Mathematics in the Middle:
Shaping the Proficiency Footprint

Thesis submitted by

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Declaration on Ethics

The research presented and reported in this thesis was conducted within the guidelines for research ethics outlined in the *National Statement on Ethics Conduct in Research Involving Humans* (1999), the *Joint NHMRC/AVCC Statement and Guidelines on Research Practice* (1997), the *James Cook University Policy on Experimentation Ethics*. *Standard Practices and Guidelines* (2001), and the *James Cook University Statement and Guidelines on Research Practice* (2001). The proposed research methodology received clearance from the James Cook University Experimentation Ethics Review Committee (approval number H3358).

(Silvia Dimarco)	(Date)

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Born Gordona, Italy, 11-11-1935
Died Babinda, Australia, 14-12-1997

Una buona mamma vale cento maestre.

Abstract

There is an undercurrent of unease seeping across Australia about sustaining a critical mass of human capital with the mathematical proficiency to successfully take up careers in mathematics, science, technology and engineering (McPhan, Morony, Pegg, Cooksey & Lynch, 2008). A key aspect of this challenge is the lack of academic rigour in the middle school mathematics classroom and the resulting impact on students' mathematical proficiency and mathematical dispositions to successfully pursue higher level mathematics courses (Prosser, 2006). It could be argued that the middle school years are the cornerstone in the provision of the mathematical proficiency that empowers students to pursue higher level mathematics courses since mathematical experiences in the middle school underpin the strategic decisions students make when considering further education involving mathematics. This research accepted the challenge of recent research (Carrington, 2002; Department of Education, Employment and Workplace Relations (DEEWR), 2008; McPhan et al., 2008; Prosser, 2006) to further explore how teachers can deepen intellectual engagement in the middle school context. The formation of a productive proficiency footprint for Australia's global well being depends upon students who are intellectually autonomous, proficient and predisposed to do and use mathematics.

This research investigated how the students and the teacher worked together in the middle school mathematics classroom and how this influenced students' mathematical proficiency and mathematical dispositions. This qualitative case study explored the question, "How do middle school mathematics teachers empower students to be proficient doers and users of mathematics?". Specifically this involved delving into how students were guided and supported by the teacher to work and think mathematically in order to construct their mathematical knowledge. Two grade 9 mathematics classrooms were observed at Amethyst College, a high school in North Queensland, over one semester in 2009. The socio-cultural and psychological perspectives within the classroom learning

community provided the theoretical framework for this study. The socio-mathematical norms of mathematical difference and mathematical argumentation comprised a lens through which classroom interactions were critically analysed. Qualitative case study methods were chosen for this research since they provided an ideal opportunity to gain an emic understanding of the uniqueness of people and programs within the realities of an educational setting (Bassey, 1999; Merriam, 1998; Stake, 1995; Tellis, 1997).

The research data reveal the reflexivity of the socio-cultural and psychological perspectives. Several core issues impacted on how teachers used their professionalism and pedagogical content knowledge to establish norms that potentially empower students into being proficient doers and users of mathematics; broadly defined these issues included: the knowledge gaps and mathematical dispositions that students brought with them to the grade 9 classroom; the teachers' epistemological beliefs, pedagogical dispositions and exhaustion and cynicism brought on by constant education reform. Streaming in the middle school and the omnipotence of the mathematics test also arose as key issues affecting students' participation in learning at Amethyst College.

The implications of the data analysis suggest that if teachers and students continue to participate as passive recipients in their respective socio-cultural domains, Australia's mathematical proficiency footprint will epitomise a largely cosmetic understanding over energised and mobilised participation. Research recommendations focus on how a re-configuration of Australia's mathematical proficiency footprint might be precipitated, and how this would enhance Australia's intellectual and social sustainability. Salient themes are building the capacity for high powered intellectual engagement in the classroom learning community and the professional learning community. Indeed the recommendations proffer the view that genuine opportunities for empowered and active participation in knowledge construction for teachers and students are what may sustain the ongoing revitalisation of middle school mathematics.

The thick description of this case study does not attempt to claim formal truths. Instead it strives to stimulate thought and reflection from the reader. In this way, the aim is to invite the reader to gain an insight into phenomenological issues, adding insight into experience and understanding.

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Chapter 1

The Power of Middle School Mathematics

> *Somebody once made the funny and effective remark mathematics was she who first is sneaking with a low frame, but soon raises her head to the heavens and walks on earth because it starts with the point and the line, but its investigations comprise heaven, earth, and universe.*
>
> *Hero, Definitiones*

1.1 Shaping the mathematical proficiency footprint

The shaping of Australia's mathematical proficiency footprint is a priority requiring action. Currently, the footprint is defined in many cases by shallow mathematical proficiencies and unproductive dispositions (DEEWR, 2008). Changing the view of mathematics so that it is used as an energy source to create and sustain intellectual and social endeavours is essential to re-shaping the proficiency footprint. The productivity of the workforce required to support the economic and civic future of Australia depends upon actions that redefine the contours of the footprint within a mathematically proficient generation. A large proportion of the actions that shape the proficiency footprint reside in mathematics education.

The literature identifies that action in the middle years is a catalyst in the process of developing students' mathematical proficiency to progress into mathematics education at senior and university levels (Carrington, 2002; Chadbourne, 2001; McPhan, et al., 2008; Prosser, 2006). This action involves effective classroom learning communities that encourage mutual engagement in rigorous mathematics through quality mathematical interactions (Prosser, 2006; Wegner, 1998). Quality mathematical interactions potentially build students' mathematical dispositions and intellectual autonomy (Kilpatrick, Swafford & Findell, 2001; Yackel & Cobb, 1996). This type of quality mathematics education in the middle school is a power socket energising the human capital necessary to build the capacity of the country to meet the challenges of the 21st century. Building

Australia’s human capital so that it has the proficiency, agency and confidence to choose and use mathematics in flexible ways hinges on mathematics being viewed as an action in the middle school classroom.

1.1.1 The implications of middle school reforms

Contemporary research literature highlights some of the implications of Australian middle schooling reforms over the last decade. One implication is that middle school reforms have contributed to the decline in the number of students who are choosing to take higher level mathematics courses (McPhan, et al., 2008). Indeed, the Australian 15 year old students who participated in the 2006 Programme for International Student Assessment (PISA) rated amongst the lowest in the world in their interest for taking senior mathematics and science subjects (Masters, 2009). The level of interest and the mathematical proficiency of the middle school students are, to some extent at least, symptomatic of the quality of education available in the classroom.

The research literature establishes that the practices existing in Australian middle school mathematics classrooms sometimes involve little more than low level routine mathematical procedures and skills with minimal evidence of mathematical thinking and reasoning (DEEWR, 2008; Stacey, 2003). It seems as though these classroom practices view the use of mathematics as a truism and “nothing is as easily forgotten as a truism” (Freudenthal, 1973, p. 16). From this perspective, mathematical experiences in the classroom become one dimensional, resulting in convergent thinking where students are focused on remembering the rules and procedures of mathematics. In this way, students are developing superficial mathematical understandings and unproductive mathematical dispositions that result in a reluctance to continue with mathematics at a higher level (Ball, 2003; DEEWR, 2008).

The evidence from the TIMSS 1999 video study underscores the fact that while curriculum documents have emphasised conceptual understanding, this has not resulted in real benefits for students' strategic competence at applying mathematics (DEEWR, 2008). Students' proficiency at applying mathematics depends upon the experiences available in the classroom learning community that promote their active engagement in the development of their intuitive understandings of mathematical ideas and concepts (Kilpatrick et al., 2001;DEEWR, 2008). Curriculum documents such as *Essential Learnings by the End of Year 9* (Education Queensland, 2007) urge that students are involved in thinking, reasoning and talking about the processes of mathematics. However, the ideals of the curriculum documents are not finding their way as effective action in the classroom. The TIMSS 1999 video study brings to light a discrepancy between the intended curriculum and the attained curriculum.

There is a climate of fatigue that surrounds schools which have endeavoured to implement the middle school curriculum reforms, so it isn't surprising that the intended curriculum and the attained curriculum may not be aligned (Luke, Elkins, Weir, Land, Carrington, Sole, Pendergast, Kapitzke, Van Kraagenoord, Moni, McIntosh, Mayer, Bahr, Hunter, Chadbourne, Bean, Alverman & Stevens, 2003; Prosser, 2006). Teachers and schools are struggling to maintain the necessary momentum to successfully implement curriculum reforms, resulting in perfunctory and superficial actions that do not foster the desired improvements to students' mathematical proficiency (Masters, 2009). One factor contributing to the struggle is that there often isn't an opportunity for a mutual understanding of curriculum reforms to develop between curricularists and teachers (Handal & Herrington, 2003). Consequently, the curriculum reforms in the middle school that have sought to bring about productive changes have, in fact, been an oppressive force shaping the proficiency footprint. Indeed, the proficiency footprint is being shaped by a generation that is less interested in mathematics and less able to choose and use mathematics (DEEWR, 2008).

1.1.2 The education revolution: shaping the proficiency footprint?

In 2008, national numeracy assessment (part of the NAPLAN test) was introduced as an accountability requirement in Australian schools, to work towards excellence in mathematics education, to build essential human capital. This was part of the *education revolution* in response perhaps to the *crisis* in mathematics education in Australia. Certainly, the government placed quality education as a priority in its policy schedule. Arising from this priority was the implementation of a National Curriculum (a priority that has been on the nation's agenda for at least 15 years) that assumes collaboration between the states and territories of Australia.

To foster the desired collaboration between the states and territories, the Council of Australian Governments (COAG) was introduced to work together with the Ministerial Council on Education, Employment, Training and Youth Affairs (MCEETYA). Furthermore, the Australian Curriculum, Assessment and Reporting Authority (ACARA) has been established to develop and implement the National Curriculum and the NAPLAN test. Questions arise though as to how these government bodies are going to improve the quality of the education occurring in our schools since they appear to be driven in part by the cooperative federalism (Reid, 2009) that is favoured by the Labor government elected into power in 2007 and again in 2010. This cooperative federalism is focused on the different tiers of government working together to solve the problems in Australian education.

The accountability agenda emerging from the NAPLAN test suggests that problem solving according to these government bodies involves consistently reminding teachers that they are now operating in a climate of quality control. A culture built around teachers who align their practice in this environment of prescriptive accountability could induce de-motivating trends such as a concentration on teaching to the test. Luke and Woods (2007, p. 16) discuss the introduction of the "quality control" of national testing as having a

> host of collateral effects that include narrowing of the curriculum, teaching to a test, teachers' deskilling and attrition, documented test score fraud and manipulation at the state and school level – with no visible sustainable effects at improving equity outcomes.

