MacNaughton, David (2008) ***Comparison of optimised composite control charts: improved statistical process control for the manufacturing industry.*** **Masters (Research) Thesis, James Cook University.**

Comparison of Optimised Composite Control Charts

Improved Statistical Process Control for the Manufacturing Industry

Thesis submitted by

David MacNaughton BE(Chem)

May, 2008

for the Degree of

Master of Science (Research)

School of Mathematics, Physics and IT

James Cook University

Statement of Access

I, the undersigned, the author of this thesis, understand that James Cook University will make this thesis available for use within the University Library and, by microfilm or other means, allow access to users in other approved libraries. All users consulting this thesis will have to sign the following statement:

> In consulting this thesis I agree not to copy or closely paraphrase it in whole or in part without the written consent of the author; and to make proper public written acknowledgement for any assistance which I have obtained from it.

Beyond this, I do not wish to place any restriction on access to this thesis.

(David MacNaughton) *(Date)*

Statement of Sources

I declare that this thesis is my own work and has not been submitted in any form for another degree or diploma at any university or other institution of tertiary education. Information derived from the published or unpublished work of other has been acknowledged in the text and a list of references is given.

(David MacNaughton) *(Date)*

Statement of Contribution by Others

The following list identifies people and institutions which have contributed to this thesis, and the contribution made.

- Professor Danny Coomans – Editing, and Primary Supervision.
- JCU High Performance Computing Centre (HPC) computing facilities.
- JCU School of Mathematics, Physics and IT: use of office and associated overheads, supply of personal computer during candidacy, printing, telephone for making contact with industry to obtain data for case studies.
- Anonymous Referees and the Editor from the Journal of Quality Technology editing for their valuable suggestions.
- David Cusack of Cement Australia Pty Ltd (CAPL) and CAPL are thanked for provision of industrial data and editing.

I would also like to thank Dr. Yvette Everingham for editing and support.

Abstract

Selecting and configuring control charts can be a difficult task. Literature has not provided evidence as to which type of composite control chart is best among composite moving average (CMA), composite exponentially weighted moving average (CEWMA) and composite cumulative sum (CCUSUM). Optimising three-component composite control charts was considered very difficult, if not impossible, to achieve. Additionally, a traditional method for comparing control charts across a domain of step shift sizes called the average ratio of average time to signal (ARATS), can lead to inconsistent conclusions. Thus, there have been insufficient methods and data published for an informed selection from composite control chart types and configurations.

This study is the first to optimise and compare two and three-component composite control charts. Distribution parameters were assumed to be unknown and were estimated from 200 observations. Software was created to automatically configure composite control charts to achieve specifications for the in-control average time to signal (ICATS) and the contribution of each of the components to false alarms, or loadings. Detection time profiles were simulated for full factorial experiments of control chart parameters using averages of at least 1,000,000 chart runs per simulation.

New performance and comparison measure were invented to complete the research. A new performance measure Mean Relative Loss (MRL) was defined and used for optimising control chart configurations. MRL compares the average time to signal (ATS) profile across a step shift domain to the profile of a reference CUSUM control chart. Average Difference Relative to the Average (ADRA) was defined to overcome the problem noted with ARATS.

Three-component CCUSUM bettered three-component CEWMA (ADRA = 5.0%) which in turn performed better than three-component CMA. Three-component CEWMA performed better than two-component CEWMA (ADRA = 5.2%). Thus it can be seen that the type of component and the number of components selected has a significant effect on performance.

This study shows how much the statistical performance of various types of optimised composite control charts can differ. Results from this study will better inform statistical quality control professionals when selecting a control chart type. The methods developed here have the further advantage of being adaptable to different assumptions and parameters. A final implication of the study is that composite control charts may now be optimised and thus fairly compared against other categories of control charts which are typically optimised in literature.

Table of Contents

List of Tables

List of Figures

Glossary and List of Acronyms

ACUSUM	Adaptive Cumulative Sum technique for control chart design
AEWMA	Adaptive EWMA technique for control chart design
ADRA	Average Difference Relative to the Average
ARARL	Average Ratio of Average Run Length
ARL	Zero-State Average Run Length
ARSSATS	Average Ratio of Steady State Average Time to Signal
ATS	Zero-State Average Time to Signal. This is equivalent to the Average Time to Signal by other authors.
CAPL	Cement Australia Pty. Ltd.
CEWMA	Composite Exponentially Weighted Moving Average
CMA	Composite Moving Average
CUSUM	Cumulative Sum
DCS	Distributed Control System
DRA	Difference Relative to the Average
EWMA	Exponentially Weighted Moving Average
Component	A component of a composite control chart scheme, i.e. one chart from a group of control charts monitoring the same quality variable, which are designed to have joint in-control specifications
FIR	Fast Initial Response
ICARL	In-Control Average Run Length
ICATS	In-Control Average Time to Signal
iid	Independently and Identically Distributed
ISO	International Standards Organisation
LCL	Lower Confidence Limit
Loading	The proportion of alarms contributed by a specific component when the monitored variable is in-control
LIMS	Laboratory Information Management System
MA	Moving Average
MRL	Mean Relative Loss

MRLMC	Mean Relative Loss Multiple Comparison
MRLPC	Mean Relative Loss Pair-Wise Comparison
MRLOCV	Mean Relative Loss to the Optimum CUSUM Vector
PC1	Principle Component - Number 1
PCA	Principle Components Analysis
PIMS	Process Information Management System
RAEQL	Ratio of Average Extra Quadratic Loss
RLE	Relative Loss Efficiency.
RLPC	Relative Loss Pair-Wise Comparison
Run Rules	Conditions for which a control chart is required to alarm such as two consecutive samples being outside of a corresponding set of control limits.
Scheme	Control chart or group of control charts
SPC	Statistical Process Control
SPSS	Trademark of the statistical software used for the data processing.
SS	Steady State
SSATS	Steady-State Average Time to Signal
SSARL	Steady-State Average Run Length
TS	Time to Signal
UCL	Upper Confidence Limit
X-Chart	An individuals control chart; equivalent to a Shewhart chart when the control limit coefficient, *h,* equals 3.0
$\overline{X}$ -Chart	Xbar control chart; a scheme which monitors the average of groups of data with a defined number of sequential samples
$\overline{X}$ -EWMA	A hybrid control charting technique comprising of Xbar and EWMA components with separate control limits for each component
$\overline{X}$ -CUSUM	A hybrid control charting technique comprising of Xbar and CUSUM components with separate control limits for each component
X-MR	Individuals – Moving Range composite control chart

List of Symbols

AlIC	Vector of component loadings (percentage contributions by each component in a composite scheme to the gross false alarm frequency)
$\mathbf{AlIC}_{\mathbf{Target}}$	
$Al_{\gamma}IC$	Loadings (percentage contributions by each component in a composite scheme to the gross false alarm frequency) for component γ
$c1$	Intercept of the secant which relates the ICATS to the ICATS search dimension variable, l
c3	Vector of intercepts of the secants which relate the loadings, **AlIC**, to the vector loadings search dimension variables, **g**
δ_{μ}	Parameter for step shift in the location of the mean, normalised
δ_{σ}	Parameter for step shift in the standard deviation, coefficient
ε_i	Random error component of a signal, at iteration i
g	Vector of component loading search dimension variables
h	Vector of control limit coefficients
h_j	Control limit coefficient for component j when distribution parameters are known
h_j '	Control limit coefficient for component j when distribution parameters are estimated from a sample of size n_{estim}
k	Reference parameter for CUSUM charts
κ	Ramp rate coefficient
l	ICATS search dimension variable
λ	Weighting parameter for EWMA charts
$m1_i$	The gradient for the SSATS search dimension variable l_i calculated for iteration i
$m3_{i,j}$	Gradients for the component loading search dimension variables $\overline{g}_{i,j}$ for search iteration i
$\lvert MR \rvert$	Absolute value of the moving range

MA_j	Moving average statistic for component j
μ_0	Mean of the in-control reference population
n	Span parameter for MA charts
n_δ	Number of nodes, location shifts at which control chart schemes are compared
$n_{effective}$	The span which would characterise a MA control chart which would have a power similar to the EWMA control chart under consideration
n_{estim}	Number of observations used to estimate the parameters of a variable
n_{trials}	Number of chart runs used in the simulation of an ICATS value
P	A control chart parameter such as λ for EWMA charts, k for CUSUM charts, and n for MA charts
s	Standard deviation estimated via the mean sum of squares based formula
σ	Altered standard deviation of monitored variable
σ_0	Standard deviation of the in-control reference population
σ_Q	Standard deviation of the EWMA statistic
t	Estimated mean from a sample of in-control observations
τ	Observation number
Y_i	Value of the monitored variable at observation i
$\mathbf{w}_i$	Response vector from a simulation. Contains the elements [ICATS, Al_1IC, Al_2IC, …., Al_vIC]

Chapter 1

Introduction

Statistical process control is a field primarily researched from the perspective of two different schools: industrial engineering and business. Control charts, the subject of this thesis, are a subset of statistical process control tools used for monitoring for deviation from a stochastic model over time. Potential applications for control charts are monitoring indicators of asset utilisation, agriculture, environment, macro-economics, community health and welfare, but control charts are most commonly applied in process and laboratory quality control within the manufacturing industry.

A major consideration for choosing the type of control chart to use for an application is detection performance. Many different control chart types have been defined since 1924 including: univariate and multivariate; individual, $\overline{X}$, simple moving average (MA), exponentially weighted moving average (EWMA), cumulative sum (CUSUM) and run rules (Montgomery,). Control chart selection and design may present a daunting set of considerations for a person wishing to implement an optimised system of control charts. Composite control charts, which offer good performance for a range of location shifts (Sparks, 2000), have insufficient comparisons available in literature to aid an informed selection. More detection power is still needed in some applications, particularly where costly off-line analysis is concerned. Methods for limiting false alarms are also required in data rich environments. This thesis makes a contribution to both of these areas.

Schemes comprising of multiple cooperating control charts monitoring a single variable are sometimes called composite control charts. Alternatively they may be called composite monitoring schemes. Composite control charts based on MA, EWMA, and CUSUM control charts have been noted (Lucas and Saccucci, 1990; Sparks, 2000, 2003; Klein, 1996, 1997) to offer good performance over a range of location shift sizes. In this thesis, composite schemes are denoted by adding "C" as a prefix to the abbreviation of the basic statistic, eg. CMA is the abbreviation for composite moving average. The primary aim of this thesis was to compare the statistical performance of CMA, CEWMA, and CCUSUM control charts for the first time over a range of location shift sizes to provide sufficient insight for informed selection from control chart options. The features of composite control charts which may facilitate use within a management structure were also explored.

1.1 Control Charts in Manufacturing

1.1.1 Australian Manufacturing Context and Motivation

In 2002, manufacturing activity represented a contribution of 13.3% to Australia's gross domestic product and a similar percentage of employment within Australia; whilst the contribution to export earnings was 47.3% (see Figure 1-1). Cost of production typically decreases as technology develops through innovations. Innovations are arguably driven by competition. Sustaining the level of Australian exports income, clearly important to maintaining the gross domestic product, requires Australians to innovate. Innovations in statistical process control may make a small contribution to the competitiveness of manufacturing and other industries in the years to come. Benefits could include increased energy efficiency via stabilised process plant operation and improved product quality. Some shortcomings in control chart technology, as noted in Section 1.2, have provided opportunities for novel and innovative works in this thesis.

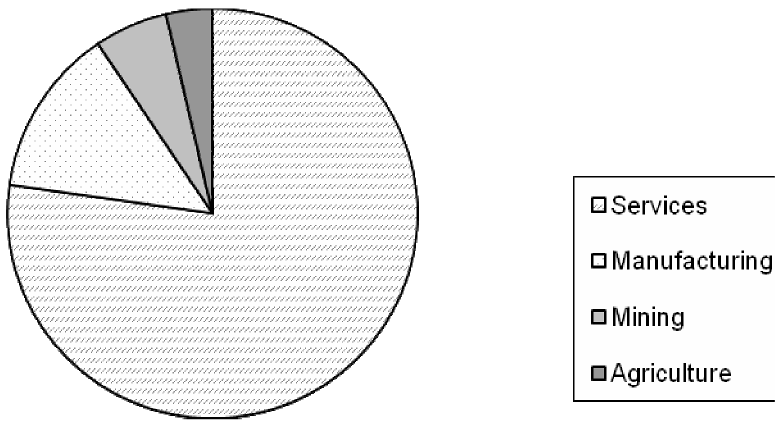

Figure 1-1. Pie chart of contribution by industry to gross domestic product in Australia, 2002. *Source: Queensland state government web page* https://www.qld.gov.au.

Please note that cement and mineral processing businesses, interest areas of the author, are grouped within the category of Manufacturing by the Australian Bureau of Statistics. Cement industry data are used as an example later in this thesis.

1.1.2 Trends and Opportunities for Statistical Process Control

Monitoring algorithms needed to be simple for the most part of the twentieth century because updating calculations and plotting of a control chart was labour intensive. Technology currently used in industry is considerably more advanced than that which was available upon the invention of the Shewhart chart in 1924 (Shewhart, 1931). Measurement, analysis and charting of process variables are mostly automated in recently commissioned continuous-process plants. Distributed control system (DCS) software is used to manage many modern process plant operations where streams of individual measurements are collected. Premium level process information management system (PIMS) software includes Matrikon's ProcessMonitor™, ProcessDoctor™ (Matricon Pty. Ltd., 2004), and Honeywell's Experion PKS (Honeywell Pty. Ltd., 2004). PIMS software are designed to access databases written by the DCS software. Features of modern PIMS software now include advanced statistical process control algorithms including multivariate projection methods such as partial least squares and principle component analysis. This advancement provides an opportunity for adoption of more complex statistical process control algorithms.

1.1.3 Purpose and Architecture of Control Charts

Control charts are used to detect changes in the distribution of a variable over time, effectively by performing serial statistical inference tests. Therefore, control charts have statistical properties conditioned to specified assumptions. Control charts differ from classical data analysis in which experimental data are analysed at the end of each screening stage. When collecting data from a continuous process plant, one may wish to detect a change in the mean of a variable as fast as possible. An inference test is needed upon every instance that a new item of data becomes available.

Constructing an individuals control chart (X-Chart) involves plotting a line-chart of the variable and marking the position of the assumed mean of the data (see Figure 1-2). Control limits are then plotted. A control limit is a boundary at which an alarm is signalled indicating a change in the local mean of the variable. For normally distributed variables the upper control limit (UCL) and lower control limit (LCL) are symmetrical about the assumed mean of the data. A simple design approach requires specifying the in-control average run length (ICARL) and then determining the required offset for the control limits from the mean to achieve that ICARL. The default design value for the margin for the control limits about the mean was historically three standard deviations (Shewhart, 1931; Nelson, 1982) giving an ICARL of 370.4. When an assay falls outside of the range between the control limits, investigation into the cause of the deviation should then commence. More elaborate control chart configurations and design procedures are discussed later.

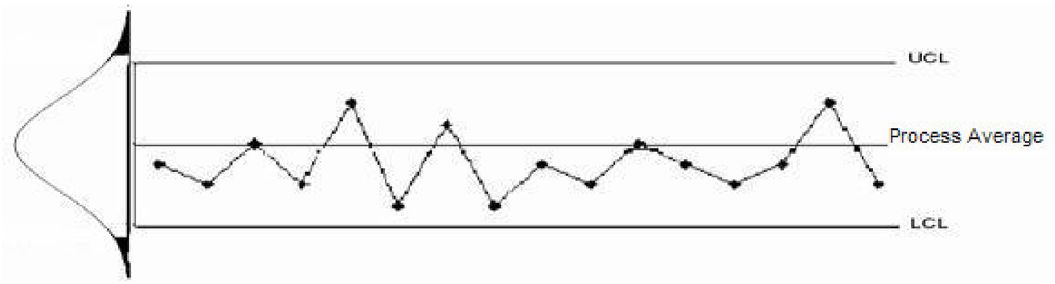

Figure 1-2. Appearance of a simple control chart. Normal curve added to demonstrate the distribution of the data. *Adapted from image supplied by Six-Sigma First (2007).*

1.1.4 Control Chart Use

Having knowledge of a shift in process values is useful because it provides the operator with a flag to search for, and to correct, the cause of the process shift. Removing the cause of the process disturbance may remove any corresponding threats to equipment longevity, plant productivity and product quality that were introduced by the process disturbance. Control charts are usually configured in a way that they alarm upon: detection of a shift in the local mean (location) of a variable; an increase in the variance, and Type I inference errors. This thesis focuses on measuring and optimising the performance of control charts for detection of shifts in the location of a variable's mean.

A Type I inference error occurs when it is concluded that a new sample is not from the population being considered when it actually is from that population (Walpole and Myres, 1989). Control charts are intended to alarm for actual changes in the distribution of a variable related to "assignable" causes. Incidental alarms related to Type I errors are not desired, but are inevitable nevertheless. Some control charts also exist for detecting a reduction in variance (MacGregor and Harris, 1993; Braun, 2003).

Alarms related to Type I inference error may be considered, in practical terms, as an event where an unlikely combination of "common causes" coincide. For a more detailed explanation, please refer to Montgomery and Woodall (1997). Frequently called "false alarms", breaches of the control limits related to Type I errors often return to a non-alarm state within a few observations. Common cause variation is accepted as part of an in-control process. Interacting with product specifications, common cause variation affects the process "capability" (Wang et al, 2000; Veevers, 1998), and may include considerable random sampling error. Common causes are usually addressed through continuous improvement programs, which may require capital investment or development of new technologies. Assignable causes are related to discrete failures that may be addressed immediately in a narrow project scope. False alarms should be minimised so that one's confidence in the control chart, hence one's alertness to assignable causes, is maintained. Reducing common cause variation requires improvement of the process and may require significant capital to purchase newer technologies. Alternatively, significant operating expenditure may be required to change of a number of operating procedures, changes which are typically based on much data and managed in a planned and non-reactive manner. Another reason that excessive false alarms are not desired is because of the over-adjustment phenomenon (see Nelson, 2003).

A control chart may not instantly alarm the effect of an assignable cause after onset. The design of a control chart can minimize detection times with consideration to an acceptable false alarm rate.

In a small fraction of cases, assignable causes may be quickly rectified by virtue of a feedback mechanism, or even by accident. Assignable causes that disappear after one observation have been called isolated special causes (Hawkins et al, 2003). Conversely, there are sustained assignable causes for which an investigation must be carried out to identify the cause of the location shift and the most appropriate way to rectify the situation. Figure 1-3 is a simplified description of the cycle of activities in which control charts play part with the intention of keeping a process predominantly in-control.

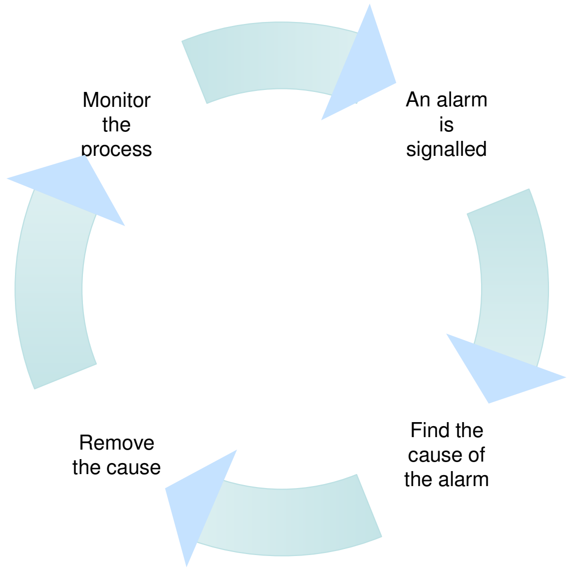

Figure 1-3. The cycle of process monitoring and correction

Plant operators and engineers may be required, by a company's quality policy, to act upon alarms generated by control charts. In reacting to an alarm, it is best to firstly diagnose the cause of the location shift using experience and operating records. Once a diagnosis is arrived at, a decision may be made to initiate restoration activities immediately. Alternatively, it may be decided to wait a period of time for a suitable maintenance window before correcting the apparent process problem. Upon restoration of the apparent cause, it might be discovered that the diagnosis was incorrect, and so the process of fault finding and restoration must be repeated. It can been seen that the total amount of time in which the process is not performing as intended is from the onset of the location shift until removal of the process shift. There are many components of time that make up this period of off-target production. Given below is a hypothetical quality control example based on 6 hourly off-line analysis. An expanded list of the sequential activities, and corresponding time intervals that occur after a serious quality problem becomes evident, may include:

- Location shift in variable. Interval between this event and sampling may be some part of 6 hours, say 3 hours.
- Sample transfer laboratory or offline analyser ~ 10 minutes
- Analysis of sample ~ 15 minutes
- Data transfer/entry into the control chart ~ 10 seconds
- Control chart Time to Signal (TS): from first location shifted data entry to detection ~ some multiple of 6hours, eg, 0, 6, 12, …hours
- Alarm signal to be noted by an Operator and commencement of action ~ 10 seconds to 20 minutes depending on other priorities
- Root cause analysis ~ 10 minutes to 2 weeks
- Management involvement and waiting time until maintenance opportunity ~ 0 seconds to 6 months.
- Engineering and operations activities to rectify the problem ~ 1 hour to 10 days.

Selection of a control chart design typically falls under the accountability of a quality manager. The basis of the control chart selection by a quality manager may consider set-up and operational cost, the efficiency in detecting excursions in quality, presentation and user friendliness.

1.1.5 Introducing Cement Quality Variables

Cement is a synthetic ingredient that is used in concrete and other building materials and is made from ground clinker, limestone and gypsum; used extensively in housing, civil structures and increasingly in roads. One measure of cement quality is the compressive strength it develops in a mortar form, a mixture of cement, sand and water. The International Organization for Standards (1989) provides a procedure for testing mortar compressive strength. Mortar compressive strength displays high variance between homogenous "control samples", having a standard deviation between 0.6 MPa and 1 MPa depending on the laboratory. Other important performance measures of mortar include the

Blaine (cm^2/gram), false set (mm), initial set (hr), final set (hr), normal consistency (mm), and 3, 7 and 28 day concrete strengths (MPa).

Typical factors affecting the mortar compressive strength include the chemical and mineral composition of the raw materials, the ratio to which they are mixed, and the particle size distribution of the ground product.

Cement Australia Pty Ltd (CAPL) provided data for use in this thesis. Figure 1-4 shows some compressive mortar strength ("ISO" as it is often referred to informally) history depicting a positive step or ramp shift at Observation 64. Compressive mortar strength sometimes increases due to increasing recirculating load in closed-circuit milling process. Control charts are applied to these data in a later chapter.

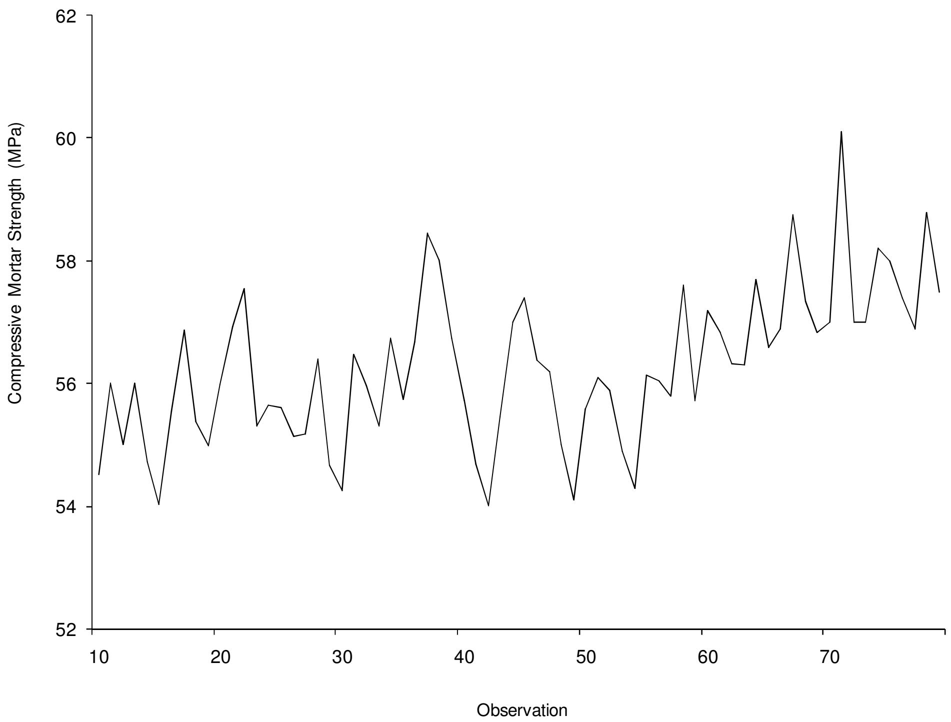

Figure 1-4. Compressive mortar strength history courtesy of Cement Australia Ltd (un-named production site and manufacturing period).

In 2003, Cement Australia's quality assurance approach was monitoring individual data using fixed width control limits arranged in warning and action zones.

1.2 Rationale for the Study

From literature, it is unclear what type of composite control chart offers the best statistical performance for a distribution of step shifts. The rationale behind this thesis relates to weaknesses in existing control chart performance measures, opportunities for optimisation of composite schemes, and developments required for making control charts designs scalable for use on a large number of variables. Listed in point form, the research is intended to cover the following knowledge base gaps:

- Some traditional assumptions in control chart studies are not representative of a typical manufacturing application.
- Few publications have used a scalar statistical measure to describe the performance of a control chart over a number of location shifts scenarios.
- Existing scalar statistical measures for control chart performance, over a number of location shifts, do not give values that are readily cross referenced between publications.
- Composite schemes have not previously been statistically optimised and compared. The effects of the type of composite scheme selected, and the number of components in a composite scheme, are not known.
- No method has been described which facilitates scaling of control chart designs according to the number of variables to be monitored by each level of company management.

Factors which made the timing of the thesis favourable include:

- Advances in computational processing rates
- Proliferation of SPC complementary software in industry

The points of rationale are expanded in the following subsections.

1.2.1 Existing Performance Measures and Comparison Techniques

A description of control chart performance over a breadth of location shifts has mostly relied on verbal descriptions and graphs (eg. Jones, Champ and Rigdon, 2001) as opposed to use of a scalar statistical measure (eg. Sparks, 2003). Often performance is described by stating the ARL for one specific location shift, for example, the ARL for a one standard deviation shift in the mean. In reality, assignable causes occur with a distribution of location shifts sizes. A standardised measure of chart performance over a distribution of location shifts is needed so that users can make a well informed design selection. To optimise control charts for a distribution of location shifts, a scalar value is required to represent the expected long-term performance. Sparks (2003) developed a performance comparison measure for a domain of step and ramp location shifts which he called relative loss efficiency (RLE). Whilst this is an important advance for control chart studies, RLE is not very suitable for use in an optimisation routine. A new statistical performance measure is required to succinctly compare control charts over a domain of step shift sizes, having a value which is readily transportable for making comparisons between publications, and which can be used for optimisation.

1.2.2 Optimised Composite Scheme Comparison

There is a gap in the knowledge base of optimum CMA, CEWMA and CCUSUM scheme performance: none of these schemes have been statistically optimised. See, for example: Sparks, 2003, on CMA; Klein, 1996, on CEWMA; Sparks, 2000, on CCUSUM; Sparks (2004) on Group of Weighted Moving Averages. In each of the publications above, a few seemingly ad-hoc designs are compared. Therefore, it is not known how much these schemes differ in performance when optimised.

Sparks (2003) compared a number of CMA schemes against EWMA and CUSUM schemes and found that the CMA scheme demonstrated fast detection for a range of location shifts. It cannot be expected that these apparently ad hoc (or semi-optimised) design results will necessarily be optimum for the domain of location shift considered. He noted that CMA schemes were favourable from the point of view that the MA statistic may be simpler to understand for less statistically trained SPC users than the CUSUM statistic. Due to the lack of local or global optimisation, further consideration of CMA schemes is warranted. Hence CMA schemes have been included in comparisons of this thesis.

CEWMA schemes with two components have been investigated by Albin, Kang and Shea (1997) who noted that CEWMA charts can detect increases in variance with favourable ICARL values. They showed the reduction in ICARL was less for an $\overline{X}$ -EWMA composite than for the $\overline{X}$ and Moving Range ($\overline{X}$ -MR) composite, but detection of large (factors greater than 2) step shifts in the standard deviation were detected similarly as fast. Therefore, optimised CEWMA schemes could potentially make range charts redundant.

Roberts (1959) suggested that, given any MA control chart, an EWMA control chart can be constructed with roughly equivalent properties. Therefore, it might also be expected that CMA and CEWMA control charts will also perform similarly when optimised. Sparks (2003) claimed that he trialled unspecified three-component CEWMA schemes which reportedly did not perform as well as CMA designs. He recommended further development of CEWMA schemes as the initial attempts were unlikely to produce an optimal design. EWMA control charts have been found to perform well in detecting ramped location shifts (Sparks 2003). CEWMA schemes, which are based on several EWMA components, may also retain this strength and similarly be efficient at trend detection. CEWMA schemes may have strengths other than performance on step location shifts that have not previously been considered. CEWMA schemes were included in the thesis to expand the knowledge base on this tool.

Finally, let us consider the potential value of optimising CCUSUM schemes. Lucas and Saccucci (1990) showed that CUSUM and EWMA schemes perform similarly, concluding that practical issues be used to decide which scheme to select. Therefore, it is reasonable to expect that CCUSUM and CEWMA schemes will also perform similarly. Hence, CCUSUM schemes were also included in the thesis.

In summary, CMA, CEWMA and CCUSUM, are all expected to perform similarly based on extrapolation of simpler concepts from literature. Some features differentiating MA, EWMA and CUSUM techniques, other than statistical or economic performance measures, have also been noted in literature. It is acknowledged that consideration of these features may assist in selection of a control chart. However, to date, no quantitative performance data based on optimisation and comparison of composite schemes has been published. Comparison of optimised composite monitoring schemes will remove all ambiguity related to the statistical performance of various composite schemes from the selection process. Possessing such information, users will be better informed on the general properties of composite schemes. This work is not intended as a substitute for detailed investigations such as economically optimising total quality cost.

1.2.3 Advances in Computational Processing Rates

Control chart properties can be derived or simulated. Simulation has been popular over a long period of time and has been used by authors such as Albin Kang and Shea (1997), Klein (1996, 1997), Jiang, Wu, Tsung, Nair and Tsui (2002), Sparks (2003), and Reynolds and Stoumbos (2004). Simulation provides a simple way of determining control chart properties, particularly in the case of composite schemes which can be complex to derive analytically. Simulation, however, does not lead to exact determination of control chart properties. The properties are estimated from a sample; therefore, a confidence region exists about each estimate. Advances in computer processing rates have permitted increased simulation sample sizes for a given processing time. Large sample size simulations were used in this thesis to distinguish control charts which have similar performance.

Decreased simulation costs has also meant that full factorial experimental designs have become feasible for investigating optimum composite designs. The advantage of optimising via full factorial designs, over advanced methods like genetic algorithms, is the option to create educational surface area plots for inclusion in the research results. One can also investigate interactions between the design parameters.

Composite control charts researched in this thesis required up to four times as many computations than do single component control charts. Advances in computer processing rates have increased the feasibility of research into control charts which are computationally demanding to research. Not only can affordable modern personal computers be used to research composite schemes, they are capable of updating and plotting the increased number of signals in a manufacturing plant which may have thousands of raw variables (being a mixture of on-line and off-line measurements).

1.2.4 Proliferation of SPC Complementary Software

Process information management system (PIMS) databases and performance management software are standard inclusions in new processing plants and a significant fraction of older plants have implemented such systems. Performance management software (see examples in Section 1.2.2) makes it easy to build control charts and the real-time computations are automatic. It is estimated that it would be economically feasible to create control charts for all controlled variables within a manufacturing company where previously only key quality variables were typically monitored in this way.

Adoption of published control chart technology by industry has been poor (Woodall and Montgomery, 1999). Poor adoption suggests that there are outstanding issues for implementation and operation of complex control charts, or lack of awareness of the availability of these techniques. There have been many innovations in the control chart field, particularly since 1980, with some very complex, and powerful tools developed. Public debate over the reasons for poor adoption, and what might be done to increase adoption, arises periodically in the

Journal of Quality Technology (for example Woodall and Montgomery, 1999; Montgomery and Woodall, 1997). Suggested reasons for poor adoption include the fact that users of control charts have very little statistical training; and some publications have purely academic merit and were never intended to be directly used in applications but are valued because they lead to more practical concepts. One particular design issue that has not been mentioned, is scaling control chart designs for process plants with vastly differing numbers of variables to be monitored.

Industry began to centrally collect data at a high frequency for a large number of variables with the adoption of DCSs from around the early 1990s. Each control chart being operated has a certain false alarm rate. An increased number of monitored control charts incur a proportional increase in the total false alarm rate. An overload of false alarms could develop if all quality variables are monitored using control charts. Personnel involved in root cause analyses of assignable causes may learn that no assignable causes exist for some alarms. Reduced motivation to rigorously investigate further alarms may then result.

Control charts may be used to generate exception reports. A large number of control charts present a logistical challenge to monitoring of quality by middle levels of management. The configuration of composite schemes may provide an opportunity to address the problem of scaling control chart designs for monitoring at different levels within a company hierarchy. Such an innovation would be a contribution to resolving practical issues experienced by industry; an issue which might otherwise cause resistance to adoption of control charts for plant-wide implementation.

1.2.5 Traditional Assumptions in Control Chart Studies

A review thesis by Woodall and Montgomery (1999) recommended that future research includes techniques for data rich and multi-step processing environments, data reduction methods, economic designs and study of the effect of estimated parameters, etc. On the subject of the effect of estimated parameters they said:

> *"Much more research is needed in this area recognising that Phase II control limits are in fact, random variables. Research shows that more data than has been traditionally recommended is needed to accurately determine control chart limits."*

The distribution parameters of monitored variables are not known in practice and must instead be estimated. Comparison of composite control charts by Sparks (2000, 2003) assumed known parameters, as have many publications. An assumption of known parameters does not reflect the situation of a typical company where control charts are applied. Conclusions regarding control chart alarm profiles for known parameters may not necessarily be consistent with an assumption of unknown parameters. No research has been published for composite control charts with estimated parameters, so it is unclear what type of composite control chart will perform best in real situations. Studies into the comparative performance of CMA, CEWMA and CCUSUM schemes based on ***estimated parameters*** have not previously been published. Optimising and comparing composite schemes in simulations where parameters are estimated is the approach used in this thesis.

1.3 Aims of the Thesis

The basic objective of this thesis was to explore composite control charts so that manufacturing end users would be sufficiently informed to select a suitable control chart. The control charts to be explored were CMA, CEWMA, and CCUSUM. The primary aim was:

Aim 1 - understand which of these composite control charts performed best over a domain of location shift sizes.

Specifically, the statistical performance was sought based on appropriate assumptions for typical manufacturing end users. That is to say, distribution parameters should be estimated rather than assumed to be known.

To achieve the primary aim, the following tasks were essential:

- Develop improved statistical measures and methods so that control chart performance could be optimised and compared for a domain of step shifts.
- Create software to derive control chart properties where existing analytical methods and software were inadequate for the task.
- Optimise composite control chart configurations (using the newly developed statistical performance measures and simulation software).
- Compare optimised composite control charts.

Secondary aims to achieve the basic objective include:
Aim 2 – determine the benefit of using three components as opposed to two.
Aim 3 – compare the performance of the control charts for ramped location shifts.
Aim 4 – identify additional opportunities that composite control charts offer over alternative control chart types.

With such insights, end users might better understand various trade-offs afforded by composite control charts when selecting a control chart to implement.

1.4 Structure of the Thesis

The structure of the thesis is as follows. Chapter 2 presents definitions and formulae. Step and ramped location shifts are defined mathematically as well as the EWMA, MA and CUSUM statistics. Chapter 3 defines the performance measures used to assess and compare control charts. Simulation of run length and alarm profiles is discussed in Chapter 4 including assumptions and specifications used, and a description of software created for the research. In Chapter 5, some insight into the basis of composite schemes is given with charts of the expected number of alarms over sequential observations from a step shift.

