QAMTAC16 Annual Conference - Big Things STEM from Maths

Title

Beyond binary thinking, knowing and teaching mathematics

Abstract

This presentation provides a framework for engaging binary thinking, knowing and teaching of mathematics (e.g., teacher-centred/ student-centred, transmission/discovery, explicit teaching/ inquiry). The framework proposes three general positions (1) oppositional, (2) equipositional, and (3) parapositional ways of thinking, knowing and teaching mathematics (Adam & Chigeza, 2014). Like grid points on a map the three general positions offer navigational markers in the complex terrain of mathematics education. The presentation illustrates potential strengths and weaknesses of these three general positions in regards to teaching measurement in Year 5 and Year 8 mathematics classrooms. The presentation calls for dissolving the binary teaching approaches that have proven divisive in mathematics education.

Background

The national attention to STEM subjects, teacher education, teacher professional development and the Australian Curriculum has reinvigorated dialogue on effective frameworks and ways of teaching mathematics. The Australian Curriculum rationale for mathematics states:

> The mathematics curriculum provides students with carefully paced, in-depth study of critical skills and concepts. It encourages teachers to help students become self-motivated, confident learners through inquiry and active participation in challenging and engaging experiences. (ACARA, 2013, para. 4)

Furthermore, the proficiency strands that describe the development and exploration of mathematics curriculum content are summarised as:

> Understanding: Students build a robust knowledge of adaptable and transferable mathematical concepts. They make connections between related concepts and progressively apply the familiar to develop new ideas.
>
> Fluency: Students develop skills in choosing appropriate procedures, carrying out procedures flexibly, accurately, efficiently and appropriately, and recalling factual knowledge and concepts readily.
>
> Problem Solving: Students develop the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively.
>
> Reasoning: Students develop an increasingly sophisticated capacity for logical thought and actions, such as analysing, proving, evaluating, explaining, inferring, justifying and generalising. (ACARA, 2013)

The rationale, content scope and sequence, and proficiency strands provide a rich background for educators to explore on how different frameworks can inform effective teaching of mathematics.

The binary-epistemic framework

One such framework is the binary-epistemic framework (Adam & Chigeza, 2014) that engages binary ways of thinking, knowing and teaching of mathematics (e.g., teacher-centred/ student-centred, transmission/discovery, explicit teaching/ inquiry). The framework proposes three general positions that offer navigational markers in the complex terrain of mathematics education: (1) oppositional, (2) equipositional and (3) parapositional ways of thinking, knowing and teaching of mathematics.

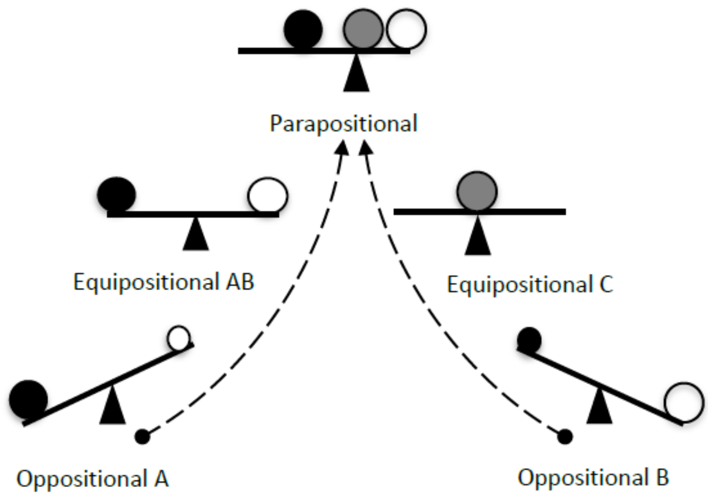

Figure 1. Binary-epistemic framework (Adam & Chigeza, 2014)

Oppositional ways of knowing and teaching

The framework proposes two oppositional ways of knowing and teaching mathematics: **Oppositional A** and **Oppositional B** which represents dispositions to choose a particular approach in opposition to its antithesis, regardless of context. **Oppositional A** is a position that reflects a relative tendency to approach mathematics knowledge as concrete, subjective, local, and teaching as student-centred, inquiry-based, collaborative, and constructivist, with an opposition to B. **Oppositional B** represents a position that reflects a relative tendency to approach mathematics knowledge as abstract, objective, universal, analytic; and teaching as teacher-centred, transmissive, and explicit, with an opposition to A.