Furthermore, Day, Flores and Viana's (2007, p. 250) research in Portugal and England found that the "greater accountability" and "public scrutiny" that accompanies "performativity agendas" contribute to a decline in job satisfaction and professional capacity for teachers. Therefore the enforcement of bureaucratic accountability agendas may cause fractures in teachers' professionalism and pedagogical practices, with students' learning the ultimate casualty.

The uncertainty that accompanies the ambiguity of political agendas diffuses the clarity of direction required to build excellence in Australian mathematics classrooms. The implications of the education revolution add to the oppressiveness currently shaping the proficiency footprint. Problematically, the shortfall of a mathematically proficient generation may indeed perpetuate the issue of superficial curriculum implementation. That is, new kinds of action in the mathematics classroom depends upon graduate and experienced teachers who are mathematically proficient so that they can create effective classroom learning communities through their professional agency. At the moment it seems as though the climate of education reform and the accountability agenda are undermining the agency and professionalism of even the most experienced and proficient mathematics teachers (Masters, 2009). Therefore, in the current culture of teaching mathematics, externally mandated curriculum reforms do not appear to be finding their way as effective actions to re-configure the proficiency footprint in a productive way for Australia's global well being.

Re-shaping the proficiency footprint involves reculturing mathematics education. This reculturing is a process involving actions that aim to bring about renewed views of mathematics, new forms of agency for teachers and students and a new focus on effective classroom practices for students' mathematical proficiency. Shaping the proficiency footprint depends upon mathematics teachers who are motivated to respond to shortfalls with effective action in their classrooms.

1.2 Quality action in mathematics education: learning from mathematical traditions

Quality mathematics education is contingent on quality teachers continually tweaking their classroom practices for the benefit of their students' mathematical thinking and reasoning capacity. There is not an exact formula to determine how teachers should teach mathematics, since what works in one classroom does not work in another (DEEWR, 2008). Teaching mathematics is and should be a complex process that demands a high level of critical thinking and critical reflection. Shaping the proficiency footprint for the 21st century requires that teachers use intellectually engaging pedagogical practices so that students can confidently choose and use mathematics in flexible and effective ways. The traditions of mathematics may be a motivating source for teachers who aim to cultivate classroom practices that can help students become better at thinking mathematically. Students' proficiency at choosing and using mathematics depends upon classroom learning communities that encourage students to think mathematically since "whoever has grasped the power of thinking will continue to exercise it" (Freudenthal, 1973, p. 8).

Mathematics has been evolving since the end of the third millennium B.C. Mathematical foundations of the Babylonians and the Greeks were built on mathematical theory before application. "Mathematics has always been ahead of its applications; it is the way of mathematics – to look for patterns of thought from which the appliers make their choice" (Freudenthal, 1970, p. 8). The last few centuries have seen mathematics used in magnificent ways such as in its application to the physics of quantum mechanics. Galileo and Newton were two of the most influential scientists and mathematicians whose powerful mathematical thinking laid the foundations that have revolutionalised our world. Sir Isaac Newton acknowledged that his mathematical capacity depended on the work of those before him:

> If I have been able to see further, it was only because I stood on the shoulders of giants.
> (Sir Isaac Newton)

Mathematics is one of the greatest achievements of civilisation and continues to be one of the most useful activities. We exist in an attention age where we can create, consume and share information and ideas instantly on the internet and through social media. The attention age urges us to become better at sorting through the profusion of information that is readily available. Indeed the stimulating era of the attention age which scaffolds upon the information age of the 21st century necessitates using mathematics to process the abundance of information available in new and unpredictable ways. However, an abundance of information becomes potentially empowering and worthy of attention only if it can be analysed in penetrating ways by looking for patterns to make meaningful mathematical deductions. So while we live in a different era, we can use Newton's common sense idea of building on the work of others to improve our capacity to think mathematically.

The essence of mathematics has evolved through not only building upon the work of others but also through disagreement, different approaches, resilience and persistence (Freudenthal, 1970). The different cultures and leading mathematicians throughout history have argued about mathematics and persisted with looking for patterns and different paths of thought. It is because of this tradition that mathematics has become a powerful tool. There were many misunderstood mathematical geniuses who did not receive recognition for their work until after they died. Number theories and the foundations of mathematics took many years to be accepted. Calculus had a tenuous beginning and there was a period of time when its development came to a standstill (Freudenthal, 1970).

Importantly, mathematicians took the time to argue about different ways of thinking about mathematics and from this emerged the possibility for deductive reasoning, mathematical rigour and new views of mathematics.

The traditions of mathematics demand that teachers take the time to view pedagogy in new ways to bring mathematical rigour to the classroom. New views of pedagogy that furnish classroom practices that place mathematics as an action, as mathematicians do, encourages students to have authorship of the mathematics they are doing. These classroom practices potentially sustain a culture where students are mutually engaged in looking for mathematical relationships to make idiosyncratic mathematical connections. These practices are proposed as a way to build a proficiency footprint better able to support the future prosperity of Australia.

1.3 The next generation of middle school reform

The research literature urges the second generation of middle school reform in Australia to bolster the middle years transition period with academic rigour (Luke, et al., 2003; Perso, 2004; Prosser, 2006). One of the identified implications of the first phase of middle schooling in Australia has been a deactivation of pedagogical goals (Carrington, 2002; Prosser, 2006). That is, teachers appear to be caught in attending to the superficial components of curriculum changes at the expense of quality pedagogical practice. Understanding more about the teacher's role in revitalising the middle school mathematics classroom with academic rigour is a vital footstep in the reform process.

Education Queensland's document*: Numeracy: Lifelong confidence with mathematics. Framework for Action 2007 -2010* places what teachers do in the classroom as a priority. Teachers in Queensland need to be critically aware of how they can implement learning opportunities that are synchronous with the Essential Learnings (Education Queensland, 2007) of the current mathematics syllabus and contemporary views on how students learn mathematics. It is becoming more widely accepted that student outcomes in mathematics depend upon the quality of the interactions available in the mathematics classroom. These interactions are steered by the classroom teacher, and the integration of subject matter knowledge, knowledge of students and pedagogical techniques are pivotal in creating opportunities for students to be engaged in doing mathematics in powerful learning communities (Ball, 2003). A powerful learning community sees the teacher and students mutually engaged in quality mathematical interactions. Shulman's (1986) conception of Pedagogical Content Knowledge (PCK) is one resource in contemplating the processes involved in reinvigorating mathematics teaching in the middle school. However, while PCK promotes the importance of teachers intersecting their content knowledge, pedagogy and knowledge of students, continually building opportunities for mutually engaging interactions in the classroom learning community is a key goal. This goal acknowledges that the mathematical actions of the students in the classroom learning community are central to building their mathematical proficiency. Therefore, for the idea of PCK to be powerful and meaningful in the middle school context, consideration may first need to be given to how all teachers can become empowered to engage effectively in the change process to continually contemplate how to improve the quality of the interactions in the classroom learning community.

The effectiveness of classroom learning communities to facilitate opportunities for quality mathematical interactions lies in the nexus between teachers' professionalism and pedagogical content knowledge and students' mathematical proficiency. The next generation of middle school reform urges that teachers step up to the challenge to become effective change agents to reactivate their pedagogical goals. Students are depending on the mathematical leadership and professionalism of teachers to build effective classroom learning communities that place mathematics as an action. Students' mathematical proficiency depends upon classroom experiences where they can learn how to find their place in the world of choosing and using mathematics.

1.4 The vocation of teaching

As a classroom teacher, with experience in middle and senior years (grades 11 and 12) mathematics and physics education in Queensland schools, I had concerns about the availability of high powered intellectual engagement for students in the middle school classroom. Physics is a science that depends on the practical application of mathematics. Mathematical rigour that encourages intellectual autonomy in the middle school is critical for students to aspire to successfully undertake higher level mathematics and physics courses. I shared the concerns discussed in the research literature about the decline in the number of students who are choosing to take senior mathematics and physics as subjects in the senior school (McPhan, et al., 2008). From my perspective, gained from tutoring senior physics and mathematics students in recent years, there is a feeling of unease about the rise in the number of very bright and capable students who appear to lack an ability to apply foundational mathematical concepts and skills.

The concerns I have about how Australia's proficiency footprint is being shaped are both professional and personal. The professional concerns are what may have prompted my interest in undertaking this research. However, the concern has become even more personal since my own children are now in their formative years of schooling.

I view teaching as a vocation that requires courage and resilience, that finds strength and focus through a willingness to blur the personal and professional. This courage involves exploring "one's ignorance as well as insight, to yield some control in order to empower the group, to evoke other people's lives as well as reveal one's own" (Palmer, 2006, p. 2). This research was part of a process that helped me explore how I might continually re-shape my own classroom practices and my professional capacity as a change agent. Taking the time and space to furnish this part of the "journey toward discovering the true sprit of teaching" (Palmer, 2006, p. 3) was integral to initiating the re-shaping of my view of teaching and learning mathematics. Finding my footing in the vocation of teaching by acknowledging the professional and the personal is a process that began in the classroom but has gained new insight and stability as a result of the actions of this research.

1.5 Microscopic view of case study methodology

The microscopic view of the case study methodology is an avenue to investigate and contemplate the effectiveness of a classroom learning community. As suggested by Shulman (2005, p. 16) we need to have "deep, rich case materials that permit us to study, analyse, slow down, review in depth and generally work through the practices and thinking of teachers" in diverse contexts. In this way, teachers may see their role as transformational, since their errors and best practice have the potential to help other teachers to adapt their knowledge and pedagogy. Significantly, this may ignite the change agency of teachers, as in other countries such as China and Japan, where teachers "work together studying their own practice, joining with other teachers in other schools" (Shulman, 2005, p. 21) to undertake ongoing and relevant professional development.

In Australia there appear to be few in-depth investigations into how the pedagogical content knowledge of teachers meld in the diverse contexts that exist in the teaching profession, and ultimately how this influences the classroom practices that directly impact the mathematical proficiency of students (DEEWR, 2008). Certainly, the literature review in the next chapter urges educators to move beyond the complex tensions that have existed over the years between subject matter knowledge overshadowing pedagogical techniques, into viewing knowledge and pedagogy through a single, subject specific lens (Ball, 2003; Shulman & Sherin, 2004). However, the influx of reform packages fired into the Australian middle school context initiates another dimension to the aforementioned tensions that requires careful consideration. The research literature highlights how the challenges of middle school reform, content and pedagogy have at times been overwhelming for teachers. There is much debate about the middle school reform agendas and their impact on the effectiveness of the classroom learning community. This case study aimed to investigate some of what is influencing the norms of practice of two middle school mathematics classrooms.