Full optimisation and comparison of three-component CMA, CEWMA and CCUSUM schemes is detailed in Chapter 6. Distribution parameters of the monitored variables were assumed to be unknown. Conclusions and recommended future directions are discussed in the Chapters 7 and 8 respectively. The appendices contain supporting data and further studies which have been set aside to streamline the key concepts of the thesis.

Chapter 2

Control Chart Definitions and Background Literature

2.1 Process and Process Disturbance Models

Random-normal independently and identically distributed (iid) processes with superimposed step and ramp location shift disturbances are the most commonly used scenarios for scheme performance comparison. The models used in this thesis for each of the disturbance types are shown below. Samples are taken at instances, i, an integer variable, and the sample at instance $i=\tau$ is the first sample that contains the shifted mean. The actual shift occurs some time between τ and $\tau-1$.

Step Shifts in the Mean:

$$Y_i = \mu_0 + \varepsilon_i \qquad \text{for } i = 1, 2, \ldots, \tau - 1$$
$$Y_i = \mu_0 + \delta_\mu \sigma_0 + \varepsilon_i \qquad \text{for } i = \tau, \tau + 1, \ldots$$

Ramp/Trend Step Shifts in the Mean (for example Davis and Woodall, 1988):

$$Y_i = \mu_0 + \varepsilon_i \qquad \text{for } i = 1, 2, \ldots, \tau - 1$$
$$Y_i = \mu_0 + \kappa \sigma_0 t_i + \varepsilon_i \qquad \text{for } i = \tau, \tau + 1, \ldots$$

For both step and ramp shifts in the mean, it is assumed that the random variation, ε is distributed as:

$$\varepsilon_i \sim N\left(0, {\sigma_0}^2\right) \qquad \text{for } i = 1, 2 \ldots$$

Step Shifts in the Variance:

$$\varepsilon_i \sim N\left(0, {\sigma_0}^2\right) \qquad \text{for } i = 1, 2 \ldots, \tau - 1$$

$$\varepsilon \sim N\left(0, \sigma^2\right) \qquad \text{for } i = \tau, \tau + 1, \ldots$$

where

$$\sigma = \sigma_0 \delta_\sigma$$

For ramp shifts in the mean, it is particularly important to be specific about when the parameter for the mean of the population actually shifts. Occurrence of an assignable cause is not restricted to uniformly spaced instances but rather occur with a continuous random distribution between sampling instances. If the disturbance is assumed to manifest infinitesimally later than $\tau - 1$, the magnitude of the ramped shift at instance τ has a specific value facilitating comparison with other studies. t is the time index for the ramp model, and is equal to 0 at $\tau - 1$, i.e.:

$$t_i = i - \tau + 1 \qquad \text{for } i = \tau, \tau + 1, \ldots$$

Traditionally, if a control chart alarms on the first sample which occurs at the same time or after a location shift, the run length is given a value of 1. However, some publications of an economic control chart nature will express an alarm on the first sample after a location shift as having a stopping time, TS = 0.

2.2 Formula for Basic Control Charts

EWMA, MA and CUSUM statistics are defined below within formulae which are in a general form for description of *j* components within a composite scheme.

2.2.1 The EWMA Statistic and Alarm Criteria

An EWMA statistic *j*, at iteration *i*, is a found by $EWMA_{i,j} = \lambda_j Y_i + (1 - \lambda_j) EWMA_{i-1,j}$ (Roberts, 1959) for some smoothing constant selected such that $0 < \lambda \le 1$, $j = 1, 2, \ldots, \text{v}$ different components in composite scheme. For $i = 1$, $Q_{i-1} = 0$. EWMA values may be warmed up for a period after $i = 1$ (see Section 2.3.4). By the central limit theorem, one could expect $EWMA_{i,j}$ to be approximately normally distributed for small smoothing coefficients regardless of the distribution of *Y*. Borror, Montgomery and Runger (1999) demonstrated that the ARL profile of EWMA control charts was robust to non-normality in the monitored variable.

When the distribution ***parameters are known***, an alarm is generated in a CEWMA scheme when any of the EWMA scheme components, *j*, alarm individually or together according to the test:

$$\left|\sqrt{\frac{(2-\lambda_j)}{\lambda_j}}\cdot\frac{(EWMA_{i,j}-\mu_0)}{\sigma_0}\right| > h_j \qquad (1)$$

Here w is the number of components each with a corresponding control limit coefficient h_j.

When the distribution ***parameters are estimated***, the positioning of control limits must be based on the sample standard deviation $\hat{\sigma}_Y$, and *t*, the sample mean. Substituting the estimated parameters into (1), one gets:

$$\left|\sqrt{\frac{(2-\lambda_j)}{\lambda_j}}\cdot\frac{(EWMA_{i,j}-t)}{s}\right| > h_j\text{'} \qquad (2)$$

where h_j', the control limit coefficient for schemes based on estimated parameters, is a function of the degrees of freedom in estimating the parameters, and the particular method of estimating the standard deviation. This identification system has been used for the MA and CUSUM components also.

2.2.2 The MA Statistic and Alarm Criteria

The MA statistic, $MA_{i,j}$, in a CMA control chart (Chen and Yang 2002, Sparks 2003) is defined as:

$$MA_{i,j} = \frac{\left(Y_i + Y_{i-1} + \ldots + Y_{i-n_j+1}\right)}{n_j}$$

Here, $MA_{i,j}$ is the moving average characterized by the span n_j; for $j = 1, 2, ..., \mathrm{v}$ components in the composite. For ***known parameters***, the control chart alarm test is:

$$\left| \frac{\sqrt{n_j} \cdot \left(MA_{i,j} - \mu_0 \right)}{\sigma_0} \right| > h_j \tag{3}$$

where h_j is the control limit coefficient for the moving average statistic MA$_j$. For an MA scheme based on ***estimated parameters*** an alarm is raised at occasion *i*, if for any *j*:

$$\left| \frac{\sqrt{n_j} \cdot \left(MA_{i,j} - t \right)}{s} \right| > h_j' \tag{4}$$

where h_j' is the control limit coefficient for the moving average statistic $MA_{i,j}$. Calculations for *t* and *s* are shown in Section 2.2.4.

2.2.3 The CUSUM Statistic and Alarm Criteria

Page (1954) developed an SPC technique which cumulates the sum of deviations from target. A two-sided CUSUM scheme requires one statistic to be calculated (Wu and Wang, 2007), one each for the control limits above and below the mean.

$$CUSUM_{i,j} = \max[\mu_0, CUSUM_{i-1,j} - \mu_0 + Y_i - k_j \sigma_0], \text{ if } CUSUM_{i-1,j} > \mu_0 \text{ or}$$

$$\left(CUSUM_{i-1,j} = \mu_0 \text{ and } \mathrm{Y}_i > \mu_0 \right).$$

or,

$$CUSUM_{i,j} = \min[\mu_0, CUSUM_{i-1,j} - \mu_0 + Y_i + k_j \sigma_0], \text{ if } CUSUM_{i-1,j} < \mu_0 \text{ or}$$

$$\left(CUSUM_{i-1,j} = \mu_0 \text{ and } \mathrm{Y}_i < \mu_0 \right).$$

and k_j is the reference value which causes the statistic to tend back to a central position of zero the variable it is statistically in-control. Zero becomes a reflective boundary (Sparks, 2000) due to the use of min and max in the formula. This gives CUSUM an advantage over MA and EWMA statistics as the more distant “memory” of random or assignable off-target runs does not cause inertia that could slow detection of present shifts in the mean to the opposite side of the target.

For ***known parameters***, and control limits which are symmetrical about the mean, the alarm condition for CUSUM components are:

$$\left| \frac{CUSUM_{i,j} - \mu_0}{\sigma_0} \right| > h_j \tag{6}$$

Where h_j is the control limit coefficient for the CUSUM component CUSUM$_j$, when parameters are known.

For ***estimated parameters,*** s is substituted for σ_0 in calculation of the CUSUM statistics in (5). An alarm condition is true if:

$$\left| \frac{CUSUM_{i,j} - t}{s} \right| > h_j \quad , \tag{7}$$

where h_j' is the control limit coefficient for the CUSUM component j when parameters are estimated.

Numerous authors have studied CUSUM schemes including Gan (1992), Koning and Does (2000), Lu and Reynolds (1999), and Sparks (2000).

2.2.4 Estimation of Dispersion in the Data

The control limits are positioned as multiples of standard deviation for each of the control charts. The standard deviation can be calculated using the traditional sample standard deviation formula, as shown in Equation 8, or via a formula based on the absolute moving range, Equation 9.

$$s = \sqrt{\frac{\sum (Y_i - t)^2}{n_{estim} - 1}} \tag{8}$$

where t is the mean of the n_{estim} observations in the in-control sample.

$$s = \frac{average(|MR|)}{1.128} \tag{9}$$

where $MR = Y_i - Y_{i-1}$ and $|MR|$ is an average of (n_{estim}-1) differenced values. The absolute moving range formula is an inefficient method for estimating the standard deviation for in-control data, but is perhaps better for data that is not truly in control.

2.3 Background Literature

A full literature review on control charts for continuous distributions of data would require several volumes (Woodall and Montgomery, 1997), with recent research topics covering the effect of parameter estimation (Bischak, 2007), data reduction (Model et al, 2002) and non-parametric techniques (Jones and Woodall, 1998), economic designs including variable sampling schemes (Vommi, Murty and Seetala, 2007), techniques for robust performance for a distribution of disturbances (Capizzi and Masarotto, 2003), time-series (Ridley and Duke, 2007; Pan and Jarrett, 2007) and change point methods (Zou, Zhang, and Wang, 2006). Discussion of literature, limited to that which is highly relevant to composite control charts and the objectives of this thesis, is continued below. The Journal of Quality technology is the most referenced journal because it is a journal that has a large proportion of papers on control charts with a theoretical content appropriate for a research degree.

2.3.1 Control Chart Phases

When it is decided to adopt a control chart for monitoring a variable, it is usually recommended to commence by retrospectively analysing the data to see if the process is in-control (eg. Bischak and Trietsch, 2007). This is called a Phase I control chart. Phase I is differentiated from Phase II partly because Phase I is retrospective and Phase II is prospective, real-time monitoring. Other differentiators are: Phase I is usually not in-control whilst Phase II is usually in-control, and the estimates for the distribution parameters are not as accurate as estimates in Phase II. Substantial effort may be required to improve operations and maintenance systems to bring the process under control requiring several iterations of data collection and parameter estimation. Once the process has been kept in-control for a period, Phase II real time monitoring can commence. Phase II charts should then be using distribution parameters estimated from data containing only common variation and not assignable causes.

Control chart schemes designed in this thesis assume estimated parameters based mostly on 200 observations (using the moving range based formula for estimating the standard deviation). Clearly, the designs will be very suitable for a scope covering Phase I to early Phase II when only 200 in-control observations, or there abouts, are available. However, the application of these designs is not as limited as the scope described above. There are usually insufficient control variables, hence insufficient degrees of freedom to be able to adjust all final and intermediate process variables to a target. As a result, the targets for many variables are determined as a consequence of decisions about control of other variables. Though, it is argued that many process plant variables targets, other than those for final product quality, need to be re-estimated and adjusted periodically. Therefore, the designs from this thesis may be considered equally applicable to Phase I and Phase II real time monitoring.

2.3.2 Composite and Adaptive Control Charts

A number of thesiss have been written on composite and adaptive control charts with performance considered in terms of robust detection of assignable causes of varying disturbance sizes. These are summarised below, commencing with previous studies on EWMA based techniques, followed by MA, then CUSUM based techniques .

Lucas and Saccucci (1990) monitored a single variable with two EWMA components concurrently to give the scheme a faster response for large step shifts. They combined the Shewhart ($\overline{X}$) and EWMA charts in a scheme to take advantage of the ARL performance of Shewhart schemes on large step location shifts. $\overline{X}$ -EWMA schemes constitute a two component CEWMA scheme with one of the smoothing constants assuming the limiting value of one. They found that the $\overline{X}$ -EWMA composite performed similarly to the $\overline{X}$ -CUSUM composite. It was noted that the control limits needed to be raised from the level used for single statistic monitoring to maintain a combined specified ICARL. A recommendation was made that the control limit coefficients for the $\overline{X}$ component be raised from approximately 3.5, the value which gives an ICARL of 500 observations in a stand alone $\overline{X}$ scheme, to “4.0 or 4.5” so that the composite scheme retained a similar ICARL.

Albin, Kang and Shea (1997) considered the $\overline{X}$ -EWMA composite with μ±3σ control limits on each component. Their scope extended to the use of run-rules and moving range (MR) charts within the $\overline{X}$ -EWMA composite and recommended that the $\overline{X}$ -EWMA be used without run-rules or MR components. The $\overline{X}$ -EWMA composite could detect increases in the standard deviation of the data, and resulted in less reduction on the ICARL than did the MR component. However, it should be noted that a Shewhart chart alone could detect a 100% increase in variance with a similar efficiency to the $\overline{X}$ -EWMA and $\overline{X}$ -Run Rules composites. When run-rules were tested, one or two rules were applied. The value of the study was to show the effect on the ICARL when additional schemes are used to monitor the same variable without altering the control limit coefficients. Advice of Lucas and Saccucci (1990) on raising the control limits was not utilised. Use of standard μ±3σ control limits resulted in non-specification of the ICARL. This confounded the effect of the components in the composite and the changing ICARL on ARL performance. However, demonstrating the effect of adding a component to a composite scheme on both the ICARL and ARL was useful information.

Klein (1996; 1997) also investigated $\overline{X}$ -EWMA and $\overline{X}$ -Run-Rules composites but with use of two to four run rules. He considered a second criterion when evaluating scheme performance for a fixed ICARL. In addition to ARL performance on different step shifts, he examined the percentiles of the in-control run length distribution. In all cases, the distribution of the $\overline{X}$ -Run-Rules composite was similar to the comparable $\overline{X}$ -EWMA with constant control limits, where fixed limits of μ±3σ were used for the $\overline{X}$ scheme. Use of time-dependent instead of fixed control limits resulted in more skewing of the in-control distribution (Klein, 1997). Both the time dependent and fixed $\overline{X}$ -EWMA schemes displayed smaller ARL values than the $\overline{X}$ -Run-Rules composite. The restriction to μ±3σ control limits for the $\overline{X}$ component of the composite may have produced sub-optimal results. Whilst simplicity was historically considered an important factor in the success of control charts, it is of interest to know what ARL would be achieved without restricting any of the control limits to a historical integer value.

Sparks (2000) investigated CCUSUM and Adaptive CUSUM (ACUSUM) schemes which were found to perform similarly. Three CUSUM components were recommended for detection of step shifts in the mean between 0.5σ and 4.0σ in size. Apparently heuristic design recommendations (with some theoretical basis) were used to choose the k values instead of optimisation. The ACUSUM method worked by adjusting the k value of a CUSUM scheme according to the optimum for the estimated step shift based on an EWMA forecast of the data. A regression model was used to find the required control limit coefficient for a given value of δ_μ (via the relationship between optimal δ_μ and the theoretical optimum value for k, $k = \delta_\mu/2$, Sparks, 2000), the limits being adjusted at each serial observation. To prevent the monitoring tool becoming excessively powerful for small step shifts, a constraint was applied to the minimum value of k. CCUSUM was found to perform better than ACUSUM at large step shifts; however, ACUSUM is sensitive to the choice of λ in the EWMA forecasting equation. Lack of optimisation and lack of a scalar performance measure for a detection of a distribution of location shifts have resulted in an incomplete understanding of the performance of CCUSUM and ACUSUM methods from Sparks' study. Nevertheless, the thesis serves as an excellent introduction to these tools demonstrating simple heuristic designs which are easy to implement.

Sparks (2003) demonstrated construction of CMA schemes and proposed a number of designs. The performance of CMA schemes in step and ramp location shift scenarios was compared to EWMA and CUSUM schemes. A comparison was yielded by measuring the relative loss efficiency (RLE) for step shifts and ramp shifts. It was found that the CMA design called "Plan 5" (design parameters are detailed in Appendix F) performed with small relative losses compared to a EWMA scheme with λ equal to 0.15 at step shifts less than that for which the EWMA was optimised, i.e. <1σ. However, the CMA scheme performed better on average over the entire 0.25σ to 4σ domain. The performance of CMA schemes compared to EWMA schemes on ramped location shifts was a different matter. An EWMA scheme with λ = 0.15 performed better than the CMA on *all* ramp location shifts from 0.005σ/observation to 0.25σ/observation. With this in mind, a CEWMA scheme may also perform well on ramped shifts if it is based on the EWMA statistic. A recommendation given in the thesis by Sparks for future work in research of CEWMA schemes was the original basis for this thesis.

Another composite control chart scheme described by Sparks (2004) is that of a group of weighted moving averages. This method applied weightings to past observations (within each component control chart of composite scheme) using two tuning parameters. These "dual controls" permitted a configuration equivalent to that of a composite exponentially weighted moving average scheme, and configurations that are more complex. A number of composite scheme designs were provided for ICARL = 400 (for known mean and standard deviation) and the performance of these schemes were compared against a CUSUM and an EWMA control chart. Unfortunately Sparks (2004) did not enlighten us with a comparison against CCUSUM (Sparks 2000) or CMA (Sparks 2003) schemes. All of these composite designs were *ad hoc* but sufficiently refined to demonstrate the advantage of composite schemes over simple control charts where efficient detection of a range of location shift sizes is required.

Adaptive EWMA (AEWMA) control charts were investigated by a number of authors including Wortham, Heinrich and Taylor (1974), Hubele and Chang (1990), Capizzi and Masarotto (2003). Capizzi and Masarotto described the AEWMA method as a smooth combination of a Shewhart and an EWMA control chart. Comparisons of ARL profiles were made between AEWMA, EWMA, CUSUM and two-component CEWMA schemes. It was concluded that the AEWMA had detection properties which were robust to varying disturbance size, and were simple in the fact that only one control chart needed to be monitored. Again, there was no formal optimisation and lack of a scalar performance measure in their study. Also of concern is the small number of simulated chart runs (10,000 per design) and lack of error analysis. Hubele and Chang's AEWMA was based on a Kalman component. The results of Hubele and Chang, and Wortham, Heinrich and Taylor, unfortunately, are probably not very relevant to readers considering ICARL=400 schemes, as insufficient constraints and specifications were applied in the design. Small and inconsistent ICARL values between 20 and 30 resulted. Adaptive control charts and run rules have not been further considered in this thesis as they do not offer a convenient hierarchical monitoring benefit (see Chapter 6, Section 6.6 for more on Hierarchical Monitoring).

2.3.3 Estimation of Parameters

In practice, population parameters, including the mean and standard deviation of a variable, are never truly known so these are estimated from previous in-control data. Jones, Champ and Rigdon (2001) investigated the effect of estimates on the EWMA scheme for independently and identically distributed (iid) data; whilst Lu and Reynolds (1999) did the same for autocorrelated processes. Several authors have also studied the effect of estimating the in-control mean and variance on ARL performance of Shewhart-type schemes including Quesenberry (1993) who researched this explicitly for individual schemes.

Jones, Champ and Rigdon (2001) found that substitution of population parameters by sample estimates can be highly unfavourable for both the in-control and out-of-control run lengths. Two-thousand observations (500 subgroups of 4) were required by EWMA schemes when using a smoothing constant of $\lambda = 0.13$, to keep the ICARL within 8% of the known-parameter based design (with a zero state ICARL = 500). This means that one needs a very large amount of iid data for estimating the mean and standard deviation of the population, to be able to use a design made for known population parameters with negligible deterioration in the ICARL specification. Two-thousand data points represent approximately six years of ***daily*** product-quality measurements from operation without occurrence of any assignable causes. Six years is a considerable delay for set up of control charts for a new process, and this data requirement *increases* when using even smaller smoothing constant values.

Jones, Champ and Rigdon (2001) found that the false alarm rate of the Shewhart scheme is less affected than EWMA schemes by estimation of parameters. This fact suggests that the ICATS of a CEWMA scheme, which has a Shewhart-like component will be less affected by estimation of parameters. The ICARL increasing and decreasing effects might cancel each other out to some degree. A list of research ideas by Woodall and Montgomery (1999) included "more research is needed on the effect of parameter estimation on control chart performance." No studies on the effect of estimation of parameters have been published for composite monitoring plans to our knowledge.

Design procedures for control charts have been proposed to manage the effect of estimating parameters. Jones (2002) suggested that the control limits be widened according to the uncertainty in the parameter estimates such that the ICARL specification is maintained. Selection of the smoothing constant should also take this uncertainty into account because small smoothing constants strongly affect the ARL performance when parameters are estimated. Parameters may then be re-estimated as Phase 1 proceeds and more in-control samples became available. Quesenberry () proposed a Q chart that was based on individual measurements or $\overline{X}$, and which has an algorithm for calculating and updating the control limits from the third observation onwards. Another option for users, in light of the effect of estimating parameters, is to accept inflated or deflated ICARL performance. As more data are accumulated, parameters may be re-estimated so the most adverse effect occurs only in phase 1 when the sample size is still small. Composite schemes lend themselves to another alternative in that components may be activated progressively as more data becomes available, thereby managing any adverse effects of estimation.

Jones, Champ and Rigdon (2001) found that the ICARL of CUSUM charts is larger when estimated parameters are used compared to known parameters. They also found that ARL values were higher when estimated parameters were used, that is, the CUSUM chart became less sensitive to both changes in the mean and variance. Studies have also been done on the performance of the change-point model with estimated parameters after the location shift (Hawkins, Qui and Kang, 2003; Zamba and Hawkins, 2006).

2.3.4 Transition to Steady-State

EWMA and CUSUM based publications vary in assumptions for the initial conditions upon simulation of control chart runs with options including steady state, zero state, and head start. It is important to note the assumed initial condition when reading thesiss as the derived ARL values are affected by this assumption.

A steady state distribution of the control chart statistic(s) is achieved in simulation by applying a warm-up where random observations are collected for a period *after* initialisation to zero state, that is when Q_0=0, until the distribution of the EWMA statistic Q_i stabilises.

Some users of control charts might argue that zero state studies are more relevant than steady state as off-specification production is more likely after plant stoppages due to misfitted assemblies and ramping up to typical process conditions. Zero state ARL values are longer than steady state values for small location shifts; therefore, the chart performance found in steady state studies may underestimate location shift detection efficiency immediately after control chart resetting. Lucas and Saccucci (1990) compared the results from both approaches and found that the difference was only 2.6% between steady state and zero state ICARL performances for EWMA schemes with h= 2.615, λ= 0.05 (for ICARL=500). The relative difference between steady state and zero state performance was less for large step shifts thus rendering the choice between methods of little practical significance.

To ensure sensitivity of control charts immediately after initialisation fast initial response (FIR) modifications to traditional control charts have been developed. Lucas and Saccucci (1990) developed a FIR scheme by using a 50% head start initialisation for EWMA schemes, that is, an initial value of the EWMA statistic that was half way between the target of the control chart and the control limit(s). Klein (1997), Rhoads, Montgomery and Mastrangelo (1996), and Steiner (1999) demonstrated use of transient control limits, on EWMA schemes, which reflect the actual variance of an in-control EWMA statistic over time, to achieve FIR. The transient control limits, in this style of FIR, are asymptotic to the steady state control limits. Rhoads, Montgomery and Mastrangelo proposed use of 50% head start in addition to transient control limits and found that this combination raised alarms quicker after scheme commencement, for out-of-control conditions, than when transient control limits are used alone. Conversely Steiner actually increased the FIR effect by making the control limits narrower for the early measurements than the limits used in previous asymptotic control limit schemes. "Asymptotic control limit FIR schemes" are simpler to construct on a spreadsheet than dual sided FIR schemes, which have additive head start terms transforming the raw variables into two different signals for separate monitoring.

Ideally, ARL values are ***always*** as small as possible for practically significant events, that is, events which are both statistically and economically significant. However, in our experience it is simply not feasible to respond to any alarms at start-up other than those which are essential to equipment protection and safety. There are often too many alarms for an operator to deal with at the time of start up. At such a time, quality is secondary to production rate ramp-up and equipment protection. EWMA schemes can have a long memory, and it makes sense to sufficiently adjust the value for Q_{i-1} after an alarm so that it does not contribute to "alarm overload" upon restart. For hot processes, it might be argued that 24 hours to 48 hours is a suitable delay prior to complete activation of all SPC monitoring tools. This may be in the transition to steady state for EWMA schemes, especially if the sampling period is long such as once per 12 hour shift. Evaluation of steady state performance is most appropriate because steady state constitutes the scenario in which alarms are desired and to which one can feasibly respond. Therefore, a steady state distribution for averaging/cumulative statistics is assumed in this thesis.

Warm-Up Runs for Steady-State Control Chart Properties

Steady-state simulation based thesiss vary in the sample size used to warm up the monitoring statistics. Albin, Kang and Shea (1997) for instance used a warm up of 35 observations whilst Sparks (2000) used 25 observations. Robinson (2007) reviewed methods typically used to decide on a suitable warm-up sample size and classified the approaches into the following: graphical, heuristic, and statistical methods and initialisation bias tests. A number of example publications are cited as users of each method. Robinson proposed an SPC approach using run rules to determine if the dispersion of a statistic is static and concluded that no one method (other than guessing) could be ruled as the superior method. Using too short a warm-up length will cause biased results. It should be noted at this point that tuning parameters which weight more heavily to past data will require a longer warm-up than tuning parameters which draw on less memory. Using too long a warm-up period is merely computationally wasteful.

Chapter 3

Performance Measurement and Comparison

3.1 Comparison and Design of Control Charts

Publications vary in the ways control charts are designed and compared. Performance measures can vary not only in the form of the performance measure calculation, but also in the location shift type and magnitude that is considered important. As performance is often optimised in the design of a control chart, performance measurement and design methods are intertwined subjects. ARL is the most common measure of control chart performance. Development of new performance measures is preceded with a detailed review of the ARL measure below.

3.1.1 Average Run Length Reviewed

The monitoring and correction cycle was discussed briefly in Section 1.1.4. In order to see if the logic of using ARL as a performance measure is sound, let us consider the elements of this cycle in more detail. The detailed model of process monitoring and correction is shown in Figure 3-1 with different contributions to off-specification production augmented. This model was developed to see if the ARL measure represents control chart performance without contamination from other contributors.

Examining Figure 3-1, “Onset of location shift” can be seen at the top of the figure. Rotating clockwise from the top dead centre of the cycle, “First sample charted after location change” is marked next, followed by “An alarm is signalled”. Consider a hypothetical example involving cement sulphate assays, in relation to the above model. Cement samples used for off-line sulphate analysis typically have a sampling period of 2 hours. A fault occurred in the gypsum dosing system, the primary source of sulphate in cement, at 12:00. The sulphate level trended down over 30 minutes but was off-target almost immediately. The first sample subsequent to the fault was at 13:00, and the control chart signalled an alarm at 16:00 for the 15:00 sample, having taken 1 hour to process the sample including data entry into the SPC system. Production was allowed to continue whilst a technician investigated the cause of the alarm. A small pile of gypsum was found on the gypsum weightometer, part of the automatic control system for gypsum dosing. The problem was corrected at 16:30 and sulphate levels trended back to the target level at the sampling station by 17:00. The total period of off-target production was 7 hours. The period related to the run length was from 12:00 to 16:00, or 4 hours. That is, the time related to the run length (RL = 3), was: (RL-1) x 2 hours = 4 hours.

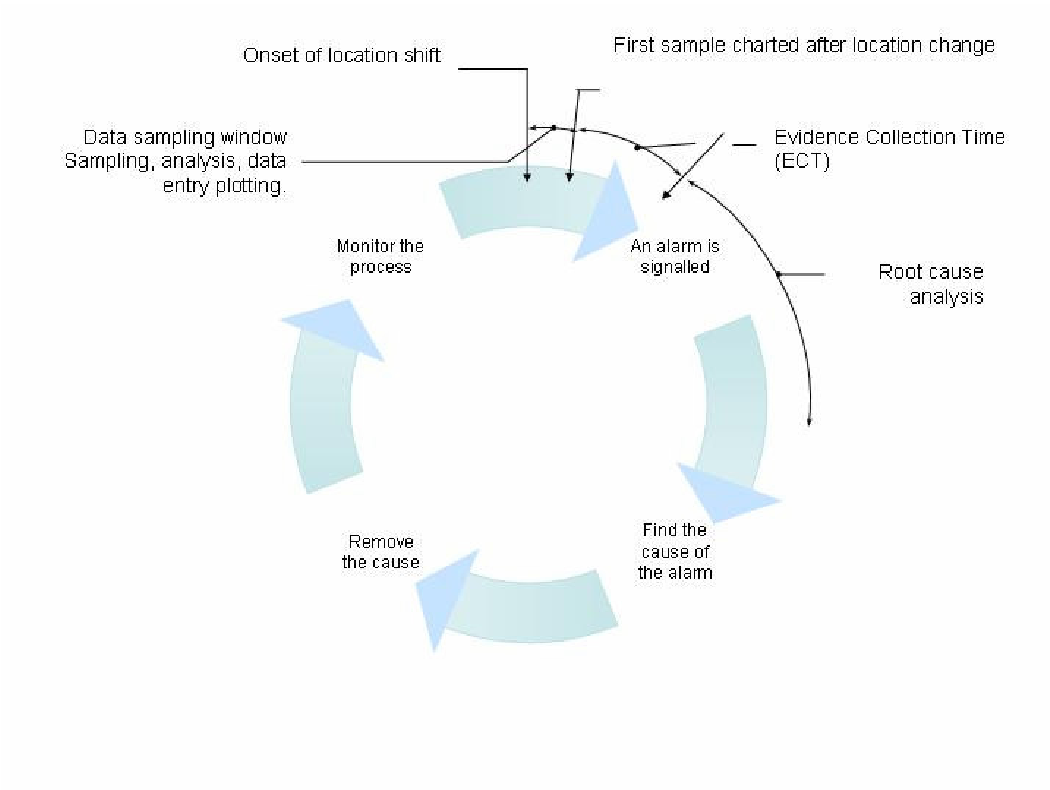

Figure 3-1. Augmented process monitoring and correction cycle.

If the 1 hour period of delay to assignable cause rectification from 12:00 to 13:00 is attributed to the sampling window, and the 1 hour period between 13:00 and 14:00 is attributed to the sample processing time, there is 2 hours yet to be accounted for. It is asserted here that this 2 hours is the only amount related to control chart performance. The run length was two: samples at 13:00 and 15:00. A run length of two multiplied by a 2 hour period, gives a period of 4 hours. The run length measure, hence ARL, inflates the apparent delay caused by the control chart by a constant of one, a constant which should instead be attributed to the sampling window and sample processing and data entry. An alternative basic performance measure to ARL, ATS, is discussed in the following subsection.

3.1.2 Performance Measures for Varying Location Shifts

ATS is a useful metric for measuring the performance of a control chart in one particular disturbance scenario. In this thesis, we are most interested in measuring the performance over an array of step shift sizes, or a domain of assessment. This chapter reviews existing statistical measures which compare the performance of control charts over a domain of step shifts. New measures are then developed including Mean Relative Loss (MRL), and Mean Relative Loss to the Optimum CUSUM Vector (MRLOCV) for measuring the individual performance of a control chart. Average Difference Relative to the Average (ADRA) is then developed for comparing the performance two control charts over a domain of step shifts.

3.1.3 Existing Design Methodologies

Design procedures often imply aspects of detection performance that are considered valuable. A classic paradigm for considering control chart performance, and a design method was proposed by Woodall (1985). He considered control chart performance in terms of SSARL performance across three regions including: in-control, indifference, and out-of-control. The in-control and indifference regions are separated at step shift magnitude θ_1, and the indifference and out-of control region is separated at step shift magnitude θ_2 (see Figure 3-2).

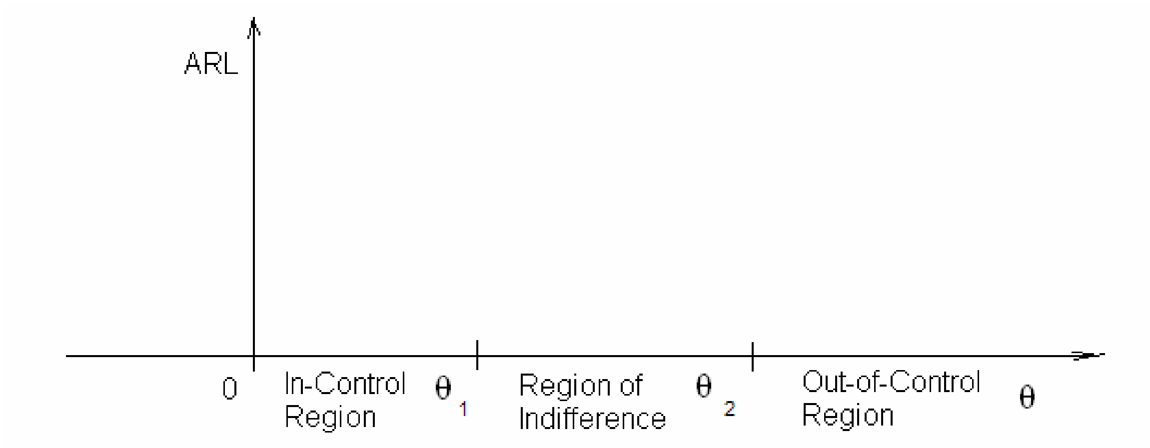

Figure 3-2. Woodall (1985)'s control regions.

Specification of either ICARL (or ICATS), or SSATS (or SSARL) at θ_2, makes schemes comparable because they have something in common. Such a specification prevents the out-of-control performance being confounded with varying in-control performance. For example, Jones, Champ and Rigdon (2001) set the ICARL to 200 assuming known parameters, in one set of their comparisons. Gan (1993) proposed a similar procedure but with the median run length as the performance measure.

Woodall (1985)'s proposed design method applied an ARL specification for the largest in-control step shift θ_1, and then optimised for the centre of the indifference region $\delta_\mu = (\theta_1 + \theta_2)/2$. Aparisi and Diaz (2007) also considered design in terms of the three regions described above, this time for EWMA schemes. They posed the optimisation problem as minimising the ARL at θ_2 subject to constraints at $\delta_\mu = 0$ and $\delta_\mu = \theta_1$.

3.1.4 Basic Performance Measures for a Simple Disturbance

SSATS and SSARL are basic statistical measures of control chart performance suitable for a simple disturbance model such as a step shift of $\delta_\mu = 1$. Consider which basic measure is most relevant to optimisation of the statistical performance of control charts. In this thesis, SSATS is related to contribution of the control chart performance to the total delay in correction of an out-of-control variable:

$$SSATS = (SSARL - 1).T$$

where T, the sampling period is 1 hour. The random period between the disturbances and the subsequent sample is not included in our SSATS measure. Authors such as Reynolds and Stoumbos (2004) considered this random component because it is important for distinguishing between the effectiveness of different sampling designs. Whilst this component is of interest in such studies, an aim of this thesis is to derive improved performance measures to distinguish the properties of different control charts without any regard to the sampling schedule.

Economic designs model cost in terms as a function of SSATS. Economically designed control charts have been designed under various assumptions (Duncan, 1956; Ohta and Rahim, 1997; Torng, Cochran and Montgomery, 1995; Das and Jain, 1997; Das, Jain and Gosavi, 1997). Total cost is probably the most relevant performance measure for any specific application. However, economic models require a lot of information which can be time consuming, if not impossible to collect (Marcellus, 2006). As cost performance models are specific to an application, optimisation results usually cannot be applied directly to other applications. Publishing research on statistical performance of control charts, however, yields beneficial general insights to many users. SSATS seems to be the most relevant basic statistical measure of performance as it is related to total cost of an out-of-control variable.

3.2 Methodology

The control regions of Woodall (1985) were simplified for this thesis, to the model shown in Figure 3-3. An in-control region was not used, but rather an in-control value defined by $\delta = 0$, for which the performance is specified. Whilst an in-control region makes good sense, specification of an ICARL value is commonly considered sufficient. For example, Robinson and Ho (1978), Lucas and Saccucci (1990), Crowder (1989) and Jones (2002) recommend specifying the ICARL which is considered appropriate for the application.