Equipositional ways of knowing and teaching

The framework proposes two equipositional ways of knowing and teaching: **Equipositional (A = B)** and **Equipositional (A + B = C)** which are characterised by a tendency to approach all mathematics knowledge and teaching with an equal measure of A and B, regardless of context. **Equipositional (A = B)** reflects an early dialogical tendency to approach mathematics knowledge and teaching using relatively polarised approaches of A and B in equal measure, regardless of context. **Equipositional (A + B = C)** reflects an early dialectical tendency to approach mathematics knowledge and teaching using an equalising "middle position" representing a balanced combination of A and B, regardless of context.

Parapositional ways of knowing and teaching

Parapositional (A <=> B) ways of knowing and teaching are characterised by (a) an understanding of the interdependent and relational nature of the different approaches, and (b) the relational, contextual, and evaluative application of these approaches for effective learning. **Parapositional (A <=> B)** reflects a relative tendency to approach mathematics knowledge and teaching using relational perspectives of A and B in a manner that is dependent on context. For example, contextual variables could include students' age, cultural background and disposition towards the subject, curriculum directives, school imperatives, and teacher's pedagogical disposition, as well as pragmatic concerns (e.g., time, space and material resources). The position represents the ability to draw on previous positions (e.g., Oppositional A or B) with an understanding of the fluidity of context, and with an adaptive ability to change positions or transform contexts, accordingly to maximize learning.

Illustration of the three general positions

The following section highlights potential strengths and weaknesses of the **Oppositional, Equipositional** and **Parapositional** ways of teaching. The illustrations relate to content descriptors and Achievement Standards from the Year 5 and Year 8 Measurement and Geometry strand in the Australian Curriculum: Mathematics.

Year 5: Content Description: Calculate the perimeter and area of rectangles using familiar metric units (ACMMG109). Achievement Standard: They use appropriate units of measurement for length, area, volume, capacity and mass, and calculate perimeter and area of rectangles.

Year 8: Content Description: Find perimeters and areas of parallelograms, rhombuses and kites (ACMMG196). Achievement Standard: Students convert between units of measurement for area and volume. They perform calculations to determine perimeter and area of parallelograms, rhombuses and kites.

Oppositional ways of knowing and teaching

Oppositional A scenario: A teacher with a disposition to this way of knowing and teaching may approach the Year 5 content descriptor (ACMMG109) as follows. When preparing an organic garden for a class-to-community project, the teacher asks if any of the students would like to take responsibility for working out the best area needed to plant 10 tomato plants in the garden, and the length of mesh fencing they would need to surround the garden area. The students are asked to work out a budget for the fencing and tomato plants and are told that they can help purchase the mesh for homework and plant the tomato plants during lunchtime. The teacher allows the students to choose a way to describe how they solved the problem (e.g., write a narrative) and then recommend a grade for their work based on the process they used to solve the problem and the effectiveness of the final product. The teacher emphasises that they must work in a group as they will all receive the same grade for their project.

Oppositional B scenario: A teacher with a disposition to these ways of knowing and teaching may approach the Year 5 content descriptor (ACMMG109) as follows. The teacher is determined that by the end of each unit "their students" will have the basic content knowledge outlined in the curriculum to solve a range of mathematical problems in the textbook and move to the next level of conceptual sophistication required in Year 6. The class files in to find a worksheet on the desk and a content descriptor and formulae written on the whiteboard: Calculate the perimeter and area of rectangles using familiar metric units. Rectangle: (Area) $A = L \times W$. (Perimeter) $P = 2L + 2W$. The teacher asks the class to read the descriptor and formulae in unison and then write them down in their exercise books. The teacher proceeds to explain the terms and demonstrate the formulae using a range of rectangles drawn on the whiteboard. The students are asked to complete the worksheet on the desk.