1.6 The power of learning communities

This research is couched in the view that students' mathematical proficiency depends upon effective action involving teachers and students being mutually engaged in a learning community. Revitalising the middle school involves pedagogy that encourages a culture of participation and views mathematics as an action. This study discusses *mathematisation* as a process that places mathematics as an action in a learning community. (Organization for Economic Cooperation and Development (OECD), 2003; Yackel & Cobb, 1996).

Mathematisation places students' mathematical proficiency at the core of classroom practice by acknowledging that students should have the opportunity to be mutually engaged in quality interactions so they may have authorship of the mathematics that they are doing. Students who are active in the construction of their mathematical knowledge depend upon the resource of teachers who can use their pedagogical content knowledge in new ways to furnish opportunities for quality mathematical interactions. Therefore, the opportunities for ownership: in mathematisation for students; and in teachers using their pedagogical content knowledge to create targets for curriculum change, have the potential to infuse equitable power relations into the middle school culture. This may be a core factor in school based revitalisation for middle school mathematics education. In this sense, the power lies with the learning communities.

Focusing on developing the power of the learning community may be a motivating source for teachers to maintain a continuity of action to sharpen their change agency. It may also be a common sense target that could redirect their energy from becoming depleted in the current climate of bureaucratic policy reform. A desired change in the culture of mathematics teaching (Hiebert & Stigler, 2004) to re-shape Australia's proficiency footprint furnished the backdrop of this research.

1.7 Case Study Research Question

The case study objective was to investigate the quality of the interactions between the teacher and students within the middle school mathematics classroom. This epistemological problem was approached by attempting to better understand how the teacher and students were mutually engaged in a “learning community” (Wegner, 1998, p. 214).

The 2007-2010 *‘Numeracy: Lifelong confidence with mathematics - Framework for Action’ document* (Education Queensland) identified *teacher knowledge and pedagogy* and *student confidence* as two of the four key priorities requiring action. This case study sought to gain insight into these priorities within the social reality of the middle school mathematics classroom. From these priorities and the national agenda which is focused on improving students’ mathematical proficiency, the following question was formulated and explored in this case study research:

- How do middle school mathematics teachers empower students to be proficient doers and users of mathematics?

1.7.1 Thesis structure

The rationale for introducing middle schooling and the implications for mathematics education are explored in Chapter 2, the literature review. The climate of policy orientated reform in Australia over the past decade and its implications are also discussed. The potential for improving academic rigour in the middle school is considered by using the research literature on teachers’ pedagogical content knowledge and opportunities for students to mathematise. Pedagogical content knowledge is examined from Shulman’s work in 1986 through to present research imploring the systematic support of professional development for teachers in this area (Mundry, 2005). The important concept of empowerment, of how teachers and students can be empowered within their socio-cultural contexts, was a salient theme identified in the research literature and this was used to map the direction of this thesis.

The methodology and methods discussed in Chapter 3 signify the research focus as understanding in context. The epistemological problem was to better understand how the teachers and students become empowered and engaged in a learning community (Wegner, 1998). The reflexivity of the socio-cultural and psychological perspectives of the classroom interactions served as the theoretical framework for this study. The methodology discusses an emphasis on examining the socio-mathematical norms of the classroom learning community, since these are considered as essential in the development of students' intellectual autonomy and a productive mathematical disposition.

This qualitative research endeavoured to produce a thick description of the microculture of two middle school mathematics classrooms at a single school, Amethyst College. Qualitative research methods were chosen since they facilitate the emic perspective to encapsulate the nuances of how the interactions in the classroom community evolved. The mathematical interactions within the middle school classroom were viewed within this study as contributing to the unfolding and reforming of students' mathematical dispositions, an important component in the development of students' mathematical proficiency (Kilpatrick, et al., 2001).

The methods used are discussed in Chapter 3 and attend to achieving internal validity through the triangulation of data. The specific design of the case study is represented by a funnel that attempts to continually refine the feasibility of data collection (Bogdan & Biklen, 2007). The psychological and socio-cultural perspectives of the classroom learning community interactions were the initial categories for data collection and analysis. Chapter 4 and Chapter 5 explore the data from the two classrooms at Amethyst College. The socio-mathematical norms of each classroom were used to explore the data in order to interpret how students were able to develop their intellectual autonomy and mathematical disposition. The analysis in Chapter 6 makes use of pattern matching and the triangulation of data. This chapter considers the initial categories of the classroom microculture across to the emic perspectives of the participants to analyse the dynamics of the variables in context.

The implications of the data analysis suggest that if teachers continue to be told by external reforms how they should do things then mathematics may well continue to be placed as a noun in the classroom. Chapter 7 discusses two key interrelated actions that see a connection between the effectiveness of the professional learning community and the opportunities for mathematisation in a culture of participation in the classroom learning community.

The first recommendation involves pedagogy that views mathematics as an action in the classroom learning community so that students can build their intuitive understanding of mathematics. This recommendation arises from the implication that developing students' mathematical proficiency at a grade 9 level depends on their ability for abstract thought so that they can participate effectively in mathematisation in the classroom learning community. The second recommendation discusses how teachers might view pedagogy in new ways in order to bring about new forms of mathematical participation in the classroom by building their change agency within an effective professional learning community.

The final chapter identifies some of the complexities involved in reculturing mathematics education. School based revitalisation that acknowledges and builds the power that lies with effective learning communities is an ongoing process rather than an event. This research is thought of as part of this process, since it sees value in building a capacity to learn by learning from others. The core agenda of this thesis is to learn more about the process of reculturing mathematics education for the benefit of students' mathematical proficiency.

Chapter 2

Literature Review

It is the supreme art of the teacher to awaken joy in creative expression and knowledge.

Albert Einstein

2.1 First phase middle schooling in Australia

The middle years have featured on the reform agenda within Australian schools for more than a decade. This reform movement was prompted by the recognition of two key problems: the transition from a student-centred, integrated approach in primary school to a subject-centred, segregated approach in secondary school; and a lack of recognition given to the educational implications of the distinct nature of the young adolescent (Carrington, 2002). Of great influence to the middle school movement in Australia was the *In the Middle Report* (Schools Council, 1993) and several other reports (Earl, 1999; Hargreaves, Earl & Ryan, 1996) highlighting the social and academic challenges precipitated by the transition into the middle years (Prosser, 2006).

Education Queensland defines grades 4 to 9 as the middle school years (Education Queensland, 2003). The espoused changes in structural arrangement have culminated in an array of middle school models across Queensland and Australia (Luke, et al., 2003). These models include: middle school within a P-12 school; middle school within a primary school; middle school within a secondary school; and autonomous middle schools (Carrington, 2002).

The evolution of middle years education has sought to create supportive environments through the realignment of school structures. Prosser (2006, p. 2) broadly summarises middle schooling in the following way:

- a separation of the middle years from the rest of the school;
- establishing teaching teams and/or sub-school groups to enhance teacher-student relationships;
- devising integrated and negotiated curriculums; and,
- using authentic assessment of rich learning tasks.

Attention has been on the development of the middle school as an "autonomous or semi-autonomous, sub-school organisation" with a "focus on pastoral care" (Chadbourne, 2001, p. 15).

The first phase of the middle school reform has demonstrated that distinguishing a middle school as a separate entity does not automatically fulfil the broader obligations of the middle school philosophy (Carrington, 2002; Prosser, 2006). Case studies in South Australia (Smyth et al., 2003, cited in Prosser, 2006, p. 8) and Victoria (Hill &Russell, 1999, cited in Prosser, 2006, p. 8) have revealed a fragmented approach to the reform process with too much focus on the structural organisation of middle schools. This follows a similar trend in the USA in the 1990s, where middle schools were more "successful at implementing structural reforms than changes in pedagogy, assessment and curriculum" (Chadbourne, 2001, p. 23).

The research within Australia reveals that the focus on structural organisation has been repressive to the middle school movement (Carrington, 2002; Chadbourne, 2001; Luke et al., 2003; Prosser, 2006). Research by Luke et al. (2003), found that acknowledging the middle school as a distinct stage does not necessarily induce the desired, improved continuity between primary and secondary school. Furthermore, they found that "effective middle years programs view the middle years as a 'first principle' status for the organisation of professional, spatial, temporal, pedagogical and epistemic capacity" and this should be a "rallying point for whole school and curriculum renewal" (Luke, et al., 2003, p. 98). This is consolidated by Carrington (2002, p. 15), who notes that greater attention given to the restructuring of pedagogical and "everyday institutional practises" improves middle school efficacy and is more facilitative for at-risk students in their progression through compulsory education, than by concentrating on school configurations alone. Moreover, Chadbourne (2001) suggests that the central tenets and practices comprising the pedagogy and curriculum of middle schooling: teaching collaboratively; integrated curriculum; authentic assessment; cooperative learning and small learning communities should be applied to a wider spectrum of students.

It has also been suggested that the educative ambitions of the middle school reform in Australia may have been diminished due to the concurrent progress of corporate management, devolution and workplace agreements for teachers (Chadbourne, 2001). Given the notion of education being economically and politically motivated, Prosser (2006) proposes that the middle school agenda could be construed as being driven by the "invisible hand of the market" (Reid, 1999, p. 193). The appointment of "generic managers" (Reid, 1999, p. 195) to make key educational decisions, with an emphasis on operating efficiently and achieving goals suggests another rationale why the first phase of middle schooling concentrated more on structural organisation and as a consequence was left to flounder in terms of its educational identity (Chadbourne, 2001; Prosser, 2006). Indeed, once again this follows a similar trend in the USA where the creation of middle schools as separate entities "has become the norm more because of societal and demographic pressures than because of scientific evidence supporting the need for a separate school for young teens" (Yecke, 2005, p. 15). The research literature generally supports the notion that placing organisational structure as a focal point can deactivate pedagogical goals.