The indifference region is defined here as $0 < \delta < \delta_A$, and no penalty was applied for varying performance across the indifference region. The out-of-control lower boundary δ_A, might be varied to reflect the needs of the application. An upper limit to the out-of-control region, δ_B, was applied and the region $\delta_A \le \delta \le \delta_B$ used as a performance assessment domain. In the examples later in this thesis, 0.5σ to 4.0σ was predominantly used as the assessment domain, as did Sparks (2003) for some of his comparisons.

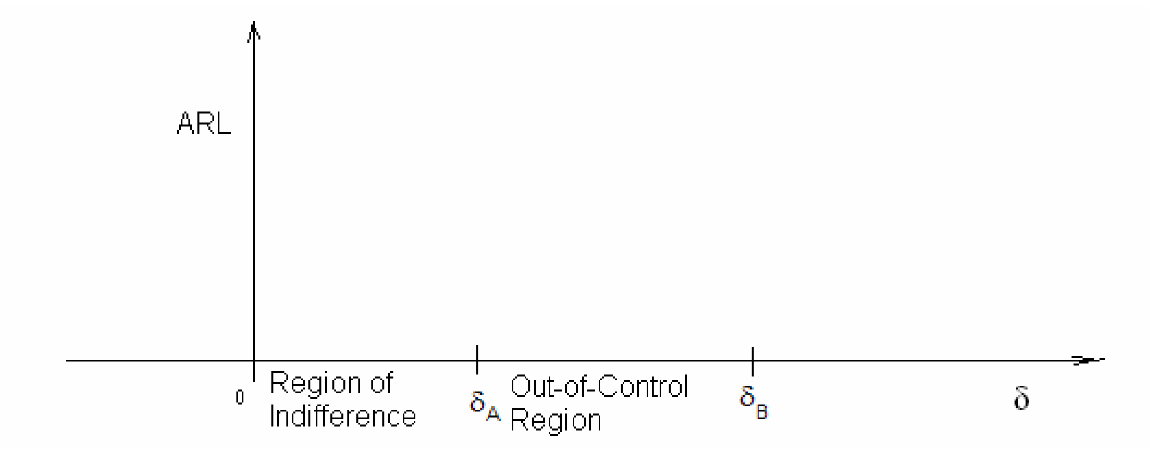

Figure 3-3. Control regions used results for scheme designs in Section 3.3 and Section 3.4.

3.3 Pair-Wise Comparison Measures for Multiple Disturbance Scenarios

One existing performance comparison measure, ARSSATS, and two new measures are discussed in this section.

3.3.1 Average Ratio of Steady State Average Time to Signal

Zhang and Wu (2006) and Wu and Wang (2007) used average ratio of steady state average time to signal (*ARSSATS*), a measure used for comparing the performance of two schemes, *a* and *b*, for multiple location shift scenarios, where the ratio of steady state average time to signal (*RSSATS*) is defined as:

$$RSSATS = SSATS_{a,\delta} \,/\, SSATS_{b,\delta}$$

and ARSSATS is defined as:

$$ARSSATS = \frac{\sum_{i=1}^{n_\delta} SSATS_{a,\delta} \,/\, SSATS_{b,\delta}}{n_\delta}$$

For example, where $SSATS_{a,\delta}$ is the SSATS for scheme *a* for some step shift scenario δ; n_δ is the total number of step shift scenarios at which the two schemes are compared.

ARSSATS is a new measure and has not been available for extensive consideration by other authors. Wu and Wang applied ARSSATS for comparison of schemes for joint step shifts in the mean and standard deviation. Let us consider some of the data presented by Wu and Wang for comparison of schemes called "3-CUSUM" and "1-CUSUM" (not described further here). Both monitoring schemes were designed to be optimum for a step shift in the mean of magnitude $\delta_\mu \sigma$, ***or*** a step shift in the standard deviation of magnitude $\delta_\sigma \sigma$. Consider the particular schemes optimised for $\delta_\mu = 2.0$ and $\delta_\sigma = 2.0$. The *ARSSATS* values (Table 4, Wu and Wang 2007) for joint small step shifts in the mean, and small step shifts in the variance, but not including pure step shifts in the mean or pure step shifts in the standard deviation, are reproduced in Table 3-1.

Table 3-1. Demonstration of ARSSATS calculation for designs by Wu and Wang (2007).

		SSATS		RSSATS	RSSATS
δ_μ	δ_σ	3-CUSUM	1-CUSUM	3-CUSUM vs1-CUSUM	1-CUSUM vs 3-CUSUM
0.3	1.3	26.5	26.3	1.008	0.992
0.6	1.3	15.4	17.4	0.885	1.130
0.9	1.3	9.33	11	0.848	1.179
0.3	1.6	11.2	10.4	1.077	0.929
0.6	1.6	8.97	8.62	1.041	0.961
0.9	1.6	6.83	6.74	1.013	0.987
0.3	1.9	6.63	6.09	1.089	0.919
0.6	1.9	5.9	5.52	1.069	0.936
0.9	1.9	5.05	4.77	1.059	0.945
			ARSSATS	1.010	0.997

The reported ARSSATS value was 1.01, as seen at the bottom of the column labelled RSSATS 3-CUSUM vs 1-CUSUM. This column describes the calculation of the ARSSATS where "3-CUSUM" is scheme *a*, and the ratio is calculated relative to "1-CUSUM", scheme *b*. The value of 1.01 suggests that "3-CUSUM" is the slower scheme on average across the different step shift scenarios. When the relativity is reversed, the ARSSATS = 0.997 suggesting that "1-CUSUM" is faster, or in other words, "3-CUSUM" is still slower. Note that the second value seems closer to unity than the first. A problem with this measure has been noted, however, as demonstrated in later Section 3.3.4.

3.3.2 Mean Relative Loss Pair Wise Comparison

The formula for Relative Loss Pair Wise Comparison (*RLPC*) is defined as follows:

$$RLPC_{a,b} = \frac{SSATS_{a,\delta} - SSATS_{b,\delta}}{SSATS_{b,\delta}}$$

and Mean Relative Loss Pair-Wise Comparison (MRLPC) is defined as:

$$MRLPC_{a,b} = \frac{\sum_{\delta=\delta A}^{\delta B} \frac{SSATS_{a,\delta} - SSATS_{b,\delta}}{SSATS_{b,\delta}}}{n_{\delta}}$$

MRLPC and *ARSSATS* are closely related. In fact,

MRLPC = ARSSATS - 1

MRLPC was considered for describing the relative loss in *SSATS* as a loss instead of a ratio.

3.3.3 Average Difference Relative to the Average

Difference relative to the average (*DRA*) is defined as:

$$DRA_{a,b} = 100\frac{\left(SSATS_{a,\delta} - SSATS_{b,\delta}\right)}{average\left(SSATS_{a,\delta}, SSATS_{b,\delta}\right)}$$

Here, we have defined *DRA* as a percentage relative loss for convenience. The Average *DRA* (*ADRA*) is defined as:

$$ADRA_{a,b} = \frac{\sum_{i=1}^{n_{\delta}} 100\frac{\left(SSATS_{a,\delta} - SSATS_{b,\delta}\right)}{average\left(SSATS_{a,\delta}, SSATS_{b,\delta}\right)}}{n_{\delta}}$$

The motivation for developing the *ADRA* measure is explained in Section 3.4.

3.3.4 Testing of Pair-Wise Performance Comparison Measures

MA(2) and MA(3) were compared using the *MRLPC* measure highlighting a problem with the *MRLPC* measure. MA(2) was shown to be optimum by the *MRL* measure, and MA(3) was shown to be optimum by the *MRLMC* measure. Surely a pair-wise comparison should clarify which scheme is better but the *MRLPC* measure was misleading. Figure 3-4 shows that MA(2) is stronger for large step shifts and MA(3) is stronger for smaller step shifts. *MRLPC* declares that both schemes are best, depending on which scheme is on the denominator, see Table 3-2.

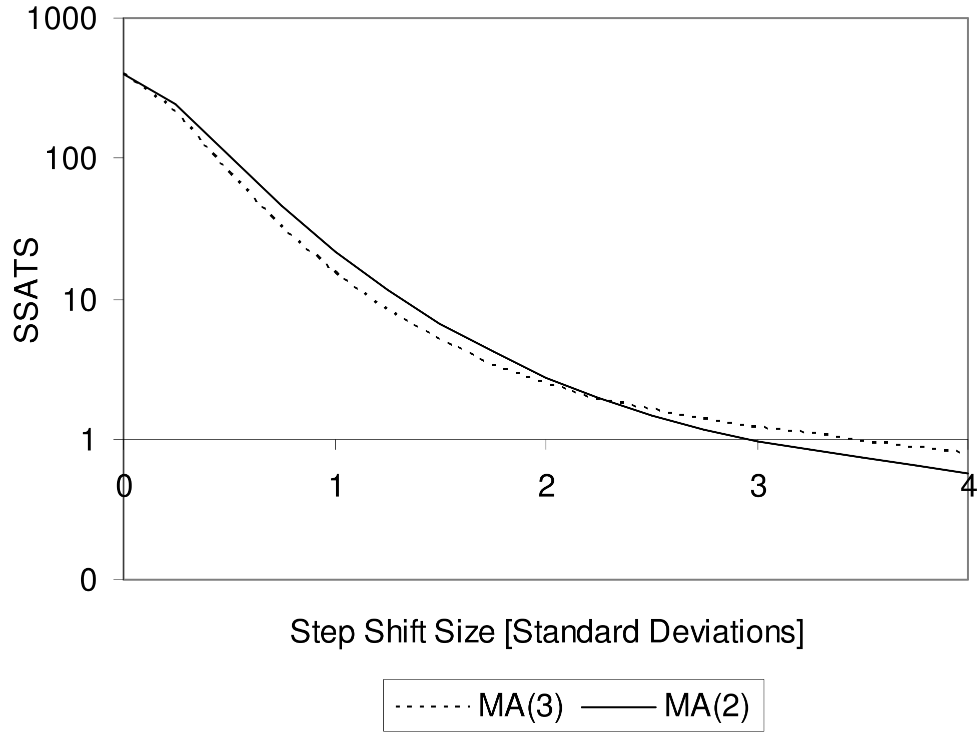

Figure 3-4. *SSATS* profiles of Moving Average Control Charts, MA(2) and MA(3).

Table 3-2. Example of a problem with *MRLPC*: MA(2) versus MA(3).

δ_μ	RLPC MA(3) relative to MA(2)	RLPC MA(2) relative to MA(3)
0.00	0.0014	-0.0014
0.25	-0.1368	0.1585
0.50	-0.2533	0.3393
0.75	-0.2985	0.4255
1.00	-0.3076	0.4443
1.25	-0.2954	0.4192
1.50	-0.2519	0.3368
1.75	-0.1963	0.2442
2.00	-0.1145	0.1293
2.25	-0.0102	0.0103
2.50	0.0999	-0.0909
2.75	0.1971	-0.1646
3.00	0.2653	-0.2097
3.25	0.3037	-0.2330
3.50	0.3256	-0.2456
3.75	0.3523	-0.2605
4.00	0.3843	-0.2776
MRLPC	0.0134	0.0578

MRLPC for MA(3) relative to MA(2) suggests that MA(3) is 1.3% better. When the *MRLPC* is calculated relative to MA(3), *MRLPC* suggests that MA(2) is 5.78% better. *MRLPC* is clearly not a reliable comparison measure, and as *ARSSATS* = *MRLPC* + 1, *ARSSATS* is also a poor pair-wise comparison measure in situation such as the example described here. Whichever term is on the denominator appears to be more favourable when using *ARSSATS* and *MRLPC* measures.

In Table 3-3, the first and second columns represent two fictitious *SSATS* profiles being compared. They are straight line profiles with the same mean, but profile B has an equivalent magnitude gradient to profile A, but negative. The third and fourth columns are *RLPC* values, first relative to profile B, then relative to profile A. Calculation of *RSSATS* is shown in the fifth and sixth columns with both directions of relativity. The seventh column shows the result for the *DRA* measure.

Table 3-3. Fictitious example of two comparable schemes.

A	B	RLPC (A-B)/B	RLPC (B-A)/A	RSSATS A/B	RSSATS B/A	DRA (A-B)/average(A,B)
1	10	-0.90	9.00	0.10	10.00	-1.64
2	9	-0.78	3.50	0.22	4.50	-1.27
3	8	-0.63	1.67	0.38	2.67	-0.91
4	7	-0.43	0.75	0.57	1.75	-0.55
5	6	-0.17	0.20	0.83	1.20	-0.18
6	5	0.20	-0.17	1.20	0.83	0.18
7	4	0.75	-0.43	1.75	0.57	0.55
8	3	1.67	-0.63	2.67	0.38	0.91
9	2	3.50	-0.78	4.50	0.22	1.27
10	1	9.00	-0.90	10.00	0.10	1.64
Average		1.22	1.22	2.22	2.22	0.00

Again, *MRLPC* declares that whichever profile is on the denominator is fastest. The calculations for these fictitious profiles suggest that the profile which is not the denominator has *SSATS* values which are 122% slower on average than the denominator profile. Clearly, both profiles cannot be relatively slower on average than each other. *ARSSATS* calculations are similarly misleading stating that the numerator is a factor of 2.22 of the denominator profile regardless of which profile assume a position on the denominator of the formula. Average ratio of average run length (*ARARL,* Zhang and Wu 2006, Wu and Tian 2005) will suffer the same short-coming as does ARSSATS, and is therefore also a dubious choice of measure for pair-wise performance comparisons. The *ADRA* measure, however, has a suitable result of zero meaning that both profiles are relatively similar on average. Reversing the relativity of the *ADRA* comparison again resulted in the same value of zero.

ARSSATS and *MRLPC* measures fail because their relative loss is calculated by scaling the absolute loss by a biased estimator of the group's mean. For example, 100 is 25% bigger than 80, but 80 is 20% less than 100, but the absolute difference is a fixed amount of 20. In this way, *ARSSATS* and *MRLPC* always overstate the relative difference when the numerator-only profile has a value which is larger (slower) at a certain step shift, and understate the relative difference when the other value is smaller. The truth in this statement can be seen in the third and fourth columns where the negative numbers have a small magnitude and the positive numbers have a large magnitude. *ADRA*, on the other hand, is not biased in the calculation of relative differences.

Now that the *ADRA* measure has been defined and tested, let us return to the problem of comparing MA(2) and MA(3) schemes. MA(3) is faster on average over the assessment domain having an *ADRA* value of -2.1% relative to MA(2). MA(3) is faster (up to 36% *DRA*) on the sub-domain 0.5σ to 2.25σ, and MA(2) is faster (up to 32% *DRA*) on the sub-domain 2.5σ to 4σ.

In the next, use of *DRA* and *ADRA* are demonstrated in a practical example for comparing a CCUSUM3 and X-MR schemes for a joint domain of mean and standard deviation step shifts.

3.3.5 CCUSUM3 compared to Amin and Ethridge's X-MR scheme using the DRA Measure

In this section, the objective is to see how competitive a three-component CCUSUM scheme is compared to an X-MR scheme (e.g. Amin and Ethridge, 1998) for detecting step shifts in the mean and step shift increases in the standard deviation. CCUSUM3D was the three-component scheme design used in the comparison and was designed to have the same specification as the X-MR scheme, *ICATS* = 500 for known mean and standard deviation parameters. The fine component accounts for 17% of all in-control false alarms, the intermediate component accounts for 41.5% and the coarse component accounts for the remainder of all in-control alarms.

The domain used for the comparison was [δ_μ = 0.0 to 2.0; increments of 0.25] and [δ_σ = 1.0 to 2.0; increments of 0.1] as the ARL data provided by Amin and Ethridge (1998) for δ_μ>2.0 and δ_σ> 2.0 provide sufficient significant figures for a comparison in this domain. Of the two X-MR designs explicitly discussed by Amin and Ethridge, the design with the most sensitivity to step shift increases in variance and the least sensitivity to step shifts in the mean was used in the comparison [M = 3.27; R = 4.57]. It was of interest whether a CCUSUM scheme, which has no component specifically intended for detecting increases in standard deviation, could detect such events as well as a composite scheme which does have a component specifically intended to detect increases in standard deviation.

SSATS results for the simulation of CCUSUM3D are shown in Table 3-4 along with the *DRA* losses of CCUSUM3D relative to the X-MR design. X-MR only out-performed CCUSUM3D by more than 1% in *DRA* terms in the case of pure variance shifts for step shifts in the standard deviation for ratios between 1.4 and 2.0. CCUSUM3D detected pure step shifts in the standard deviation almost as well as did the X-MR scheme, but CCUSUM3 is much better at detecting pure step shifts in the mean, and joint step shifts in the mean and step shift increases in the standard deviation.

CCUSUM3D has a peak advantage over the X-MR scheme for a pure step shift of 0.75σ with a *DRA* of -160%. The advantage that CCUSUM3D has for detecting large pure step shifts in the mean rapidly dissipates for increasingly large step shifts in the variance when these two different disturbances occur simultaneously. However, the CCUSUM3D scheme is still 24% better (*DRA*) for joint step shifts of [δ_μ = 2, and δ_σ = 2].   *ADRA* was calculated on the modified assessment domain which included all values ***except*** the shaded cells of Table 4 [$0 \le \delta_\mu \le 0.25$ ***and*** $1.0 \le \delta_\sigma \le 1.2$]. Overall, CCUSUM3D performed much better on the assessed domain, with *ADRA* = -50%, than did the X-MR scheme.

Table 3-4. *SSATS* values derived for CCUSUM3D for known parameters. *DRA* values are relative to X-MR scheme (design parameters: M = 3.27; R = 4.57). CCUSUM3D had reference values and control limit coefficients: k_1 =0.35, h_1 =8.6615; k_2 =1.0, h_2 =2.9776; k_3 =1.8, h_3 =1.5477; Al_1IC=17.0%; Al_2IC=41.5%; Al_3IC=41.5%.

	SSATS DRA [%]	δ_σ 1	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
δ_μ	0.00	500.196	190.061	91.399	51.852	32.951	22.794	16.705	12.832	10.214	8.372	7.017
		-0.8	-2.1	-1.1	0.1	**1.7**	**0.0**	**3.1**	**2.6**	**2.1**	**2.1**	**1.7**
	0.25	148.231	90.446	57.778	38.628	27.097	19.852	15.121	11.950	9.657	7.999	6.762
		-97	-62	-36	-21	-11	-9	-2	-1	-0.4	0.0	0.9
	0.50	37.127	31.961	26.731	21.777	17.779	14.431	11.860	9.845	8.283	7.100	6.120
		-153	-117	-82	-56	-36	-28	-15	-11	-7	-5	-3
	0.75	17.085	15.945	14.513	12.947	11.453	10.053	8.780	7.684	6.760	5.956	5.269
		-160	-132	-101	-76	-54	-44	-28	-21	-14	-12	-10
	1.00	10.192	9.660	9.093	8.364	7.727	7.072	6.455	5.886	5.358	4.841	4.446
		-157	-132	-106	-84	-63	-54	-37	-29	-22	-20	-14
	1.25	6.751	6.451	6.127	5.818	5.476	5.147	4.831	4.530	4.234	3.943	3.658
		-149	-127	-104	-84	-66	-58	-42	-34	-26	-24	-18
	1.50	4.657	4.521	4.357	4.197	4.053	3.880	3.683	3.503	3.361	3.191	3.029
		-139	-119	-98	-81	-65	-59	-45	-37	-29	-27	-20
	1.75	3.336	3.275	3.196	3.110	3.059	2.943	2.885	2.765	2.689	2.592	2.502
		-128	-109	-91	-77	-62	-57	-44	-39	-32	-27	-21
	2.00	2.462	2.423	2.406	2.370	2.343	2.287	2.263	2.198	2.153	2.103	2.043
		-115	-99	-83	-70	-59	-54	-43	-37	-33	-28	-24

3.4. Individual Scheme Performance Measures for Multiple Disturbance Scenarios

Ideally, individual performance measures can express the effectiveness of a particular design via a "standardised" value that can readily benchmarked against other studies. Four individual measures of scheme performance are described in this section including one existing measure, relative loss efficiency (*RLE*). These measures are equally applicable to univariate and multivariate control charts, for comparing performance over a domain of disturbance scenarios.

3.4.1 Relative Loss Efficiency

Sparks (2003) used the *RLE* measure in his investigation of composite moving average (CMA) schemes. *RLE* measures the relative difference in ***ARL*** for scheme *r* compared to the best of a series of *p* different schemes. To determine how well the scheme performed over a domain of location changes, he added together the loss terms for a number of different location shift scenarios:

$$RLE = \frac{\sum_{i=1}^{n_\delta} \frac{SSARL_{r,\delta} - \min\left(SSARL_{l,\delta}\right)}{\min\left(SSARL_{l,\delta}\right)}}{n_\delta}$$

where *r* is the identity of the scheme in question with $l = 1, 2, ..., p$ and $l \neq r$; and *l* is the identity of the various schemes being compared with *r*. Further, δA was a step shift of 0σ and δB is the largest step shift considered in the evaluation.

RLE indicates, by its relative magnitude, a desirable scheme within a group of schemes. By looking at the equation for *RLE*, we can see that the *RLE* value for a scheme depends on:

1) the type and number of the schemes to which it is compared,
2) the range of the deterministic shift magnitudes applied, and
3) the number of different step shifts within this range that have been applied and contributed to the summed relative loss efficiency.

RLE values change as more schemes are added to a comparison. A performance measure that changes as an optimisation routine progresses (for one fixed scheme design) is not very suitable for use in typical optimisation methods.

The magnitude of a calculated *RLE* value does not help the user understand the difference in performance of a control chart because the magnitude is sensitive to the diversity of other schemes which are compared. Any *RLE* value quoted in a publication does not universally indicate the performance of that design; it only indicates its **rank** within the designs compared in a specific study. That is to say, *RLE* cannot be classified as a standardised individual performance measure (neither is it a pair wise performance comparison measure). Owing to the deficiencies noted above, three new performance measures are developed as follows.

3.4.2 Mean Relative Loss Multiple Comparison

Mean Relative Loss Multiple Comparison (*MRLMC*) was developed from the *RLE* measure. When calculating the relative loss multiple comparison (*RLMC*), *SSATS* is substituted for *ARL* into the *RLE* measure:

$$RLMC = \frac{SSATS_{r,\delta} - \min_l \left(SSATS_{l,\delta} \right)}{\min_l \left(SSATS_{l,\delta} \right)}$$

and, then *RLMC* is divided by the number of step shift scenarios at which the comparison is made to calculate *MRLMC*, where

$$MRLMC = \frac{\sum_{i-1}^{nodes} \frac{SSATS_{r,\delta} - \min_l \left(SSATS_{l,\delta} \right)}{\min_l \left(SSATS_{l,\delta} \right)}}{n_\delta}$$

and where n_δ is the number of different levels of δ at which the *SSATS* values are assessed. *SSATS* was used in the *RLMC* formula to make the comparison focused on detection performance related to the choice of the control chart without the delay time due to sampling and analysis of the first sample. The effect of the number of step shifts used to compare control charts is mostly removed by dividing by the number of step changes at which the comparison is made.

Like *RLE*, *MRLMC* is not a standardised individual performance measure because the value changes depending on which schemes are included in the comparison. The $\min_l\left(SSATS_{l,\delta}\right)$ part of the function causes *MRLMC* to be sensitive to the diversity of the competing designs included in the calculation.

3.4.3 Comparison of CCUSUM3B to Koo and Ariffin's run rules using MRLMC

To demonstrate use of *MRLPC*, let us compare a CCUSUM scheme having three components to run rules schemes designed by Koo and Ariffin (2006). Koo and Ariffin published data for two-component run rules schemes which included the components: an Individuals Chart, and either a two-of-two or a two-of-three rule. A number of such designs with *ICARL* = 370 were described, each differentiated by the number of in-control false alarms generated by each component when operated in isolation. To measure *MRLMC* performance, *SSARL* data of the run rules schemes was converted to *SSATS* by subtracting 1.

The assessment domain lower boundary was set to δ_A = 0.6σ because this was the closest level to 0.5σ simulated in the publication by Koo and Ariffin. An assessment domain upper boundary of δ_B = 4.0σ was used. The design of CCUSUM3B was derived for the input specifications *ICATS* = 370 assuming known parameters. CCUSUM3B was not optimised, but is expected to be a reasonably good design for the assessment domain based on experience. The *RLMC* profiles are shown in Table 5 for the odd-numbered run rules schemes from the Koo and Ariffin study, relative to the CCUSUM3B scheme.

Table 3-5. CCUSUM3B and various run rules schemes by Koo and Ariffin (2006) using the *MRLMC* performance measure. The control limit coefficients for CCUSUM3B were: k_1=0.35, h_1=8.207; k_2=1.0, h_2=2.8231; k_3=1.8, h_3=1.4538; Al_1IC=17%; Al_2IC=41.5%; Al_3IC=41.5%. CLC1 is the control limit coefficient for the individuals component, and CLC2 is the control limit coefficient for the two-of-two or three-of-three rule.

δ_μ	CCUSUM -3B		Two-of-Two(A)		Two-of-Two (C)		Two-of-Two (E)		Two-of-Three (A)		Two-of-Three (C)		Two-of-Three (E)	
	CLC1		3.4		3.6		3.8		3.4		3.6		3.8	
	CLC2		1.843		1.81		1.792		1.986		1.955		1.94	
	SSATS	*RLMC*	*SSATS*	*RLMC*	*SSATS*	*RLMC*	*SSATS*	*RLMC*	*SSATS*	*RLMC*	*SSATS*	*RLMC*	*SSATS*	*RLMC*
0	370.635	0.006	372.22	0.011	372.84	0.012	371.69	0.009	372.13	0.010	371.94	0.010	368.33	0.000
0.2	170.912	0.000	280.48	0.641	281.24	0.646	277.38	0.623	277.78	0.625	275.46	0.612	274.03	0.603
0.4	52.907	0.000	155.5	1.939	154.78	1.926	152.58	1.884	147.85	1.795	144.25	1.726	142.9	1.701
0.6	23.97	0.000	80.97	2.378	79.83	2.330	79	2.296	75.57	2.153	73.59	2.070	72.8	2.037
0.8	14.184	0.000	44.3	2.123	43.77	2.086	43.65	2.077	40.57	1.860	39.86	1.810	39.47	1.783
1	9.508	0.000	25.4	1.671	25.45	1.677	25.38	1.669	23.24	1.444	22.78	1.396	22.55	1.372
1.2	6.822	0.000	15.38	1.254	15.51	1.274	15.55	1.279	14.31	1.098	14.05	1.060	14.08	1.064
1.4	5.033	0.000	9.68	0.923	9.75	0.937	9.85	0.957	9.13	0.814	8.98	0.784	9.03	0.794
1.6	3.807	0.000	6.48	0.702	6.48	0.702	6.6	0.734	6.05	0.589	6.01	0.579	6.04	0.587
1.8	2.929	0.000	4.41	0.506	4.48	0.530	4.52	0.543	4.52	0.543	4.16	0.420	4.17	0.424
2	2.288	0.000	3.14	0.372	3.21	0.403	3.27	0.429	3.27	0.429	3.03	0.324	3.09	0.351
2.2	1.818	0.000	2.32	0.276	2.38	0.309	2.45	0.348	2.45	0.348	2.31	0.271	2.36	0.298
2.4	1.459	0.000	1.8	0.234	1.87	0.282	1.94	0.330	1.94	0.330	1.81	0.241	1.87	0.282
2.6	1.177	0.000	1.4	0.189	1.49	0.266	1.58	0.342	1.58	0.342	1.44	0.223	1.51	0.283
2.8	0.949	0.000	1.12	0.180	1.21	0.275	1.28	0.349	1.28	0.349	1.19	0.254	1.25	0.317
3	0.763	0.000	0.9	0.180	0.98	0.284	1.07	0.402	1.07	0.402	0.98	0.284	1.06	0.389
4	0.221	0.000	0.27	0.222	0.34	0.538	0.42	0.900	0.42	0.900	0.34	0.538	0.43	0.946
5	0.031	0.000	0.05	0.613	0.07	1.258	0.11	2.548	0.11	2.548	0.07	1.258	0.11	2.548
	MRLMC	0.0000		0.8008		0.8495		0.9040		0.8287		0.7325		0.7804

It can be seen from Table 3-5 that CCUSUM3B was the fastest scheme at all measurement nodes on the assessment domain (*MRLMC* = 0). Two-of-Three (C) appears to be the next best scheme, although there is no clear advantage for two-of-three schemes generally over two-of-two schemes. Unfortunately, one cannot determine how much slower the other schemes based on *MRLMC* values. Hence, a more refined individual performance measures is required where more insight is desired.

3.4.4 RLMC Profiles within a Node-Optimised Set

The effect of the number of nodes experienced by the *RLE* measure was effectively removed in the *MRLMC* comparison by averaging, assuming that sufficient comparison nodes are included to be representative. However, at least two factors can adversely affect the standardisation of *MRLMC* values, namely:

1) The optimisation of schemes for the nodes
2) The type of schemes included in the comparison

To explain these two considerations, an example is given below for applying *MRLMC* to EWMA schemes.

An un-optimised four-component EWMA scheme CEWMA4D (*ICATS* = 400, having contributing components: [$\lambda_1 = 0.055$, $h_1 = 2.9849$; $\lambda_2 = 0.3$, $h_2 = 3.2286$; $\lambda_3 = 0.55$, $h_3 = 3.2841$; $\lambda_4 = 1.0$, $h_1 = 3.3259$]) was assessed using the *MRLMC* individual performance measure. Single-component EWMA control charts, based on the following smoothing coefficient values [0.055, 0.1, 0.13, 0.2, 0.25, 0.3, 0.4, 0.45, 0.5, 0.6, 0.64, 0.8, 0.85, 0.9] and *ICATS* = 400, were used as the reference profiles. Figure 3-5 shows the *RLMC* profiles for CEWMA4D and some of the reference single-component EWMA control charts. It can be seen that CEWMA4D is a robustly fast scheme across the assessment domain, but is never the fastest scheme at any measurement node in the comparison.

The selection of reference schemes has an impact on the interpretation of an MRLMC value as an individual performance measure. If the selections of schemes included in a *MRLMC* comparison are not node optimised, a zero *RLMC* value means indicates the fastest control chart ***of those considered*** for a step shift of the size under consideration. The interpretation for a zero *RLMC* value becomes the fastest control chart possible within the EWMA class, for the step shift size under consideration. Therefore, a control chart with *MRLMC* = 0 is the fastest control chart in the class being considered, at all of the nodes in the assessment domain.

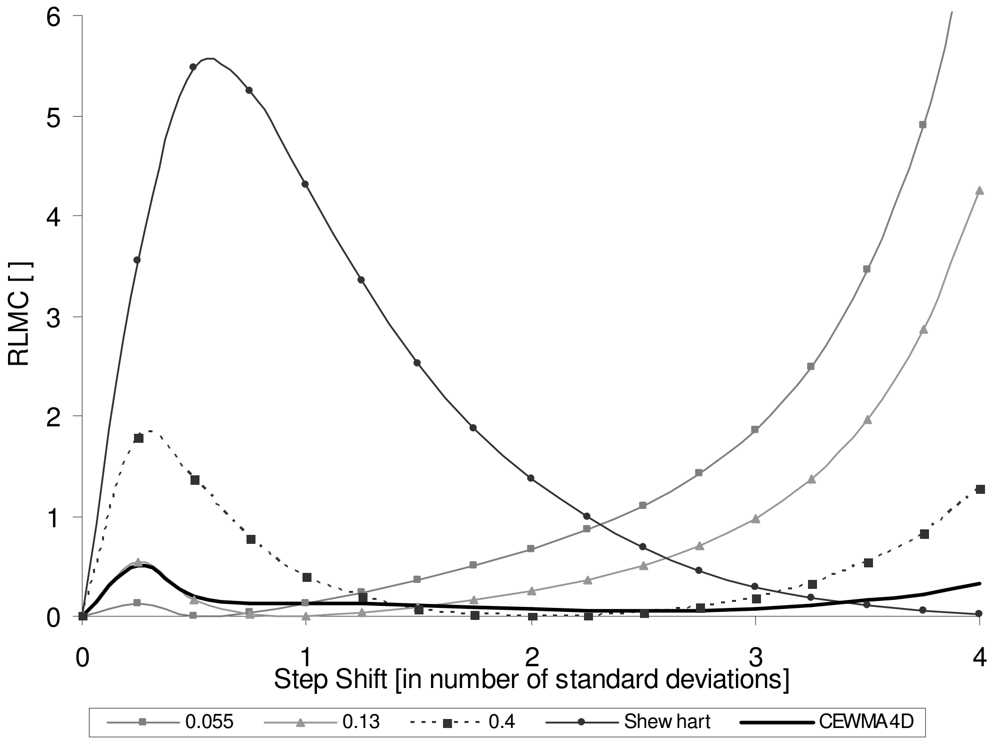

Figure 3-5. *RLMC* profiles for EWMA(λ) type schemes where λ is defined in the legend and CEWMA4D is defined by the component design [$\lambda_1 = 0.055$, $h_1 = 2.9849$; $\lambda_2 = 0.3$, $h_2 = 3.2286$; $\lambda_3 = 0.55$, $h_3 = 3.2841$; $\lambda_4 = 1.0$, $h_4 = 3.3259$]. *RLMC* data was calculated from a set of schemes which included EWMA control charts based on the following smoothing coefficients [0.055, 0.1, 0.13, 0.2, 0.25, 0.3, 0.4, 0.45, 0.5, 0.6, 0.64, 0.8, 0.85, 0.9], with *ICATS* = 400. *SSATS* performance was for schemes designed assuming known parameters in all cases.

When a background of node optimised reference schemes is included in a comparison, *MRLMC* offers a unique and useful interpretation. In this example, CEWMA4D had an *MRLMC* of 12.6% suggesting that this scheme was only 12.6% slower in *SSATS* terms than a node optimised EWMA scheme on the assessment domain. If a universally meaningful *MRLMC* performance measure is to be defined, it would require that a standard and large set of control charts are included in the comparison. *MRLMC* can demand significant data handling and interpretation efforts unless certain restrictions are place on the reference profiles set. The next measure is developed to reduce data handling demands and remove the ambiguity from the interpretation of the resulting performance measure value.

3.4.5 Mean Relative Loss to the Optimum CUSUM Vector

Mean Relative Loss to the Optimum CUSUM Vector (*MRLOCV*) is defined as:

$$MRLOCV = \frac{\sum_{\delta=\delta A}^{\delta B} \frac{SSATS_{\delta} - OCV_{\delta}}{OCV_{\delta}}}{n_{\delta}}$$

where OCV is a vector of *SSATS* results from a particular set of CUSUM schemes. Each value in the vector is from a different CUSUM scheme which was designed for *SSATS* = 400 and optimised for that particular location shift scenario (step shift size). In other words, OCV is a vector of the CUSUM scheme node-optimal *SSATS* values. The SSATS profile for OCV for step shifts is shown in Table 3-6.

MRLOCV is a performance measure related to a standard reference profile, thus an *MRLOCV* value has a meaning or interpretation that could potentially facilitate comparison of the performance across various design focused papers. Instead of including many node-optimised vectors in a comparison, only a single vector, OCV, needs to be handled. Comparison with just one other vector gives a firm quantity that does not change as new schemes are measured and added to a dataset.

A performance measure referencing a vector of node-optimised EWMA or MA *SSATS* values could also be defined. OCV was selected as the basis for the reference vector because CUSUM schemes are more efficient than MA and EWMA schemes.

Table 3-6. Optimum CUSUM reference vector, OCV, used for *RLOCV* calculations. The vector was created from profiles of 15 different CUSUM schemes.

δ_μ	SSATS	δ_μ	SSATS
0.00	400.	2.25	1.611
0.50	24.67	2.50	1.250
0.75	13.20	2.75	0.953
1.00	8.210	3.00	0.714
1.25	5.541	3.25	0.528
1.50	3.916	3.50	0.377
1.75	2.856	3.75	0.266
2.00	2.122	4.00	0.179

3.4.6 Mean Relative Loss

Mean Relative Loss (*MRL*) is defined as:

$$MRL = \frac{\sum_{\delta=\delta A}^{\delta B} \frac{SSATS_\delta - C_\delta}{C_\delta}}{n_\delta}$$

where the vector C_δ is the SSATS for a step shift in the mean δ for CUSUM control chart based on estimated parameters (n_{estim} = 200) with k = 1.1, and h = 2.2908. The *SSATS* profile for C_δ for step shifts is shown in Table 3-7.

