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# Transient natural convection in a differentially heated cavity with and without a fin on the sidewall

**Thesis submitted by**
**Feng XU, B.Sc., M.Sc.**
**in September 2006**

**for the degree of Doctor of Philosophy**
**in the School of Engineering**
**James Cook University**

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# Acknowledgements

Many people were involved in this thesis, and I gratefully appreciate their assistance here.

First of all, the help from my two supervisors, Prof. John Patterson and Dr. Chengwang Lei, is gigantic and is gratefully appreciated. In our regular weekly meetings, many good suggestions from them enabled me to take many shortcuts in my study, and many critical questions also made my thinking more logical. I have no doubt that these discussions will benefit my future research as well. Furthermore, marvelous ideas in many publications by them and co-authors enlightened me on my studies. Their direct and indirect help is acknowledged in depth.

I am also grateful to the technical staff in the workshop. In particular, Mr. Curt Arrosmith, Mr. Stuart Petersen and Mr. David Kauppila helped me enormously with my experiments.

It is a pleasure to thank Prof. Steve Armfield for kindly discussing some interesting results in my thesis.

I really appreciate the help from Ms. Helen Shorthouse who arranged and did many things for me when I attended those conferences.

Many thanks also go to my office mates: Mr. Suvash Saha and Dr. Li Shu. They gave me considerable assistance in many aspects when I quite desired.

I am pleased to thank the Australian government, James Cook University and the Australian Research Council, who sponsored all of my tuition and living cost as well as research cost during my study.

Finally, my thanks also go to my wife and daughter whose support allowed me to pay almost all of my attention to my study.

# Abstract

Transient natural convection in a differentially heated cavity is a typical model of many flow processes in the ocean and atmosphere, and has extensive applications in industrial systems for which it is of significance to enhance or depress the heat transfer through the cavity. Therefore, transient natural convection with and without a fin on the sidewall of a differentially heated cavity is experimentally and numerically investigated in this thesis. One of the main objectives of this study is to investigate the effectiveness of the fin for the purpose of enhancing the heat transfer through the heated sidewall.

The shadowgraph technique is employed to visualize transient natural convection in a rectangular cavity, and thermistors are used to measure the time series of the temperatures at different locations adjacent to the hot sidewall. The numerical simulations in this thesis focus on the above experimental model. The SIMPLE scheme based on the finite volume method is adopted to solve the two dimensional governing equations using the commercial FLUENT software package.

The present flow visualizations demonstrate that the transition of the natural convection flow in the cavity from the start-up may be classified into three stages: an initial stage, a transitional stage and a quasi-steady stage. The initial stage includes the growth of the thermal boundary layer and the Leading Edge Effect. The correlation of the thickness of the thermal boundary layer with time obtained from the present experiments is consistent with that of a previous scaling analysis. In the transitional stage, the separation and trailing waves of the horizontal intrusion are observed and eventually a double-layer structure of the thermal boundary layer is visualized in shadowgraph images. The traveling waves of the inner layer in shadowgraph images become clear in the quasi-steady stage, and temperature measurements indicate that this is because the amplitude of the traveling waves in the thermal boundary layer increases as the stratification of the fluid in the cavity is enforced.

The corresponding numerical simulations validated by the experiments show that the bright strips of the double-layer structure in shadowgraph images correspond to the extrema of the second derivative of the temperature. In fact, the stratification of the fluid in the cavity results in a temperature distribution adjacent to the sidewall with a maximum and a minimum of the second derivative of the temperature, which in turn leads to an opposing thermal diffusion directing to the temperature minimum.

The flow visualizations on transient natural convection with a fin on the sidewall of the cavity show that the transition from the start-up is likewise classified into three stages with variations from those without the fin, which also depend on the dimension of the fin. Two fins of different geometric parameters, one small square fin and the other large thin fin, are considered in the experiments. In the initial stage, a lower intrusion front appears underneath the fin and later bypasses the fin. The lower intrusion front bypassing the small square fin reattaches to the downstream thermal boundary layer, but the one bypassing the large thin fin moves upwards and strikes the intrusion under the ceiling. Although there are some variations of the transition of the thermal boundary layer flows between the cases with and without a fin, the double-layer structure of the thermal boundary layer appears in all cases in the quasi-steady stage. In particular, the presence of the large thin fin may cause the separation and vortex shedding of the thermal flow around the fin, which trigger the instability of the downstream thermal boundary layer.

The numerical simulations of transient natural convection with a fin on the sidewall of the cavity are consistent with the above-mentioned experimental results. The numerical results confirm that the transient strong convection flows such as the lower intrusion front and the oscillations of the thermal flow around the fin may enhance the heat transfer through the sidewall. Furthermore, the numerical results demonstrate that the enhancement of the heat transfer is dependent on the parameters of the fin, such as the dimension and position of the fin and the number of fins, with a maximum enhancement of 33% observed in the present simulations.