2.2 The young adolescent

An initial driving influence of the middle school movement was the need to focus on the uniqueness of the young adolescent by considering their emotional and social well-being. The research findings suggest that the pastoral care programs implemented in middle schools have created the desired supportive environments for the young adolescent (Carrington, 2002; Luke et al., 2003; Perso, 2004; Prosser, 2006). However, the research also identifies that "social support is necessary, but of itself, not sufficient to achieve improvements in students' outcomes" (Carrington, 2002, p. 5). A focus on pastoral care within middle school has been "one-dimensional" and what has been overlooked is "the need to support the transition in an academic sense" (Perso, 2004, p. 29). This premise underlies Yecke's (2005) report *Mayhem in the Middle* concerning middle schooling in the USA. The report pertinently summarises the view that:

> Too many educators see middle schools as an environment where little is expected of students either academically or behaviourally, on the assumption that self-discipline and high academic expectations must be placed on hold until the storms of early adolescence have passed. The sad reality is that by the time those storms have dissipated, many students are too far behind to pick up the pace and meet current state academic requirements, much less the challenging expectations of federal laws such as No Child Left Behind. (Yecke, 2005, p. 17)

The literature in Australia and the USA cautions against "supplanting academic rigour" with too great a focus on the emotional development of young adolescents (Yecke, 2005, p. 29). In the assessment of the first phase of middle schooling within Australia, Luke et al. (2003, p. 12) suggest that the "second generation of middle schooling must …respond to [the] criticisms of the first generation of middle schooling by fostering academic and intellectual rigour". Carrington (2002), on the other hand, challenges the next phase of middle school reform to use the wealth of knowledge yielded from the first phase of middle schooling to consider not only the academic and developmental needs of young adolescents but also the increasing diversity of their world. This is summarised by Knobel and Lankshear (2003, p. 80, cited in Prosser, 2006, p. 9):

> Pedagogy and curriculum cannot be 'hostaged' to every change in cultural tools and uses that appear on the horizon. At the same time, if certain limits to learners' affinities, allegiances, identities and prior experience are transgressed, even 'successful' learners will decline the offer made by formal education.

In essence, the literature suggests that the line of distinction between school curriculum and the omnipresent, persuasive multiplicity of youth culture needs to be blurred, but not at the expense of academic expectations.

2.3 A decade of curriculum reform in Queensland schools

Simultaneous to the middle school movement, there have been ongoing curriculum reform initiatives that have played out in Queensland schools over the past decade. A brief chronological view is presented in Table 1:

Table 1: Chronology of Queensland Curriculum initiatives

Year(s)	**Curriculum initiative**
1998 - 2006	Outcomes-based Key Learning Area (KLA) syllabuses
1999	Queensland School Reform Longitudinal study
2000-2004	New Basic Trial
2001-2003	Assessment and Reporting Taskforce
2001-2008	Year 1-10 Curriculum Framework
2002	Productive Pedagogies

2004	Mathematics Core Learning Outcomes (CLO)
2005 ongoing	Queensland Curriculum Assessment and Reporting (QCAR) Framework
2007 -2010	Numeracy Framework
2008 ongoing	P-12 Curriculum Framework
2009	Learning P-12

(Adapted from Masters, 2009, p. 48)

The outcomes-based syllabus (1998 – 2006) was taken up in all sectors of state and non-state schools. The mathematics syllabus was one of the eight KLAs and it had eight general learning outcomes and five strands each with two or three topics. Each of the eleven topics had six levels and so the KLAs for mathematics involved 66 Core Learning Outcomes (CLO) (Masters, 2009). The CLOs were meant to be supported by discretionary learning outcomes to "broaden understandings and provide opportunities for students to pursue interests and challenges" (Mathematics: Core Learning Outcomes, 2004, cited in Masters, 2009, p. 43). Teachers used their judgments to assess students' demonstration of aspects of learning outcomes. An evaluation of this system resulted in a common criticism that "the reliance on the use of individual CLOs to organise curriculum resulted in a crowded curriculum and fragmentation of the areas of study" and a large number of CLOs "resulted in loss of cognitive depth and growth" (Masters, 2009, p. 44).

Thus the QCAR Framework (2005 ongoing) developed out of the limitations identified within the KLA outcomes-based syllabus. The QCAR framework has five components (Education Queensland, 2008):

- Essential Learnings Years P – 9: identifies *Ways of Working* to develop *Knowledge and Understanding*.
- Standards: a common frame of reference and a shared language to describe student achievement.

- Assessment Bank: supports the everyday assessment practices of teachers through access to a range of online quality assessment tools.
- Queensland Comparable Assessment Task (QCAT) provides information on what students know, understand and can do in a selection of Essential Learnings; intended to promote consistency of teacher judgments across the state.
- Guidelines for reporting: a common framework to assist in the consistency of reporting across the state.

The overarching intention of the Essential Learnings was to allow schools the autonomy to develop their own school curriculum programs to attend to the diverse needs that exist in their context. Furthermore, the QCAR framework acknowledges that curriculum, pedagogy, assessment and reporting need to be aligned (Masters, 2009). However, the ongoing curriculum reforms in Queensland schools appear to have resulted in a fatigue that may have undermined the potential available in the QCAR framework.

2.4 Revitalising the middle school reform

Curriculum reform exhaustion is being felt within the Australian middle school. (Luke et al., 2003; Prosser, 2006). In Queensland schools, Masters' (2009, p. 49) report discussed views that highlight some of the consequences of ongoing curriculum transformations over the past decade:

- There has been too much curriculum 'churn' in recent years, with schools having to respond to too many changes.
- The school curriculum is overcrowded and the tasks teachers perform often detract from their professional work.
- There has been a loss of focus on the basics with teachers being required to spend time on a wide range of other topics and issues.
- An excessive focus on data/test outcomes can distract attention from broader school curriculum.

The current climate of fragmented reform, together with the attempt to implement an integrated curriculum with key learning areas and authentic assessment, finds many middle schools struggling to meet educational aims. This is evidenced by research showing that “traditionally strong students are at best only being maintained” (Prosser, 2006, p. 9). The latest literature recognises that the middle school agenda in Australia needs school-based revitalisation, with well-defined approaches attending to the heterogeneous, social and economic conditions that exist within specific contexts. Additionally, in the current environment of accountability and standards, intellectually engaging pedagogy to improve student engagement, achievement and proficiency is paramount (ACARA, 2009; Carrington 2002; McPhan et al., 2008; Prosser, 2006). Specifically, there is an urgent need for “higher order intellectual engagement in literacy and numeracy by members of target groups in order for all to access employment and to pursue improved life pathways through school to post-compulsory study, work and community life” (Luke et al., 2003, p. 7). Yecke (2005), in reference to middle schooling in the USA, strongly suggests uprooting “the anti-academic mindset that drives it” (p. 35) and focusing “first and foremost on students’ acquisition of essential academic skills and knowledge” (p. 7). Certainly, improving students’ mathematical proficiency arises as an urgent need within Australia’s latest educational reform movement, the education revolution, which is cast with the development of the Australian National Curriculum (ACARA, 2009).

2.5 Defining mathematical proficiency in the latest reform in Australian education

The *Shape of the Australian Curriculum: Mathematics* (ACARA, 2009) places mathematical proficiency as a key strand, coupled to the key strand of content. The focus of the mathematical proficiency strand is described in terms of how the content is to be embedded within the curriculum. Terms such as understanding, fluency, problem solving and reasoning are used to elaborate how the mathematical content may become mathematical action so that students are “thinking and doing mathematics” (ACARA, 2009, p. 7). This is similar to the

QCAR framework Essential Learnings (Education Queensland, 2008, p. 1) that encourages students to explore mathematics as a way of thinking by using the Ways of Working framework. Within this framework were similar action terms involving how students should do mathematics in order to become mathematically proficient. In this sense, it could be construed that the continual reforms are little more than the same ideas repackaged. While the ideas are good, in that they acknowledge that students need opportunities to make connections between the mathematical concepts and how they are applied, how this actually occurs in the classroom and how this idea of mathematical proficiency is assessed becomes an overarching concern.

Certainly, it now appears to be well established that proficiency in mathematics involves not only a knowledge base, but also using the knowledge base in flexible ways. Setting standards of proficiency within the curriculum is important since it is from these standards that the teacher is assumed to base their pedagogical practice. Of course arising from this is the assessment of mathematical proficiency. How these standards are set and assessed are critically important (Schoenfeld, 2007) to the classroom practices. Schoenfeld (2007a, p. 12) discusses the influence of assessment and refers to the phenomenon of "WYTIWYG…What You Test Is What You Get".

In 2008, Australia introduced the National Assessment Program – Literacy and Numeracy: the NAPLAN test as a precursor to the development of the National Curriculum. The Numeracy test assesses the mathematical proficiency of Australia's students in grades 3, 5, 7 and 9 using multiple choice tests: calculator and non-calculator. The test may be considered as a tracking system of a student's progress in school mathematics and may be used as a reflection of their mathematical proficiency. The views of various stakeholders: governments; schools; principals; teachers and parents have played out in the media, particularly in regard to the resulting accountability agenda of the high stake test. NAPLAN has produced wide spread debate. However, how the NAPLAN test contributes to revolutionalising mathematics education is in itself debatable.

Some suggest that assessing mathematical proficiency using a multiple choice test is “difficult or impossible” (Burkhardt, 2007, p. 78). In terms of mathematical proficiency Burkhardt (2007, p. 78) makes the important point that “nobody who knows mathematics thinks that short multiple-choice items really represent mathematical performance”. Perso (2009, p. 13) discusses how the questions on the NAPLAN test are “limited in their capacity to assess the reasoning that is part of strategic and contextual numeracy”. Moreover, “a pen and paper test, particularly a multi-choice test, is unable to test reliably students’ confidence and disposition to use mathematics” (Perso, 2009, p. 13). Essentially, the results from the NAPLAN test have limited validity in terms of understanding students’ mathematical proficiency since it is difficult to know if students have strategically reasoned through the problems. Therefore, it is difficult for teachers to use the NAPLAN test for diagnostic purposes, to realign their classroom practices, without evidence of students’ working out that shows their thinking and reasoning. Indeed anybody who uses mathematics knows that the wrong answer can be attained due to simple errors even though the thinking and reasoning process is robust. Similarly, some students can make a lot of lucky guesses when choosing an answer, without thinking and reasoning.

Another issue arising from high stake tests is that it sometimes results in teachers drilling content and procedure at the expense of the conceptual development of mathematical ideas through problem solving. Schoenfeld (2007a, p. 12) discusses how in the lead up to high stake tests in the USA, some teachers “felt they had to focus on skills that were related to items on the test…hence what they were teaching-in some cases for weeks or months-did not reflect the practices they wished to put in place”. Schoenfeld (2007) also discusses a positive of high-stake tests as being increased curriculum and therefore instructional time devoted towards mathematics. However, in terms of students’ mathematical proficiency, this type of instructional practice may reap little more than short term results. Therefore, a significant concern emerges about the development of the mathematical proficiency strand of the National Curriculum since the NAPLAN test appears to be an assessment that may have contributed to the setting of the standards. In this sense what might be written in the policy documents about students' mathematical proficiency might be considered as superficial rhetoric that becomes removed from the actual classroom practice. Consequently, the great expectations of the new goals within the *Shape of the Australian Curriculum: Mathematics* (ACARA, 2009) become blurred and may not bring about revolutionary changes to classroom practice for the desired improvement of students' mathematical proficiency.

