Strain Influence Factors for Footings on an Elastic Medium

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ABSTRACT

Shallow foundations in granular soils are designed such that the settlements are within tolerable limits. Schmertmann's (1970) method is one of the most rational methods for computing settlements of footings in granular soils, and is commonly used world-wide. The method relies on a strain influence factor that varies with depth. Schmertmann et al. (1978) proposed separate strain influence factors for axi-symmetric and plane strain loading situations, representing circular and strip footings. The objective of this paper is to revisit Schmertmann's influence factors using FLAC and FLAC^{3D}, explicit finite difference codes used widely in geotechnical modelling, and the theory of elasticity. Linear elastic and non-linear elastic constitutive models were used in the analysis. The influence factors derived are compared with those proposed by Schmertmann. For square and rectangular footings, the problem becomes three-dimensional and therefore FLAC^{3D} was used in the analysis. The strain influence factors are developed for footings with breadth/length ratios of 0.25, 0.50, 0.75 and 1.0. The strain influence factors for the rectangular footings are presented along with those for the circular and strip footings. The effect of Poisson's ratio was also investigated. While verifying the original strain influence factors, the new factors proposed for the rectangular footings will be valuable in the design of shallow foundations on granular soils. The use of non-linear elastic constitutive model is more realistic than the traditional linear elastic model, and the differences are discussed.

Keywords: shallow foundation, settlement, granular soil, strain influence factor

1 INTRODUCTION

In the design of shallow foundations, two major criterions are taken into consideration- bearing capacity and settlement. If the foundation is resting on granular soil, settlement is believed to be more critical than bearing capacity in most cases. Usually, an acceptable limit of 25 mm settlement is maintained in the design of shallow footings. In case of cohesionless soil, it is hard to get undisturbed soil sample which creates difficulty in determining compressibility of the soil mass. As a result, a large number of settlement prediction methods are available in the literature for footings on granular soil, much more than cohesive soils. Schmertmann (1970) proposed a settlement prediction method which is based on cone penetration test results and relies on strain influence factor which is a function of depth. This method is widely used by geotechnical engineers all over the world for its simplicity and reliability. Burland and Burbridge (1985) proposed a semi-empirical method for settlement calculation which is being more commonly used recently.

The concept of strain influence factor is straightforward and simple. If a uniform pressure *q* is applied over a large area on an elastic half space, the resulting strain at any depth *z* becomes q/E_z. If the load is applied over a limited width *B*, the resulting strain at a depth *z* along the centreline will obviously be less and can be expressed as:

$$\varepsilon_z = \frac{q}{E_z} I_z \qquad (1)$$

where, E_Z = elastic modulus at depth *z*

I_Z = influence factor at depth *z*

The strain influence factor can be used to determine the vertical settlement *s* of shallow footing resting on granular soil by:

$$s = C_1 C_2 q_{net} \sum_{2B}^{0} \frac{I_z d_z}{E_z} \qquad (2)$$

where, C_1 = embedment depth correction factor

C_2 = time correction factor

After careful observation of theoretical and experimental results, Schmertmann (1970) proposed a simplified 2*B*-0.6 diagram as shown in Figure 1(a). This shows that the influence factor is 0 at the foundation level, increases linearly to peak at 0.6 at a depth of 0.5 *B*, and then decreases linearly to 0 at a depth of 2*B*.

To account for the effect of foundation shape on settlement, Schmertmann et al.(1978) modified the 2*B*-0.6 diagram as shown in Figure 1(b). For square and circular footing, the value of I_Z at the footing level is 0.1; it reaches its peak at a depth of 0.5*B* and reduces to zero at 2*B*. In case of strip footing, I_Z value is 0.2 at foundation level, peaks at *z=B* and becomes zero at *z=4B*. The influence factor diagram for a rectangular foundation can be obtained by interpolating between these two. The peak value of influence factor can be calculated by:

$$I_{z,peak} = 0.5 + 0.1\sqrt{\frac{q_{net}}{\sigma'_{vo}}} \qquad (3)$$

where, q_{net} is the net applied pressure and σ'_{v0} is the overburden pressure at the depth where peak occurs.