Table 3-7. CUSUM reference distribution, C_δ, used for RL calculations. CUSUM design parameters are k = 1.1, and h = 2.2908, and parameters were estimated from 200 observations.

δ_μ	SSATS	δ_μ	SSATS
0.00	400.	2.25	1.644
0.50	78.4	2.50	1.250
0.75	30.6	2.75	0.978
1.00	14.05	3.00	0.763
1.25	7.54	3.25	0.601
1.50	4.604	3.50	0.463
1.75	3.080	3.75	0.349
2.00	2.198	4.00	0.260

The above CUSUM chart has an *SSATS* = 400, thus is most suitable for a standard reference for other control charts with and *SSATS* = 400. This particular configuration of a CUSUM control chart was chosen because it is approximately optimised for the midpoint of the comparison domain.

3.4.7 Comparison of the Performance Measures

The question arises, how does selection among *RLE, MRLMC, MRLOCV* and *MRL* measures affect optimisation? These performance measures were compared for the MA control charts as shown in the Table 3-8.

Table 3-8. *MRL* and *RLE* performance for Optimum MA(n) schemes.

MA Span, n	RLE	MRLMC	MRLOCV	MRL
1	1.395	1.611	1.980	1.057
2	0.553	0.799	1.069	***0.456***
3	0.381	***0.755***	***1.036***	0.498
4	***0.348***	0.851	1.158	0.636
5	0.359	0.980	1.316	0.790
7	0.424	1.250	1.640	1.084
8	0.464	1.381	1.796	1.221
12	0.627	1.842	2.344	1.689
28	1.120	3.092	3.820	2.911
30	1.169	3.211	3.960	3.026

Table 3-8 demonstrates the *RLE, MRLOCV* and *MRL* measures are not consistent in which MA control chart is optimum on the assessment domain. The optimum *MRL* result was for MA(2), and the optimum *RLE* measure was from MA(4). The effect of using *MRL*, which is a comparison measure based on *SSATS*, was to indicate a coarser component as the optimum scheme as compared to *RLE* which is based on the *SSARL* measure. This is because the *SSATS* measure is smaller than the *SSARL* measure. Effectively, *MRL* weights the small *SSATS* values, which corresponding to large step shifts, more heavily than larger *SSATS*. As smaller span are optimum for large step shifts, a smaller span is optimum for the *MRL* measure than the optimum found by the *RLE* measure.

MRLMC and *MRLOCV*, however, did agree that the MA(3) scheme is optimum for the assessment domain. *MRLMC* and *MRLOCV* are different because the *MRLMC* effectively referenced a node-optimal vector of MA *SSATS* values, whereas, *MRLOCV* referenced a node-optimal vector of CUSUM *SSATS* values. Also, the reference scheme profiles in *MRLMC* were based on known parameters, but the OCV was based on parameters estimated from 200 observations using the absolute moving range formula to estimate standard deviation. There were sufficient schemes included in the *MRLMC* calculation so that the *SSATS* values were approximately node-optimal. It has not been considered whether the agreement between *MRLMC* and *MRLOCV* can be expected always. The large difference in the tuning parameter between a MA(2) and a MA(3) schemes may have contributed to the common finding for the optimum tuning parameter.

In Section 3.4 it was shown that MA(3) was 2.1% faster than MA(2) on average according to the *ADRA* measure. From this information, we might conclude that the *MRLOCV* measure is a better individual performance measure to use when optimising design configurations.

3.4.8 *MRL* and *MRLOCV* used for Optimisation of Composite Schemes

To demonstrate use of individual performance measures *MRL* and *MRLOCV*, two-component CCUSUM designs were optimised. This optimisation exercise also demonstrated the affect of the performance measure selection on optimised composite scheme configuration. A three-factor, four-level full factorial experimental design, that is a 4^3 design, was performed on schemes with a specification of *ICATS* = 400 when parameters are estimated from 200 observations and Equation 1 is used to estimate the standard deviation. The levels used in the experimental design are shown in Table 3-9. Table 3-10 shows a slice of results for a loading on the fine component of Al_1IC = 25%.

Table 3-9. Experimental design levels for two-component CCUSUM scheme optimisation

Factor	Levels
Al_1IC [%]	10, 20, 25, 30
k_1	0.3, 0.4, 0.5, 0.6
k_2	1.0, 1.2, 1.4, 1.6

Table 3-10. *MRL* and *MRLOCV* performance for two-component CCUSUM schemes at Al_1IC = 25% slice of the experiment lattice

	MRL, *MRLOCV*	k_1 0.2	0.4	0.6	0.8
k_2	1.0	-0.068	-0.071	-0.068	-0.046
		0.208	0.207	0.242	0.314
	1.2	-0.102	-0.121	-0.114	-0.083
		0.168	0.146	0.188	0.276
	1.4	-0.089	**-0.128**	-0.124	-0.090
		0.182	**0.136**	0.177	0.273
	1.6	-0.048	-0.109	-0.114	-0.081
		0.230	0.157	0.190	0.288

The optimum two-component CCUSUM configuration according to both the *MRL* and *MRLOCV* measures was [Al_1IC = 25%, k_1= 0.4, k_2 = 1.4]. Identical results from both the *MRL* and *MRLOCV* measures suggest that optimisation is not particularly sensitive to the performance measure used when optimising two-component CCUSUM schemes. It has not been established whether optimisation of other composite schemes, such as three-component CCUSUM schemes, will be more sensitive to the choice of performance measure than shown here.

3. 5 Absolute versus Relative Performance Measures

So far, only relative performance measures have been discussed. In *ADRA* and *MRLOCV*, performance is relative to some profile. Absolute losses could also be considered when measuring individual performance or in pair-wise comparisons. Small step shifts are relatively slow to be detected by control charts. As a result, the absolute differences between the *SSATS* values for various schemes tend to be most divergent for small step shifts. Unweighted absolute loss measures mainly represent the performance on small step shifts hence optimisation attempts on unweighted absolute loss measures will be optimised for smaller step shifts than optimised unweighted relative loss measures. In the absence of detailed cost data, relative loss measures seem to be more appropriate for optimisation purposes than absolute loss measures as they will produce designs which are suitably tuned for moderate to large step shifts which are more likely to harm product quality. A comparison measure called Ratio of Average Extra Quadratic Loss (*RAEQL*, Reynolds and Stoumbos 2004) is effectively a weighted absolute loss measure. *RAEQL* weights large step shifts in the mean and/or variance proportionately to the size of the step shift. *RAEQL* would be a reasonable choice of performance measure if it also applied a weighting factor for the expected frequency of various disturbance sizes. Assuming that the weighting for the relative consequence of larger step shifts will be offset by a higher frequency of step shifts which are smaller in size, it would not be wise to introduce one of these weightings without the other. Relative loss performance measures are likely to have more generally applicability than absolute loss measures. Absolute loss measures have not been explored in this thesis.

One might argue that there is a risk that the relative performance measures such as *MRLOCV* and *MRL* measures might over-represent relative differences that are too small to be addressed in practice. Imagine a relative loss of 30% at large step shifts. A process operator cannot practically benefit from a warning that is 30% less delayed in the evidence collection window if the absolute benefit is only 0.01 observation (taken two hourly) on average. The benefit would be 0.01 [observation] × 2 [hours/observation] × 3600 [seconds/hour] × 0.3 [-] = 21 seconds. Cleary, twenty-one seconds is not long enough to do much. This is not a valid argument because even though the differences in SSATS might be 0.01 observations, effectively a ratio of 99:1 for 0 and 2-hour delay events. Something practical can usually be achieved by an operator within a two-hour period, this typical margin being provided by avoidance of a one-sample delay afforded by superior scheme design.

3. 6 Discussion and Conclusions on Statistical Measures of Control Chart Performance

Economic control charts have the potential to best optimise control chart configurations for specific applications, but are complex to build. Statistical performance measures are convenient for researching the effect of control chart design factors for general situations, and a number of such measures have been reviewed and developed in this thesis. One needs to be careful when choosing a statistical performance measure for researching a control chart as it has been shown that the increasingly popular *ARSSATS* may produce misleading results. Schemes look more favourable when they are represented in the denominator of the *ARSSATS* formula compared to when they are represented in the numerator. A new measure called *ADRA* was proposed as a measure for comparing the performance of two control chart schemes, for a number of location shift scenarios. *ADRA* does not demonstrate any hysteresis based on which position it takes in the performance measure formula.

Statistical performance measures based on *SSATS* better represent the economic advantage of a monitoring scheme than measures based on *SSARL*, neglecting the imperfections of any assumptions relating the statistical measure to the economic performance. When the *SSARL* measure was substituted for *SSATS* in the *MRLMC* measure, the effect on optimisation was to produce a control chart configuration which was more sensitive to smaller step shifts and slightly detuned for larger step shifts. In a more general context, however, selecting between individual performance measures based on *SSARL* or *SSATS* may be somewhat pedantic if there is not a good understanding of economic factors of the problem at hand. *ADRA* is a superior pair wise comparison measure and MRLOCV is a superior individual performance measure for a domain of disturbances as compared to *SSARL* and *SSATS* measures.

Demonstrations using the new performance measures yielded insights regarding the performance of composite CUSUM schemes to alternative control charts. The MRLMC measure showed that a three-component CUSUM scheme performed better on average in detecting step shifts in the mean on an assessment domain from 0.5σ to 4.0σ. Also, a three-component CUSUM scheme performed better on average than an X-MR scheme over a domain of pure and joint step shifts in the mean and step shift increases in the standard deviation as compared to an X-MR scheme. The X-MR scheme performed slightly better in the case of pure step shift increases in the standard deviation. However, the three-component CUSUM scheme performed much better pure step shifts in the mean and joint step shifts in the mean and variance increases. A CUSUM-MR scheme might reasonably be expected to perform better for a joint step shift of [δ_μ = 2, and δ_σ = 2] than both X-MR and CCUSUM3 schemes. Further research could be performed to verify this hypothesis.

Two different performance measures have been derived which are suitable for optimising monitoring schemes for specified domain. Both *MRL* and *MRLOCV* agreed upon the optimum configuration for a CCUSUM2 composite schemes in an example. However, *MRLOCV* was shown to produce a better design than *MRL* according to the *ADRA* measure, when optimising single-component MA schemes. *MRLOCV* optimisation resulted in a design which was more efficient small step shifts in the mean as compared to the *MRL* measure. Development of individual performance measures is still in its infancy, and further investigation is recommended to determine if any measure is globally superior for measuring performance and optimising control chart designs.

3.7 Rationale for Defined Assessment Domain Boundaries

In this subsection, rationale for the nominated values of lower and upper assessment domain boundaries is presented. Universal values for the bounds of an assessment domain are not advocated in this section, but rather, considerations that a SPC system designer needs to consider when assigning assessment domain boundaries for individual quality variables to be monitored.

3.7.1 Defining the lower boundary of the out-of-control region, δA

Defining δA might be aided by several considerations. It is asserted that small location shifts are not feasible to rectify and control charts should not be tuned for sensitivity to small location shifts. Firstly, by way of support for this assertion, small shifts in the mean of a quality variable will be less problematic for a customer than large shifts. Hence, small shifts offer less benefit for correcting the problem than do large shifts. One's ability to find the cause of a small location shift in quality is an important consideration for the definition of the limit to practical significance. Variables that have a weak effect on quality in their normal operating range are less likely to have an online measurement device. Again, decisions to include monitoring devices such as on-line analysers are typically based on benefit to cost ratios or net present worth. In the absence of on-line analysers, assignable causes would require a manual sampling investigation which may be relatively time consuming. In a processing plant situation, users tend to have incomplete records and frequently rely on the memories of various contractors, operators and managers when trying to identify causative events that coincide with quality problems. Diary-records are often minimal in detail, and memory of these events is limited in duration. Records do not exist for every event a manual adjustment to a ventilation damper. Another three practical reasons as to why there should be a limit to the power of the fine component include "risk of over-correction", "management of priorities" and "cost of distraction". These concepts are further developed below.

Risk of Over-Correction

Upon an alarm, process operators may feel compelled to make an adjustment to whatever adjusting device is related and available, even if the assignable cause cannot be found. If the adjustment to the location of the variable exceeds the size of the original disturbance, or is in the wrong direction, the variation of the variable from target will increase. As an assignable cause with a small effect is less likely to be correctly identified, an alternative adjusting device is more likely to be used with an incorrect adjustment magnitude.

Management of Priorities

In relation to management of priorities, an assignable cause with a large effect presents a large threat to quality. Therefore, alarms arising from small shifts in a variable are less important than alarms for larger threats. If an alarm from a small effect requires staff to complete some quality assurance documentation, time is robbed from project execution time that is otherwise available for eliminating priority assignable causes. A strategically refined continuous improvement program tackles root causes that most grossly harm product uniformity. Projects to improve process stability should prioritise resolution of assignable causes which have "large" effects, say greater than 2σ initially, until large shifts become rare.

The phase of control chart operation and the type of variable will determine the appropriate value for δA. Chemical process plants, for instance, are complex and can be frequently out-of-control to the extent of continual wandering. Laboratories, however, constitute a process that is simple in comparison to a process plant. Large homogenous batches of material are held in laboratories for the purpose of checking the reproducibility of results from analytical equipment. "Control-samples" are frequently taken from these batches and analysed to see if the calibration or preparation procedures are in-control. Control-samples have comparatively few triggers of location shift compared to variables in process plant streams. As control-samples are not subject to frequent large location shifts, laboratory staff can logistically address smaller location shifts. This being true, a smaller value of δA would be appropriate for laboratory control samples than for plant variables.

Cost of Distractions

Just as searching for false alarms incur expenses that do not reap the intended reward, so can alarms on trivial shifts. Assuming that the causes of small location shifts cannot easily be found and corrected, searching for these alarms has an unfavourable expected benefit to cost ratio.

3.7.2 Defining the upper boundary of the out-of-control region, δB

An interest when doing this thesis was to compare optimised composite schemes to better understand the potential performance of the different types of components, for location shifts sizes typical of those considered in publications. In this thesis, $\delta B =$ 4.0 was used because this is typical for many studies. No logical reason is proposed as to why a larger value should not be used. It is acknowledged that *MRL* is dependent on the values δA and δB which are subjectively assumed. However, as fallible as the measure may be, *MRL* provides valuable insight into the performance potential of different schemes for hypothetical situation. Naturally, however, a designer can use any value when customising a design for a specific variable. One would need specific information on the likely distribution of location shifts magnitudes to be encountered to get a more meaningful performance measure. An economic design might be used if sufficient information is available.

Chapter 4

Determining Control Chart Properties

A publication titled "Design and optimisation aids for composite control charts" (MacNaughton and Coomans, 2009) describes the software created and used for the thesis. Simulation and specification seeking algorithms are described in detail, as are detailed instructions on how to navigate about the graphical user interface to achieve control chart simulation and design functions. The following sections of this chapter cover the content of the publication without repeating content from previous chapters.

4.1 Precedence for use of Simulation Software

Software is generally used to determine the properties of control charts in quality control literature. Luceno and Puig-Pey (2002) created a FORTRAN Computer Program (Luceno and Puig-Pey, 2002) for computing the run length distribution for CUSUM control charts. Turner, Sullivan and Batson (2001) demonstrated software for retrospective analysis of a change point within individual observations. Aparisi and García-Díaz (2007) used a genetic algorithm to optimise a control regions problem for single-component exponentially weighted moving average (EWMA) schemes and made that software freely downloadable. Wu and Wang (2007) released a program for optimisation of the parameters of a single-component CUSUM scheme which monitors a weighted statistic of the average and squared deviation of the observations. The statistic was designed for efficient detection of step shifts in the mean and variance, and derivation of results was based on the Markov chain model. None of the software described above is suitable for design of composite schemes with multivariable in-control performance specifications. Unfortunately, the Markov chain approach has not previously been applied to composite control charts (Wu and Wang, 2007), and neither has the integral equation approach.

Simulation is an option for estimating an empirical run length distribution and has been used in the study of EWMA related schemes by Albin, Kang and Shea using 30,000 trials in 1997; Klein 40,000 trials in 1997; Jiang, Wu, Tsung, Nair and Tsui 160,000 trials in 2002; Sparks 100,000 trials in 2003; Reynolds and Stoumbos a combination of 100,000 and 1,000,000 trials in 2004, per derived average run length value. One can see from the above chronology that simulation sizes have generally increased over time. The simulations in this thesis typically have 4,000,000 trials.

4.2 Simulator Overview and Assumptions

A simulation program "Composite Monitoring Schemes" with a graphical user interface (Figure 4-1) was created by the master's candidate to ensure ease during extensive simulation work. The executable program is MS Windows 2000, and XP compatible. A restricted freeware, beta quality version of the program may be downloaded from www.jcu.edu.au/~jc133757/index.htm.

The software was created using MS Visual C++ which is a language and compiler used for creation of Microsoft Windows compatible programs which can have graphical user interfaces. C++ is an object orientated language (the basis of MS Visual C++) which facilitates sharing and reuse of code. Software can be written using classes which encapsulate code and data. In fact, the class for the random normal number generator "StochasticLib" was acquired on the World Wide Web, provided courtesy of Fog (2003).

Assumptions built into the simulator include:

- Monitored variables are identically, independently and normally distributed.
- Scheme performance for step shifts from 0.5σ to 4.0σ in the mean is considered important.
- Steady state distribution of statistics which reference in-control process history (achieved through 100 warm-up observations).

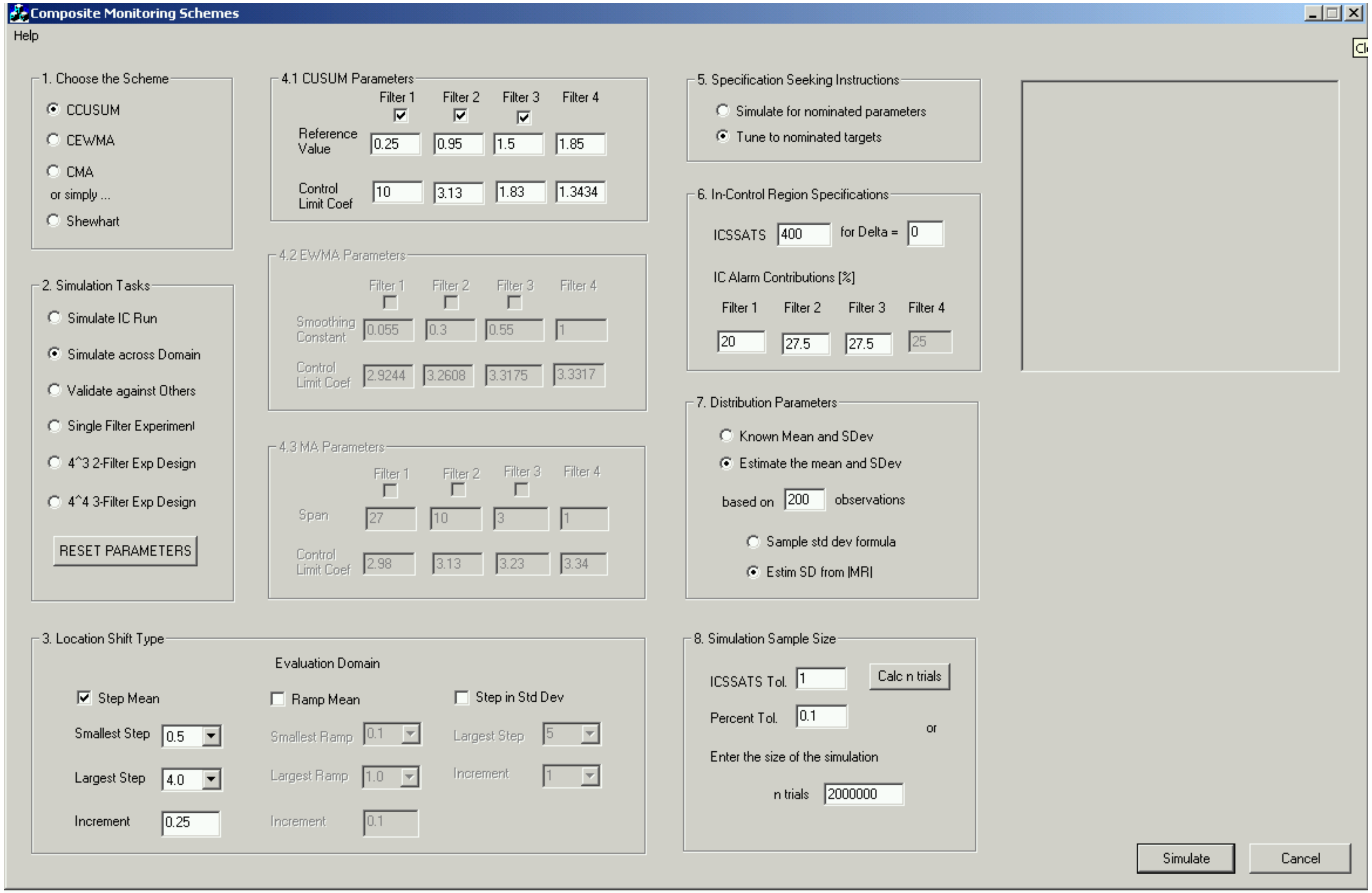

Figure 4-1. The graphical user interface of the simulator software created for the thesis.

The warm-up is a series of random values generated to fully develop the distribution of the observed statistic. Only upon the last observation of the warm-up, immediately prior to the step shift, the components of the composite schemes are tested against their respective control limits. If the value of the statistic is not within the upper and lower control limits, then the simulation is discontinued and not considered in the performance statistics. A normal distribution truncated at the upper and lower control limits results for the warmed up statistic to be monitored at $i = 1$.

The interface permits distribution parameters to be known or estimated from a specified number of observations.

The software was validated against ARL profiles published by authors such as Sparks (2000, 2003), Quesenberry (1993), and Lucas and Saccucci (1990). All validation was based on steady state simulation as this is the only form of simulation for which the software was configured. Results were generally within 1% of other published results except where the other publications used small simulation sizes. Details of the validation method and the results are described in Appendix D.

A flow chart describing the simulation of a set of steady state control chart runs is found in Figure 4-2. The key C++ classes hosting the simulation algorithms class inheritance structure are described in Appendix E.

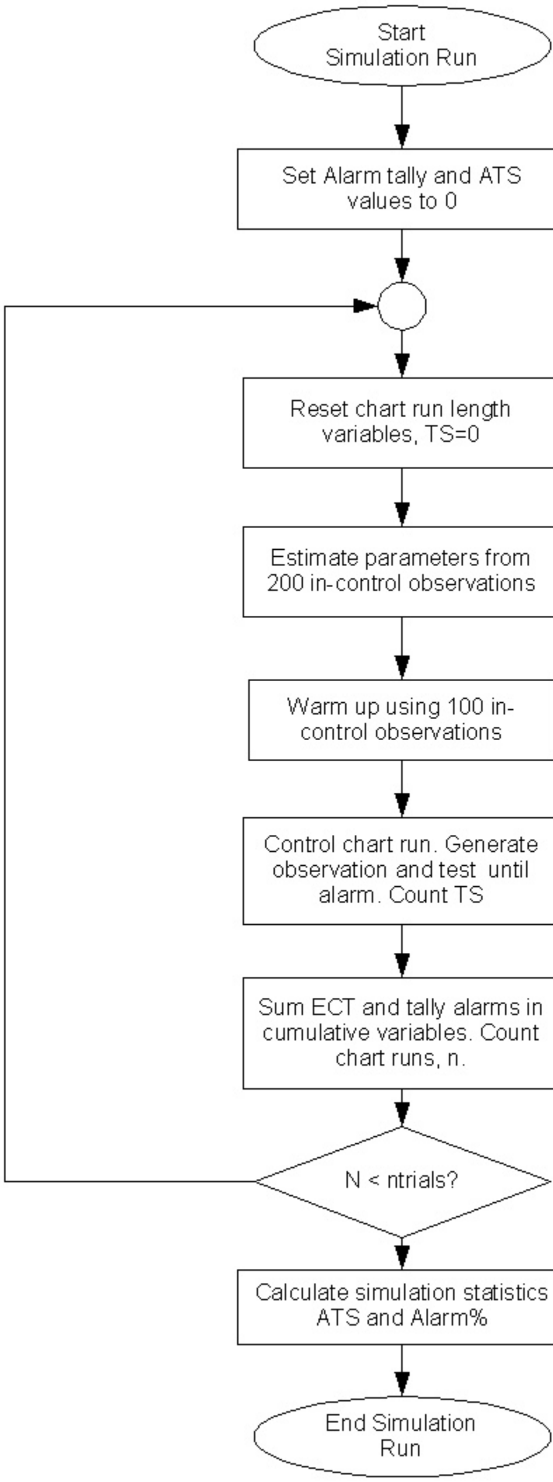

Figure 4-2. Flow chart for the simulation of ATS values from *ntrials* x chart runs. TS is the Time to Stop for a single chart run.

4.3 Modelling the Contribution of Individual Components

Design of a three-component control chart, which has control limits that are symmetrical about the process average, requires six design decisions to be made: three tuning parameters and three control limit coefficients. If we wish to specify the tuning parameters directly, three design decisions remain. For the remaining design decisions we have the option of specifying three control limit coefficients, but if "loadings" are introduced one may study design of composite schemes in a more general fashion. The loading for component γ, $AL_{\gamma}IC\%$, is defined here as the percent contribution of component γ to the overall false alarms rate. That is, the control limits for each of the schemes can be designed such that the corresponding component contributes some specified proportion of the overall number of false alarms in the simulation. Consider a three-component composite control chart for example. Simulating 1,000,000 in-control trials may generate an overall count of 1,500,000 alarm signals. This is because two or three components can alarm simultaneously upon the same chart stopping observation. Of the overall count, Component 1 may contribute 400,000 alarm signals. Thus Component 1 would have a loading of $100\% \times \frac{400{,}000}{1{,}500{,}000} = 26.7\%$. Clearly, a loading will always be in the domain 0%-100%. Optimum loading for a component is expected to remain fairly constant when ICATS specifications are increased or decreased. Another example: given some performance criteria, the optimum loading of Component 1 may be 13% when ICATS = 400, and the optimum loading may be 9% when ICATS = 2,000. Now consider an example where the control limit coefficients are scaled directly. Control limit coefficients may range from zero to infinity. Furthermore, the relationship between control limit coefficients and ICATS was found to be non linear by this author. A further example: say the control limit coefficient value h_1 = 2.9013 is optimum when ICATS = 400, but when ICATS = 2,000, h_1 = 3.6120 optimum. Further, h_1 = 3.6870 is hypothetically found to yield poor performance. Thus, it can be seen that it could be difficult to anticipate control limit coefficients in the vicinity of the optimum. Loadings are more intuitive and values in the vicinity of the optimum can be anticipated more easily.

4.4 Algorithm for Seeking In-Control Specifications

The specifications applied in this thesis are multivariable in nature with the ICATS and loading variables all being dependent on the control limits of the individual, interacting control chart components. ICATS increases as the value of the control limit coefficients. Conversely, loadings decrease as the value of the corresponding control limit coefficient decreases. Further, a change in one of the control limit coefficient changes all of the loadings, not just the corresponding loading.

A form of secant method was used in the program to seek the control limit coefficient needs to achieve specified ISATS and loading. The algorithm is similar to a basic secant method (Black, 2004) with additional constraints and features to ensure that the convergence is robust to data containing sampling error. Economy is gained by using reduced simulation sizes for three of the four stages of approach to the solution.

The algorithm was formulated with a response vector $\mathbf{w}_i$ having the following elements $[ICATS, Al_1IC, Al_2IC, …., Al_vIC]_i$. Control limit coefficients are the independent variables affecting the response vector elements. Secant methods find the value of the response for some initial value of the independent variable then perturb the independent variable and the second value for the response vector is recorded. A formula for the secant which joins the co-ordinates of the two observations on the function is found. A straight line relationship is assumed between the most recent two coordinates [response, independent variable] and the secant is projected to a new estimate of the independent variable required to achieve the response target. Due to the multivariable nature of the problem, the response-independent variable relationships were not defined directly. In stead, search-dimension variables were introduced to de-correlate the affects of the control limit coefficients. This formulation approach permits convergence of all response vector elements (ICATS and loadings) simultaneously.

The search-dimension independent variable for ICATS is the scalar l_i, which is given an initial value $l_0 = 1$. Component loadings use the search dimension variable vector $\mathbf{g}_i$ whose elements are individual factors for the control limit coefficients of the respective components. Initial values are assigned $g_{i,j} = 1$ for each element *j*.

The algorithm commences by simulating charts with the initial values, $\mathbf{h}_0$, for the control limits of the v components to generate the response vector $\mathbf{w}_0$. A second response vector $\mathbf{w}_1$ is generated by perturbing the search-dimension variables and then the gradients are calculated.

The gradient $m1_i$, for the ICATS search dimension variable l_i, is calculated for iteration i:

$$m1_i = \frac{ICATS_i - ICATS_{i-1}}{l_i - l_{i-1}}$$

and the intercept for the corresponding secant is:

$$c1 = ICATS_i - m1_i.l_i$$

The vector of gradients, $\mathbf{m3}_i$, relating $\mathbf{AlIC}_i$ to the component loading search-dimension variables in $\mathbf{g}_i$, is calculated:

$$\mathbf{m3}_i = (\mathbf{AlIC}_i - \mathbf{AlIC}_{i-1})/(\mathbf{g}_i - \mathbf{g}_{i-1})$$

and the vector of intercepts for the corresponding secants is:

$$\mathbf{c3}_i = \mathbf{AlIC}_i - \mathbf{m3}_i \cdot \mathbf{g}_i$$

The secant method projects successive estimates for the control limits, $\mathbf{h}_i$, required to achieve the in-control specifications using the following formulae:

$$\mathbf{g}_{i+1} = (\mathbf{AlIC}_{\mathbf{Target}} - \mathbf{AlIC}_i)/\,\mathbf{m3}_i + \mathbf{g}_i$$

and

$$l_{i+1} = (ICATS_{\mathbf{Target}} - ICATS_i)/\,m1_i + l_i$$

At this point, the iteration history is indexed via temporary memory locations. The iteration index for the following step becomes i. The secant method is repeated with the new vector of control limit coefficients $\mathbf{h}_i$, being calculated from the search-dimension variable values using the formula:

$$\mathbf{h}_i = l_i \cdot \mathbf{g}_i \cdot \mathbf{h}_0$$

In the first level of the specification seeking algorithm, 10,000 chart runs are simulated to estimate the parameters for the secants. The algorithm iterates toward the solution by repeatedly measuring the gradients, projecting new control limits and re-simulating. Once the tolerances in the response vector for the stage are achieved, the algorithm proceeds to the next stage with an increased simulation size and smaller tolerances. At the fourth stage the user input tolerances are applied. Convergence to a solution has proven to be reproducible when starting from different initial values.

When using a simple secant method random error in the response variable causes inaccurate gradient estimation, particularly after a relatively small increment in the independent variable. The quasi-secant method varied from the simple secant method by inclusion of stability enhancing features.

4.5 Using the Software

4.5.1 The Main Interface

The software interface is arranged across eight groups of controls for the user to set up the simulation instructions and occupy with parameters and specifications. Below, the control groupings are explained under headings that match the text on the interface (in bold text).

a) **Choose the Scheme.** Choosing the control chart type is permitted in the first group of radio buttons. A choice of CCUSUM, CEWMA, CMA and Shewhart are given. Single component schemes are basically one-component composite schemes which may be selected through the corresponding parent scheme. For example, a CUSUM scheme can be selected by choosing the CCUSUM scheme radio button and un-checking three of the four components.

b) **Simulation Tasks.** Next, in the second interface grouping, the simulation task needs to be specified. The Simulate IC Run radio button is selected when only an in-control scenario needs to be simulated. If one also wishes to find the ATS for various deterministic events then the second option Simulate Across a Domain needs to be selected.

A validation routine is accessible via the Simulation Task group to validate the software in real time. Published ARL profiles are stored within the code and a text file opens at the end of the validation routine showing the relative differences of the results compared to other authors.

Four-level full factorial experiments for step shifts in the mean can be selected for two or three-component schemes. A lattice of tuning parameters is then defined from a child window when the simulation is initiated. The experiment is based on the scheme type selected in "1. Choose the Scheme".

c) **Location Shift Type.** Location shift type and size can be specified in the third group of controls only when Simulate across a Domain or Experiment Design is selected in the second group of controls. Step shift and ramp shift in the mean, and shift in the variance can be selected. When the checkboxes for these options are checked, the user can then enter the required domain boundaries and increment size in the respective text boxes. Pre-coded values can be selected from a drop-down list, or original values entered. If step shift in the mean and step shift in the variance are both selected, the pure and joint combinations of both step shift types will be simulated.

d) **Parameter Definitions.** The fourth interface group permits specification of the tuning parameters and control limits. Controls are arranged in three subgroups by scheme basis: CUSUM, EWMA and moving average (MA). Checkboxes enable the user to select a composite scheme with between one and four components. Composites based on a combination of different statistics, for example EWMA plus CUSUM, are not permitted.

e) **Specification Seeking Instructions.** Simulation by "Simulate IC Run" or "Simulate across Domain" can be done either for entered parameters and control limit coefficients, or according to ICATS and IC Alarm Contribution (loading) specifications. In the latter option, loading specifications are entered into the IC Alarm Contribution edit-boxes.

The control limit coefficients in the Scheme Parameters group are used as initial values and an algorithm iteratively seeks toward the specifications.

f) **In-Control Specification** accepts positive values for the ICATS and loadings specifications. Controls within this group are only active when Tune to Specifications is selected in Specification Seeking Instructions.

g) **Distribution Parameters** is the control group which permits the user to choose between known and estimated distribution parameters (mean and standard deviation of the monitored variable). The standard deviation can be estimated using the traditional sample standard deviation formula or via the absolute moving range based formula as described in Chapter 3.

h) **Simulation Sample Size.** User tolerance specifications for the simulated results need to be entered when the Specification Seeking Instruction "Tune to nominated targets" is selected. There is a button here for the ICATS to be calculated from 15 simulations of 1000 chart runs per simulation. The ICATS is displayed on the output text pane (above the "Simulate" button), and an estimate of the simulation size required for attaining the tolerances within 2 standard deviations of the mean is output to the "ntrials" field. Because ICARL (hence ICATS) values for EWMA control charts are approximately geometrically distributed (Gan, 1993), the search can become fail to converge when a simulation size of less than 100,000 is used.

4.5.2 The Experimental Design Dialog Box

In Step b) "Simulation Task" (in Section 4.5.1), a series of control chart configurations completing an experimental design can be initiated. If an experimental design task is selected, the dialog box as shown in Figure 4-3 pops up when the Simulate button is clicked.

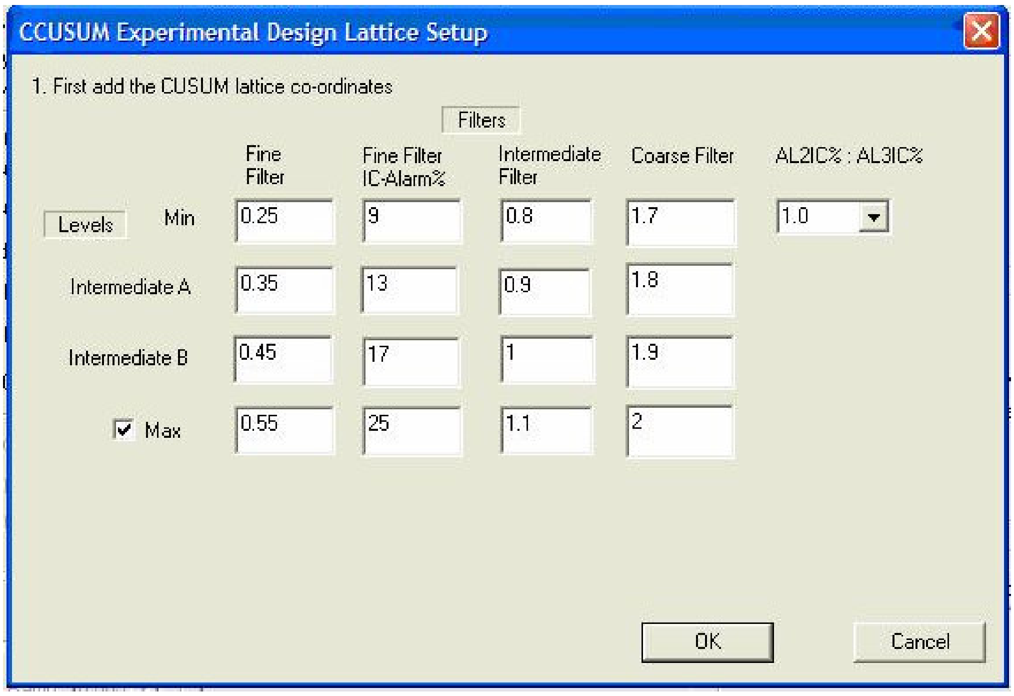

Figure 4-3. Dialog box for defining a three- to four-level, full factorial, CCUSUM3 performance measurement experiment.