From the viewpoint of research in mathematics education, mathematics assessment should encompass a continuum of mathematical content and process with an emphasis on conceptual understanding and how students think mathematically (Ball 2003; Schoenfeld, 2007). In this sense, mathematical proficiency is described in the USA by five interconnecting strands:

- *conceptual understanding*: comprehension of mathematical concepts, operations, and relations
- *procedural fluency*: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- *strategic competence*: ability to formulate, represent, and solve mathematical problems
- *adaptive reasoning*: capacity for logical thought, reflection, explanation, and justification
- *productive disposition*: habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one's own efficacy.

(Kilpatrick, et al., 2001, p. 116)

By acknowledging how the strands are interconnected, the true nature of mathematical proficiency may emerge. Understanding and continually reflecting upon classroom practices that endeavour to develop mathematical proficiency becomes a critical target for mathematics teachers. Especially relevant to the classroom practices in the Australian middle school context might be the idea of interweaving student's mathematical disposition to the four other strands. However, the essential element of a student's mathematical disposition appears to be missing from the key descriptors of mathematical proficiency in the *Shape of the Australian Curriculum: Mathematics* (ACARA, 2009).

Acknowledging the idea of developing a productive mathematical disposition is essential to empowering students to pursue mathematics at a higher level. This process involves effective classroom practices "designed to engage students who have traditionally avoided or not performed well in mathematics in school" (Ball, 2003, p. 80). This requirement has been acknowledged by ACARA (2009, p. 9):

> Although there are challenges at all years of schooling, participation is most at threat in Years 6–9. Student disengagement at these years could be attributed to the nature of the curriculum, missed opportunities in earlier years, inappropriate learning and teaching processes, and perhaps the students' stages of physical development.

However, as suggested by Ball (2003, p. 10) "if teachers hold a restricted view of proficiency and are not themselves proficient in mathematics as well as in teaching, they cannot bring their students very far toward current goals for school mathematics". Problematically, mathematics has been shaped within the Australian National Curriculum through "restricted time frames for consultation with the profession" (Reid, 2008, p. 18). So it isn't unforeseeable that some mathematics teachers might be grappling with what the goals for mathematics education in Australia are and how these are to be attained.

Indeed, sound pedagogical practices in the middle school mathematics classroom are the linchpin that may redirect the alignment of students' mathematical dispositions and therefore develop the concept of mathematical proficiency in productive ways for students. In response to research evidence, (Hayes, Mills, Christie & Lingard, 2006; Luke et al., 2003) there has been a strong shift in the middle school movement to the examination of pedagogy. The *Productive Pedagogies* project (Hayes et al., 2006) comprehensively researched the pedagogical techniques of Australian middle school teachers and revealed inconsistencies in pedagogical techniques within schools and across the country. Concerns were also expressed about student involvement, confidence and achievement. This research highlights the need to develop effective, intellectually engaging pedagogy and a curriculum relevant to developing students' mathematical proficiency (Carrington, 2002; Prosser, 2006).

2.6 Teachers in the middle school

The research and literature on the first phase of middle schooling have exposed complex tensions within the middle school framework. Part of these tensions can be attributed to the expectations placed on teachers. Teachers were expected to work collaboratively in teaching teams to implement an integrated and negotiated curriculum, produce rich learning tasks and authentic assessments (Main, 2007; Prosser, 2006). They were required to become "skilled change agents" (Datnow, Hubbard & Mehan, 2002, p. 62) within the structural changes of the reform process. However, teachers have been traditionally regarded as "semi-professionals and recipients of reform policies rather than the change-makers" (Collay, 2006, p. 2). The different stages involved in implementing the middle school reform, coupled with the complexities of the new practices and a lack of awareness of how to effectively manage conflict have resulted in disillusioned teachers (Main, 2007). Thus, it is not surprising that the first generation of middle schooling teacher teams had difficulty maintaining the momentum of the reform (Main & Bryer, 2007; Prosser, 2006).

The literature purports that compared to traditional practices, middle schooling results in work intensification for teachers (Carrington, 2002; Chadbourne, 2001; Prosser, 2006). The higher workload is attributed to factors such as: the need for greater teacher collaboration and professional development; more subject area meetings owing to an integrated curriculum; extra time spent on lesson preparation due to teaching multiple subjects and a focus on pastoral care of early adolescents requiring more frequent parental contact (Carrington, 2002; Chadbourne, 2001; Prosser, 2006). Confounding things even further is the lack of compensation, in terms of time, money or career advancement, given the onus placed on middle school teachers to develop strong professional communities. Chadbourne (2001, p. 6) suggests that "to attract, develop and retain high quality teachers, middle schooling needs to offer teachers career advancement opportunities". Additionally, second generation middle schooling within Australia also needs to develop with an awareness of the struggle teachers have over their "pedagogical identity" so that they can

> maintain their role as curriculum designers and not be merely technicians; sustain critically reflective learning communities of colleagues and friends; and not succumb to pedagogies of resentment that are driven by a logic of deficit views of students and their communities.
> (Prosser, 2006, p. 13)

Therefore, in order for the next phase of middle schooling to gain the desired momentum, teachers should be encouraged to be part of the revitalisation process from the outset.

2.7 Pedagogy and subject expertise

Dynamic culture and pedagogy within any educational delivery system are contingent on the powerful resource of professional teachers with subject expertise (Hammond & Ball, 1997). The greater the mastery teachers have of their subject area, the greater ability they have to teach for understanding and promote the desired higher order thinking processes in their students (Carrington, 2002; Chadbourne, 2001; Hayes et al, 2006; Prosser, 2006; Yecke, 2005). The literature undeniably supports Stodolsky's (1998, cited in Chadbourne, 2001, p. 17) opinion that

> the more subject expertise teachers have, the more they can: devise challenging and engaging learning tasks for students within their subject; provide clear and powerful explanations of complex concepts within their subject; and teach for understanding and higher order thinking within their subject.

Unfortunately though, the realities of the Australian middle school have contributed to a deficit of teachers with robust subject matter expertise (Chadbourne, 2001; McPhan et al., 2008; Prosser, 2006). The research conducted by Luke et al. (2003, p. 23), found that within middle school mathematics classrooms "many teachers do not have the specific mathematics training and knowledge necessary to facilitate the development of mathematical concepts over time". Principals note the lack of qualified, enthusiastic mathematics teachers to teach within the middle school (Ingvarson, Beavis, Bishop, Peck & Elsworth, 2004; Jasman & Martinez, 2002; Lovat, 2003; McPhan, et al., 2008). The concern is well founded since the

> quality of student learning outcomes is directly dependent on the quality of the teacher; and, the essential components of effective teaching are command of subject, and knowledge of and capacity to implement effective pedagogical practices. (NSW Department of Education and Training, 2000, cited in Lovat, 2003, p. 2)

The research associates the fragility of subject development and effective pedagogy to middle schooling realities such as: expert teachers with subject specialities tend to remain in the senior school (grades 10, 11, 12), with some refusing to teach within the middle school; primary school teachers who teach in middle schools have generalist knowledge rather than specific content knowledge; the generalist approach neglects the intricacies inherent in specific subjects; integrated curriculum themes take "precedence over subjects" and a focus on the pastoral care needs of students "eclipses arrangements for meeting teachers' need to develop and maintain subject expertise" (Chadbourne, 2001, p. 29). Similarly, in the USA, Yecke (2005) states that there are not enough middle school teachers who have the necessary subject expertise to engage students in effective and intellectual learning.

The Australian Teacher Education Association (Jasman & Martinez, 2002) emphatically advocates the need for teachers to teach within their area of expertise since "this is the only way to ensure quality and equity of education for all Australian children, particularly in subjects such as science, maths and ICT, which provide high-stake capital in the knowledge economy and current job market" (Jasman & Martinez, 2002, p. 9). Furthermore, these researchers discuss research suggesting that teaching outside their subject areas places excess stress on teachers and limits quality teaching and learning opportunities for their students, especially for "students who are currently disadvantaged by schooling" (Jasman & Martinez, 2002, p. 9). In actuality, as mentioned earlier, catering for the key middle school objectives can limit the accessibility of teachers with subject expertise. For example, catering for smaller teaching teams that are required to teach across subject disciplines within the middle school may result in subject specialist teachers choosing to teach their subject area only in the senior school. Therefore, Chadbourne (2001) argues cogently that it is important to preserve the initial rationale of the middle school movement, while also maintaining the strength of the traditional school's subject based curriculum leadership. Indeed, Jasman and Martinez (2002), suggest that given the shortage of expert subject teachers,

teachers need to be retrained, at the cost of their employer, to improve their competency within specific curriculum areas if they are to teach in the middle school. Research findings from McPhan et al. (2008) accentuate the need to retain and attract "degree-qualified mathematics teachers in primary and secondary teaching", furnished through collaborative efforts, research and incentives from stakeholder groups such as governments, educational authorities and schools. This is also discussed by Lovat (2002), who refers to quality teaching as being "the single greatest parameter for attention of teacher education personnel, teaching unions and employing systems in the current era" (p. 2). The literature unequivocally prioritises placing quality teachers, with proficient, subject specific, pedagogical techniques in the revitalised middle school classroom.

2.8 Mathematics in the middle school

To remain globally competitive and meet projected needs, Australia requires a legion of students pursuing careers in science, mathematics, technology and engineering. This concern pervades all levels of mathematics learning, from the fundamental to the enriched. The middle years of schooling are formative in the provision of the mathematical confidence and mathematical literacy required for students to participate in senior level and university mathematics courses (McPhan, et al., 2008). Mathematical literacy in this respect encompasses "the functional use of mathematics in a narrow sense as well as preparedness for further study, and the aesthetic and recreational elements of mathematics" (Organization for Economic Cooperation and Development, 2003, p. 25). Mathematical confidence refers to students having the mathematical disposition to use their mathematical knowledge and skills when they need to.

