proofs and values of numerical constants, together with a set of exercises aimed at reinforcing the material in the text, and a comprehensive list of references to books and original papers.

Additional General Reading Materials

- A good introductory text on statistical mechanics: D.V. Schroeder, "Thermal Physics" (Addison-Wesley, Longman, 2000).

- A good introductory background to kinetic theory can be found in the following article: E.D.G. Cohen, *Amer. J. Phys*. 61:524, 1993 (Sections I and II A,B,C only).
- A widely used text for graduate level statistical mechanics: K. Huang, "Statistical Mechanics" 2nd Edition (Wiley, 1987), especially Chapters 3–5.
- A favourite classical mechanics text: H. Goldstein, "Classical Mechanics", 2nd Edition (Addison-Wesley, 1980).
- Graduate level texts dealing with charged particles in gases:
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