In summary, both numerical and experimental results show that the fin may significantly change the transient natural convection flows and improve the heat transfer through the cavity if properly configured. This research has great potential for industrial applications in which the enhancement of heat transfer is desirable.

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# Nomenclature

| | |
|---|---|
| $A$ | aspect ratio of the cavity |
| $a$, $b$ | coefficients of the discretized equation (3.25) |
| $\vec{A}_e$ | surface area vector enclosing the control volume |
| $c_p$ | specific heat capacity at certain pressure |
| $D$ | depth of the cavity (in the $z$-direction) |
| $f$ | frequency of oscillations |
| $F(\phi)$ | function of $\phi$ |
| $F_i$ | body force |
| $F_x$, $F_y$, $F_z$ | components of body force (in the $x$-, $y$- and $z$-directions) |
| $Gr$ | Grashof number |
| $g$ | acceleration due to gravity |
| $H$ | height of the cavity |
| $h$ | heat transfer coefficient of the sidewall |
| $H_{fin}$ | heat transfer rate of the hot sidewall with a fin |
| $H_{nofin}$ | heat transfer rate of the hot sidewall without a fin |
| $\hat{H}$ | normalized difference of the heat transfer rate of the hot sidewall between the cases with and without a fin |
| $I_0$ | illumination intensity entering the object |
| $I_e$ | light intensity exiting the object |
| $I_s$ | light intensity on the screen |
| $k$ | thermal conductivity |
| $L$ | width of the cavity |
| $l$ | distance between the object and screen |
| $n$ | refractive index |
| $Nu$ | Nusselt number |
| $Nu_{cen}$ | Nusselt number on the vertical centerline of the cavity |
| $p$ | pressure |
| $Pr$ | Prandtl number |

| | |
|---|---|
| $q'$ | local heat flux of the sidewall |
| $Ra$ | Rayleigh number |
| $R_\phi$ | residual of $\phi$ |
| $S$ | temperature gradient in the center of the cavity |
| $s_0$, $s_1$ | series number of the control volume |
| $S_\phi$ | sources of $\phi$ in the general transport equation (3.15) |
| $T$ | temperature |
| $t$ | time |
| $T_0$ | temperature at the initial time |
| $T_4$, $T_5$ | temperature readings of Thermistors 4 and 5 |
| $T_c$ | temperature of the cold sidewall |
| $T_h$ | temperature of the hot sidewall |
| $u$, $v$ | velocity components |
| $V$ | volume of the control volume (or area of the control surface for 2D) |
| $\vec{v}$ | velocity vector |
| $v_{max}$ | maximum of $y$-direction velocity in the thermal boundary layer |
| $x$, $y$, $z$ | coordinates |
| $x_{ex}$, $y_{ex}$, $z_{ex}$ | coordinates of the light ray exiting the object |
| $x_{in}$, $y_{in}$, $z_{in}$ | coordinates of the light ray entering the object |
| $x_s$, $y_s$ | coordinates of the light ray on the screen |

Greek symbol

| | |
|---|---|
| $\alpha_r$ | under-relaxation factor |
| $\beta$ | thermal expansion coefficient |
| $\Delta I$ | variation of the light intensity |
| $\Delta\vec{s}$ | displacement vector from the upstream surface centroid to the edge |
| $\Delta T$ | temperature difference between hot and cold sidewalls |
| $\Delta t$ | time step |
| $\Delta_x$ | displacement in the $x$-direction |
| $\Delta_y$ | displacement in the $y$-direction |
| $\Delta\phi$ | variation of $\phi$ |
| $\delta_T$ | thickness of the thermal boundary layer |
| $\phi$ | quantity in the general transport equation (3.15) |

| | |
|---|---|
| $\bar{\phi}_e$ | average of $\phi$ on the edges |
| $\phi_{old}$ | value of $\phi$ in the previous iteration |
| $\Gamma_\phi$ | diffusion coefficient for $\phi$ in the general transport equation (3.15) |
| $\kappa$ | thermal diffusivity |
| $\mu$ | dynamitic viscosity |
| $\nu$ | kinematic viscosity |
| $\rho$ | density |
| $\rho_0$ | density at the initial time |
| $\tau$ | time from the start-up to the steady state |
| $nb$ | number of edges enclosing the control volume |

Subscript:

| | |
|---|---|
| $e$ | edge of the control volume |
| $i, j$ | tensor indices |
| $n$ | normal to the edge |
| $nb$ | number of edges enclosing the control volume |

Superscript:

| | |
|---|---|
| $n$ | time level |