In their report, *Maths, Why Not?,* McPhan et al. (2008, p. 2) investigated the question, "Why is it that capable students are not choosing to take higher-level mathematics in the senior years of schooling?". The findings by McPhan et al. (2008) support Ingvarson et al. (2004) on the effect of the teacher's role in the classroom. Their respective studies reveal that teachers are central to improving student engagement in and disposition toward mathematics. In particular, the studies found that quality mathematical experiences in the middle school underpin the strategic decisions students make about pursuing further education involving mathematics. Moreover, what happens in the middle school mathematics classroom is fundamental to the correlates of mathematical literacy that encourage further study in mathematics and related attitudes such as “self-confidence, curiosity, feelings of interest and relevance, …and the desire to do or understand things that contain mathematical components” (OECD, 2003, p. 26). Problematically, there exists a “condition” of middle school mathematics that results in “discontinuities” at the interface of senior schooling and middle schooling (Ridd, 2004, cited in McPhan et al., 2008, p. 20). The inconsistencies contained within the curriculum and between middle school mathematics pedagogy and curriculum were identified by McPhan et al. (2008). These include: a lack of academic rigour, the insufficient treatment of fundamental mathematical ideas, and inaccessible content resulting in limited opportunities for student ownership of tasks. Carrington (2002, p. 12) also identified these deficits and suggests that greater attention be accorded to developing “stronger pedagogical knowledge, repertoires and intellectual rigour” within the middle school classrooms. In addition, Stacey (2003) identified that the middle school mathematics classrooms in Australia were steeped in what she refers to as the “shallow teaching syndrome”, where the focus was on high repetition of low complexity problems and on students following procedures without reasons (cited in DEEWR, 2008, p. 30).

The effect of the inconsistencies identified by McPhan et al. (2008) is evidenced by Belward, Mullamphy, Read and Sneddon (2007) who discuss the decline over the last “10 to 15 years in the mathematical ability” (p. 842) of students entering university courses requiring mathematics. One of the factors they discuss as contributing to this decline is what they believe is a “lack of consistent mathematics background from secondary school” (p. 843). They surmise that the reform efforts in mathematics education that focus on making the mathematics curriculum more palatable to students through an emphasis on real-life situations detracts from learning the essential core knowledge and procedures in mathematics. Correspondingly, the comparative study conducted by An, Kulm and Wu, (2004) into mathematics teachers' pedagogical content knowledge in China and the United States found a marked difference and a resulting impact on teaching practice. In terms of teaching practice, the study found that the Chinese mathematics teachers relied on traditional practices and reasoning to develop mathematical procedures and conceptual knowledge. The teachers in the USA focused on activities promoting creativity and inquiry to develop concepts in mathematics and this resulted in what the authors described as a "lack of connection between manipulatives and abstract thinking and between understanding and procedural development" (An et al., 2004, p. 170). Indeed, it seems a balance needs to be returned to the mathematics curriculum. This balance would see a focus on developing conceptual understanding of key ideas and skills in mathematics, nurtured within a range of situations from real-life to purely mathematical. It has been established that suitably qualified, effective mathematics teachers are the critical catalyst schools require to bring about such a balance (Hayes et al., 2006).

The literature surmises that effective mathematics teachers require the ability to intersect their content knowledge with pedagogical techniques (Ball, 2000). That is, teachers must have "well thought out, conceptually sound, and rigorous approaches to teaching" mathematics if they are "to enable students to construct their knowledge" (Jasman & Martinez, 2002, p. 9). Significantly, the literature does indicate a somewhat paradoxical notion that teachers with advanced coursework degrees in mathematics are not inevitably efficacious in the mathematics classroom. In fact, research by Ball et al. (2001, cited in Ingvarson et al., 2004, p. 19) and Wilson and Floden (2003, cited in Ingvarson et al., 2004, p. 19) suggest that a higher level of exposure to traditional teaching techniques in mathematics "may actually imbue teachers with pedagogical images and practices that hinder their teaching", so much so that they are unable to "unpack mathematical content for students" (Ingvarson et al., 2004, p. 19). Therefore, "it is not just what mathematics teachers know, but how they know it and what they are able to mobilize mathematically in the course of teaching" (Ball, 2000, p. 243) that is important. A recurring trend within the literature is that effective teachers can understand the mathematics from diverse pedagogical perspectives so that they may respond effectively to the diverse needs presented by students in the classroom.

At the nucleus of restoring quality teaching to middle school mathematics is the consideration of how the essential knowledge base of the mathematics teacher is encapsulated in Shulman's (1986) conception of Pedagogical Content Knowledge (PCK). In the document, Numeracy: Lifelong Confidence with Mathematics – Framework for Action 2007 – 2010, Education Queensland identifies teacher knowledge and pedagogy as one of the four key priorities, fundamental to improving students' outcomes in mathematics and numerical confidence. The Mathematics in Australian Reform-Based Learning Environments (MARBLE) project in Tasmania also identified pedagogical content knowledge as an essential focal point within curriculum reform (Watson, Beswick, Brown, Callingham, 2007). Numeracy leadership is another key priority within the Education Queensland action plan. Here, teachers are encouraged to build their capacity as change agents by taking the challenge to engage in collaborative research partnerships so that they can have greater input into curriculum design, inform planning and bring the desired balance back to mathematics education. However, it is important to understand and implement the priorities within the Education Queensland action plan through a synthesis of the micro and macro perspectives, to generate the desired mathematical proficiency of students. From a macro perspective, mathematics teachers need to be efficacious change agents to inform mathematics curriculum planning. For this to be meaningful, teachers require a profound understanding and insight into how students learn mathematics. Therefore, from a micro perspective, teachers must possess effective and adaptable pedagogical content knowledge.

2.9 Pedagogical Content Knowledge

The notion of Pedagogical Content Knowledge has been built upon since Parr (1888) initiated the idea that in each subject there exists a special type of knowledge required for instruction (Bullough, 2008). Shulman (1986) built upon this initial conceptualisation of PCK in consultation with colleagues in the *Knowledge Growth in Teaching* project. This project examined how teachers acquire and develop their subject matter expertise and how this impacts their teaching. The project was innovative at that time since it shifted the perspective of teacher development and education from a narrow focus on classroom management and lesson organisation to the examination of the intersection of pedagogy and content. It is important to note that Shulman (1986) highlighted that in terms of research in teacher education the pendulum has always been swinging between focusing on content or pedagogy. In contrast, the *Knowledge Growth in Teaching* project examined PCK through a single knowledge lens, synthesising the three knowledge bases: subject matter knowledge, pedagogical knowledge and knowledge of context. In this way the project strove to address a perceived imbalance by properly blending teacher capacities in pedagogy and content. Shulman (1986, p. 6) suggested that

> The key to distinguishing the knowledge base of teaching lies at the intersection of content and pedagogy, in the capacity of a teacher to transform the content knowledge he or she possesses into forms that are pedagogically powerful and yet adaptive to the variations in ability and background presented by the students.

Since Shulman's (1986) work with PCK, the approach has been broadened to encompass particularities within subject disciplines. This is best summarised by Ball et al. (2007, p. 3):

> The continuing appeal of the notion of pedagogical content knowledge is that it bridges content knowledge and the

> practice of teaching, assuring that discussions of content are relevant to teaching and that discussions of teaching retain attention in content.

The research literature acknowledges the importance of PCK, pursued through a practical, subject specific approach.

In mathematics, PCK is the type of content knowledge distinguishing the mathematics teacher from the mathematician, statistician or accountant. Many authors concur that knowing mathematics well does not necessarily qualify one to teach (Ball et al., 2007; Battista, 2001; Shulman, 1986; Ticha & Hospesova, 2006; Turnuklu & Yesildere, 2007). This specific characteristic was brought to the fore by Shulman (1986, p. 13) who suggested that

> the teacher is not only master of procedure but also of content and rationale, and capable of explaining why something is done. The teacher is capable of reflection leading to self knowledge, the metacognitive awareness that distinguishes draftsman from architect, bookkeeper from auditor. A professional is capable not only of practising and understanding his or her craft, but of communicating the reasons for professional decisions and actions to others.

It is the distinction between "knowing to teach mathematics" and "knowing mathematics" that characterises an effective mathematics teacher (Turnuklu & Yesildere, 2007, p. 1). The notion of subject matter knowledge or mathematical content knowledge MCK for teaching is the "single factor which seems to have the greatest power to carry forward our understanding of the teacher's role" (Elbaz, 1983, p. 45, cited in An et al., 2004, p. 146). This is supported by Bromme (1994, p. 75) who stated: "the fusing of knowledge coming from different origins is the particular feature of the professional knowledge of teachers as compared with the codified knowledge of the disciplines in which they have been educated". Furthermore, "within a given context, teachers' knowledge of content interacts with the knowledge of pedagogy and students' cognition and combines with beliefs to create a unique set of knowledge that drives classroom behaviour" (Fennema & Franke, 1992, p. 162). Indeed research suggests that teachers with more mathematical content knowledge facilitate improved learning opportunities for their students and therefore improved problem solving performance (Swafford, Jones, & Thornton, 1997). While the research acknowledges the importance in the distinction of the content knowledge of the mathematics teacher, it requires broader recognition if it is to have the desired impact on mathematics education.

An essential feature of mathematical content knowledge MCK is one of teachers developing a sense of "trajectory of a topic over time…to develop its intellectual core in students' minds and capacities so that they eventually reach mature and compressed understandings and skills" (Ball, 2000, p. 246). Shulman (1986, p. 10) touched upon this idea and suggested that teachers require a "familiarity with the topics and issues" within a subject area that spans the years. A recent study by Ball, et al. (2007, p. 42) highlights the importance and possible scope of "horizon knowledge". They define this as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 42). As suggested by Papick (2011, p. 389) "mathematics teachers should deeply understand the mathematical ideas (concepts, procedures, reasoning skills) that are central to the grade levels they will be teaching and be able to communicate these ideas in a developmentally appropriate manner". An example of this is in the development of a student's understanding of fractions. A study conducted in Finland on the development of understanding and self-confidence in mathematics (Grades 5-8) found that "fractions become an increasingly important predictor for future achievement" in mathematics (Hannula, Maijala & Pehkonen, 2004, p. 23). Mathematics teachers need to be acutely aware of how to allow students to develop a clear and comprehensive understanding of fractions. Furthermore, students should develop the skills to manipulate fractions as a fundamental, mathematical conceptual tool. The notion of horizon knowledge reinforces the urgent requirement to have specialist mathematics teachers within the middle school, who are acutely aware of how mathematics topics relate to further learning and real life contexts (Chadbourne, 2001; McPhan, et al., 2008; Prosser, 2006). The literature acknowledges the need for further research into horizon knowledge and its implications for mathematics education.

The literature describes PCK in terms of various components. However, the aim of the tools of effective PCK is about creating opportunities for students to become mathematically literate in order to "process, interpret and communicate numerical, quantitative, spatial, statistical" and "mathematical information, in ways that are appropriate for a variety of contexts, and that will enable them to…participate effectively in activities that they value" (Evan, 2000, cited in Hoogland, 2004, p. 1). Effective PCK has the potential for teachers and students to feel mathematically powerful (Lott Adam, 1997). Power in this respect "generates responsibility and responsibility encourages and enables children to engage in meaningful learning experiences" (Lott Adams, 1997, p. 2), and this encourages teachers to create meaningful learning experiences, enhancing their curriculum leadership potential.