The "4^4 Experimental Design" function uses four nested loops to increment through the levels of the experiment. All permutations of the experimental levels are achieved as required for a full factorial design, setting the composite scheme parameters and in-control specifications for each simulation in turn. At the centre of the nested loops, a specification seeking search algorithm is called. The search algorithm repeatedly calls the simulation algorithm, manipulating the control limits until the in-control specifications are reached. The resulting design parameters and SSATS profile are stored in an array. When all of the design configurations in the lattice have been executed, the MRL and MRLOCV performance data is calculated and written along with the design configuration into AnalysisFile.txt.

The fifth design variable is AL_2IC:AL_3IC, the ratio of the loadings on the intermediate and the coarse components. To collect performance results for four levels of this design variable, one seeks to run another three instances of the software for the "4^4 Experimental Design" task. Execution of the different levels can be done in series or parallel with different values for AL_2IC: AL_3IC. If you choose to do these simulations in parallel, run the software from different folders (directories, on the storage device) then merge the four resulting AnalysisFile.txt files into one file. Finally, import AnalysisFile.txt into a spread-sheeting software package and analyse the results. A "pivot table" is convenient for this purpose.

4.5.3 Simulator Output Data Files

Three different text files can be created by the software, described as follows.

Simulator_Output.txt. All Simulation Tasks options produce a file called Simulator_Output.txt. Simulator_Output.txt is created in the operating directory when the Simulate button is depressed, or opened if it already exists. This file can be used to track detailed information on the intermediate solution when seeking toward the specifications hence can be used for fault finding if convergence is not achieved. This file also contains detail of the proportion of alarms triggered by each of the components for different location shifts.

AnalysisFile.txt is the database output of the experimental design simulation functions of the software and contains one composite control chart design and set of performance descriptors per row. The columns of this file are:

- dICATS, the target ICATS for the simulation
- dICBoundary, the step shift size to which the dICATS is specified. Typically dICBoundary = 0.
- dNcomponent, the number of components in the composite
- dType, the type of composite scheme: CMA, CEWMA, CCUSUM
- dNestim, the number of observations from which the distribution parameters are estimate, *if the parameters are estimated.*
- dEstimMethd, the method of estimating distribution parameters with formulas specified as follows: 'SD' = std dev formula; 'MR' = moving range formula; 'KNOWN' = the parameters were not estimated but rather assumed to be known.
- Al1%, the targeted loading for the fine component.
- AL2toAL3, the targeted ratio of loadings for the intermediate and coarse components
- P1, P2, P3, the tuning/reference parameters for each component
- H1, H2, H3, the control limit coefficients for each component
- A1, A3, the actual component loadings,
- MRL
- MRLOCV
- ATS(dICBoundary), SSATS(0.5), SSATS(0.75), …, SSATS(4.0); 0.25 increments in the step shift size; the number in brackets is the step shift size.

The data for each variable are in columns suitable for importing into a statistical analysis or database query program. One may choose to transpose a subset of the text file data to arrange the data in a format more typical of that seen in publications. **Validation.txt** is created when running the validation simulation task and is written in the operating directory. It shows the software's results and the relative difference compared to the other authors after adjusting by -1 to equate to SSATS terms.

4.6 An Application to Industrial Data

A variable sometimes considered important in cement quality is the 28-day compressive mortar strength. It is desired to prospectively monitor individual observations of the data to see if the mortar compressive strength is in-control. If a control chart which has ICATS = 400 is used for the analysis, there should be few, if any, false alarms generated by the dataset. Design of a four-component CCUSUM scheme using the Composite Monitoring Schemes computer program is demonstrated below for a cement quality problem. The resulting scheme design is referred to as CCUSUM4a and it is applied to 28-day compressive mortar strength data.

4.6.1 Distribution Parameters: 28-day Compressive Mortar Strength

28-day compressive mortar strength data was shown in Figure 1-4 of Chapter 1.
To create a control chart, an estimate of the mean and standard deviation is required from the full dataset of 236 weekly observations. Table 4-1 summarises the descriptive statistics from the dataset. There seems to be a shift at approximately observation 66, thus the mean of the first location is estimated from the first 65 observations.

Table 4-1. Compressive Mortar Strength Dataset Statistics.

Statistic	Result	Sample Size
Stand. Dev.	1.22	236
$average(\lvert MR \rvert)$	0.880	235
$average(\lvert MR \rvert)/1.128$	0.780	
$\overline{Y}_1$	55.91	65
$\overline{Y}_2$	57.84	34
$\overline{Y}_1 - \overline{Y}_2$	1.93	
$\delta_\mu = (\overline{Y}_1 - \overline{Y}_2)/s$	2.47	

The control charts in this thesis are designed assuming no autocorrelation exists. Departures from the identically and independently assumption run the risk of altered in-control and out-of-control run lengths. However, small amounts of autocorrelation have small impacts on the ARL properties. In fact Wardell, Moskowitz and Plant (1992) found that standard control charts can even perform better than common cause control (CCC) and special cause control (SCC) charts, which especially designed for autoregressive moving average (ARMA) data, in certain circumstances. EWMA(λ=0.3) was better at detecting small shifts in the mean (up to 2 std.dev.) and large shifts when the autocorrelation factor was negative and the moving average factor was positive, than did CCC and SCC charts. With these facts in mind, any trivial amount of autocorrelation in the cement quality data has been disregarded for the purpose of demonstrating our CCUSUM3 control chart.

It can be seen that the estimated standard deviation which uses the regular standard deviation formula (1.22) is lower than the estimate based on the absolute moving range formula (0.78). Absolute moving range is less sensitive to changes in location than are squared deviations from target when the location of the data varies sufficiently. Due to the apparent heterogeneity of the location of the data and the fact that individual observations are not being grouped, the absolute moving range based formula was considered the most appropriate method for estimating the standard deviation of the in-control process. In the example, the sample sizes for the mean and standard deviation differ. The standard deviation has been estimated from 236 absolute moving range values, and the mean of the first location has been estimated from 65 observations. For simplicity, a single sample size assumption of nestim = 150, a compromise between the two different samples sizes, will be applied. A procedure for using the software to design the control limit coefficients is given next.

4.6.2 Using the Software to Design a Control Chart

The procedure given below solves the control limit coefficients required to achieve specified in-control performance for a CCUSUM scheme using Composite Monitoring Schemes computer program. Simulation processing times will be in the order of 20 minutes on a Pentium Core Duo 3.0 GHz IBM compatible personal computer. Using a running instance of the software, the steps are:

1. **"Choose the Scheme"** - Set to the "CCUSUM" option.
2. **"Simulation Task"** - Set to the "Simulate across Domain" option.
3. **"Location Shift Type"** Check "Step Mean" and select a domain of [0.5, 4.0; 0.25 Increment].
4. **"CUSUM Parameters**" Ensure that all four components are activated, enter reference values, and initial values for the control limit coefficients. In the control chart examples below, CUSUM reference values of 0.25, 0.95, 1.5 and 1.85 were used.
5. **"Specification Seeking Instructions**" - Select "Tune to nominated targets".
6. **"In-Control Region Specifications**" - Accept the default values of 400 and 0 for ICATS and Delta respectively. Change the IC Alarm Contributions (loadings) to 10, 30, 30, 30 for Components 1-4 respectively.
7. **"Distribution Parameters"** - In the example, "Estimate the mean and SDev" was selected, "based on **150** observations". Further, "Estimate SD from |MR|" was selected.
8. **"Simulation Sample Size"** - Enter the simulation tolerances here (ATS Tol = 1.5 and PCNT Tol = 0.15 were applied).
9. **"Simulate"** - Click the "Simulate" button and wait approximately 20 minutes for a dialog box to indicate that the simulations have been completed and converged to target.
10. "**Close the application and review the results"** - The results will be found in the file called "Simulator_Output.txt" in the same directory as the executable. An extract of the output file is shown below in Table 3.
11. **"Construct Control Charts using the Design**" - Take the converged control limit coefficients solution from the simulator output file and construct a control chart. The component values and alarm points for the 28-day Compressive Mortar Strength monitoring example are shown in Table 4.

Columns of Table 4-2 show the perturbations and secant projections of the control limit coefficients and the effect on ATS; however, some content available from Simulator_Output.txt was omitted for simplicity. Data from Simulator_Output.txt not shown in Table 4-2 includes the loading responses to the combination of control limit coefficients applied. Examining Table 4-2, it can be seen that 10 secant iterations were required to converge within tolerance of the targets. Only the last of the fifteen responses to the initial values, Level 0, is used in the secant method. Three iterations were applied at Level 1 and one iteration at Level 2. Forty-five replicate simulations, using the intermediate solution achieved at Level 2, were sampled to re-estimate the standard deviations of ICATS and the loading for Component 1, prior to proceeding to Level 3. One iteration was performed at Level 3, and then four iterations at Level 4 to converge within tolerance of the simulation targets.

Table 4-2. Control Chart Design Convergence Results.

Extracted from Simulator_Output.txt. CUSUM reference values were 0.25, 0.95, 1.5 and 1.85 for Components 1-4 respectively.

ATS	δ_μ	δ_σ	h_1	h_2	h_3	h_4	nrejects	ntrials	Level
15 trials of 1000 chart runs									
310.269	0	1	12	2.8	1.8	1.2	31	1000	0
270.898	0	1	11.4075	2.6618	1.7111	1.1993	311	10000	1
403.353	0	1	12.3295	2.9072	1.674	1.3026	275	10000	1
394.323	0	1	12.2062	2.8826	1.7611	1.2896	273	10000	1
395.479	0	1	12.2384	2.869	1.7444	1.3014	850	33782	2
45 trials of 1000 chart runs.									
397.669	0	1	12.2232	2.843	1.746	1.3132	5197	197703	3
393.953	0	1	12.1876	2.8462	1.7419	1.3101	23497	879063	4
394.708	0	1	12.0433	2.8533	1.744	1.3116	24012	879063	4
412.374	0	1	12.2592	2.8743	1.7578	1.3218	22742	879063	4
400.352	0	1	12.1808	2.854	1.7477	1.3163	23361	879063	Result
37.083	0.5	1	12.1808	2.854	1.7477	1.3163	6600	293021	.
17.681	0.75	1	12.1808	2.854	1.7477	1.3163	5065	219766	.
10.416	1	1	12.1808	2.854	1.7477	1.3163	4043	175813	.
6.602	1.25	1	12.1808	2.854	1.7477	1.3163	3338	146511	.
4.376	1.5	1	12.1808	2.854	1.7477	1.3163	2801	125580	.
3.041	1.75	1	12.1808	2.854	1.7477	1.3163	2455	109883	.
2.197	2	1	12.1808	2.854	1.7477	1.3163	2114	97674	.
1.638	2.25	1	12.1808	2.854	1.7477	1.3163	2050	87906	.
1.236	2.5	1	12.1808	2.854	1.7477	1.3163	1765	79915	.
0.947	2.75	1	12.1808	2.854	1.7477	1.3163	1685	73255	.
0.716	3	1	12.1808	2.854	1.7477	1.3163	1490	67620	.
0.527	3.25	1	12.1808	2.854	1.7477	1.3163	1472	62790	.
0.383	3.5	1	12.1808	2.854	1.7477	1.3163	1314	58604	.
0.279	3.75	1	12.1808	2.854	1.7477	1.3163	1239	54941	.
0.185	4	1	12.1808	2.854	1.7477	1.3163	1216	51710	Result

4.6.3 Generating the Composite CUSUM Control Chart

Next, the lower control limits (LCL) and upper control limits (UCL) are calculated as shown in Table 4-3 using the descriptive statistics (Table 4-1) and the control limit coefficients (from the design produced as per Table 4-2).

Table 4-3. Composite Control Chart Design CCUSUM4a for 28-day Compressive Mortar Strength (Cement Quality) Example.

	Component			
i	1	2	3	4
k_i	0.25	0.95	1.5	1.85
h_i	12.18	2.854	1.7477	1.3163
$h_i\sigma$	9.5008	2.2262	1.3633	1.0268
LCL	46.40	53.67	54.53	54.87
UCL	65.40	58.12	57.26	56.92

The CUSUM statistics for CCUSUM4a's components and the respective upper control limits from the observation of the step shift are shown below in Figure 4-4. The red observation marker indicates the point at which the component of the composite scheme first alarm. Component 2, 3 and 4 alarm concurrently for the first time at observation 67 and Component 1 first alarms at observation 69.

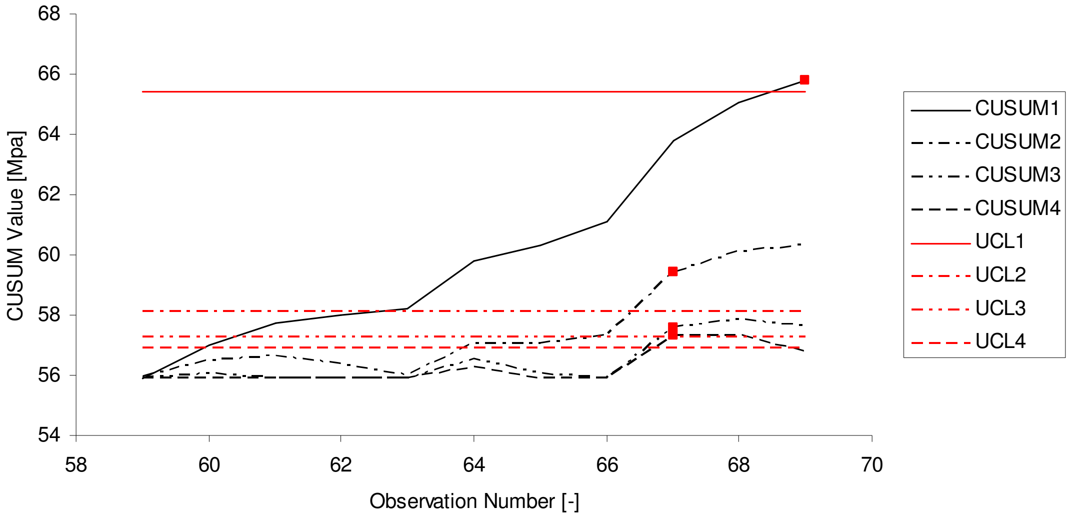

Figure 4-4. Chart Run for CCUSUM4a on 28-day Compressive Mortar Strength, Cement Quality Example.

4.7 Concluding Remarks on Significance of the Software

It is hoped that wider use of composite control charts will be encouraged by the free beta software may be used to design charts for user specifications. The software includes functions for experimental lattices where users may enter the ICATS specification and levels for the tuning parameters. The logistics for researchers wishing to use optimised composite schemes as a benchmark for other control chart methods have been greatly improved.

Chapter 5

Understanding Tuning Parameter Optimisation

Optimisation of a control chart via the tuning parameter affects the statistical power of the component, which in a moving-average type component, is effectively related to the amount of evidence used to distinguish the out-of-control population from an in-control population. For a MA component, as the span (n) is increased, the power of the component increases. Maximising power is not our primary concern, but rather minimising the average detection time of the monitoring tool. As n becomes greater than the number of observations elapsed since a step shift, more in-control data might dilute the location shifted data as captured in the moving average statistic.

For example, a MA scheme based on a span of 2, i.e. MA(2), has an ATS of 1.58 for a step shift of 4σ, whilst MA(15) has an ATS of 1.87 (see Table B-1 of Appendix B). MA(15) reacts slower to a step shift of 4σ compared to the MA(2) scheme because the additional 13 observations in the span increase the weighting of in-control history in the statistic.

Hunter (1986) described the slow reaction of control charts which are tuned to have a high power, as the scheme suffering from the "memory" of past observations. Large spans however are required to develop sufficient power to detect small shifts competitively. As a result, each different span is optimal for a unique location shift size. Combining moving-average type schemes with different spans in a composite scheme might be a way to achieve best detection times over a broad range of non-stationary scenarios.

5.1 EWMA Performance over Time

Considering the performance of a EWMA scheme over time from the onset of a location shift helps one to understand the benefit of using multiple components in a composite scheme over a basic scheme. Each component draws different amounts of statistical inference power, and this can be likened to multiple memory levels.

The probability of detecting a location shift is detected by a control chart is a function of time. Sparks (2003) graphically demonstrated for MA schemes how the probability of detecting a step shifts increased to some maximum. The probability reached its maximum when the number of observations elapsed since a step shift reaches a value equal to the span n, the number of observations in the moving average statistic. In a similar fashion, the conditional probability of detecting a step shift upon a new observation when using EWMA schemes is explored. The probability, conditional on the scheme not already being in alarm, increases with each observation from the onset of the step shift until some maximum probability is reached. Unlike the simple moving average, the EWMA reaches its maximum probability of detection smoothly. The probability profile of EWMA schemes are shown below in Figure 5-1 and Figure 5-2 for step shifts of 1σ and 2σ respectively.

In Figure 5-1, it can be seen that at TS = -1 the probability of correctly detecting of a step shift for all EWMA control charts is zero. At TS = 0 the first measurement that is made after the deterministic shift, the probability of detection of the shift has risen above zero. The probability of the EWMA statistic with λ = 0.55 rose fastest initially, but by the third observation at TS = 3, it has become the least probable alarm to signal. EWMA(λ=0.55) has reached a steady-state detection probability at approximately TS = 5. EWMA(λ=0.05) has the best detection probability when TS $\geq$ 10. In Figure 5-2, a similar pattern can be observed for detecting a 2σ step shift except probabilities climb much faster than for the 1σ step shift in the mean. Similar observations can be made by inspecting MA scheme detection probability curves for various spans Sparks (2003).

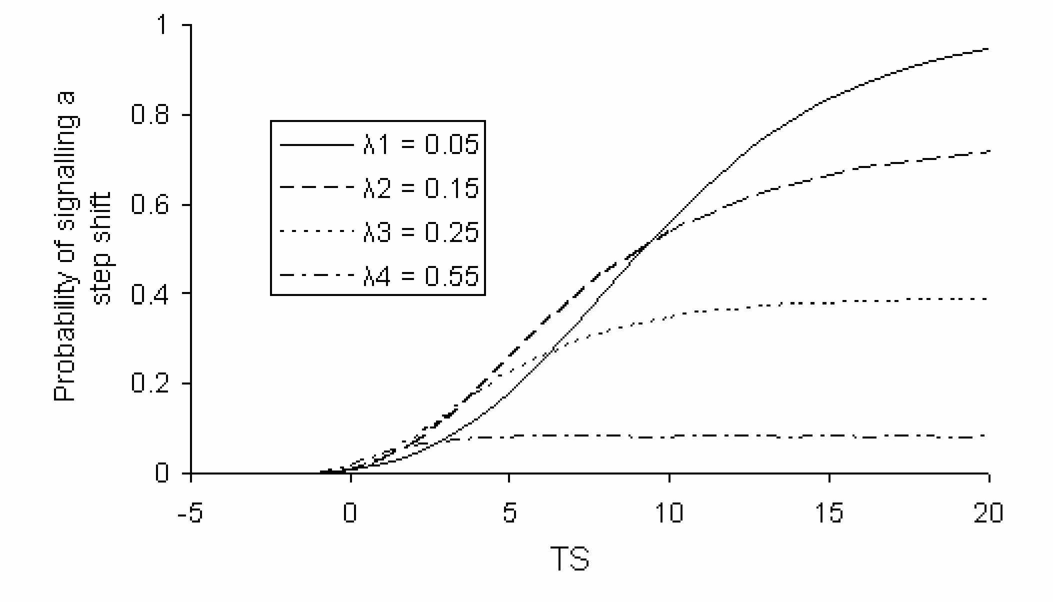

Figure 5-1. The probability of detecting a step shift using EWMA schemes with λ = 0.05 to 0.55, for a step shift of 1 σ, ICARL = 400 observations, based on 100,000 simulated chart runs.

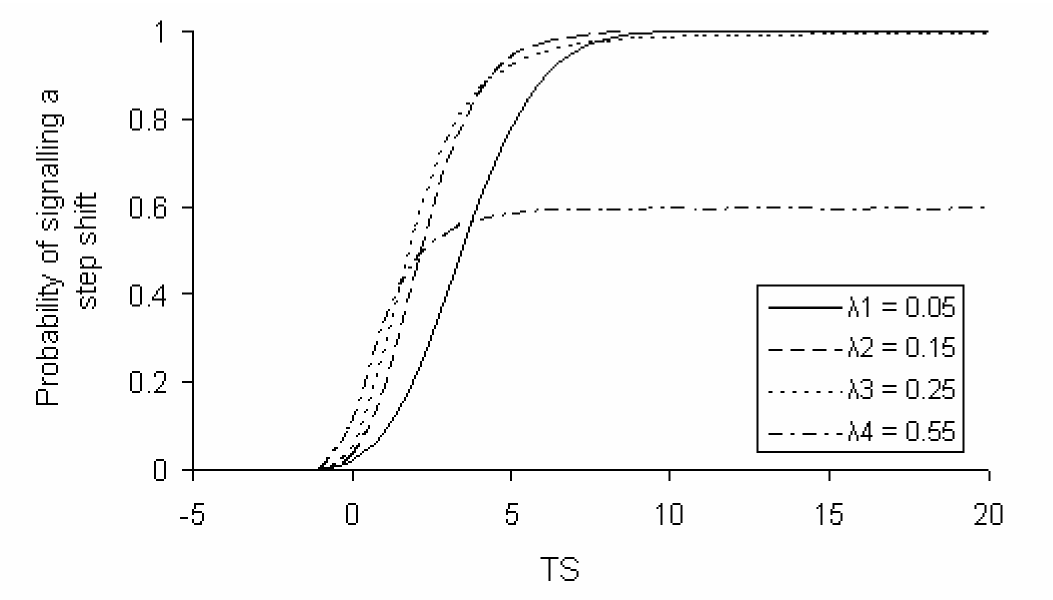

Figure 5-2. The probability of detecting a step shift using EWMA schemes with λ = 0.05 to 0.55. for a step shift of 2 σ with ICARL = 400 observations, based on 100,000 simulated chart runs.

Statistical power to distinguish samples not only increases as the difference between the means increases, but also as sample size increases. Figure 5-1 and Figure 5-2, demonstrated that the shortest memory EWMA chart (i.e. λ=0.55) initially had the highest probability of detecting a step shift because at TS = 1 its value contains the smallest weighting to past in-control observations. Long memory control charts have a larger effective sample size, hence more statistical power than short memory control charts. Prior to reaching steady state detection probability, the contribution of out-of-control data to the EWMA value decreases as TS increases. The control chart which has superior power (longest memory, i.e. λ=0.05) gains advantage over the other control charts as more observations are incorporated and the misleading influence of past in-control data diminishes.

Thus far, this discussion on detection probabilities has only considered the performance of the control chart at each new observation as though the collective result for the current and previous observations is unimportant. The ATS performance of a scheme relates to the cumulative probability of detection or the expected number (N) of alarm signals. Figure 5-3 shows the expected number of alarms after a step shift of 1σ simple signalled by EWMA schemes with $\lambda_1 = 0.05$ $\lambda_2 = 0.15$ and $\lambda_3 = 0.25$ and $\lambda_4 = 0.55$. It can be seen that EWMA ($\lambda_2 = 0.15$ and $\lambda_3 = 0.25$) reach an expected N = 1 earlier than the control charts which had smaller and larger λ's. That is to say, EWMA control charts which have a medium memory may have better average performance than control charts with either greater or lesser memory.

Figure 5-4 reveals that when the deterministic step shift is increased to 2σ, larger values of the smoothing constant are required for optimisation than for the 1σ step shift scenario. That is, $\lambda_3 = 0.25$ and $\lambda_4 = 0.55$ reached a cumulative probability of 1 before the smaller smoothing coefficients.

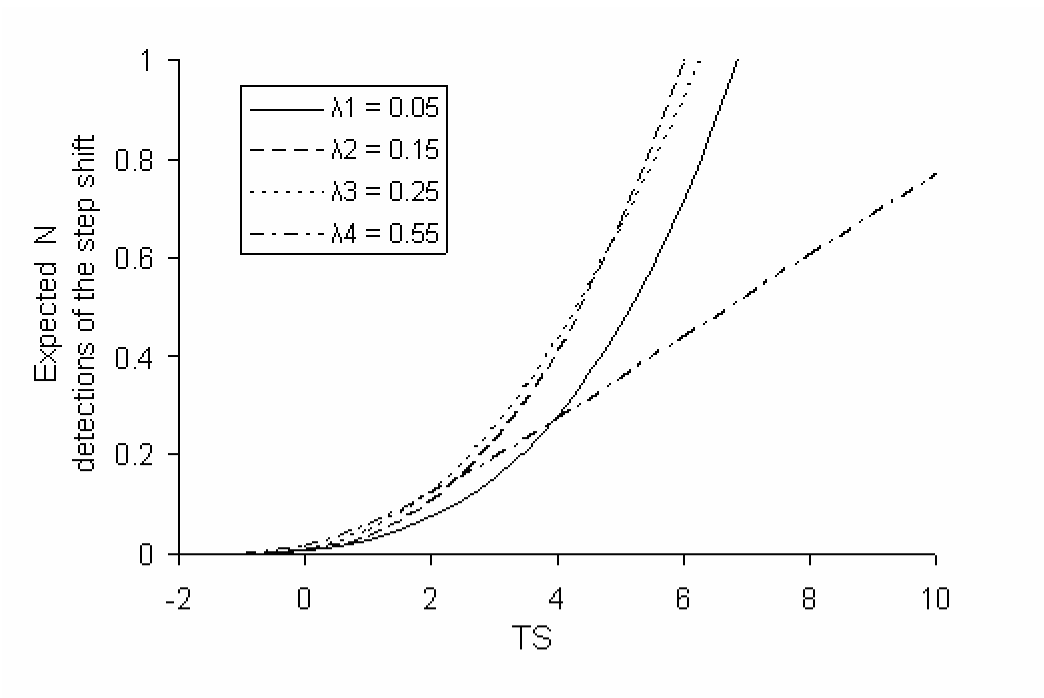

Figure 5-3. The expected number of alarms for detecting a step shift of 1 σ using EWMA schemes with λ = 0.05 to 0.55, with ICARL = 400 observations, based on 100,000 simulated chart runs.

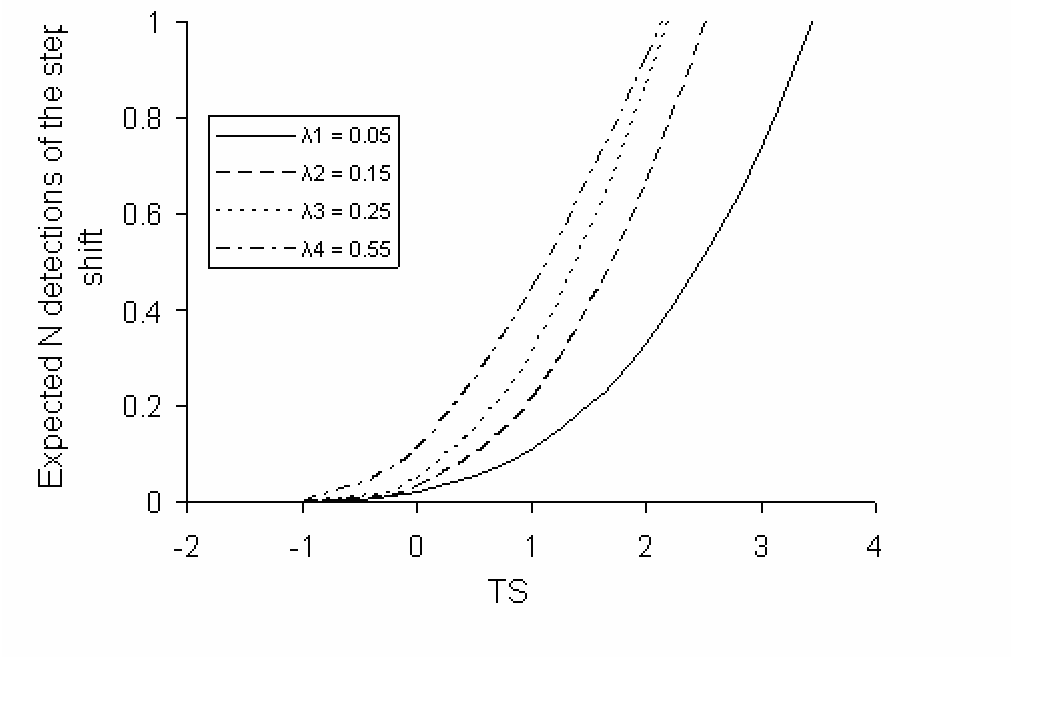

Figure 5-4. The expected number of alarms for detecting a step shift of 2 σ using EWMA schemes with λ = 0.05 to 0.55, with ICARL = 400 observations, based on 100,000 simulated chart runs.

Figure 5-1 and 5-2 demonstrated that EWMA control charts which had larger smoothing constants had high probabilities of detecting out-of-control process at each observation relative to the other control charts. However, control charts which had a small smoothing constant ultimately developed higher probability of detecting an out-of-control process at each observation. Figure 5-3 and 5-4 considered the collective performance over several observations showing that optimal chart memory is a trade-off between the power of the EWMA statistic and rate at which the influence of past in-control observations diminishes. Figure 5-3 and 5-4 also demonstrated that the optimum smoothing coefficient will depend on the size of the step shift disturbance. Fortunately for the statistical quality control practitioner, one is not restricted to just one control chart design. Therefore, he (she) does not need to have exact knowledge of the size of step shifts to be encountered in the future. Several control charts optimised for different step shift sizes can be used concurrently to monitor a single variable, as demonstrated in Chapter 6.

Different control charts have different performance characteristics. Classical single component control charts have a single tuning parameter that permits optimisation for a specific location shift magnitude. The relative performance of the optimised scheme falls away quickly as the magnitude of the shift varies from the design basis ***value***. A composite control chart achieves fast detection of assignable causes across a broad domain of step or ramp shifts, by monitoring a single variable using several components simultaneously. Each component draws on a different distribution of past data with the result that the expected performance of the control chart remains close to the optimum value at any point across the design basis ***domain***. Composite schemes have better overall performance across a domain than a single component scheme (e.g. CMA, Sparks 2003), but single component schemes may perform better for a small sub-domain about a particular location shift magnitude.

Basic MA, EWMA and CUSUM schemes are now compared to help support conclusions to be made about the difference in performances between various composite schemes in Chapter 7.

5.2 Literature on Comparisons of Basic Control Charts

Various references state that MA and EWMA, and EWMA and CUSUM control charts can be designed to have similar performance. Improved computational capabilities permit these statements to be verified through simulation with more accuracy than previous studies. Roberts (1959) suggested that MA and EWMA schemes which are linked by a common variance term would lead to similar ARL properties, which can be determined according to the formula:

$$n_{effective} = \frac{2-\lambda}{\lambda} \tag{10}$$

Equation (10) can be derived from equations for the variance of the EWMA and MA statistics. Let us call this calculated n the effective span of the EWMA scheme with smoothing constant λ.

Now let us consider EWMA and CUSUM control charts. Lucas and Saccucci (1990) stated that EWMA and CUSUM schemes can be tuned to have sensitivity and performance so similar that they stated: "nonstatistical criterion could be used to decide which particular procedure should be used". Whilst a small difference in performance may not lead to improved quality control due to the number of other practical considerations, there is no reason not to constantly improve all challenges to good quality production (Woodall and Montgomery, 1999). In the following section, results are first discussed in terms of ATS values.

5.3 MA, EWMA and CUSUM Comparisons

Simulations showed that EWMA schemes could be optimised to have smaller ATS values than the MA scheme for step shifts of various magnitudes. An example of a curve used for optimizing the tuning parameters of a CUSUM scheme for a step shift of 0.5σ is shown in Figure 5-5. The optimum steady-state ATS values for MA, EWMA and CUSUM schemes were found at the minima of the ATS versus tuning parameter curves. Results are summarized in Table 5-1 for step shifts of 0.25, 0.5, 0.75, 1 and 3σ. Full ATS profiles are found in Tables B-1 and B-2 of Appendix B.

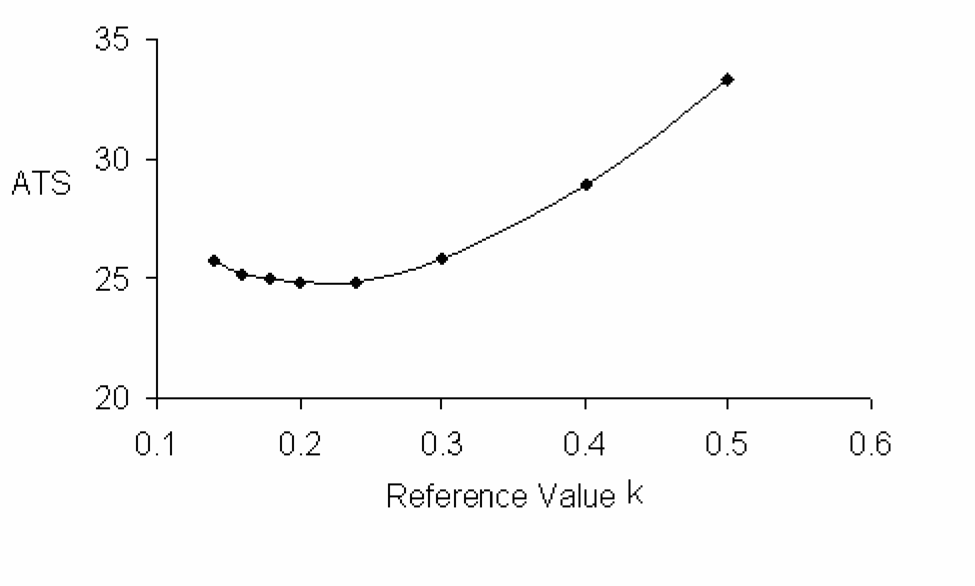

Figure 5-5. Optimal ATS for the CUSUM Control Chart for Step Shift of 0.5σ, with ICATS = 400.

Table 5-1. Comparison of optimised MA control charts relative to optimised EWMA control charts.

δ	ATS MA	ATS EWMA	Relative Difference in ATS [%]	Relative Difference in ARL [%]
0.5	27.0	25.50	6.1	5.7
0.75	14.55	13.73	6.0	5.6
1.0	9.06	8.574	5.6	5.1
3.0	0.961	0.801	20	8.9

At the five discrete step change scenarios investigated, the steady-state EWMA scheme had detection times at least 5% faster than MA schemes. For example, in the 0.5σ step shift scenario, the optimum EWMA scheme achieved an ATS of about 25.5 whilst the MA scheme achieved an ATS of about 27 which was 6.1%

slower. The relative difference in ATS values jumps from approximately 6% for step shifts of 1σ, to 20% for step shifts of 3σ.

Table 5-2. Comparison of optimised CUSUM control charts relative to optimised EWMA control charts.