Terzaghi et al. (1996) suggested a simpler influence factor diagram as shown in Figure 1(c) . They proposed I_Z = 0.2 at footing level and peak of 0.6 at 0.5*B* depth for all footings. The depth of influence (Z_I) was kept same as Schmertmann et al. (1978) for circular and strip footing but for rectangular footing, it should be interpolated by:

$$Z_I = 2B[1 + \log(\frac{L}{B})] for L/B \le 10 \qquad (4)$$

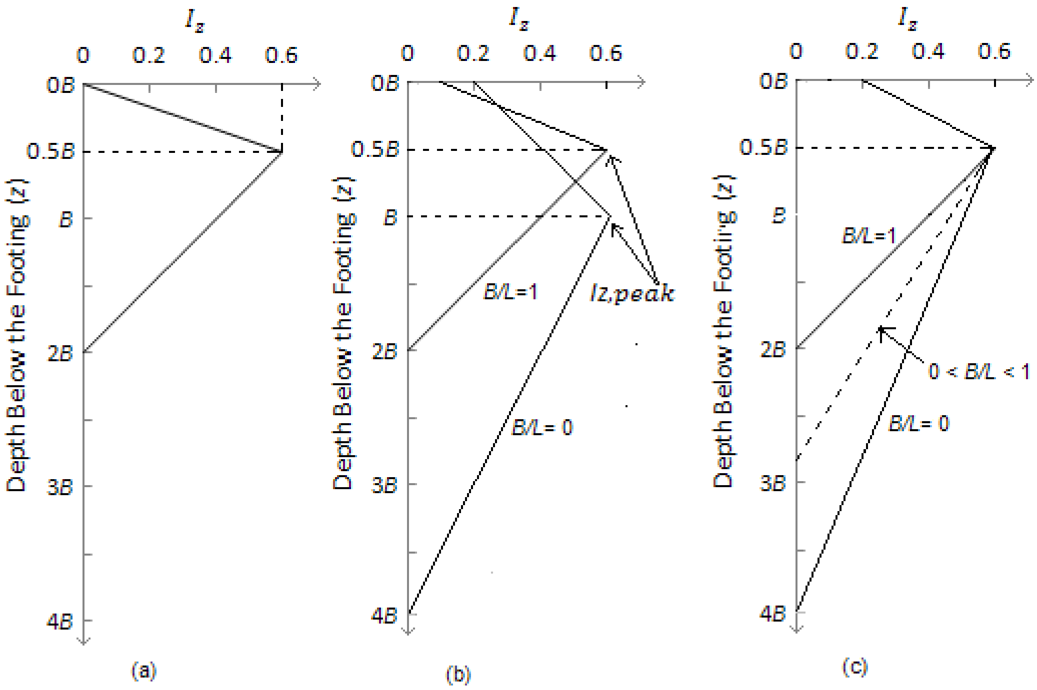

Figure 1. Strain influence factor diagrams- a) Schmertmann (1970), b) Schmertmann et at. (1978), c) Terzaghi et al. (1996) (adapted after Sivakugan and Das 2010)

Mayne and Poulos (1999) proposed a spreadsheet integration technique to obtain the strain influence factor at various depths to calculate the foundation settlement. This technique can be used in settlement calculation on homogeneous to non-homogeneous soils having finite to infinite soil layer thicknesses.

Despite the popularity of Schmertmann's strain influence factor method, it is very conservative (Sivakugan et al. 1998) and lacks accuracy (Tan and Duncan 1991). So, there is plenty of scope to work further on the influence factor diagrams, thus improving the settlement prediction method. In this paper, the authors used linear and non-linear elastic models in FLAC^{3D} and FLAC to derive strain influence factors for all regular foundation shapes. Also, the depth to maximum strain, depth of influence and the effect of Poison's ratio was investigated.

2 STRAIN INFLUENCE FACTORS ON AN ELASTIC MEDIUM

2.1 Linear elastic analysis

The authors used explicit finite difference codes FLAC and FLAC^{3D} and the elastic theory to revisit Schmertmann's strain influence factors. Influence factors for all regularly shaped footings (including circular, square, rectangular and strip footings) were derived using linear elastic model. FLAC was used to model axi-symmetric and plane strain loading conditions and FLAC^{3D} to model square and rectangular cases. The strain influence factors were developed for footings with breadth/length ratios of 0.25, 0.5, 0.75 and 1.0. The modelling was done keeping the horizontal and rectangular boundaries 6.0 *B* away from the centreline of the footing and the footing width was fixed at 1.0 m. The elastic modulus was taken as 30 MPa and Poisson's ratio (*v*) was fixed at 0.2 for all cases. The footings were placed on ground surface and a uniform pressure of 100 kPa was applied.

Using FLAC and FLAC^{3D}, vertical and horizontal stresses were obtained at various depths along the centreline below the footing, which were then used to calculate the vertical strain using the constitutive relationship of Hooke's Law:

$$\varepsilon_z = \frac{1}{E_z}[\sigma_z - \nu(\sigma_x + \sigma_y)] \tag{5}$$

where, ε_z and E_z are the vertical normal strain and elastic modulus respectively at a depth *z* below the centreline of the footing, and $\sigma_x, \sigma_y, \sigma_z$ are the stresses along *x*, *y* and *z* directions.