2.10 Mathematical pedagogical content knowledge for mathematical literacy

The document produced by Education Queensland Numeracy*: Lifelong Confidence with Mathematics – Framework for Action 2007 – 2010*, places the PCK of teachers as a key priority. The desired improvement in the PCK of teachers is to facilitate the successful implementation of appropriate strategies to promote the mathematical literacy and confidence of students in the classroom and beyond. However, some of the research literature acknowledges that mathematical literacy is "hard to acquire and hard to teach" (De Lange, 2002, p. 78). Part of the problem is in the definition of what it means to be mathematically literate. De Lange (2002) discusses the term mathematical literacy and how it is tossed around and narrowly defined by some, with too great an emphasis on the quantitative aspects of mathematics. A broader approach needs to be considered when mathematical literacy is defined in order to encompass “basic mathematical literacy [that is] a level expected of all students up to age 15 or so, independent of their role in society” and “advanced mathematical literacy” defined by how students need to fit into their community of practice (De Lange, 2002, p. 81).

Mathematical literacy involves a crucial capacity to use mathematical knowledge to creatively respond to a variety of non–routine, real life situations relevant to an individual’s life. Romberg (2001, p. 5) refers to the “interplay” between the ideas and procedures of mathematics and its functions as being able to “mathematise”. A starting point for mathematising in the classroom is to consider the preconceptions and misconceptions of students, since “if their initial understanding is not engaged, they may fail to grasp new concepts and information that are taught, or they may learn them for the purpose of the test but revert to their preconceptions outside the classroom” (Romberg, 2001, p. 8). Additionally, the challenge for the teacher is “how to create classroom experiences so that a student’s understanding grows over time?” (Romberg, 2001, p. 8). The OECD (2003) acknowledges mathematisation as a fundamental process that educators should put into practice to improve the mathematical literacy of their students. The mathematisation cycle framework (OECD, 2003, p. 38) is described for teachers in the following way:

1. Start with a problem situated in reality;
2. Organise it according to mathematical concepts and identify the relevant mathematics;
3. Gradually trim away the reality through processes such as making assumptions, generalising and formalising, which promotes the mathematical features of the situation and transforms the real world problem into a mathematical problem that faithfully represents the situation;

4. Solve the mathematical problem; and
5. Make sense of the mathematical solution in terms of the real solution, including identifying the limitations of the solution.

Specifically though, this framework appears to fit with the mathematical modelling cycle which is described by Yoon, Dreyfus & Thomas (2010, p. 144) as follows:

> The modelling cycle begins in the real world, where one determines which pieces of information in the real context are relevant to the problem. Next, one interprets the relevant information in the real world mathematically to create a mathematical model. This model is then used to find a mathematical result, which is in turn interpreted back into the real world context. The fitness of the model is then assessed, and if necessary, the cycle begins again in pursuit of a model with a better fit.

Furthermore Yoon, Dreyfus and Thomas (2010, p. 145) characterise mathematisation as “interpreting the structural aspects (i.e. the objects, relations, actions, patterns, regularities, assumptions etc.) in a real world system, and expressing this structure in a mathematical model using mathematical representations”.

In order to forge opportunities for mathematisation, teachers should have the mathematical foresight and confidence in their pedagogical skill to depart from the traditional daily classroom routines that Romberg (2001, p. 8) discusses as consisting of three segments, “a review, presentation, and study/assistance”. Teachers whose classrooms revolve around these routines tend to “rely on unmodified subject matter knowledge most often directly extracted from the text or curriculum material” (Turnuklu & Yesildere, 2007, p. 11) and “tend to make broad pedagogical decisions without assessing students’ prior knowledge, ability levels, or learning strategies” (Cochran, 1997, p. 2). Research evidence indicates that teachers must have an extensive knowledge of mathematics teaching, curriculum and content so that they can “make transformation from one form to another” (An et al., 2004, p. 148). It can be expected that the interactions between a teacher and their students become mathematically powerful (Lott Adams, 1997) when the teacher is pedagogically confident in their mathematical ability.

Mathematics teachers need guidance and collaborative, professional support in order to articulate the integration of their subject matter knowledge and pedagogical techniques. The qualitative study by Ball, Thames and Phelps (2007) *Content Knowledge for Teaching: What Makes It Special?* examined the mathematical demands of teaching and creating classroom environments conducive to mathematising. The authors admit their surprise at how much “purely mathematical knowledge was required” by mathematics teachers, even in “everyday tasks” such as “assigning students work, listening to student talk, grading” (p. 30) or when “considering what numbers are strategic to use in an example” (p. 29). A consensus has been emerging among researchers and teacher educators that developing mathematics teachers’ PCK requires a greater focus on core mathematical reasoning (An et al., 2004; Ball, et al., 2007; Battista, 2001; Koirala, Davis & Johnson, 2007; Turnuklu & Yesildere, 2007).

From their study, Ball et al. created a practical set of domains of “content knowledge for teaching” (2007, p. 42) mathematics, embedding within it Shulman’s (1986) initial categories of subject matter knowledge and pedagogical content knowledge. The domains elaborated upon Shulman’s (1986) work and concentrated on the act of teaching more so than the PCK dimensions presented by earlier researchers such as Fennema and Franke (1992, cited in Turnuklu & Yesildere, 2007) and Bromme (1994). The domains (Ball et al., 2007) may be summarised as follows:

Domain 1: Common Content Knowledge (CCK). This is the “mathematical knowledge and skill used in settings other than teaching” (but still required by teachers) (p. 32).

Domain 2: *Specialised Content Knowledge (SCK*). The authors were particularly interested in this domain since it was the “mathematical knowledge and skill uniquely need by teachers in the conduct of their work” requiring “unique mathematical understanding and reasoning” (p. 34). Furthermore, the authors suggest that “teachers require knowledge beyond what is being taught to students” (p. 34). Examples of what teachers routinely need to be able to do in the classroom are (Ball et al., 2007, Figure 3, p. 34):

- Presenting mathematical ideas.
- Responding to students’ ‘why’ questions.
- Finding an example to make a specific mathematical point.
- Recognising what is involved in using a particular representation.
- Linking representations to underlying ideas and to other representations.
- Connecting a topic being taught to topics from prior or future years.
- Explaining mathematical goals and purposes to parents.
- Appraising and adapting the mathematical content of textbooks.
- Modifying tasks to be either easier or harder.
- Evaluating the plausibility of students’ claims (often quickly).
- Giving or evaluating mathematical explanations.
- Choosing and developing useable definitions.
- Using mathematical notation and language and critiquing its use.
- Asking productive mathematical questions.
- Selecting representations for particular purposes.
- Inspecting equivalencies.

Domain 3: Knowledge of Content and Students (KCS). This domain refers to the requirement of teachers knowing and anticipating student conceptions and misconceptions (p. 36).

Domain 4: Knowledge of Content and Teaching (KCT). This final domain involves teachers "sequencing particular content" and "making instructional decisions". The task of teaching mathematics thus requires "an interaction between specific mathematical understanding and understanding of pedagogical issues that affect student learning" (p. 38).

The comprehensive domains presented by Ball et al. (2007) would be a useful foundation in a collaborative, professional learning process for mathematics teachers aiming to enhance the micro perspective of their numeracy leadership as desired in the Education Queensland Action plan.

The aforementioned domains are a step towards creating the desired "coherent theoretical framework" (Shulman, 1986, p. 9). However, "the bridge between knowledge and practice remains inadequately understood…and underdeveloped" (Ball, et al., 2007, p. 3). The report by Ball et al. (2007) indicates that despite the concept of PCK shaping contemporary research, its "potential remains insufficiently exploited" (p. 4), with the conceptual core initiated by Shulman and his colleagues not evolving in ways to effectively enhance practice. Indeed, "what seems most important is knowing the mathematics actually used in teaching" (Ball et al., 2007, p. 45). Correspondingly Mason and Spence (1999, cited in Potari, Zachariades, Christou & Pitta-Pantazi, 2008, p. 2) suggest that "mathematical and pedagogical knowledge constitutes not only knowing that, knowing how, knowing why but also knowing to act and knowing to act in the moment". Hence, teachers need to understand more than the topics in mathematics. They need to "support and optimize" (Battista, 2001, p. 29) the students' construction of mathematical ideas through effective classroom practices.

The *Realistic Mathematics Education* (RME) (Freudenthal, 1991, cited in Zulkardi, 2004, p. 2), framework has the potential to evolve the PCK of teachers and enhance practice to support the "constructive processes" (Battista, 2001, p. 29) involved in mathematics. RME advocates that mathematics must be close to the life world of the students. "The word, 'realistic', refers not just to the connection with the real world, but also refers to problem situations which are real in students' minds" (Zulkardi, 2004, p. 2). Lott Adams, (1997, p. 2) advises that this "relevancy, gives children a platform from which they can construct their own mathematical knowledge". The organisation of mathematics education in this way involves a process of "guided reinvention" (Zulkardi, 2004, p. 2). Treffers (1987, cited in Zulkardi, 2004, p. 3) discusses the use of "horizontal and vertical mathematisation" within the RME framework.

Horizontal mathematisation involves students devising mathematical strategies that allow them to conceptualise and solve a real life situation. Open - ended investigations and the effective use of oral and written communication in the classroom are avenues for horizontal mathematisation to occur. These tasks encourage mathematical literacy since students have opportunities to: describe; identify; formulate and visualise the mathematical problems in their own way; discover relations and regularities; recognise isomorphisms in different problems and transfer real life problems into mathematical problems (Romberg, 2001; Zulkardi, 2004). Vertical mathematisation involves moving within the world of mathematical symbols. Teaching students to independently read and interpret the mathematics is a catalyst in this process. Students need to gain the autonomy to confidently represent a situation using formulas, refine models and ultimately make mathematical generalisations (Zulkardi, 2004).

The process of reinvention involves using the activities of horizontal mathematisation to gain a model and then vertical mathematisation to end up with a mathematical solution involving strategies that may be applied to other purely mathematical or real life problems. RME differs from a constructivist approach, since it is a guided reinvention, where teachers require profound PCK to didactically guide students through the levels of thinking required. Students should feel ownership of the mathematical concepts and this initiates the mathematical power required to enhance mathematical literacy and the self-efficacy to be mathematically confident (Lott Adams, 1997).