δ	CUSUM ATS	Relative Difference in ATS [%]	Relative Difference in ARL [%]
0.5	24.8	-2.7	-2.6
0.75	13.22	-3.7	-3.5
1.0	8.25	-3.8	-3.4
3.0	0.716	-10.6	-4.7

5.4 Conclusions on Comparisons Basic Control Charts

A reason that the difference in performance between MA and EWMA schemes is so high at step shifts of 3σ is due to the integer nature of the MA tuning parameter. MA control charts cannot be finely optimised when the optimal span is small. For a 3σ step shift the optimum span was found to be n=2 and the optimal smoothing coefficient for the EWMA scheme for the same step shift was $\lambda = 0.65$. Using Equation (10), it can be seen that the comparable MA scheme, according to Roberts, would have an effective span of n=2.077. Of course this must be rounded to the nearest integer for a MA scheme, MA(2), resulting in deviation from the optimal span by approximately 4%. Run rules would also be expected to perform inefficiently as their "tuning parameter" is typically also an integer.

For control charts optimised for detection of single step shift scenario: CUSUM is superior to EWMA. The reflective boundary k prevents the inertia problem Woodall, Hoerl, Palm and Wheeler (2000) from developing where a random chart run occurring on one side of the mean can cause a delayed response of a moving average statistic (MA, or EWMA) to an assignable cause which causes a shift to values on the other side of the mean.

Another feature of the CUSUM statistic that differentiates it from the MA and EWMA statistic warrants discussion. CUSUM might also be better understood by considering the form of a one-sample, one-sided t-test with the following hypothesis:

$H_{0:}$ $x_1 - t \leq k.s$
H_1: $x_1 - t > k.s$

where $x_1 = CUSUM_{i-1} + x_i$; and k is chosen such that k.s is considered to be just practically significant. For step shifts larger than *k.s*, the consequences of losing of control of the monitored variable become more likely. Use of a non-zero value for *k* permits a focus on non-trivial step shift magnitudes. The MA and EWMA statistic lack the flexibility to specify a non-zero k value. Consequently, MA and EWMA statistics always test very small step shifts and must have their performance “detuned” to have an acceptable SSATS. “Detuning” is achieved by having wide control limits, which in turn harms the detection performance for all non-zero step shifts.

It is interesting to note the how use of ATS affects the relative difference in performance as compared when ARL is used. It can be seen that the relative differences in ARL values are substantially less than the relative difference for ATS values for step shifts of 3σ, particularly in the case of the comparison between MA and EWMA schemes. Optimisation for a broad domain step shift sizes based on ARL values instead of ATS values would result in different composite designs.

Chapter 6

Optimisation of Composite Control Charts

6.1 Introduction

Two-component and three-component control charts have not previously been optimised; therefore, the full potential of these schemes is unknown. In this chapter, CEWMA2, CEWMA3, CMA3 and CCUSUM3 schemes are optimised and compared with each other, using the performance measures developed in Chapter 3. It is intended to publish the material in this chapter after successful publication of Chapter 3 and 4.

6.2 Methodology

The optimisation method used was direct search of a discrete lattice (Carlyle, Montgomery and Runger, 2000) created by a full factorial experimental design. It is a simple technique which is costly in terms of the number of results that need to be generated. When the optimum was found at a boundary of the original search lattice, additional levels were added to the lattice to prevent constrained optimisation. Simulations were replicated to ensure at least two standard errors (refer to Appendix A - Error Analysis) differentiated the apparent optimum result from the other results at the inner hyper-cube of the lattices. Response surface modelling (e.g. Wu and Hamada, 2000) was not applied as such a technique would produce misleading results, over-fitting to data points insufficient in number for such a degree of variance.

Full lattice simulation provided an optimum solution despite interaction terms existing between the design parameters. For example, there is interaction between the values of the tuning parameter and Al_1IC. As the fine component becomes coarser, a larger value for Al_1IC becomes locally optimal. Generation of a full lattice also permitted data for presentation in surface-area graphics. A lattice with three-factors at four-levels, full factorial design, that is, a 4^3 design was used to optimise CEWMA2 schemes. The levels used for the experimental designs are shown below in Table 6-1. Three-component composite schemes were simultaneously optimised with respect to up to 5 parameters at four levels, that is, a 4^5 design.

Table 6-1. Levels used in optimisation of the CEWMA2, CMA3, CEWMA3 and CCUSUM3 schemes. Al j IC refers to the percentage of alarms attributable to component j when the monitored variable is in an in-control state.

Scheme	Values for the different levels
CEWMA2	
λ_1	[0.1, 0.15, 0.25, 0.30]
λ_2	[0.55, 0.7, 0.8, 0.95]
Al_1IC [%]	[15, 30, 45, 60]
CEWMA3	
λ_1	[0.05, 0.08, 0.12, 0.15]
λ_2	[0.35, 0.4, .43, 0.48]
λ_3	[0.9, 0.93, 0.96, 1.0]
Al_1IC [%]	[9, 12, 17, 25]
Al_2IC:Al_3IC	[1, 1.222, 1.5, 2.333]
CMA3	
n_1	[7, 8, 9,10]
n_2	[2, 3, 4]
n_3	[1]
Al_1 [%]	[15, 20, 25]
Al_2IC:Al_3IC	[0.667, 1, 1.222]
CCUSUM3	
k_1	[0.25, 0.35, 0.45, 0.55]
k_2	[0.8, 0.9, 1.0, 1.1]
k_3	[1.7, 1.8, 1.9, 2.0]
Al_1IC [%]	[9, 13, 17, 25]
Al_2IC:Al_3IC	[1, 1.222, 1.5, 2.333]

6.3 Optimised Scheme Configurations

The configurations of the optimum composite schemes from the simulated lattices are shown in Table 6-2. Whilst the response surface for MRL values to designs in the region of the optimum schemes are shown in Figures 6-1 to 6-4.

Table 6-2. Configuration and performance of optimised composite schemes. P_χ refers to the tuning parameter of the respective component of the respective composite scheme.

	CEWMA2	**CMA3**	**CEWMA3**	**CCUSUM3**
MRL	-0.055	-0.050	-0.096	-0.137
P_1	0.25	9	0.12	0.35
P_2	0.8	2	0.48	1
P_3	N.A.	1	1	1.8
h_1	3.0925	3.2025	3.2512	8.8138
h_2	3.1021	3.1546	3.1025	2.7295
h_3	N.A.	3.2007	3.1869	1.3856
Al_1IC [%]	45.0	20.0	17.0	13.0
$Al_2IC:Al_3IC$	N.A.	1	1.22	1
Al2 [%]	55.0	40.0	46.5	43.5
Al3 [%]	N.A.	40.0	36.6	43.5
δ_μ	ATS			
0	400.5	399.5	399.7	400.4
0.50	58.1	64.0	47.8	39.2
0.75	21.87	23.35	18.93	17.16
1.00	10.89	11.264	10.283	9.959
1.25	6.520	6.785	6.539	6.445
1.50	4.397	4.681	4.511	4.382
1.75	3.159	3.421	3.233	3.090
2.00	2.350	2.529	2.378	2.242
2.25	1.787	1.864	1.772	1.670
2.50	1.369	1.368	1.331	1.255
2.75	1.050	1.013	1.006	0.958
3.00	0.798	0.748	0.756	0.726
3.25	0.597	0.550	0.563	0.545
3.50	0.441	0.409	0.411	0.402
3.75	0.318	0.296	0.294	0.292
4.00	0.227	0.209	0.204	0.202

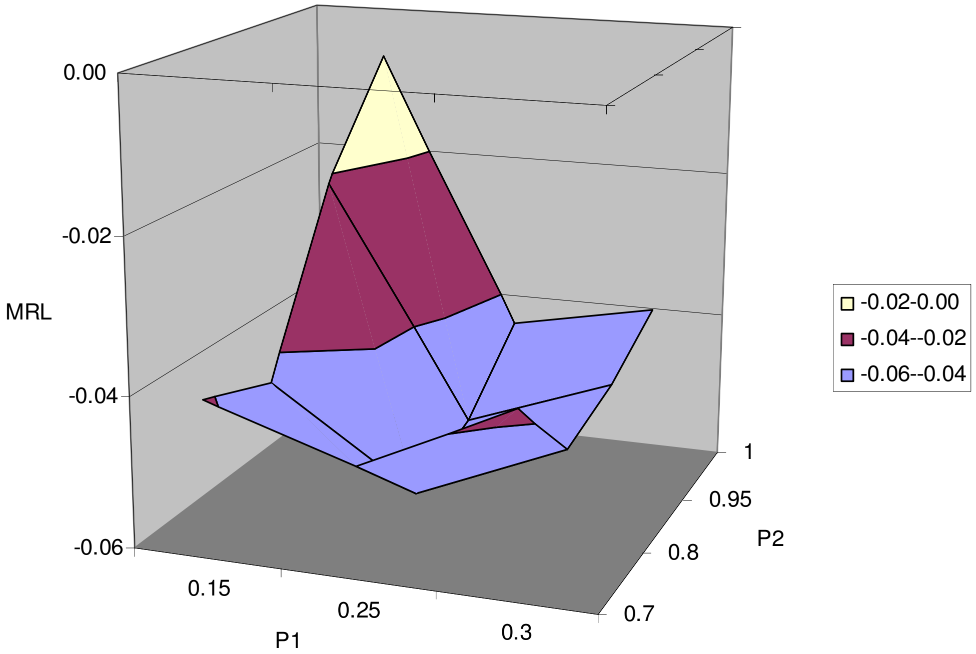

Figure 6-1. Surface plot of MRL values for CEWMA2 designs, $Al_1IC = 45\%$.

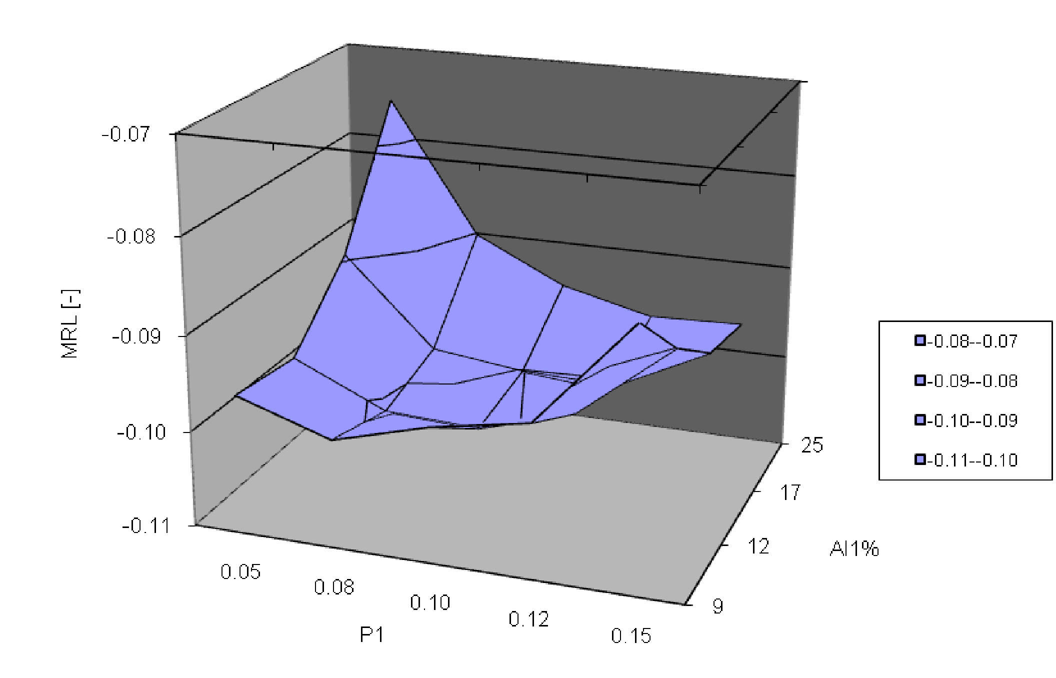

Figure 6-2. Surface plot of MRL values for CEWMA3 designs. $\lambda_2 = 0.43$, $\lambda_3 = 0.96, Al_2IC : Al_3IC = 1.222:1$.

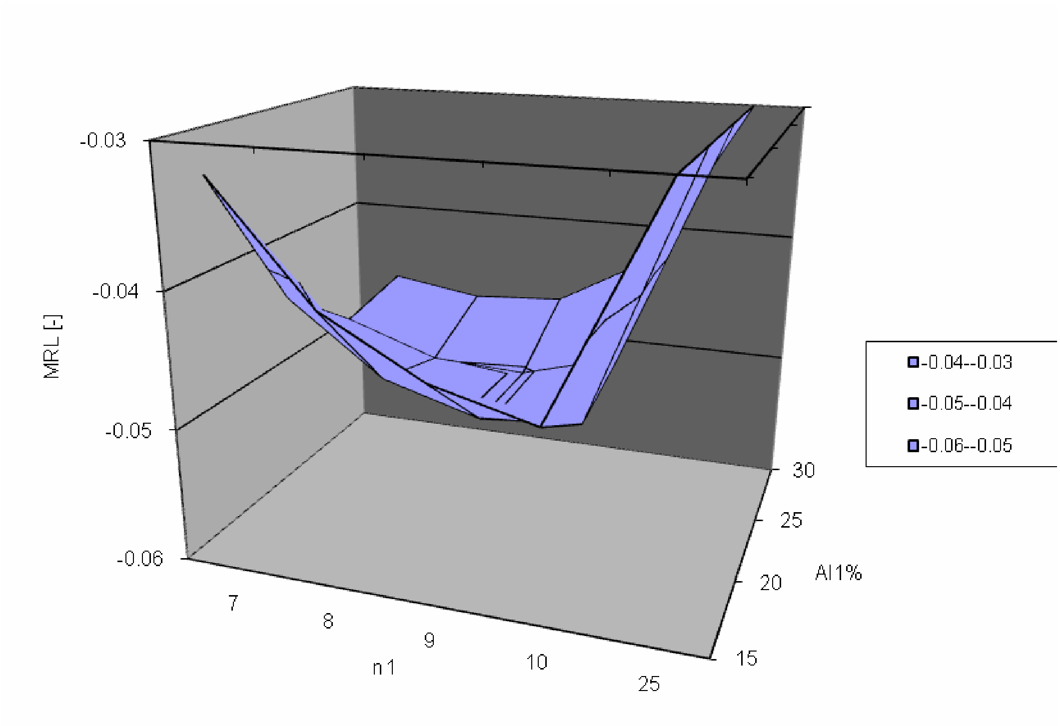

Figure 6-3. Surface plot of MRL values for CMA3 designs. $n_2 = 2$, $n_3 = 1, Al_2IC : Al_3IC = 1:1$.

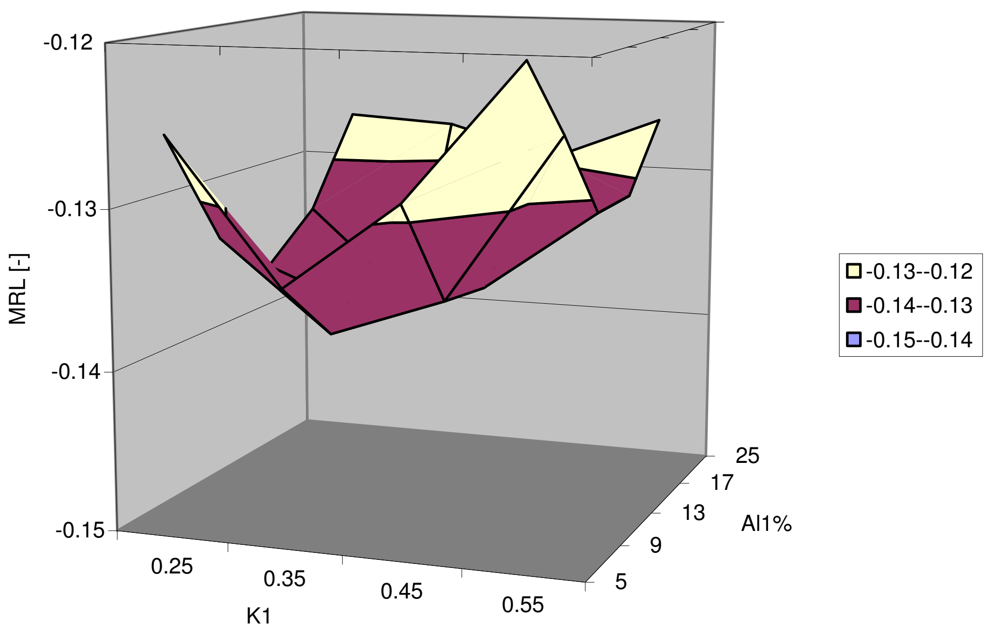

Figure 6-4. Surface plot of MRL values for CCUSUM3 designs. $k_2 = 1.0$, $k_3 = 1.8, Al_2IC : Al_3IC = 1:1$.

6.4 Effect of the Number of Components

Prior to optimising and comparing different composite schemes, it was decided to investigate how sensitive a CEWMA scheme was to the number of components used (CEWMA was formally the focus of the thesis). No published literature to date has demonstrated the additional benefit for use of three components in CEWMA schemes as opposed to two components. For CCUSUM schemes, however, Sparks (2000) recommended three components for a domain of 0.5σ to 2.0σ, and four components for good performance across a 0.5σ to 4.0σ step shift domain. Optimisation of four-component schemes was considered to be excessive in scope for a thesis and unnecessary for comparing CCUSUM, CEWMA and CMA schemes.

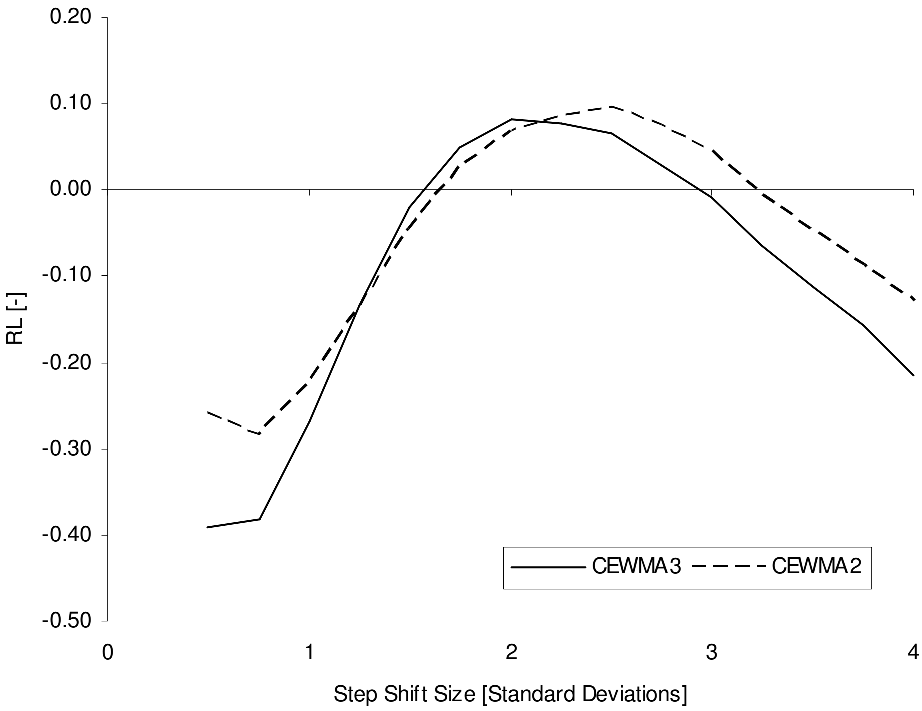

Figure 6-5. The RL profiles of optimum CEWMA2 and CEWMA3 schemes, relative to the reference CUSUM scheme (k=1.1, h=2.2908).

The optimum CEWMA3 scheme was 5.2% faster overall (ADRA) than the optimum CEWMA2 scheme. CEWMA3 is faster than CEWMA2 from 0.5σ to 1σ, and from 2.25σ to 4σ. Both two-component and three-component schemes provide a broader domain of good detection performance than that achieved by the reference CUSUM scheme. However, the composite schemes are approximately 7%-8% slower (RL ×100%) than the reference scheme around the centre of the assessment domain (2.0σ to 2.25σ). The additional component served to increase the performance at small and large step shifts on the domain investigated. Optimised CEWMA2 and CEWMA3 schemes perform similarly for step shifts between 1.25σ and 2σ.

Figure 6-1 shows the MRL data in the vicinity of the optimum CEWMA2 scheme. Some sublevels were subsequently simulated but the MRL performance was indistinguishable from that determined at the major levels according to the error bars calculated in Table 6-3. Table 6-3 summarises the error analysis details in Appendix A for up to two replicates of each lattice point. The best design for a CEWMA2 scheme, on the assessment domain, was found to have the parameters [λ_1 = 0.25, λ_2 = 0.80, Al_1IC = 45%] based on 3 simulations of that lattice point. Interpolation of the main lattice at [λ_1 = 0.20, 0.35, 0.4; λ_2 = 0.75, 0.9, 1.0; Al_1IC = 40%, 50%] did not produce any results which were significantly better.

The best CEWMA3 scheme was found to have the parameters [λ_1 = 0.12, λ_2 = 0.48, λ_3 = 1.0, Al_1IC = 17%, Al_2IC : Al_3IC = 55:45]. Some of the other designs in the vicinity of the optimum are shown below in Figure 6-2.

Table 6-3. Error bars for simulated MRL results.

1.96×Std.Error		Number of simulations 1	2	3
Type of Scheme	CMA3	0.0026	0.0021	0.0017
	CEWMA3	0.0024	0.0017	0.0014
	CCUSUM3	0.0024	0.0017	0.0014

6.5 Comparison of Three-Component Composite Schemes

Analysing the relative loss profiles in Figure 6-6, it can be seen that the simple CUSUM reference distribution is at most only 2% faster (DRA) than CCUSUM3 at 2σ, which is in the vicinity of a 2.2σ for which the reference scheme was optimised. Despite the strength of the reference CUSUM scheme at 2σ, it is much slower than the composite schemes outside of the 1.5σ to 3σ region. The optimised CMA3 scheme is generally the weakest three-component scheme across the domain. It performs more strongly than the reference scheme at small and large step shifts, approximately 22% faster (DRA) at the boundaries of the domain. CCUSUM3 has its most notable advantage over the reference CUSUM scheme at 0.5σ (DRA = -40%).

CCUSUM3 is consistently the fastest composite scheme across the domain of comparison. The region of most divergent relative loss performance *between the composite schemes* is at 0.5σ where CCUSUM3 is better than CEWMA3 by approximately 20% (DRA). A large spread also exists between the schemes at 2σ where CCUSUM3 is better than CEWMA3 by approximately 6% (DRA). Basically, the optimised CCUSUM3 scheme significantly outperforms the optimised CMA3 and CEWMA3 schemes at small and moderate step shift sizes within the assessment domain. There is little difference between the composite schemes from 3σ to 4σ. CCUSUM3 is 5.0% faster overall (ADRA) relative to CEWMA3.

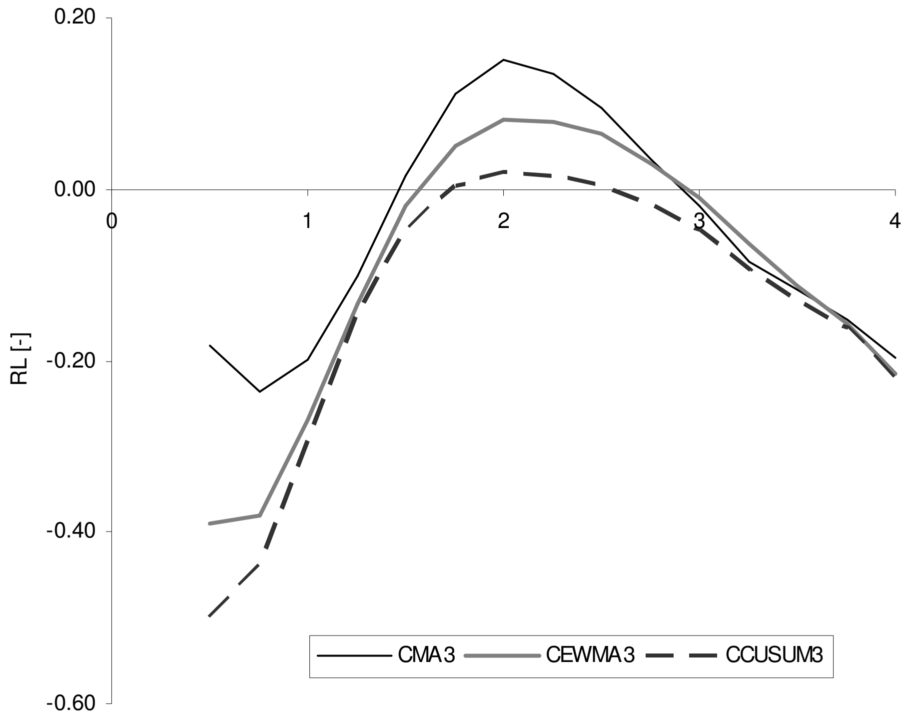

Figure 6-6. RL profiles for optimum CMA3, CEWMA3 and CCUSUM3 control charts, relative to the reference CUSUM scheme (k=1.1, h=2.2908).

Sparks (2000) recommended three components for CCUSUM schemes with k values of 0.375, 0.5 and 0.75 for good performance across a 0.75σ to 1.6σ step shift domain. He did not claim, however, that these parameters were optimum under any assessment criteria. His scheme is compared with the optimum CUSUM3 as shown in Appendix F. However, as he was targeting good performance on a smaller sub-domain, a fair performance comparison cannot be made.

An important observation was made by Lucas and Saccucci (1990) in their comparison of EWMA and CUSUM schemes designed for a step shift of 1σ:

> "Our comparisons showed that the ARL's for the EWMA are usually smaller than the ARL's of the CUSUM up to a value of the shift near the one that the scheme was designed to detect. Beyond this shift, the ARL's of the EWMA are larger than the ARL's of the corresponding CUSUM."

From the above quotation, one might expect increasing efficiency of optimised CCUSUM schemes relative to optimised CEWMA schemes for increasing step shift size. A pattern such as that described by Lucas and Saccucci above cannot be seen in Figure 6-6. Performance of the CCUSUM3 and CEWMA3 schemes actually converges in the proximity of 4σ step shifts. Our design method was not to optimise for a single point, however, but rather optimising for an out-of-control region. Therefore, the observation of converging performance does not contradict the observation made by Lucas and Saccucci. In fact, the superior performance of the CCUSUM3 scheme over the CEWMA3 scheme for an out-of-control region is consistent with their observation.

Another possibility one may consider after the observation drawn in the above quotation by Lucas and Saccucci (1990) is that a CUSUM-EWMA composite may offer superior performance to a two-component CCUSUM scheme. Our initial 3-level full factorial experiments found no evidence of this, rather a preliminary indication on a domain of step shifts was somewhere between the performance of CCUSUM2 and CEWMA2 composites. Rigorous investigation into this matter is nominally outside the scope of this thesis.

6.6 Ramp Location Shift Performance

The optimised three-component composite control chart designs from Section 6.2 (see Table 6-2 for parameter values) were compared for ramped location shift scenarios. CEWMA3 performed slower for smaller ramp coefficients, but in the order of 2% better (DRA = -1.7%) for a ramp rate of 0.125 σ /observation. CMA3 performed slower than CCUSUM3 for all ramp coefficients investigated, although the difference was not significant for a ramp rate of 0.125 σ /observation. The results are shown in Table 6-4 below.

Table 6-4. Comparison of optimized three-component composite schemes on ramp location shifts relative to the CCUSUM3 scheme.

Ramp	CCUSUM3	CEWMA3		CMA3	
Coefficient	ATS	ATS	DRA [%]	ATS	DRA [%]
0.005	82.634	86.575	4.658	93.345	12.173
0.020	35.479	36.031	1.544	38.439	8.009
0.045	21.289	21.116	-0.816	22.05	3.512
0.080	14.608	14.372	-1.629	14.802	1.319
0.125	10.780	10.597	-1.712	10.872	0.850
0.180	8.298	8.219	-0.957	8.463	1.969
0.245	6.590	6.540	-0.762	6.808	3.254

The ramp coefficients were chosen to be similar to that used by Sparks (2003). CEWMA seems favourable because it was faster for more significant ramp coefficients, but it is not clear from literature how much practical influence such small shift rates have on plant operations. Overall CEWMA3 and CCUSUM3 are similar so should not be considered a deciding factor between the two schemes. CMA is statistically the least favourable option of those compared for detection of ramp shifts.

6.7 Hierarchical Monitoring

Exception reports can succinctly inform the managers of a manufacturing facility (asset), by summarising control chart alarms. For an asset within a large multinational company, normal business activities are often managed by a person under the job title Operations Manager (or General Manager). Plant operating personnel are typically divided into several plant areas and a laboratory lead by area Managers and Laboratory Manager respectively. Each plant area may then be further divided into several process areas operated by a dedicated team who are lead by a Superintendent. Line management for one such process area is highlighted in Figure 6-7, a simplified organisational chart for manufacturing plant operating teams.

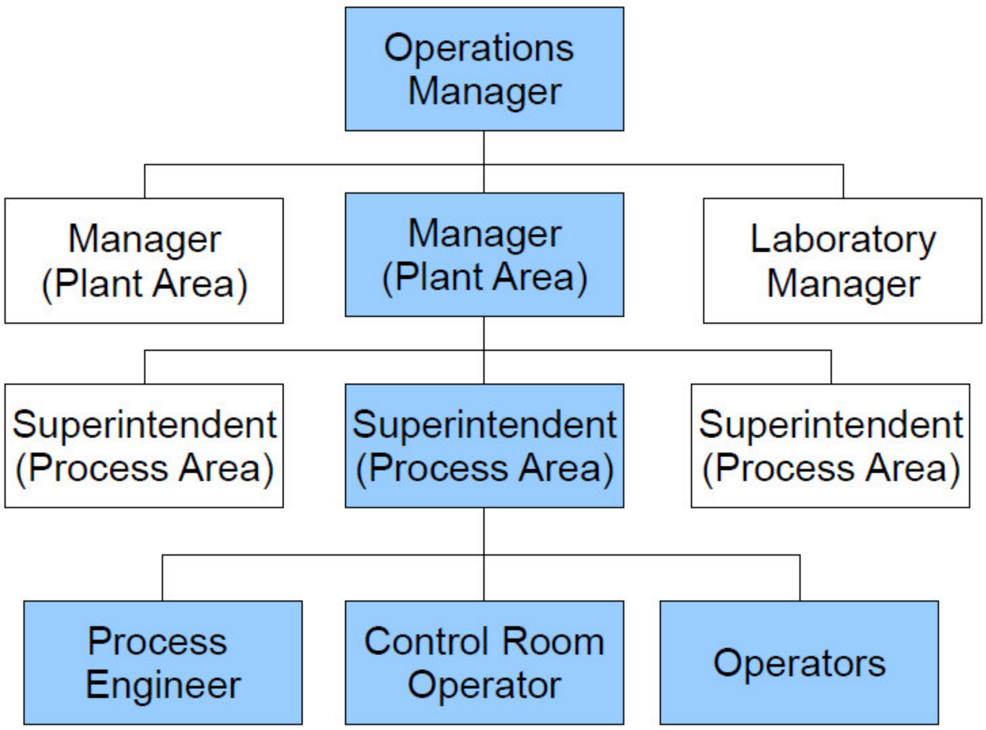

Figure 6-7. Simplified organisational chart showing the process and laboratory components for a large manufacturing plant operation.

The differentiated locus of responsibility of personnel in the hierarchy of an organisation is an important consideration when designing tools and methods to assist operating and management.

Process Engineers and Control Room Operators (who often have an overlapping scope of focus in monitoring) should frequently monitor an array of critical and non-critical operating data (process input, control and output variables). Monitoring is usually via visual inspections, taking samples, measurement and analysis of samples and viewing tables and trends for online instruments and laboratory analysis results. Detection of process disturbances should trigger activities including: identifying causes of the disturbance, assessing risk, identifying potential corrective actions and formulating an action plan.

Closely monitoring the value of all variables in a process area would employ such numbers of engineers that a low benefit to cost ratio from their employment may result. One way of reducing this workload at a process engineer's level is to closely monitor only key variables for the process area, and process variables that are out of their normal range. Exception reports generated by composite control charts may be used for this purpose. In this scenario, the composite control charts should be designed to optimise the trade-off between detection sensitivity and annoyance frequency (defined here as the collective rate of false alarms and alarms for trivial process disturbances for the entire set of control charts configured for the process area). That is, these control charts are to be optimised for use by the Process Engineer.

Superintendents manage the performance of individuals and coordinate their team to achieve the following goals: cost effectively produce quality outputs subject to safety, equipment longevity and legal constraints. Furthermore, they need to liaise with superintendents of upstream and downstream process areas to coordinate operations. Whilst superintendents usually have process engineering technical skills and responsibilities to achieve the above goals, they have less time available to review operating data relative to their process engineer(s) who have mostly technical responsibilities. Hence, the Superintendent would be best served by an exception report that is more succinct relative to the exception report reviewed by the Process Engineer.

Managers are accountable for the economic performance of their plant area, and complying with established business processes. They are responsible for coordinating and coaching several superintendents and liaising with other plant area, safety, human resources and maintenance managers. When costly or dangerous process disturbances occur, access to accurate operating data is useful. The information can be used to objectively coach superintendents in problem solving and to consult with pier managers and the Operations Manager. As mangers have a higher proportion of non-technical responsibilities as compared to superintendents, a comparative reduction is warranted in the amount of operating data reviewed in a set period. The method of reducing technical data should also account for the multiple process areas in his (her) locus of accountability.

An Operations Manager coordinates and coaches several managers, and administers business systems to support production of a quality product, achieving cost effectiveness and asset longevity within the safety and legal constraints mentioned previously. To achieve this, one might argue, an operations manager only needs to monitor the key inputs and outputs for the overall process, and hire good managers.

It can be seen that inclusion of different amounts of operating data is appropriate in exception reports reviewed by managers at different levels within an organisation. To reduce the amount operating data, a method is required. Several options are clearly evident: certain variables may be deselected based on lower criticality; certain variables may be deselected according to the specific technical responsibilities not included within the respective employee's job description; finally, the sensitivity of the composite control charts may be reduced. The suitability of these information reduction methods is explored below for each level of management.

For a superintendent, only "problem" areas need to be bought into focus; a "problem" being defined as a variable of sufficient business criticality sufficiently disrupted from its ideal operating range (which hopefully equates to the normal operating range about which control charts are centred). However, it can be difficult to apply a heuristic as to which variables are critical to the business as criticality depends on the size of the departure from the (multivariate-) normal operating range. Thus it would be advisable for the Superintendent to monitor all variables in their process area via exception, but using control charts which are designed to be less sensitive to small disruptions.

For managers who oversee several process areas, several options for further reducing this data seem logical. Intra-process area inputs and outputs within the plant area may be deselected, monitoring only at the plant area boundaries. However, timely notification of inappropriate levels in critical operating variables or extremes in quality at the process area interfaces may be helpful. This information may prompt the manager to coordinate the superintendents and coach them in problem solving. Thus, de-selection of all intra-process area variables is not advisable. Instead, a combination of de-selection and desensitising is recommended. Composite control charts for critical operating variables may be desensitised for small step shifts (relative to those used by the superintendents), and non-critical variables should be deselected.

Which method of desensitising control charts is best? One option is to spread the control limits further apart. Ideally, however, when a significant process disturbance occurs, and all levels of management receive the same notices on exception reports and view identical control charts. All staff working from “the same page” enhances communication. Composite control charts offer a way of differentiating the annoyance rate and sensitivity to small step shifts that does not require viewing completely different control charts. As each control chart component contributes to the annoyance rate, monitoring a subset will reduce this annoyance rate. The control chart components which are most effective at detecting small step shifts (see Chapter 5) seem the most logical components to deselect when reduced sensitivity to small step shifts is required.

The Operations Manager and plant area managers may view a subset of the control chart components from each composite scheme. In fact, exception reports for a plant area manager might best be based only on the coarsest component of all the composite control chart components. The Operations Manager is probably well advised to further deselect most operational variables except for the few most critical for the whole facility.

Timely provision of critical information is supportive of loss mitigation actions. In fact, economic losses and legal non-conformances are potentially avoidable if the information is sufficiently timely. Electronic exception reports based on control charts are capable of transferring critical operating data **and** other business performance measures directly to an operations and plant area managers. As the information transfer is not triggered by indirect feedback from a production rate or quality crisis, the information transfer is arguably completed in a timely manner.