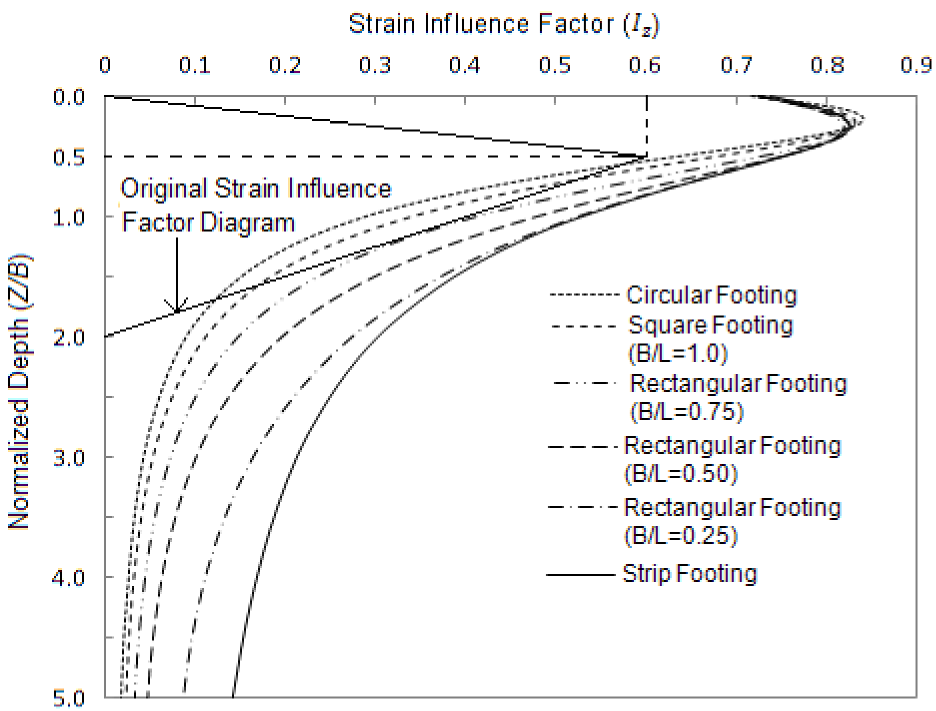

Figure 2. Strain influence factor diagrams obtained from linear elastic analysis

The influence factor was then obtained by:

$$I_z = \frac{\varepsilon_z E_z}{q} \tag{6}$$

Figure 2 shows the strain influence factor diagrams for footings of various shapes obtained from linear elastic analysis. The diagrams show some variations when compared to Schmertmann's (1970) originally proposed influence factor diagram as shown in the figure. Unlike the original diagram, the influence factors range between 0.72-0.74 at the base of the footing, peak at around 0.83 at a depth of 0.2-0.25 and extend to a greater depth. The strain influence factors extend to depth well below 4*B* proposed by Schmertmann for strip footing, and the factors are significantly larger. For rectangular footings also, there are noticeable strains at depths of z = 2*B* to 4*B*, and beyond. The shape of the strain influence factor plot was very similar for all footing shapes. The strain influence factor diagrams obtained from linear elastic analysis do not vary with Young's modulus, but their value changes with Poisson's ratio which is discussed in the next section.

2.2 Effect of Poisson's ratio

There are some difficulties involved in laboratory triaxial testing (for example, capping problems, seating errors, non-uniformity of stress etc.) which result in higher Poisson's ratio value, ranging from 0.25-0.45 (Lo Presti1995). Nowadays, these can be avoided by mounting local strain devices at midlevel of soil specimen and measuring strain internally (Tatsuoka and Shibuya 1992). Tatsuoka et al. (1994) showed that the drained value of Poisson's ratio for elastic continuum solutions ranges from 0.1 to 0.2 in sands. Therefore, the authors derived the strain influence factors for v = 0.1 and 0.2 using linear elastic model in FLAC. Figure 3 shows the effect of Poisson's ratio on the strain influence factors in circular and strip footings.