Potential strengths and weaknesses

The weakness of binary oppositional dispositions is that they neglect their own weakness and the relative strengths and co-dependence of their relational opposites. For example, the **Oppositional A** scenario represents a deliberate pedagogical embrace of concrete, practical, student-centred ways of teaching; and subjective, interpretivist ways of knowing. The teacher engages the children through practical and authentic projects (e.g., the community garden) and does incidental mathematics (e.g., garden bed perimeter) along the way. The strength of the approach is its immediate concreteness and the potential to engage students (they can look busy). However, when abstraction is sacrificed for, rather than explicitly derived from concreteness, it can paradoxically limit students' preparation to engage deeply with new and different concrete problems. It can also limit students' preparation to engage with the Year 8 content descriptor (ACMMG196) - Find perimeters and areas of parallelograms, rhombuses and kites.

In terms of the proficiency strands, the approach tends to immerse children in authentic projects without sufficiently scaffolding, or at least complementing, the immersion with conceptual knowledge, procedural knowledge, or de-cluttered representations. The primary criticism of this oppositional approach is not that it may not work and be beneficial for some students; rather, it is that it unnecessarily excludes other positions from a pedagogical repertoire that may ultimately be used to engage more students over a longer period of time, and can potentially limit students' preparation to engage with higher mathematics learning in later years.

The counter-problem is illustrated in the **Oppositional B** scenario. Here, the pedagogical emphasis is on abstraction, with the presumption that knowledge of the abstraction will automatically be activated in multiple concrete contexts. So, the teacher uses rote methods, worksheets with abstract representations and theory-testing at the expense of more concretised pedagogies used in the **Oppositional A** scenario. The strength of the approach is in the immediate measurability and accumulation of content knowledge and its application to abstractions. However, when concreteness is sacrificed for abstraction, abstraction loses its meaning as there is little to abstract from or return to, in order to check the accuracy of the abstraction or when preparing to engage with higher mathematics learning in later years.

Equipositional ways of knowing and teaching

Equipositional (A = B) scenario: This approach is characterised by a "balance in all things" approach. As much as is possible, almost mechanically, the teacher divides the class time between different learning areas, and different pedagogical approaches and does not deviate from this division. For example, a Year 5 class begins a mathematics session by listening to an explanation of the content descriptor "Calculate the perimeter and area of rectangles using familiar metric units". They copy definitions of key terms (i.e., area, perimeter, rectangle), rote learn formulae and then individually complete pre-set questions requiring students to calculate the area and perimeter of different rectangles. On the half time, the teacher signals "student time", whereupon the children work in groups with a computer to find a global application of the measurement of area and perimeter and a personally relevant application of the same.

Equipositional (A + B = C) scenario: This position attempts to find the common ground of different pedagogies and combine them into a single approach. The Year 5 class enters the room to find pieces of grid paper with centimetre intervals on each desk. On another handout is the formula for finding the area of a rectangle, the formula for the perimeter of a rectangle, and an example of each. The teacher proceeds to explain the terms and demonstrate the procedure of finding the area using grid paper and then using a formula. The students are then paired and taken into the school playground where they are given the task of recording the perimeter and area of any rectangles they can find. The teacher moves between the pairs during the session to explain the content knowledge that students are applying.

Potential strengths and weaknesses
While perhaps a development in a cognitive sense, the equipositional dispositions can also be pedagogically destructive. They have the strength of drawing from both poles, but the weakness of ignoring the contextually dynamic and differentiated nature of "dynamic equilibriums". As such, the attempt to target learning through mechanically equal pedagogical divisions, is as effective as shooting arrows mechanically left, right, straight or randomly at, a randomly moving target. As such, student time may turn into a "pooling of ignorance" for some students who do not adequately grasp the conceptual knowledge to apply. Similarly, the "teacher time" may be wasted on other students who would better learn and appreciate the abstractions through the more concurrent exploration of concrete contexts. Likely, the students may be good at calculating the area and perimeter of rectangles in the school-yard or with a single piece of grid paper with centimetre intervals, but struggle to move fluidly between different units of measurement; or beyond the school-yard fence; or engaging with higher mathematics learning in later years.