Some preliminary comparisons have been completed for the ratio of false alarms from an eight-component scheme versus only the coarsest component, but are not reported in this thesis. The concept of hierarchical monitoring has been introduced without rigor or referencing merely to demonstrate the potential of composite control charts over adaptive control charts, justifying the significance of these studies. Further development of the hierarchical monitoring methods is recommended to interested researchers.

Chapter 7

Conclusions

In this chapter, the results of the thesis are evaluated for satisfaction of the thesis aims. Interpretations and limitations are discussed, then justification and significance of the thesis are considered.

7.1 Satisfaction of Thesis Aims

The basic objective of this thesis was to explore composite control charts so that manufacturing end users would be better informed to make a control chart selection. This objective could be fulfilled in terms of the primary and secondary aims:

Primary Aim

Aim 1 – determine which composite control chart performed best over a domain of location shift sizes when distribution parameters were estimated.

Secondary Aims

Fulfilling the following secondary aims would further satisfy the basic objective:

Aim 2 – determine the benefit of three-component over two-component schemes.

Aim 3 – compare the performance of the control charts for ramped location shifts

Aim 4 – identify additional opportunities that composite control charts offer over the alternatives.

To achieve these aims, the following tasks were completed:

- Improved statistical measures were developed in Chapter 3, for optimising and comparing control chart performance over a domain of step shifts. The results as summarised in Section 7.1.1.
- Software was created to derive control chart properties where existing analytical methods and software were inadequate, as described in Chapter 4. A high level review of the software is given in Section 7.1.2.

- Composite control chart configurations were optimised (Chapter 6, Section 6.3) using the simulation software and newly developed statistical control chart performance measures; and
- In Chapter 6, the composite control charts were optimised (Section 6.3) and compared (Section 6.5) ***satisfying Aim 1 of the thesis***, as summarised in Section 7.1.3.

Also in Chapter 6, composite control charts were explored in some depth completing the secondary aims of the thesis. Sections 6.4 explored the effect of the number of components for two- and three-component schemes ***satisfying Aim 2***; Section 6.6 investigated the comparative performance for ramped location shifts, ***satisfying Aim 3***. Section 6.7 investigated an idea for Hierarchical Monitoring which demonstrated a special capability of composite control charts, thus ***satisfying Aim 4.***

7.1.1 Develop Improved Methods of Control Chart Performance Measurement

Historically, much control chart statistical and economic optimisation has been completed for a specific step shift (say in increase of one standard deviation to the mean). Control chart design configurations and design heuristics have also been recommended for good performance over a domain of step shifts. To quantitatively optimise a control chart for a domain of step shifts a suitable performance measure is required. No satisfactory performance measure had previously been defined for a domain of step shifts. In this thesis, new measure MRLOCV and MRL have been defined. MRL and MRLOCV are true performance measures which permit optimisation. Performance comparison measures, such as relative loss efficiency (RLE), destabilise an optimisation routine as every performance value changes as each new performance response vectors is measured and incorporated. A new performance comparison measure was also developed in this thesis, Average Difference Relative to the Average (ADRA), for comparing the performance of two control charts across a domain of step shifts. ADRA is easier to interpret than RLE, and gives consistent values.

7.1.2 Create Software to Derive Control Chart Properties

Insights from this study into the performance of optimised composite control charts are unique owing to the large amount of data simulated. Over 10,000 control charts configurations were simulated in total completing several full factorial experimental lattices with replicates. A computer program called "Composite Monitoring Schemes" was developed by the student as part of the thesis. The software applied a novel algorithm based on the secant method to solve the control chart configuration achieving certain performance specifications. Historically, control chart configurations were solved manually by trial and error (in the case of simulation studies). Simulating data points for an experimental design is labour intensive in the absence of code to solve specifications. Creation of the software was essential to completing this large study in the student's remaining worklife.

7.1.3 Optimise and Compare Composite Schemes (Aim 1)

Optimised three-component CMA, CEWMA and CCUSUM schemes were compared. An optimised CCUSUM3 scheme proved to be the best composite by a significant amount and was 5.0% faster (ADRA) relative to the optimum CEWMA3 design. Performance advantages were most notable in the lower and middle portions of the assessment domain.

7.1.4 Determine the Benefit of using Three Components over Two (Aim 2)

The performance of composite schemes as a function of the number of components was investigated. Two-component and three-component CEWMA schemes (CEWMA2, CEWMA3) were optimised and compared. A reasonable reduction in detection times of 5.2% (ADRA) was achieved by employing the third EWMA component in a CEWMA scheme, although with a smaller marginal benefit than with an addition of the second component.

7.1.5 Compare the Performance of the Control Charts for Ramped Location Shifts (Aim 3)

CEWMA3 perform better than CCUSUM3 and CMA3 schemes. However, the difference in performance between CEWMA3 and CCUSUM3 was insignificant.

7.1.6 Identify other Opportunities of Composite Control Charts (Aim 4)

Hierarchical monitoring, using exception reports based on composite control charts, was described for process and business monitoring at increasingly elevated ranks within a manufacturing organisation. Components of composite control charts may be deselected to reduce the flux of information presented to plant area and operations managers. The components which have the most sensitivity to small step shifts were recommended for de-selection in the reports viewed by plant area and operations managers.

7.2 Interpretations and Limitations

7.2.1 Nominated Assumptions

Optimisation of designs required certain assumptions. Parameters were estimated from in-control samples to ensure that the performance comparisons were meaningful in real situations. Assessment domain boundaries have been nominated. These selections have been made without broad consultation with the manufacturing industry. Unfortunately, any particular sample size assumption or selection of assessment domain boundaries is hardly appropriate for applications universally. Never-the-less, this study has demonstrated that CCUSUM3 performs the best of the composite schemes compared on a 0.5σ to 4.0σ step shift assessment size domain. Furthermore, there is no apparent reason suggesting that the superior performance of CCUSUM3 will not persist when schemes are optimised for broader or narrower assessment domains. In fact, the CCUSUM3 performs better than CMA3 and CEWMA3 at these boundaries. Regardless, our results would be more relevant to an end user if the research applied assumptions specific to their particular application.

Weightings for the relative frequency of various step shift sizes and for the economic business consequence of the various step shift sizes also need to be considered in practice. Using MRL, which has a weighting of unity for economic and frequency parameters, may not be optimal for a specific application. Regardless of this fact, all composite schemes were optimised using a common performance measure. Therefore, it has been demonstrated in this thesis that CCUSUM3 schemes are superior to CEWMA3 and CMA3 schemes under at least one criterion. It seems likely that CCUSUM3 schemes will also be superior to CEWMA3 and CMA3 schemes when optimised for a specific application with known economic and frequency parameters.

7.2.2 Algorithm Complexity

Difficulty of control chart setup and interpretation are important considerations for the end user. Algorithm complexity should not be allowed to harm adoption of good control chart tools; interpretability at the user interface is paramount. This raises the question of whether it is practical to use a CCUSUM3 in preference to CEWMA3 based purely on statistical superiority. CUSUM transformations of data do not appear as similar to the original data as does EWMA smoothing. However, with a small amount of experience using CUSUM "trends", it is expected that process engineers and control room operators will find the CCUSUM3 chart informative and not confusing. ***With the 5.0% advantage (ADRA) has over CEWMA3, CCUSUM3 should be adopted in preference to CEWMA3 and CMA3 composite control charts.*** If any training is required to make the tool effective, the cost is expected to be viable.

The practicality of adopting composite schemes warrants further scrutiny at this juncture. A relevant anonymous aphorism states: "simplicity *is* efficiency." Simplicity in statistical monitoring tools may be considered in two parts: simplicity in presentation of the user interface, and simplicity of the algorithm. The appearance of control charts at the user interface needs to be simple for easy interpretation by control room operators. Composite schemes are complex in terms of the number of different transforms of the raw signal that are created and the associated control limits which must be designed. Simplicity is a less critical issue for specialist software designers than it is for general users. The level of complexity is acceptable for the control systems engineers who would implement the technology. When using the control charts, computers would automatically execute the calculations upon receipt of new data, thus there is little of this complexity passed onto the control room operator.

Control charts offer benefit to a user only when combined with follow-up activities. Uniformity of process plant operations is influenced by factors other than increasing the efficiency of control charts. In series with detecting a disturbance to operational uniformity, identifying the assignable cause and correcting the process are also needed to regain uniform operation. Both identification and correction activities need to be done efficiently to reduce out-of-control durations. Ultimately, the method described in this thesis for control chart performance measurement, comparison and optimisation of composite control charts, may make a contribution to producing cheaper, higher quality products. However, the contribution will be relatively limited where there is low performance in cause determination and correction.

7.3 Justification and Significance of the Research

With recent control chart publications covering topics such as generalisation of advanced multivariate, non-parametric and data mining techniques, one might wonder if research into performance measures and composite control chart optimisation is justifiable. Multivariate, non-parametric and data mining techniques offer increased (or decreased) power as required for decision making; however, those techniques are not the only option for improving detection performance. Until now research into multivariate techniques, for example, has not facilitated process monitoring in a typical hierarchical organisational structure. Composite schemes offer good detection efficiency across a domain of step shift sizes, and can be used in a way to support monitoring from different levels within a company. Furthermore, composite control charts may also be used in conjunction with multivariate, non-parametric and data mining techniques. Thus, the research of this thesis was well justified, potentially stimulating further research in hierarchical monitoring.

Massive amounts of simulated data were required to optimise these control charts. Optimisation was previously considered to be practically impossible, most likely due to the manual nature of designing control chart configurations to meet performance specifications. Development of the software "Composite Monitoring Schemes" was a significant change as compared to the approach used by previous researchers of composite control charts. The software used a novel robust secant-method algorithm automating much of the previous manual work.

Creation of the software thus permitted compilation of the significant quantity of simulated data hence the findings of this thesis.

Composite control charts are not necessarily widely used. As described above, they have been difficult to design and optimise in the past. However, unrestricted use of the freeware computer program (accessible via the internet – see Chapter 4) may promote increased usage of composite control charts. It is likely that the future impact of this thesis will largely depend on developments in hierarchal monitoring which may be based on composite control charts.

There is inherent complexity in the setup of a composite control chart. Despite the amount of underlying detail, composite control charts may be presented in a moderately simple form applying sensible interface design standards. Given typical manufacturing process plant software, it is argued that simplicity is no longer a dominating control chart design requirement if the scheme can be presented simply. Therefore, the data and methods of this study have potential for significant outcomes in industry.

Considering the costs of implementing a system of control charts, Wu and Wang (2007) suggested that it is less easy to implement composite schemes as compared to single component schemes. It is argued here that the overhead cost per variable would be small when implementing a large system of control charts. Potentially, process information management system (PIMS) software could be configured to efficiently "build" composite schemes. Additionally, the builder tools could also: 1) provide control chart designs which are scaled to achieve a net false alarm rate specification; 2) adjust detection sensitivity according to the risk level; and 3) achieve specified ICATS performance for different levels of management audience. The reduction in the rate of false alarms, for a given detection efficiency, is likely to outweigh the cost of upgrading to composite schemes. It is estimated that PIMS software is now possessed by the majority of manufacturing companies. Hence adoption of a CCUSUM scheme having three or more components is now very plausible, if not feasible. It is concluded that the contributions of this thesis to manufacturing, and perhaps other industries, has potential to be of immediate significant economic value if adopted.

Chapter 8

Recommendations

After this research, a set of additional questions come to mind. Uncertainty remains regarding composite scheme design parameters which are typically appropriate for industry. One also wonders what hierarchical monitoring methodologies would best benefit the manufacturing industry. Specifically, upon commencement of this thesis the following information could not be found in literature:

- What alternative assumptions (assessment domain, sample size for estimating parameters) would best reflect the context in manufacturing? Discussed in Section 8.1.
- What combination of tools and methods would best support monitoring from several levels of management within an organisation? Discussed in Section 8.2.
- What would be the most effective format for presenting CCUSUM3 schemes, whether by graphics or exception reporting? Discussed in Section 8.3.

Additionally, the following opportunities are of interest to the author of this thesis:

- Development of cause identification and correction tools, see Sections 8.4.
- Integration of composite control charts with advanced multivariate, non-parametric and data-mining methods, as discussed in Section 8.5.

Further explanation of these opportunities is offered below.

8.1 Optimisation for Alternative Assumptions

It is not known whether the assumptions applied in this thesis reflect conditions typically found in industry. Besides the question of the assessment domain definition, there is the question of model weightings. A model of the typical frequencies of step shifts and business costs as a function of the step shift size could be built from a random selection of industrial examples. It is recommended that managers, engineers and process operators from continuous manufacturing process plants be interviewed, and plant data examine to answer these questions.

A composite control chart design dataset has been created for future consultancy work, containing full SSATS profiles for various assumptions (see Appendix C for a list of the assumption combinations available). A pre-simulated design dataset is particularly useful for accelerating future studies. The design and performance dataset may be used with frequency and cost parameters weightings to find the optimum composite control chart configurations for a specific industry application (economic design). Economic designs may be optimised quickly without time consuming simulation of SSATS performance.

Further benefit may then arise from expanding the consultancy design dataset. One such opportunity is to expand the sample sizes used to estimate the mean and standard deviation. In particular, the student is interested in heterogeneous sample sizes for estimation of the mean and standard deviation.

8.2 Monitoring Needs within an Organisation

Control charts have traditionally been designed for monitoring from only one level of an organisation, usually at the "factory floor" level. The following question highlights why traditional control chart designs do not suit organisation-high implementation: "What false alarm rate is acceptable at middle management ranks?" Monitoring of many variables from several departments can lead to exceptionally high net false alarm rates. At middle management levels, such a system would be inoperable unless something is done to component out less important events and false alarms.

To develop new control charts to function within a tailored monitoring system, the following organisation needs need to be understood:

- Are periodic reviews of quality and process control preferred at middle management level or a review of alarms as they are generated?
- Which monitoring schemes formats are considered desirable by various levels of management within a typical organisation?
- Does use of common alarm criteria for all monitored variables significantly enhance the effectiveness of a quality system?

These questions could be raised in a survey after trialling variations of hierarchical monitoring methods at a number of manufacturing companies.

8.3 Form of CCUSUM Presentation

It is recommended that the ease of interpreting CCUSUM3 schemes be verified. Visual enhancement of CCUSUM3 schemes may be required to make the tool effective. If the overlapping CUSUM components are confusing, allow the user to choose which CUSUM statistic(s) and corresponding control limits are displayed using graphical user interface buttons. The presence of an alarm from a statistic which is not displayed can be indicated on the line plot by uniquely colouring the raw data according to whatever component(s) are in alarm. A separate real-time alarm list can be used to confirm the details of component(s) in alarm. Alternatively, alarms can be reported in the form of an exception report, avoiding the need for training control room operators to interpret CCUSUM3 graphics. It is recommended that combinations of the above ideas be trialled and developed further with industry participants. The objective being, to maximise the effectiveness of composite control charts.

8.4 Identification and Correction Tools

Statistical and/or rule based diagnostics might be used to analyse root causes and automatically report out-of-control variables and corrective actions with the use of a trouble-shooting database. Electronic automation of these tasks not only reduces the labour intensiveness of managing quality, but also reduces the required experience level of the process engineers and operators who are employed for a given level of effectiveness.

8.5 Composite Control Charts for Multivariate Techniques

Several cooperating multivariate control charts such as principal component scores, multivariate EWMA (MEWMA), multivariate (MCUSUM) or Hotellings T^2 could simultaneously monitor a variable. The software used in this thesis can easily be modified to simulate chi square and other distributions as required. See Lowry and Montgomery (1995) for a review thesis on multivariate control charts.

Appendices

Extra investigations were performed outside of the central theme of the thesis and have been included in the appendices to enhance the flow of the central concepts.

Appendix A - Error Analysis

The objective of this appendix is to determine error bars for simulation results to describe a symmetrical 95% confidence interval surrounding each experimental design lattice point.

Table A-1. CMA3 Error Analysis.

Data

n1	n2	n3	h1	h2	h3	A1%	A2%	A3%	MRL
9	2	1	3.2035	3.1546	3.2011	20.0	40.0	40.0	-0.0491
9	2	1	3.2020	3.1542	3.2004	20.0	39.9	40.0	-0.0510
9	2	1	3.2037	3.1538	3.2004	19.9	40.0	40.0	-0.0496
9	2	1	3.2022	3.1548	3.2015	20.1	39.9	40.0	-0.0507
9	2	1	3.2043	3.1547	3.2017	20.0	40.0	40.0	-0.0490
9	2	1	3.2034	3.1540	3.2012	20.0	40.1	39.9	-0.0497
9	2	1	3.2016	3.1544	3.2015	20.1	40.0	39.9	-0.0493
9	2	1	3.2035	3.1548	3.2010	20.0	40.0	40.0	-0.0494
9	2	1	3.2034	3.1539	3.2006	20.0	40.0	40.1	-0.0499
9	2	1	3.2035	3.1544	3.2010	20.0	40.0	40.0	-0.0490
9	2	1	3.2031	3.1553	3.2013	20.1	39.9	40.0	-0.0504
9	2	1	3.2032	3.1544	3.2013	20.0	40.0	40.0	-0.0501
9	2	1	3.2033	3.1546	3.1999	20.0	40.0	40.0	-0.0499
9	2	1	3.2036	3.1535	3.2003	19.9	40.1	40.0	-0.0510
9	2	1	3.2028	3.1539	3.2005	20.0	40.0	40.0	-0.0511
9	2	1	3.2031	3.1542	3.2009	20.0	40.0	40.0	-0.0480
9	2	1	3.2035	3.1555	3.2014	20.0	39.9	40.0	-0.0497
9	2	1	3.2018	3.1541	3.2013	20.1	40.0	39.9	-0.0512
9	2	1	3.2032	3.1548	3.2019	20.0	40.0	39.9	-0.0488
9	2	1	3.2036	3.1545	3.2012	20.0	40.0	40.0	-0.0474
9	2	1	3.2039	3.1549	3.2011	20.0	40.0	40.0	-0.0493
9	2	1	3.2029	3.1540	3.2009	20.0	40.0	40.0	-0.0494
9	2	1	3.2036	3.1544	3.2010	20.0	40.0	40.0	-0.0504
9	2	1	3.2041	3.1541	3.2008	20.0	40.1	40.0	-0.0488
9	2	1	3.2028	3.1545	3.2005	20.0	39.9	40.1	-0.0488
9	2	1	3.2025	3.1539	3.2006	20.0	40.0	40.0	-0.0504
9	2	1	3.2019	3.1542	3.2004	20.1	39.9	40.0	-0.0502
9	2	1	3.2025	3.1553	3.2017	20.1	39.9	40.0	-0.0507
9	2	1	3.2027	3.1541	3.2001	20.0	40.0	40.0	-0.0509
Average			**3.2031**	**3.1544**	**3.2009**				**-0.0498**
Standard Deviation			**0.00070**	**0.00048**	**0.00051**				**0.00095**

Error Analysis

	S.E	**±1.96*S.E.**
For one observation of the MRL from each design	0.0013	0.0026
For two observations of the MRL from each design	0.0011	0.0021
For three observations of the MRL from each design	0.0009	0.0017

Table A.2 CEWMA3 Error Analysis.

Data

k1	k2	k3	h1	h2	h3	A1%	A2%	A3%	MRL
0.12	0.48	0.93	3.2430	3.1002	3.1738	17.0	45.7	37.4	-0.0945
0.12	0.48	0.93	3.2422	3.1004	3.1740	17.0	45.6	37.3	-0.0946
0.12	0.48	0.93	3.2417	3.0992	3.1728	17.0	45.7	37.3	-0.0967
0.12	0.48	0.93	3.2445	3.1005	3.1742	16.9	45.7	37.4	-0.0943
0.12	0.48	0.93	3.2410	3.0991	3.1739	17.1	45.7	37.3	-0.0971
0.12	0.48	0.93	3.2411	3.0995	3.1728	17.0	45.6	37.4	-0.0949
0.12	0.48	0.93	3.2430	3.1005	3.1736	17.0	45.6	37.4	-0.0949
0.12	0.48	0.93	3.2427	3.1001	3.1747	17.0	45.7	37.3	-0.0958
0.12	0.48	0.93	3.2425	3.0999	3.1738	17.0	45.7	37.4	-0.0957
0.12	0.48	0.93	3.2413	3.1004	3.1751	17.0	45.7	37.3	-0.0947
0.12	0.48	0.93	3.2407	3.0991	3.1735	17.0	45.6	37.3	-0.0964
0.12	0.48	0.93	3.2420	3.0997	3.1731	17.0	45.6	37.4	-0.0964
0.12	0.48	0.93	3.2418	3.1000	3.1742	17.0	45.6	37.3	-0.0966
0.12	0.48	0.93	3.2423	3.0996	3.1738	17.0	45.7	37.3	-0.0948
0.12	0.48	0.93	3.2424	3.0998	3.1738	17.0	45.7	37.4	-0.0962
0.12	0.48	0.93	3.2416	3.0995	3.1728	17.0	45.6	37.4	-0.0955
0.12	0.48	0.93	3.2420	3.1000	3.1735	17.0	45.6	37.4	-0.0954
0.12	0.48	0.93	3.2421	3.0990	3.1731	16.9	45.7	37.3	-0.0949
0.12	0.48	0.93	3.2434	3.1004	3.1754	17.0	45.7	37.3	-0.0941
0.12	0.48	0.93	3.2415	3.0992	3.1739	17.0	45.7	37.3	-0.0950
0.12	0.48	0.93	3.2403	3.0994	3.1725	17.1	45.6	37.3	-0.0945
0.12	0.48	0.93	3.2426	3.1005	3.1741	17.0	45.6	37.4	-0.0944
0.12	0.48	0.93	3.2415	3.0997	3.1739	17.0	45.6	37.3	-0.0962
0.12	0.48	0.93	3.2422	3.1005	3.1739	17.0	45.6	37.4	-0.0941
0.12	0.48	0.93	3.2417	3.0986	3.1737	17.0	45.7	37.3	-0.0958
0.12	0.48	0.93	3.2429	3.0999	3.1734	17.0	45.6	37.4	-0.0964
0.12	0.48	0.93	3.2427	3.1006	3.1740	17.0	45.6	37.4	-0.0952
0.12	0.48	0.93	3.2425	3.1002	3.1740	17.0	45.7	37.4	-0.0953
0.12	0.48	0.93	3.2422	3.1001	3.1737	17.0	45.6	37.4	-0.0951
Average			**3.2421**	**3.0998**	**3.1738**				**-0.0954**
Standard Deviation			**0.00086**	**0.00055**	**0.00065**				**0.00085**

Error Analysis

	S.E	**±1.96*S.E.**
For one observation of the MRL from each design	0.0012	0.0024
For two observations of the MRL from each design	0.0009	0.0017
For three observations of the MRL from each design	0.0007	0.0014

Table A-3 CCUSUM3 Error Analysis.

Data

k1	k2	k3	h1	h2	h3	A1	A2	A3	MRL
0.35	1	1.75	8.7892	2.7224	1.4386	13.0	43.5	43.6	-0.1391
0.35	1	1.75	8.7856	2.7233	1.4395	13.0	43.5	43.5	-0.1388
0.35	1	1.75	8.7843	2.7229	1.4393	13.0	43.5	43.5	-0.1389
0.35	1	1.75	8.7835	2.7231	1.4390	13.0	43.5	43.5	-0.1400
0.35	1	1.75	8.7861	2.7252	1.4395	13.1	43.4	43.5	-0.1393
0.35	1	1.75	8.7804	2.7235	1.4385	13.0	43.4	43.5	-0.1394
0.35	1	1.75	8.7865	2.7226	1.4384	13.0	43.4	43.5	-0.1387
0.35	1	1.75	8.7881	2.7231	1.4386	13.0	43.5	43.5	-0.1398
0.35	1	1.75	8.7785	2.7223	1.4394	13.1	43.5	43.4	-0.1394
0.35	1	1.75	8.7831	2.7227	1.4389	13.0	43.4	43.6	-0.1397
0.35	1	1.75	8.7830	2.7226	1.4389	13.0	43.5	43.5	-0.1398
0.35	1	1.75	8.7888	2.7221	1.4386	12.9	43.6	43.5	-0.1393
0.35	1	1.75	8.7909	2.7228	1.4389	13.0	43.5	43.5	-0.1395
0.35	1	1.75	8.7868	2.7232	1.4390	13.0	43.5	43.5	-0.1388
0.35	1	1.75	8.7878	2.7235	1.4391	13.0	43.5	43.5	-0.1378
0.35	1	1.75	8.7839	2.7223	1.4385	13.0	43.5	43.5	-0.1400
0.35	1	1.75	8.7870	2.7233	1.4390	13.0	43.5	43.5	-0.1386
0.35	1	1.75	8.7860	2.7230	1.4388	13.0	43.5	43.5	-0.1377
0.35	1	1.75	8.7940	2.7228	1.4393	13.0	43.5	43.6	-0.1372
0.35	1	1.75	8.7849	2.7230	1.4390	13.0	43.5	43.5	-0.1381
0.35	1	1.75	8.7871	2.7237	1.4393	13.1	43.5	43.5	-0.1406
0.35	1	1.75	8.7825	2.7222	1.4386	13.0	43.5	43.5	-0.1395
0.35	1	1.75	8.7862	2.7234	1.4392	13.0	43.5	43.5	-0.1399
0.35	1	1.75	8.7880	2.7239	1.4395	13.0	43.4	43.5	-0.1374
0.35	1	1.75	8.7898	2.7245	1.4398	13.0	43.4	43.5	-0.1389
0.35	1	1.75	8.7896	2.7244	1.4397	13.0	43.5	43.5	-0.1393
0.35	1	1.75	8.7858	2.7233	1.4391	13.0	43.5	43.5	-0.1397
0.35	1	1.75	8.7813	2.7219	1.4384	13.0	43.5	43.5	-0.1387
0.35	1	1.75	8.7872	2.7228	1.4383	13.0	43.5	43.6	-0.1405
Average			**8.7861**	**2.7231**	**1.4390**				**-0.1391**
Standard Deviation			**0.00329**	**0.00075**	**0.00041**				**0.000855**

Error Analysis

	S.E	±1.96*S.E.
For one observation of the MRL from each design	0.0012	0.0024
For two observations of the MRL from each design	0.0009	0.0017
For three observations of the MRL from each design	0.0007	0.0014

Appendix B - MA and EWMA ATS Profiles

This appendix contains the tables for MA and EWMA ATS profiles discussed in Chapter 3. Population parameters are assumed to be known and comparisons are made for ICATS = 400. The comparison domain was $\delta_A = 0.5$ to $\delta_B = 4$ for all measures.

Table B-1. ATS profiles and MRLMC comparison of MA control charts for a selection of designs from MA(1) to MA(30).

	Span, n																			
	30		28		12		8		7		5		4		3		2		1	
δ	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC
0.00	400.	0.001	400.	0.002	400.	0.002	400.	0.001	400.	0.002	400.	0.001	400.	0.000	400.	0.001	400.	0.000	400.	0.001
0.25	80.4	**0.000**	82.0	0.020	115.7	0.439	139.6	0.736	148.4	0.845	172.8	1.149	190.4	1.368	213.5	1.655	247.4	2.076	302.	2.755
0.50	27.18	0.005	27.05	**0.000**	32.66	0.208	40.12	0.483	43.57	0.611	54.29	1.007	63.35	1.343	77.57	1.868	103.9	2.841	165.3	5.113
0.75	16.76	0.135	16.37	0.108	14.77	**0.000**	16.60	0.124	17.69	0.198	21.58	0.461	25.28	0.712	31.85	1.157	45.40	2.074	85.51	4.791
1.00	12.63	0.375	12.29	0.338	9.234	0.005	9.184	**0.000**	9.391	0.023	10.72	0.167	12.27	0.336	15.15	0.649	21.88	1.382	45.52	3.957
1.25	10.12	0.646	9.845	0.601	6.968	0.133	6.225	0.012	6.149	**0.000**	6.387	0.039	6.961	0.132	8.214	0.336	11.66	0.896	25.30	3.115
1.50	8.403	0.920	8.177	0.868	5.726	0.308	4.834	0.104	4.626	0.057	4.377	**0.000**	4.491	0.026	5.039	0.151	6.736	0.539	14.67	2.351
1.75	7.166	1.202	6.966	1.141	4.858	0.493	4.029	0.238	3.802	0.168	3.362	0.033	3.254	**0.000**	3.366	0.034	4.188	0.287	8.882	1.730
2.00	6.229	1.533	6.058	1.464	4.214	0.714	3.472	0.412	3.246	0.320	2.771	0.127	2.562	0.042	2.459	**0.000**	2.777	0.129	5.522	1.246
2.25	5.483	1.834	5.329	1.754	3.694	0.909	3.036	0.569	2.841	0.468	2.387	0.234	2.145	0.109	1.935	**0.000**	1.955	0.010	3.558	0.839
2.50	4.890	2.347	4.765	2.261	3.281	1.246	2.690	0.841	2.511	0.719	2.098	0.436	1.857	0.271	1.607	0.100	1.461	**0.000**	2.347	0.606
2.75	4.402	2.805	4.285	2.704	2.936	1.538	2.400	1.074	2.246	0.941	1.864	0.611	1.638	0.416	1.385	0.197	1.157	**0.000**	1.546	0.336
3.00	4.000	3.162	3.886	3.044	2.657	1.765	2.161	1.249	2.008	1.089	1.666	0.734	1.464	0.523	1.216	0.265	0.961	**0.000**	1.039	0.081
3.25	3.652	4.255	3.541	4.095	2.412	2.471	1.959	1.819	1.825	1.626	1.503	1.163	1.310	0.885	1.086	0.563	0.833	0.199	0.695	**0.000**
3.50	3.362	6.261	3.255	6.030	2.207	3.767	1.786	2.857	1.657	2.579	1.363	1.944	1.182	1.553	0.973	1.102	0.734	0.585	0.463	**0.000**
3.75	3.102	9.104	3.011	8.808	2.025	5.596	1.630	4.309	1.511	3.922	1.235	3.023	1.07	2.485	0.879	1.863	0.650	1.117	0.307	**0.000**
4.00	2.872	13.579	2.790	13.162	1.867	8.477	1.500	6.614	1.384	6.025	1.127	4.721	0.971	3.929	0.796	3.041	0.575	1.919	0.197	**0.000**
MRLMC		3.211		3.092		1.842		1.381		1.250		0.980		0.851		**0.755**		0.799		1.611

Table B-2. ATS profiles and MRLMC comparison of EWMA control charts for a selection of designs from EWMA(0.02) to EWMA(1), page 1/2.

	λ																	
	0.02		0.055		0.1		0.13		0.2		0.25		0.3		0.35		0.4	
δ	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC
0.00	400.	0.001	400.	0.001	400.	0.000	400	0.001	400.	0.002	400.	0.001	400.	0.001	400.	0.001	400.	0.002
0.25	66.27	**0.000**	75.19	0.135	91.34	0.378	102.2	0.541	126.5	0.909	142.3	1.146	157.3	1.373	171.4	1.586	184.2	1.780
0.50	27.50	0.078	25.50	**0.000**	27.50	0.078	29.61	0.161	36.07	0.415	41.44	0.625	47.33	0.856	53.86	1.112	60.31	1.365
0.75	16.93	0.234	14.17	0.033	13.73	**0.000**	14.01	0.021	15.62	0.138	17.22	0.254	19.27	0.404	21.60	0.573	24.24	0.766
1.00	12.09	0.410	9.585	0.118	8.711	0.016	8.574	**0.000**	8.806	0.027	9.282	0.083	9.987	0.165	10.88	0.269	11.94	0.393
1.25	9.319	0.603	7.141	0.229	6.241	0.074	5.972	0.028	5.812	**0.000**	5.912	0.017	6.129	0.055	6.479	0.115	6.922	0.191
1.50	7.557	0.816	5.638	0.355	4.792	0.151	4.512	0.084	4.210	0.012	4.162	**0.000**	4.200	0.009	4.313	0.036	4.499	0.081
1.75	6.328	1.047	4.628	0.497	3.846	0.244	3.576	0.157	3.238	0.048	3.125	0.011	3.091	**0.000**	3.102	0.004	3.154	0.020
2.00	5.414	1.316	3.894	0.666	3.182	0.361	2.922	0.250	2.580	0.104	2.452	0.049	2.375	0.016	2.338	**0.000**	2.342	0.002
2.25	4.717	1.631	3.344	0.865	2.690	0.500	2.451	0.367	2.116	0.180	1.981	0.105	1.888	0.053	1.836	0.024	1.802	0.005
2.50	4.161	2.000	2.914	1.101	2.314	0.668	2.088	0.505	1.773	0.278	1.637	0.180	1.539	0.110	1.476	0.064	1.428	0.030
2.75	3.715	2.508	2.573	1.430	2.018	0.906	1.808	0.707	1.506	0.422	1.375	0.298	1.277	0.206	1.207	0.140	1.153	0.089
3.00	3.345	3.176	2.291	1.860	1.778	1.220	1.577	0.969	1.297	0.619	1.170	0.461	1.072	0.338	0.998	0.246	0.940	0.174
3.25	3.031	4.155	2.055	2.495	1.578	1.684	1.393	1.369	1.125	0.913	1.000	0.701	0.907	0.543	0.838	0.425	0.779	0.325
3.50	2.769	5.640	1.856	3.451	1.413	2.388	1.239	1.971	0.979	1.348	0.866	1.077	0.775	0.859	0.704	0.688	0.640	0.535
3.75	2.543	7.861	1.691	4.892	1.268	3.418	1.108	2.861	0.867	2.021	0.753	1.624	0.663	1.310	0.590	1.056	0.527	0.836
4.00	2.348	11.423	1.540	7.148	1.146	5.063	0.991	4.243	0.764	3.042	0.656	2.471	0.566	1.995	0.493	1.608	0.430	1.275
MRLMC	2.860		1.676		1.118		0.913		0.638		0.530		0.461		0.424		0.406	

Table B-2. ATS profiles and MRLMC comparison of EWMA control charts for a selection of designs from EWMA(0.02) to EWMA(1), page 2/2.

	λ																	
	0.45		0.5		0.6		0.64		0.8		0.85		0.9		0.95		Shewhart	
δ	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC	ATS	RLMC
0.00	400.	0.000	400.	0.002	400.	0.001	400.	0.001	400.	0.002	400.	0.001	400.	0.001	400.	0.001	400.	0.001
0.25	197.6	1.981	208.5	2.146	231.	2.491	239.	2.610	270.	3.070	279.	3.207	286.	3.326	294.	3.437	302.	3.556
0.50	67.22	1.636	74.55	1.923	90.25	2.539	96.8	2.796	125.4	3.918	135.2	4.302	145.3	4.698	154.6	5.062	165.2	5.478
0.75	27.08	0.973	30.46	1.219	38.06	1.773	41.54	2.026	57.94	3.221	64.15	3.674	70.72	4.152	77.85	4.672	85.63	5.239
1.00	13.20	0.539	14.71	0.715	18.24	1.127	19.95	1.327	28.74	2.352	32.36	2.774	36.21	3.223	40.57	3.732	45.48	4.305
1.25	7.475	0.286	8.158	0.404	9.867	0.698	10.77	0.853	15.41	1.651	17.45	2.002	19.67	2.385	22.29	2.836	25.32	3.357
1.50	4.74	0.139	5.048	0.213	5.906	0.419	6.332	0.521	8.879	1.133	10.01	1.406	11.33	1.721	12.90	2.099	14.69	2.529
1.75	3.249	0.051	3.389	0.096	3.823	0.237	4.045	0.309	5.466	0.768	6.094	0.972	6.840	1.213	7.745	1.506	8.868	1.869
2.00	2.37	0.014	2.426	0.038	2.627	0.124	2.753	0.178	3.505	0.499	3.870	0.655	4.319	0.847	4.872	1.084	5.545	1.372
2.25	1.793	**0.000**	1.804	0.006	1.898	0.059	1.938	0.081	2.356	0.314	2.575	0.436	2.833	0.580	3.158	0.761	3.559	0.985
2.50	1.402	0.011	1.387	**0.000**	1.405	0.013	1.428	0.030	1.632	0.177	1.755	0.265	1.895	0.366	2.089	0.506	2.328	0.678
2.75	1.115	0.053	1.086	0.025	1.059	**0.000**	1.064	0.005	1.159	0.094	1.219	0.151	1.299	0.227	1.409	0.331	1.544	0.458
3.00	0.899	0.122	0.859	0.072	0.811	0.012	0.801	**0.000**	0.830	0.036	0.852	0.064	0.896	0.119	0.963	0.202	1.036	0.293
3.25	0.727	0.236	0.683	0.162	0.630	0.071	0.614	0.044	0.588	**0.000**	0.602	0.024	0.618	0.051	0.655	0.114	0.692	0.177
3.50	0.587	0.408	0.547	0.312	0.478	0.146	0.460	0.103	0.421	0.010	0.417	**0.000**	0.424	0.017	0.442	0.060	0.462	0.108
3.75	0.474	0.652	0.428	0.491	0.363	0.265	0.344	0.199	0.294	0.024	0.288	0.003	0.287	**0.000**	0.295	0.028	0.305	0.063
4.00	0.378	1.000	0.334	0.767	0.268	0.418	0.247	0.307	0.199	0.053	0.192	0.016	0.189	**0.000**	0.191	0.011	0.194	0.026
MRLMC	0.408		0.430		0.527		0.585		0.950		1.116		1.307		1.534		1.796	

Appendix C - Composite Scheme Design Dataset

The designs dataset generated to date contain the quantity of designs as listed below in Table C-1. These designs are not available publicly but are intended for use in consultancy work.