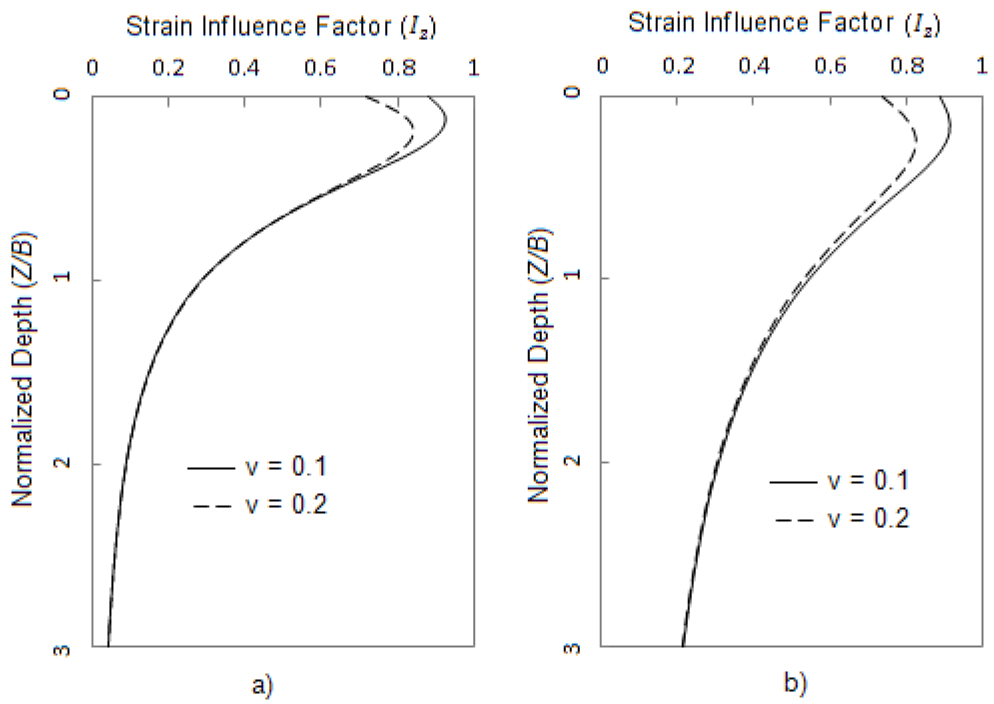

Figure 3. Effect of Poisson's ratio on strain influence factor diagrams- a) circular footing, b) strip footing

2.3 Non-linear elastic analysis

Hyperbolic nonlinear elastic soil model in FLAC was used to investigate the variation of vertical strain with depth. The nonlinear elastic soil model is based on the hyperbolic stress-strain relationship proposed by Kondner and Zelaska (1963):

$$(\sigma_1 - \sigma_3) = \frac{\varepsilon}{\frac{1}{E_i} + \frac{\varepsilon}{(\sigma_1 - \sigma_3)_{\max}}} \quad (7)$$

where: $(\sigma_1 - \sigma_3)_{\max}$ = asymptotic value of stress difference

ε = axial strain

E_i = initial tangent modulus

Figure 4 shows the vertical strain distribution below the centreline of a circular footing resting on the surface of a homogeneous granular soil. Three different loading conditions were considered- 0.5, 1.0 and 1.5 times the working load of the soil (one-sixth, one-third and half of the bearing capacity of the soil). The result show that the depth of maximum vertical strain occurs at a depth of 0.3 *B* below the footing for all cases. This is little higher than what was obtained from linear elastic modelling (0.2 *B*) but less than Schmertmann's (1970) simple triangular approximation (0.5 *B*). Figure 4 shows that $I_{z,peak}$ occurs at 0.3*B* in nonlinear elastic analysis.

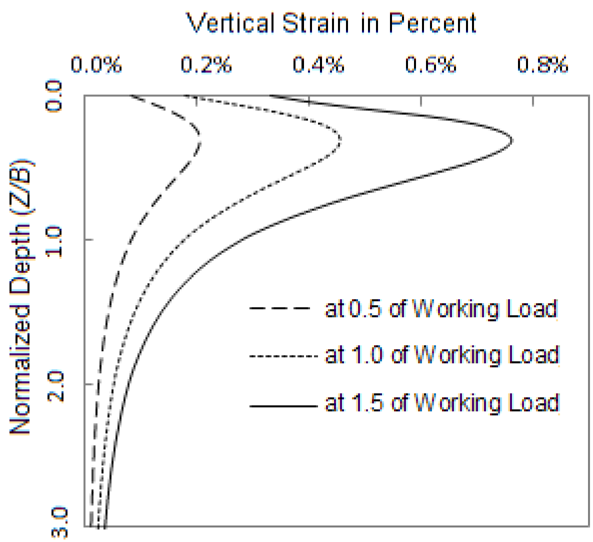

Figure 4. Vertical strain at different loading conditions in nonlinear elastic analysis

3 CONCLUSION

Strain influence factor diagrams for footings of various shapes (strip, circular, square, rectangular) were presented using linear elastic models in FLAC and $FLAC^{3D}$. The diagrams were then compared with Schmertmann's (1970) simple triangular approximation. Unlike the original strain influence diagram, the proposed diagrams start at 0.72-0.74 at footing level, rises up to 0.83 at 0.2*B*-0.25*B* depth and extend to a greater depth. Effect of Poisson's ratio on the diagrams was discussed and presented graphically for circular and strip footings. Also a simple hyperbolic nonlinear model was used to investigate the depth at which maximum vertical strain occurs. The result shows that the peak occurs at 0.3*B* (whereas $I_{z,peak}$ occurs at 0.2*B* in linear elastic modelling) for axi-symmetric loading at any stress level.

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