Parapositional ways of knowing and teaching
Parapositional (A<=>B) scenario: This position is characterised by a pedagogical fluidity or dexterity that can evaluate and select one or more ways of teaching and knowing from a range of possibilities, with a differentiated knowledge of context, individual student needs and practical limitations.
The Year 5 class chatters expectantly, each holding a rectangular object brought from home or chosen from around the classroom. The teacher tries to involve students' home-lives but is mindful that some students may be marginalised. The teacher then links the children's objects to many more things (concrete, abstract, local and global) that have rectangles. By the end of the explanation the children have contributed almost twenty more rectangular objects. Having given students the opportunity to understand intuitively the relative size of different rectangles, the teacher reveals some notes on the whiteboard and challenges the students to deepen and sharpen their ability to measure rectangles. Calculate the perimeter and area of rectangles using familiar metric units
Rectangle: (Area) $A = L \times W$. (Perimeter) $P = 2L + 2W$.

The children explore different ways of representing the formula, using different letters or words. For example, for area one student suggests, "space inside the rectangle = long line x short line" and writes it as "S = LL x SL". When the children have annotated their notes on the basis of the discussion, they are given time to explain how to find the area of rectangles using different types of grids to a friend. The teacher uses this time to check for and modify understanding with individual students. During the remaining week, the teacher finds many incidental opportunities to integrate and check students' understanding of the content descriptor in-context. It is this pedagogical fluidity or dexterity that enables many of the students to develop "the confidence to use the familiar to develop new ideas, and the "why" as well as the "how" of mathematics" (National Curriculum Board, 2009, p. 6).

Potential strengths and weaknesses

It is the parapositional disposition that encompasses most of these strengths and avoids most of the weakness, most of the time. Thus, the teacher is relatively more responsive to the stability and dynamism of classroom contexts "gauging" and differentiating students' pedagogical needs, even within a single lesson, yet within a relatively stable scaffold. The teacher provides concrete examples that flow between local and global contexts. And deliberately and explicitly derives abstractions from these broad concretisations and encourages the children to do the same with new innovations and creations. This prepares children to engage deeply with new and different concrete problems and move fluidly between different units of measurement and beyond the school-yard fence. It also better prepares the students to engage with future learning the Year 8 content descriptor (ACMMG196) - Find perimeters and areas of parallelograms, rhombuses and kites.

This is an approach to mathematics curriculum "that emphasises many physical models and representations—pictorial, manipulative, verbal, real-world, and symbolic—[and] is more successful in aiding students' development of conceptual understanding" (Reys et al, 2004, p. 285). The teacher draws on and draws out students' prior knowledge to draw them in to formal learning. The teacher applies in practice what Bobis et al (2013) describe in theory; "The realisation that children already possess a great deal of knowledge before formal instruction occurs has caused many educators to reconsider how children learn mathematics" (p. 6). In terms of the proficiency strands, this approach to teaching is both integrative and dissociative. Thus, the proficiencies understanding, fluency, problem-solving and reasoning are parts of the same whole that can be temporarily coordinated or separated for the most adaptable result.

Conclusion

This presentation has provided a framework for engaging binary thinking, knowing and teaching of mathematics (e.g., teacher-centred/ student-centred, transmission/discovery, explicit teaching/ inquiry) and proposed three general positions (1) oppositional, (2) equipositional, and (3) parapositional ways of knowing and teaching mathematics as grid points on a map that offer navigational markers in the complex terrain of mathematics education.. The presentation illustrated potential strengths and weaknesses of these three general positions in regards to teaching measurement in Year 5 and Year 8 mathematics classrooms. The presentation argued that the parapositional way of knowing and teaching mathematics encompasses most of the strengths and avoids most of the weakness, most of the time.

References

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