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# **An Experimental Investigation into the Effect of Interface Friction on Bagasse Compaction between Grooved Steel Platens**

Thesis submitted by

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BE (Mech.), Universidad Nacional de Trujillo, Perú

in May 2005

for the degree of Master of Engineering Science  
in the School of Engineering (Mechanical Engineering)  
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# Abstract

Modern factory crushing units process prepared sugar cane through sets of counter-rotating grooved rolls. A typical unit in Australia would process in excess of 600 tonnes of material per hour. Throughput and extraction performance is strongly dependent on material behaviour, the geometry and surface condition at the roll bagasse interface. Factories use welding procedures to arc roughen the tips of grooves in an effort to increase friction. Although industry procedures appear ad hoc it is clear that some level of roughness is crucial to performance. A similar statement can be made in respect to roll grooving given the wide variation in adopted practice. This project involved an experimental investigation into the effects of interface friction on bagasse compaction between grooved steel platens. An apparatus was developed for use in the SOE MTS testing facility. A factorial design experiment involving 105 tests randomised in blocks was conducted to discover the interaction between friction (the dependent variable) and groove angle, compaction, and roughness (independent variables). The results indicate that roughness, groove angle and compaction significantly affect friction coefficient. While roughness and groove angle contribute to increase friction coefficient, compaction causes a marked decrease. Observations on samples of bagasse exhibiting pure shear suggest that the frictional forces generated at the interface cannot be sustained by the shear strength of bagasse. Comparisons between friction coefficient and shear coefficient showed that the friction coefficient values approach the shear coefficient values under particular geometric and loading conditions. An empirical model was developed to explore variables. The effect of groove angle, degree of roughness (location and size of roughened asperity) and sample compaction on friction has been ascertained.

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# Symbols

$A$	Cross-sectional area
$A_a$	Apparent area
$A_t$	true area
$C$	Variable compaction
$C_c$	Compression ratio
$C_f$	Filling ratio
$D$	Roller diameter
$D_m$	Mean Diameter
$E$	Elastic modules
$F$	Force
$F_c$	Compressive force, yield function in cap region
$F_n$	Normal force
$FR$	filling ratio
$F_s$	Yield function in Drucker-Prager shear region
$F_t$	Tangential force, yield function in transition region
$G$	Variable groove angle
$G_b$	Bagasse shear modules
$H$	Height of bagasse penetration
$K$	Bulk modulus
$K_o$	Plasticity constant
$L$	Length
$M$	Mass
$N$	Reacting normal force
$N_s$	Number of responses
$N_x, N_y$	Reacting normal force components
$R$	Variable roughness, experimental parameter
$R_a$	Average roughness
$R_q$	Root mean square
$R_x, R_y$	Reacting forces components

$T$	Time
$V_f$	Volume of fibre
$V_j$	Volume of juice
$V_o$	Initial no gas volume of cane
$W$	Gravitational force
$c$	Cohesion coefficient
$c_w$	Cohesion coefficient on the wall
$d$	material cohesion
$d_{\tau'}$	Shear stress differential
$d_{\gamma}$	Shear strain differential
$d_{\sigma'}$	Normal stress differential
$d_{\varepsilon_v}$	Volumetric strain differential
$d\tau'$	Differential of shear strain
$d\sigma'$	Differential of normal stress
$\dot{\varepsilon}$	Shear strain of liquid film
$f$	Fibre fraction
$f_a$	Friction force due to adhesion
$f_f$	Frictional force
$g$	Acceleration due to gravity
$h$	Height of asperity
$h_d$	Hardness
$h_m$	Height of bagasse blanket
$i$	initial condition
$k$	Number of levels
$m_f$	Final mass of prepared cane or bagasse
$m_i$	Initial mass of prepared cane or bagasse
$p$	Hydrostatic pressure
$q$	Von Mises equivalent stress
$s_t$	Tangential speed
$v$	Peripheral speed
$w$	Work opening
$\alpha$	Angle of nip, cap transition parameter

$\alpha_a$	Asperity angle
$\alpha_i$	Initial contact angle
$\hat{a}$	Treatment number
$\beta$	Neutral plane angle, internal friction coefficient
$\lambda$	Specific density
$\gamma$	Compaction
$\gamma_{bss}$	Bagasse shear strain
$\gamma_{zx}$	Engineer's strain
$\dot{\gamma}_f$	Liquid film shear strain
$\delta$	Displacement differential
$\varepsilon_e$	Elastic strain
$\varepsilon_p$	Plastic strain
$\varepsilon_v$	Volumetric strain
$\varepsilon_{xz}, \varepsilon_{zx}$	Pure shear strain
$\zeta$	Asperity tip radius
$\eta$	Angle of an inclined wall
$\theta$	Groove angle
$\theta_s$	Shear plane angle
$\mu$	Friction coefficient
$\mu_{pp}$	Porous pressure
$\mu_s$	Static friction
$\mu_w$	Friction coefficient on the wall
$v$	Specific volume
$\pi$	Dimensional product
$\rho_c$	Density of cane
$\rho_f$	Density of fibre
$\rho_j$	Density of juice
$\rho_o$	No gas density of prepared cane
$\sigma$	Total normal stress

$\sigma'$	Normal effective stress
$\sigma_Y$	Yield stress
$\sigma_x, \sigma_y, \sigma_z$	Normal stress components
$\sigma_1, \sigma_2, \sigma_3$	Normal stress components
$\sigma_{sd}$	Standard deviation of asperities heights
$\tau, \tau'$	Shear stress
$\tau_1, \tau_2, \tau_3$	Shear stress components
$\tau_w$	Shear stress on the wall
$\tau_{xy}, \tau_{yx}$	Internal shear stress
$\phi$	Angle of friction
$\phi_w$	Angle of wall
$\phi'$	Effective angle of internal friction
$\psi$	Plasticity index.