Table C-1. Scope of composite control chart design and performance dataset.

dICATS	dICBoundary	dNcomponents	dNestim	dEstimMthd	#Designs
CCUSUM					
1200	0	3	200	SD	136
1200	0	3	100	SD	12
400	0	3	200	MR	2486
400	0	3	200	SD	1161
400	0	3	100	SD	786
400	0	2	200	MR	397
400	0.25	3	200	SD	1099
400	0.25	3	200	MR	1209
200	0	3	200	MR	1025
200	0	3	100	MR	1076
CEWMA					
400	0	3	200	MR	929
400	0	2	200	MR	220
CMA					
400	0	3	200	MR	347
Total					10883

Appendix D - Validation of the Software

The raw data for validation tests of the software are described in this appendix.

The thesis software was validated against ARL profiles published by authors such as Sparks (2000, 2003), Quesenberry (1993), and Lucas and Saccucci (1990). All validation is based on steady-state simulation as this is the only form of simulation for which the software is currently configured. All functions of the code relating to simulation and determination of run length performance for each of the three statistics were validated via the following simulation runs:

- EWMA on step shift with known parameters
- CMA on step shifts with known parameters
- CMA on ramp shift with known parameters
- CCUSUM with known parameters
- Shewhart chart based on estimated parameters

Results were generally within 1% with some minor exceptions. Descriptive statistics were also used to confirm that the data was distributed normally. In the following validation results tables, the relative difference between the results from the thesis software, and that of the published data are calculated:

$$\text{Rel Diff} = \frac{(\text{Thesis Result-Other Result})}{\text{Thesis Result}}$$

Validation results are shown below in Tables D-1 to D-5.

Table D-1. Validation of EWMA ATS on known parameters by comparsion against Lucas and Saccucci (1990).

δ	ATS	Rel diff
0.00	482.279	-0.008
0.25	80.187	-0.006
0.50	26.873	-0.005
0.75	15.028	0.002
1.00	10.163	-0.004
1.50	6.026	-0.001
2.00	4.134	-0.011
2.50	3.138	-0.001
3.00	2.468	-0.005
3.50	1.999	-0.011
4.00	1.690	0.006

Table D-2. Validation of CMA step and ramp ATSs on known parameters by comparsion against Plan 5 (Sparks 2003).

δ	ATS	Rel diff
0.00	397.697	0.001
0.25	123.073	0.001
0.50	33.738	0.004
0.75	14.717	-0.002
1.00	8.672	-0.005
1.50	4.456	0.001
2.00	2.777	0.000
2.50	1.802	0.008
3.00	1.200	-0.001
3.50	0.888	-0.018
4.00	0.733	-0.003

κ	ATS	Rel diff
0.000	396.295	-0.007
0.005	82.757	0.003
0.010	53.766	0.002
0.025	29.728	0.023
0.050	18.918	-0.004
0.075	14.593	0.001
0.100	12.153	0.002
0.200	7.623	-0.006
0.250	6.613	-0.003

Table D-3. Validation of two sided CUSUM ATSs on known parameters by comparsion against Sparks (2000).

δ	ATS	Rel diff
0.00	393.254	0.005
0.25	124.015	0.012
0.50	33.549	-0.006
0.75	14.447	-0.003
1.00	8.359	0.002
1.50	4.143	0.003
2.00	2.545	-0.006
2.50	1.766	-0.014
3.00	1.310	-0.008

Table D-4. Validation of estimated parameters results for individuals Shewhart Chart ATSs with n_{estim} = 100, by comparsion against Quesenberry (1993).

δ	ATS	Rel diff
0.00	586.100	0.023
1.00	58.762	-0.001
2.00	6.270	0.018
3.00	1.101	0.001
4.00	0.205	0.026
5.00	0.020	-0.080

Table D-5. Validation of normally distributed data created using the C++ class StochasticLib class (Fog, 2003) which is based on the Mersenne Twister algorithm.

Descriptive Statistics

	N	Mean		Std.	Skewness		Kurtosis	
	Statistic	Statistic	Std. Error	Statistic	Statistic	Std. Error	Statistic	Std. Error
V1	999960	-.0008	.00100	1.00160	-.007	.002	.074	.005
Valid N (listwise)	999960							

Comments on the Descriptive Statistics

- Slight Kurtosis is noted.
- Standard deviation is approximately equal to zero.
- Mean is well within two standard errors distance from zero.

Decision

Accept as valid pseudo normal random number generation as mean and standard deviation are as specified and the skewness and kurtosis are acceptable.

Appendix E - Software Details

E.1 Seeking to Specification Block Diagram

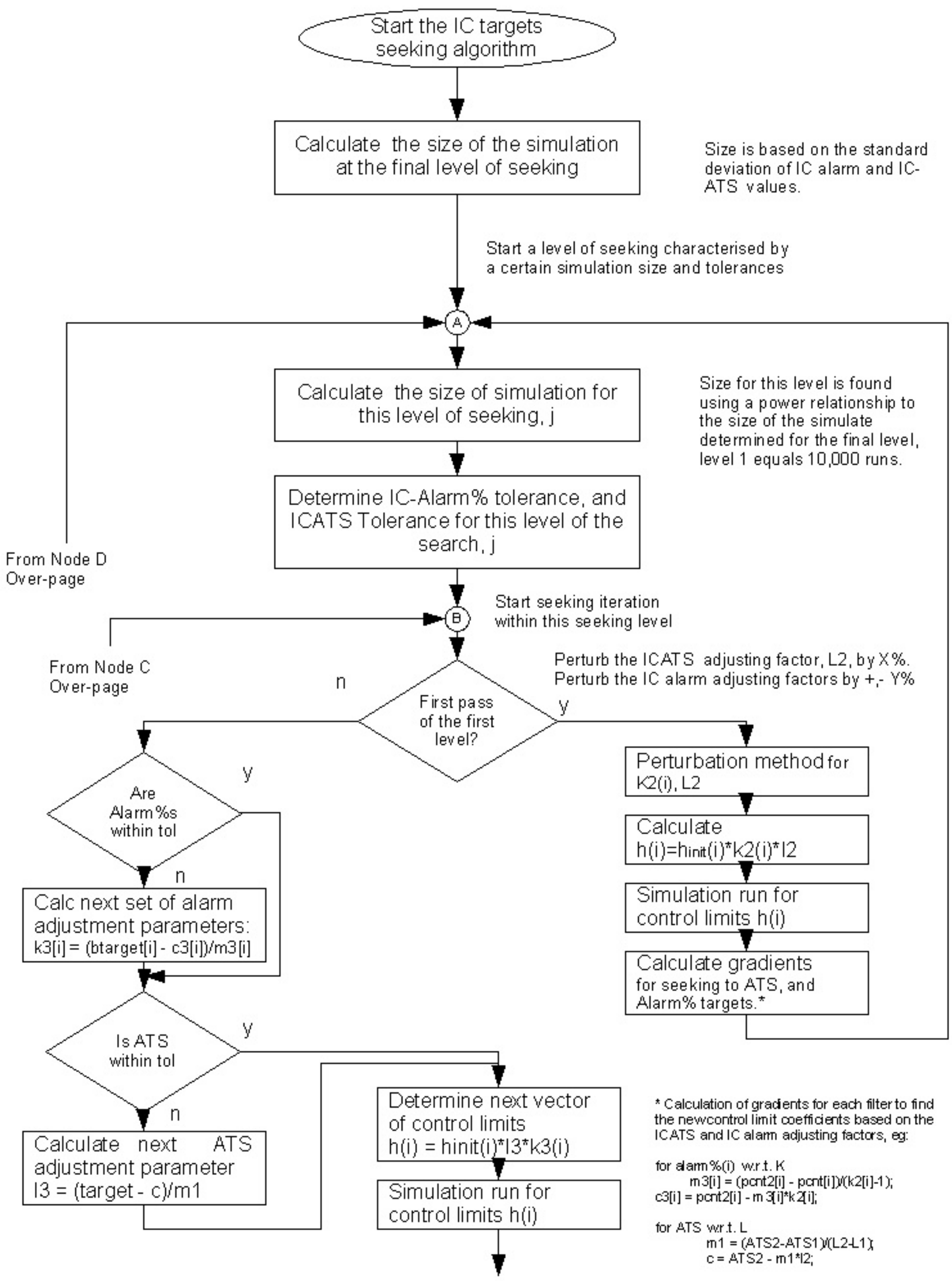

Figure E-1. Specification seeking algorithm. Page 1 of 2.

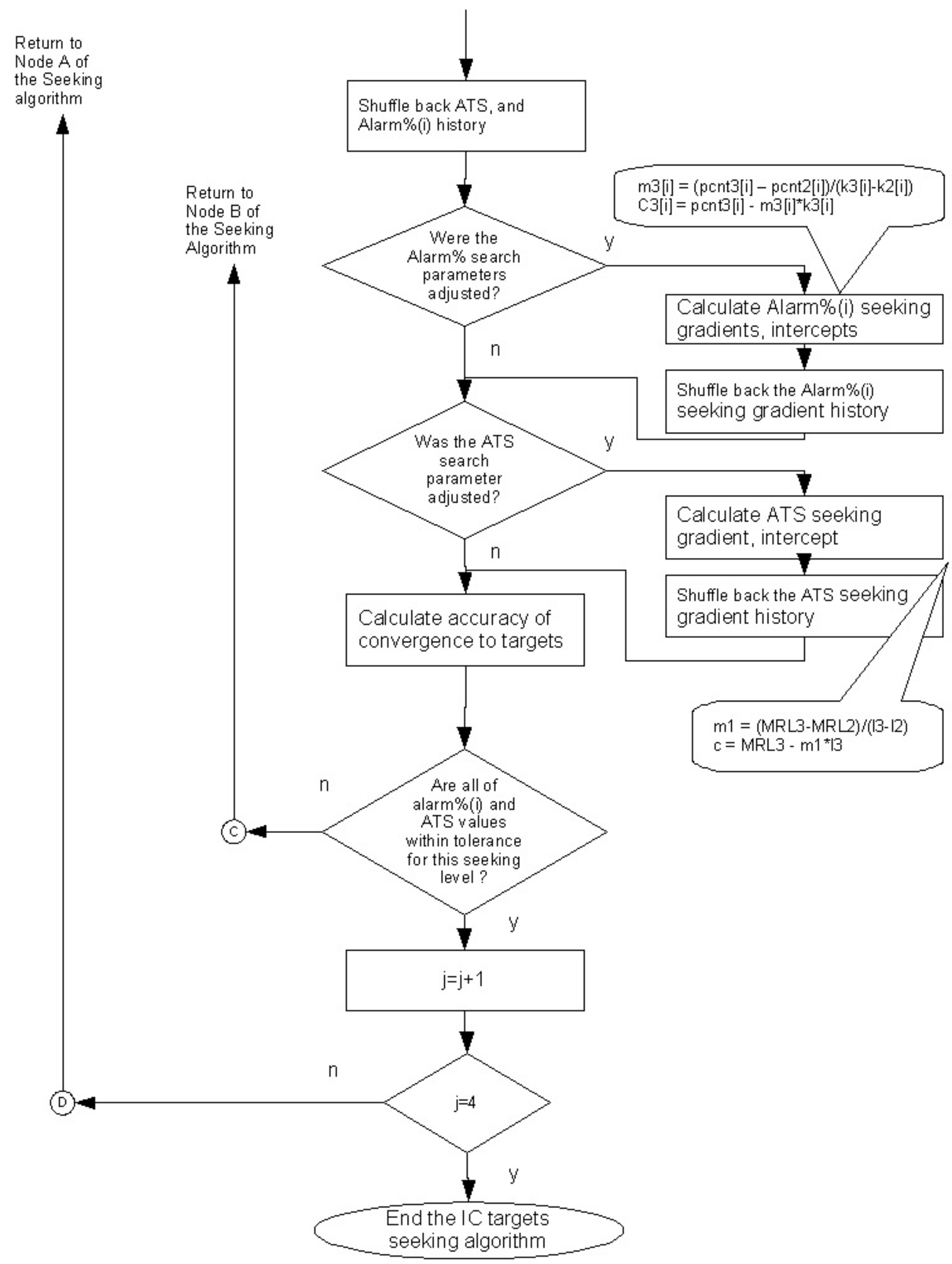

Figure E-1. Specification seeking algorithm. Page 2 of 2.

E.2 C++ Class Inheritance Structure

The inheritance structure and the specification seeking flow sheet are contained in this appendix. The graphical user interface class creates an instance of class Tune upon the "Simulate" mouse Onclick event. Tune draws on member variables and member functions of the parent classes stdev and ATS. Simulator is the highest level in the inheritance structure created in this thesis. Simulator includes StochasticLib and MS Visual C^{++} Windows utility classes. The algorithms for seeking to ICATS targets; indexing through ramp and step location shifts; indexing through experimental design lattices; validation; comparison and preparation of MRL data output, are performed from the member functions of Tune. stdev is used to calculate the standard deviations of ICATS and component loading values "IC Alarm Contributions [%]". The standard deviations are used to size the simulations sufficiently to achieve convergence with user tolerance specifications. The class inheritance structure is shown in block diagram in Figure E-2.

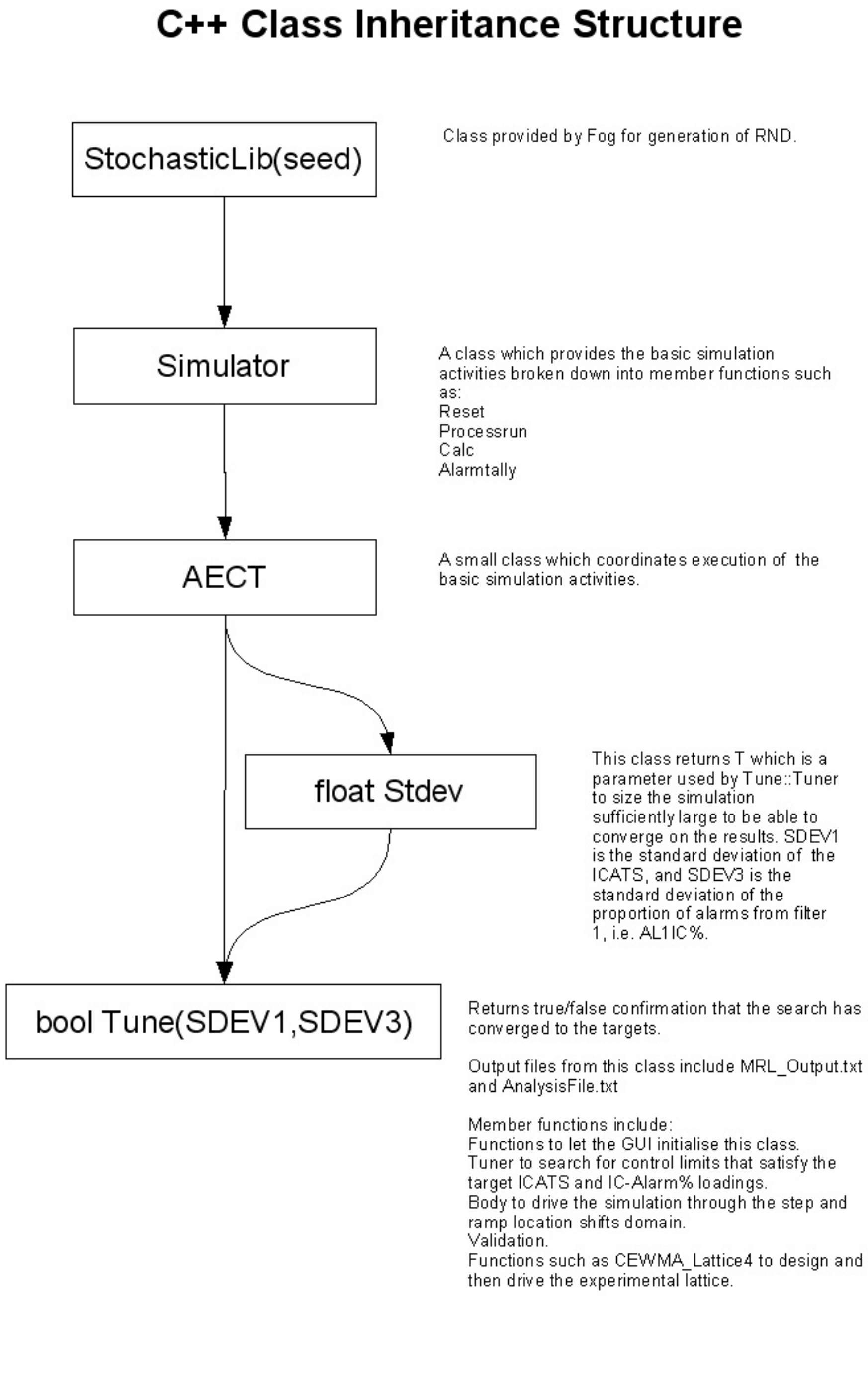

Figure E-2. C++ class inheritance structure.

Appendix F - Comparison of Designs for Different Assessment Domains

The objective of this appendix is to understand the performance of an optimal three-component CCUSUM scheme in the context of un-optimised schemes published in literature. A three-component CCUSUM design published by Sparks (2000) was used for comparison. Sparks' design used nearly equal loadings on the three components, between 30% and 35% each. Unfortunately, the design by Sparks has an ICATS = 422 when parameters are estimated from 200 observations using the absolute moving range formula, 5% larger than the optimized CCUSUM3 scheme which was designed for ICATS = 400. A new CCUSUM3 design, CCUSUM3a (parameters given in Table F-1) was simulated with the same k_j values and loadings as the optimum design, but for a specification of ICATS=422. Strictly speaking, CCUSUM3a has not been optimised, but it is unlikely to be too dissimilar from the optimum configuration.

Table F-1. Comparison of Sparks' CCUSUM3 scheme relative to the optimum CCUSUM3 scheme, CCUSUM3a.

	Sparks	CCUSUM3a
ADRA	21.5%	
k_1	0.375	0.35
k_2	0.5	1
k_3	0.75	1.8
h_1	6.6758	8.8981
h_2	5.2851	2.754
h_3	3.6848	1.3993
Al_1IC [%]	34.19	12.95
$Al_2IC:Al_3IC$	0.867	1
Al_2IC[%]	30.57	43.46
Al_3IC [%]	35.24	43.59

Examining the ATS profiles in Figure F-1, Sparks' design was up to 17% faster (DRA) for step shifts from 0.5 σ to 1.75 σ, but up to 170% slower at larger step shifts of 4σ. Overall, CCUSUM3a was 21.5% faster (ADRA) relative to Sparks' plan.

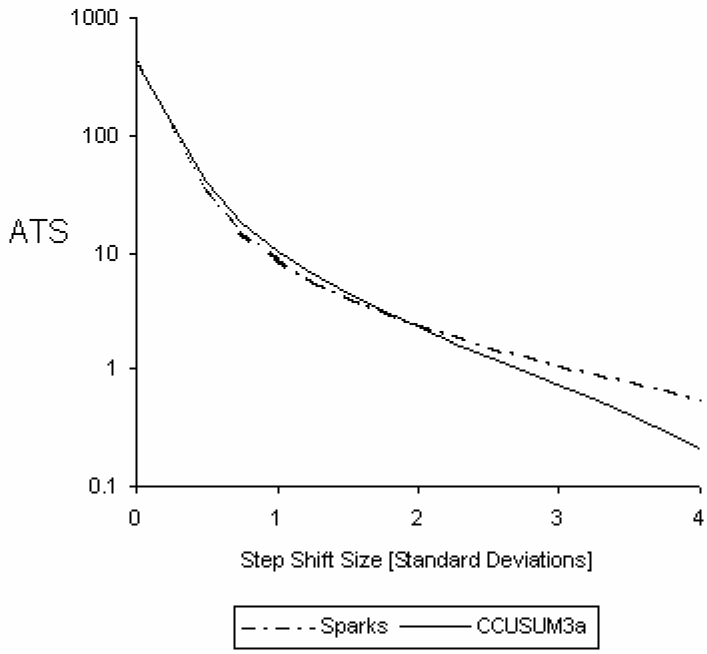

Figure F-1. ATS profiles for CCUSUM3a and a CCUSUM3 scheme by Sparks (2000).

It should be acknowledged that Sparks' plan was designed for a domain of 0.75σ to 1.5σ, and not 0.5σ to 4.0σ as was our CCUSUM3 design, so it is hardly a fair comparison. The comparison does demonstrate, however, how sensitive scheme performance is to the selection of reference values and loadings, and to the assessment domain. A comparison of Sparks' plan relative to the optimised CCUSUM3 scheme (ICATS = 400, described in Chapter 6) scheme was also conducted and it is noted that the findings were similar values despite the differing ICATS specifications.

Appendix G - Four-Component CEWMA Designs for Various ICATS Targets

The objective of this appendix is to provide a set of CEWMA designs for range of ICATS specifications. From the designs, users may assess how sensitive ICATS responses are to changes in control limit coefficients. CEWMA4 designs based on ***known*** parameters with ICATS = 100 to 1000 and their ATS profiles are shown below in Table G-1. The ATS profiles of individual components are also investigate in this appendix.

Table G-1. Designs for CEWMA4A schemes with ICATS = 100 to 1000, for known parameters.

		ICATS Specifications				
		100	200	400	700	1000
Parameters		Control Limit Coefficients, h_j				
λ_1	0.055	2.4092	2.7081	2.9849	3.1893	3.3115
λ_2	0.3	2.7464	2.9969	3.2286	3.4006	3.5065
λ_3	0.55	2.8351	3.0683	3.2841	3.4481	3.5485
λ_4	1.0	2.8940	3.1177	3.3259	3.4845	3.5824
	δ	ATS				
	0.00	99.8	199.8	400.2	700.1	999.8
	0.25	45.1	69.3	104.8	145.18	178.6
	0.50	18.68	24.62	31.49	37.71	42.14
	0.75	10.24	12.92	15.83	18.20	19.79
	1.00	6.429	8.010	9.694	11.09	11.96
	1.25	4.341	5.383	6.482	7.408	8.004
	1.50	3.076	3.793	4.567	5.196	5.609
	1.75	2.240	2.757	3.308	3.766	4.069
	2.00	1.658	2.059	2.470	2.812	3.038
	2.25	1.246	1.554	1.880	2.138	2.314
	2.50	0.934	1.178	1.439	1.647	1.787
	2.75	0.698	0.900	1.106	1.273	1.393
	3.00	0.516	0.675	0.840	0.983	1.077
	3.25	0.374	0.500	0.637	0.753	0.832
	3.50	0.264	0.361	0.475	0.572	0.631
	3.75	0.182	0.260	0.346	0.424	0.476
	4.00	0.120	0.177	0.243	0.305	0.349

In Table G-1, loadings for all designs were specified such that: Al_1IC = 20%, Al_2IC = 27.5%, Al_3IC = 27.5%, Al_4IC = 25%. These loadings have not been optimized. Rather, the designs reflect the low loadings on fine and coarse components mimicking the configuration of the optimum CEWMA3 design, as determined in Chapter 6.

References

Albin, S. L., Kang, L. and Shea, G. (1997). "An X and EWMA Chart for Individual Observations." Journal of Quality Technology **29**(1): 41-48.

Amin, R. W. and Ethridge, R. A. (1998). "A note on individual and moving range control charts." Journal of Quality Technology **30**(1): 70.

Aparisi, F. and García-Díaz, J. C. (2007). "Design and optimization of EWMA control charts for in-control, indifference, and out-of-control regions." Computers & Operations Research **34**(7): 2096.

Bischak, D. P. and Trietsch, D. (2007). "The Rate of False Signals in ... Control Charts with Estimated Limits." Journal of Quality Technology **39**(1): 54.

Black, P. E. (2004). *secant search, in Dictionary of Algorithms and Data Structures [online]*. Paul E Black, ed. U.S. National Institute of Standards and Technology.17 December 2004 (accessed 17 April 2007). Available from http://www.nist.gov/dads/HTML/secantsearch.html

Borror, C. M., Montgomery, D. C. and Runger, G. C. (1999). "Robustness of the EWMA chart to non-normality." Journal of Quality Technology **31**: 309-316.

Braun, W. J. (2003). "The LR-chart: An alternative to the MR-chart." Quality Engineering **16**(1): 87.

Capizzi, G. and Masarotto, G. (2003). "An adaptive exponentially weighted moving average control chart." Technometrics **45**(3): 199.

Carlyle, W. M., Montgomery, D. C. and Runger, G. C. (2000). "Optimization problems and methods in quality control and improvement." Journal of Quality Technology **32**(1): 1.

Chen, Y.-S. and Yang, Y.-M. (2002). "An extension of Banerjee and Rahim's model for economic design of moving average control chart for a continuous flow process." European Journal of Operational Research(143): 600-610.

Crowder, S. V. (1989). "Design of Exponentially Weighted Moving Average Schemes." Journal of Quality Technology **21**(3): 155.

Das, T. K. and Jain, V. (1997). "An economic design model for X charts with random sampling policies." IIE Transactions **29**(6): 507.

Das, T. K., Jain, V. and Gosavi, A. (1997). "Economic design of dual-sampling-interval policies for X charts with and without run rules." IIE Transactions **29**(6): 497.

Davis, R. B. and Woodall, W. H. (1988). "Performance of the Control Chart Trend Rule Under Linear Shift." Journal of Quality Technology **20**(4): 260-262.

Duncan, A. J. (1956). The economic design of the xbar chart used to maintain current control of a process. Journal of the American Association**:** 228-242.

Fog, A. (2001).*Non-Uniform Random Number Generators (stocc.h and stoc1.cpp).* Agner Fog, 2001-2007 (accessed 1 July 2003). Available from http://www.agner.org/random/stocc.htm

Gan, F. F. (1992). "Exact run length distributions for one-sided CUSUM schemes." Statistica Sinica **2**: 297-312.

Gan, F. F. (1993). "An optimal design of EWMA control charts based on median run length." Journal of Statistical Computation and Simulation **45**: 169-184.

Hawkins, D. M., Qiu, P. and Kang, C. W. (2003). "The change point model for statistical process control." Journal of Quality Technology **35**(4): 355-366.

Honeywell Pty. Ltd. (2004). Experion PKS, Honeywell Pty Ltd.(Accessed Jan 2004). Available from http://hpserb.honeywell.com/cultures/en-US/Products/Systems/ExperionPKS/default.htm

Hubele, N. F. and Chang, S. I. (1990). "Adaptive Exponentially Weighted Moving Average Schemes Using a Kalman Component." IIE Transactions **22**(4): 361.

Hunter, J. S. (1986). "The Exponentially Weighted Moving Average." Journal of Quality Technology **18**(4): 203-210.

ISO (1989). ISO 697:1989 Methods of testing cements - Determination of strength. Geneva, International Organization for Standards.

Jiang, W., Wu, H., Tsung, F., Nair, V. N. and Tsui, K.-L. (2002). "Proportional Integral Derivative Charts for Process Monitoring." Technometrics **44**(3): 205.

Jones, L. A. (2002). "The statistical design of EWMA control charts with estimated parameters." Journal of Quality Technology **34**(3): 277-288.

Jones, L. A., Champ, C. W. and Rigdon, S. E. (2001). "The performance of exponentially weighted moving average charts with estimated parameters." Technometrics **43**(2): 156-166.

Jones, L. A. and Woodall, W. H. (1998). "The performance of bootstrap control charts." Journal of Quality Technology **30**(4): 362.

Khoo, M. B. C. and Ariffin, K. N. b. (2006). "Two Improved Runs Rules for the Shewhart X Control Chart." Quality Engineering **1**(18): 173-178.

Klein, M. (1996). "Composite Shewhart-EWMA statistical control schemes." IIE Transactions **28**: 475-481.

Klein, M. (1997). "Modified Shewhart-exponentially weighted moving average control charts." IIE Transactions **29**: 1051-1056.

Koning, A. J. and Does, R. J. M. M. (2000). "CUSUM charts for preliminary analysis of individual observations." Journal of Quality Technology **32**(2): 122-132.

Lowry, C. A. and Montgomery, D. C. (1995). "A review of multivariate control charts." IIE Transactions **27**(6): 800-810.

Lu, C. W. and Reynolds, M. R. (1999). "EWMA Control Charts for Monitoring the Mean of Autocorrelated Processes." Journal of Quality Technology **31**: 166-188.

Lucas, J. M. and Saccucci, M. S. (1990). "Exponentially weighted moving average control schemes: properties and enhancements with discussion." Technometrics **32**: 1-29.

Luceno, A. and Puig-Pey, J. (2002). "Computing the run length probability distribution for CUSUM charts." Journal of Quality Technology **34**(2): 209-214.

MacGregor, J. F. and Harris, T. J. (1993). "The exponentially weighted moving variance." Journal of Quality Technology **25**(2): 106.

MacNaughton, D. J. and Coomans, D. (2009). "Design and Optimization Aids for Composite Control Charts." Quality Engineering **21**: 33-43.

Marcellus, R. (2006). "On Statistical Process Control Design." The Quality Management Journal **13**(1): 43-57.

Matricon (2004). Process Doctor, Matricon Pty. Ltd.(Accessed Jan 2004). Available from http://www.matriconopc.com

Model, F., Konig, T., Piepenbrock, C. and Adorjan, P. (2002). "Statistical process control for large scale microarray experiments." Bioinformatics **18**(1): 155.

Montgomery, D. C. and Woodall, W. H. (1997). "A discussion on statistically-based process monitoring and control." Journal of Quality Technology **29**(2): 121.

Nelson, L. S. (1982). "Control Charts for Individual Measurements." Journal of Quality Technology **14**(3): 172-173.

Nelson, L. S. (2003). "Is your process being adjusted too frequently?" Journal of Quality Technology **35**(1): 115-116.

Ohta, H. R., M.A. (1997). A dynamic economic model for an xbar control chart design. IIE Transactions**:** 481-486.

Page, E. S. (1954). "Continuous Inspection Schemes." Biometrika **41**: 100-115.

Pan, X. and Jarrett, J. (2007). "Using vector autoregressive residuals to monitor multivariate processes in the presence of serial correlation." International Journal of Production Economics **106**(1): 204.

Quesenberry, C. P. (1991). "SPC Q Charts for Start-Up Processes and Short or Long Runs." Journal of Quality Technology **23**: 231-224.

Quesenberry, C. P. (1993). "The effect of sample size on estimated limits for Xbar and X control charts." Journal of Quality Technology **25**: 237-247.

Reynolds, M. R. and Stoumbos, Z. G. (2004). "Control Charts and the Efficient Allocation of Sampling Resources." Technometrics **46**(2): 200-214.

Rhoads, T. R., Montgomery, D. C. and Mastrangelo, C. M. (1996). "A Fast Initial Response Scheme for the Exponentially Weighted Moving Average Control Chart." Quality Engineering **9**: 317-327.

Ridley, D. and Duke, D. (2007). "Moving -window spectral model based statistical process control." International Journal of Production Economics **105**(2): 492.

Roberts, S. W. (1959). "Control charts based on geometric moving averages." Technometrics **42**(1): 97-101.

Robinson, P. B. and Ho, T. Y. (1978). "Average Run Lengths of Geometric Moving Average Charts by Numerical Methods." Technometrics **20**(1): 85-93.

Robinson, S. (2007). "A statistical process control approach to selecting a warm-up period for a discrete-event simulation." European Journal of Operational Research **176**(1): 332.

Shewhart, W. E. (1931). Economic Control of Quality Manufactured Product. NY, Van Nostrand, New York.

Six-Sigma-First (2007). X-Chart [online], Six-Sigma First.

Sparks, R. (2000). "CUSUM Charts for Signalling Various Location Shifts." Journal of Quality Technology **32**(2): 157-171.

Sparks, R. (2003). "A group of moving averages control plan for signaling varying location shifts." Quality Engineering **14**(4): 519-532.

Sparks, R. (2004). "A group of weighted moving averages: An efficient plan for monitoring a range of process changes." International Journal of Production Research **42**(13): 2627-2638.

Steiner, S. H. (1999). "EWMA Control Charts with Time-Varying Control Limits and Fast Initial Response." Journal of Quality Technology **31**(1): 75-86.

Torng, J. C.-C., Cochran, J. K. and Montgomery, D. C. (1995). "Implementing statistically constrained economic EWMA control charts." Journal of Quality Technology [H.W. Wilson - AST] **27**: 257.

Turner, C. D. J., Sullivan, J. H. and Batson, R. G. (2001). "A program for retrospective change-point analysis of individual observations." Journal of Quality Technology [H.W. Wilson - AST] **33**(2): 242.

Veevers, A. (1998). "Viability and capability indexes for multiple response processes." Journal of Applied Statistics **25**(4): 545-558.

Vommi, V., Murty and Seetala, S. N. (2007). "A simple approach for robust economic design of control charts." Computers & Operations Research **34**(7).

Walpole, R. E. and Myres, R. H. (1989). Probability and Statistics for Engineers and Scentists. NY, Macmillan.

Wang, F. K., Hubele, N. F., Lawrence, F. P., Miskulin, J. D. and Shahriari, H. (2000). "Comparison of three multivariate process capability indices." Journal of Quality Technology **32**(3): 263.

Wardell, D. G, Moskowitz, H, and Plante, R D. "Control Charts in the Presence of Data Correlation." Management Science. 38(8), pp. 1084-1105.

Woodall, W. H. (1985). "The statistical design of quality control charts." The Statistician **34**: 155-160.

Woodall, W. H., Hoerl, R. W., Palm, A. C. and Wheeler, D. J. (2000). "Controversies and contradictions in statistical process control." Journal of Quality Technology **32**(4): 341.

Woodall, W. H. and Montgomery, D. C. (1999). "Research issues and ideas in statistical process control." Journal of Quality Technology **31**(4): 376-386.

Wortham, A. W., Heinrich, G. F. and Taylor, D. (1974). "Adaptive exponentially smoothed control charts." International Journal of Production Research **12**: 683-690.

Wu, J. C. F. and Hamada, M. (2000). Experiments - Planning, Analysis, Parameter Design Optimisation. NY, John Wiley & Sons, Inc.

Wu, Z. and Tian, Y. (2005). "Weighted-loss-function CUSUM chart for monitoring mean and variance of a production process." International Journal of Production Research **43**(14): 3027-3044.

Wu, Z. and Wang, Q. (2007). "A single CUSUM chart using a single observation to monitor a variable." International Journal of Production Research **45**(3): 719–741.

Wu, Z. and Wang, Q. (2007). "A single CUSUM chart using a single observation to monitor a variable." International Journal of Production Research **45**(3): 719-741.

Zamba, K. D. and Hawkins, D. M. (2006). "A multivariate change point model for statistical process control." Technometrics **48**(4): 539-549.

Zhang, S. and Wu, Z. (2006). "Monitoring the process mean and variance using a weighted loss function CUSUM scheme with variable sampling intervals." IIE Transactions **38**: 377-387.

Zou, C., Zhang, Y. and Wang, Z. (2006). "A control chart based on a change-point model for monitoring linear profiles." IIE Transactions **38**(12): 1093.