There are many strategies available to teachers that support the development of effective PCK in order to generate opportunities for their students to mathematise. At the core of this is that teachers acknowledge the ongoing development of their classroom practice. Classroom practice specific to mathematics teaching and learning involves more than just mathematical knowledge and skill (Ball, 2003). Mathematical practice refers to

> mathematical representation, attentive use of mathematical language and definitions, articulated and reasoned claims, rationally negotiated disagreement, generalizing ideas, and recognizing patterns.
> (Ball, 2003, p. 30)

Problematically though, Ball (2003) hypothesised that these mathematical practices are not consistently fostered in the mathematics classroom. Consequently, this undermines the capacity of the classroom learning community to improve the students' mathematical proficiency. Thus, in order to uncover and understand how students might choose and use mathematics proficiently, Ball (2003, p. 36) suggests that there needs to be an "emphasis on investigating mathematical practices…to design systematic opportunities for students (and teachers) to develop the learning resources needed to build a system" to implement meaningful learning opportunities that involve learning and doing mathematics as a social activity in the classroom. Certainly, examining the classroom practices of the teachers and how they use their PCK as a resource to furnish meaningful learning opportunities for the classroom learning community is a focus that would "yield crucial insights that are needed to close the broad gap between those few who become mathematically proficient and the many who do not" (Ball, 2003, p. 35).

However, while it may be acknowledged that the PCK of the teachers is essential, there needs to exist within schools a collaborative, supportive environment where teachers build positive cultures around an automatic process of action and reflection so that they can use their PCK in effective ways to enhance students' mathematical proficiency. This process is crucial to teachers becoming numerate leaders and effective change agents within the diverse embedded contexts that exist within the education system.

2.11 Reflection to improve pedagogical content knowledge

In an effort to bridge the gap between mathematical knowledge and practice, and improve the quality of the interactions that occur within the classroom, the literature on PCK ranks critical pedagogical reflection as one of the key facets of teacher competence and professionalism (An, et al., 2004; Brookfield, 1995; Cochran, 1997; Goodell, 2000; Mundry, 2005; Shulman, 1986; Turnuklu & Yesildere, 2007). Participation in effective reflection facilitates opportunities for teachers to shift from perceptions to deliberate and rationalised action: "it is possible to treat reflection connected with interpretation of teaching/learning situations as the best way to develop the teachers' professional way of thinking and to present practical didactical theory" (Slavik, 2004, p. 1 cited in Turnuklu & Yesildere, 2007, p. 133). Cochran (1997, p. 2) describes critical reflection and interpreting subject material as crucial, so that the teacher:

> finds multiple ways to represent the information as analogies, metaphors, examples, problems, demonstrations, and/or classroom activities; adapts the material to students' developmental levels and abilities, gender, prior knowledge, and misconceptions; and finally tailors the material to those specific individuals or groups of students to whom the information will be taught.

Moreover, as initiated by Shulman (1986), the teaching process involves transforming subject knowledge so that it may be accessible to the student.

The research encourages teachers to allow their PCK to remain in a state of flux so that it may be continually restructured. Varying contexts in mathematics education mean there is no singular or distinct best method of teaching. Thus, the process of reflection enables teachers to utilise their "wisdom of practice" to create a "veritable armamentarium of alternative forms of representation" (Shulman, 1986, p. 9) and allows the integration of PCK to be constructed along a "continuum" (Veal & MaKinster, 1999, p. 11). In their report, An et al. (2004, p. 149) suggest that teaching can be considered as a "divergent" or "convergent" process. A divergent approach to teaching is described as being centred on curriculum and subject knowledge. Teaching as a convergent process places the student at the focus, where the teacher reflects on students' thinking in the classroom. Teaching for understanding involves: "building on students' mathematical ideas, addressing students' misconceptions, engaging students in mathematics learning, and promoting students' thinking mathematically" (An et al., 2004, p. 149). This reflective, convergent process has the potential to span the divide between content and pedagogy and the "fragmentation of practice" which "leaves teachers on their own with the challenge of integrating subject matter knowledge and pedagogy in the context of their work" (Ball, 2000, p. 241). Some research literature places "qualified pedagogical reflection on the same level as the other kinds of competence and considers it as a determining feature of the teacher's professionalism" (Turnuklu & Yesildere, 2007, p. 133). Thus, teachers should allow for their pedagogical knowledge to be continually restructured by scaffolding on students' conceptions, while always spiralling around positive mathematical learning outcomes for their students.

2.12 Reflection for scaffolding students' prior knowledge

Many studies indicate that building on students' prior knowledge and "knowledge of students' cognitions" is "one of the important components of teacher knowledge" since "learning is based on what happens in the classroom" (Turnuklu & Yesildere, 2007, p. 2). Dickerson and Dawkins (2002, p. 1) highlight the importance of teachers understanding "subject matter and pedagogical theory well enough to be able to identify particular points within the subject matter that learners find easy or difficult, and go on to explain why in each case". As researchers have refined and revised Shulman's (1986) model of PCK, Cochran (1997, p. 2) identifies "teachers' knowledge of students' abilities and learning strategies…and prior knowledge of the concepts to be taught" as being "especially visible in the last decade due to literally hundreds of studies on students' misconceptions in science and mathematics". Indeed, in their treatise, "Taxonomy of PCK" attributes, Veal and MaKinster (1999, p. 10) assert that the knowledge of students "has more significance compared to pedagogical knowledge" and that only "after a teacher develops a solid understanding" of their students can they employ powerful pedagogical techniques "appropriate to the student, domain, or concept" (Veal & MaKinster, 1999, p. 10). Observations of Chinese mathematics classrooms by An et al. (2004, p. 166) found that "teachers spent at least one-third of the time reviewing prior knowledge", since they feel that "using prior knowledge not only helps students to review and reinforce the knowledge being taught but also helps them to picture mathematics as an integrated whole rather than as separate knowledge" (An et al., 2004, p. 165). Accordingly, Goodell (2000, p. 49) reinforces a common theme by suggesting that:

> understanding in mathematics requires action on the part of the learner in the form of making connections to other things she or he already knows; that teachers have a critical role in promoting understanding through the ways in which they organise classroom instruction and assessment; and that reflection is a vital part of this process.

It is important to note that pedagogy which embeds within itself a consistent, critical reflection on students' prior knowledge and conceptions, is crucial to shifting the stagnating "deficit view of students" (Prosser, 2006, p. 13) mentioned earlier in this review.

Certainly, the rise of student misconceptions in the mathematics classroom may be paralleled to Veal and MaKinster's (1999, p. 11) claim that content knowledge and knowledge of students are "embedded in one another because student errors and misconceptions are more easily recognized when a teacher knows the content topics and concepts". This is supported by Potari et al. (2008, p. 3) who claim that the mathematics teacher needs specialised mathematical knowledge to allow effective reflection upon students' solution methods and to "transform classroom communication to a real mathematical communication". Therefore, this reiterates the urgent need to have qualified, subject specialists within the classroom (Jasman & Martinez, 2002; Ingvarson et al, 2004; Lovat, 2003; McPhan et al., 2008).

One of the aims of mathematics education is to encourage students (and teachers) to be mathematically literate, life long learners. "The ability to exercise mathematical knowledge rests upon reflectivity" and teachers should aim to "provide opportunities [for students] to reflect on their mathematical learning" so that they can "create, ponder, and extend ideas in mathematics" (Lott Adams, 1997, p. 2). In order to fully appreciate what students are thinking, Davies and Walker (2007) explored the notion of *Teaching as Listening*. They investigated how four teachers listened to and interpreted students' ideas. This study evolved in response to education reforms calling for pedagogical practices to acknowledge students' articulations on mathematical ideas. A significant finding of their research was that

> the teachers' content knowledge became a central organiser for the lessons and a defining feature of effective teaching. The depth of teachers' content knowledge – both subject matter knowledge and pedagogical content knowledge – mediated their enactment of effective listening practices. (Davies & Walker, 2007, p. 236)

Furthermore, Davies and Walker (2007) found that teachers who weren't prepared to listen to and understand their students' thinking tended to "minimize or dismiss it, by imposing their own understanding" (p. 230). Correspondingly, An et al. (2004) found that teachers with effective PCK could extract distinct meanings from students' responses. This then allowed them to scaffold on students' prior knowledge to overcome the various challenges present in mathematical concepts. This evidence, once again places the PCK of the teacher as a powerful tool in realising positive mathematical literacy outcomes for students.

Broadening Davis and Walker's (2007) approach of Teaching as Listening, Steele (2005) uses discourse analysis to examine mathematical pedagogical conversations between teachers. Steele (2005) delineates the epistemological distinctions between pedagogy and mathematics and discusses the conditional nature of the act of teaching and reasoning about teaching. The examination of discussions between teachers found that "the knowledge base that one teacher develops may be vastly different from the knowledge base of another teacher" (Steele, 2005, p. 296). Moreover, Steele's (2005) findings reinforce that "while mathematics has accepted rules and structures for reasoning, pedagogy is an inherently interpretive act, where teachers' reasoning is filtered through their personal frame, built from their experiences and values in the classroom and their beliefs about teaching" (p. 321). Steele's (2005) discussion of pedagogy as an inherently interpretive act, provides some insight into why it has been difficult to fine tune the coveted "clearer sense of categories of content knowledge for teaching" (Ball et al., 2007, p. 46) discussed earlier in this review.

2.13 Professional development through joint reflection

The research literature validates the notion that reflection can be a useful professional development vehicle "promoting the teacher as a life long learner" (Veal & MaKinster, 1999, p. 11). Striving to teach for understanding by using reflective practice to develop their tacit knowledge encourages teachers to gather vital information to inform their practice so that "they will have ten years of experience rather than having one year of experience ten times over" (Goodell, 2000, p. 58). School-based professional development requires teachers to routinely and jointly investigate their teaching practices. However, this requires "systematic and systemic school district support" (Battista 2001, p. 30). Indeed, research suggests that in countries with students who are achieving comparably higher in mathematics than students in the USA, teachers are afforded a significant amount of time to work together on "joint planning and curriculum development, pursuing classroom-relevant research, participating in ongoing teacher-led study groups" and offering "demonstration lessons to one another" (Battista, 2001, p. 58). The key priority of Numeracy Leadership in Education Queensland's Framework for Action 2007- 2010 suggests that there needs to be structures in place to foster a culture that focuses on "professional development; resources, time and space for teachers to reflect on and be effective in their practices" (Education Queensland, 2007, p. 5). While support may be encouraged in terms of time and resources, teachers are the essential component, and therefore should understand the processes of this professional development and its possible merits.

A practical example proving to be mutually beneficial is placing novice teachers with mentor teachers in “content focused mentoring” (Mundry, 2005, p. 10). This “content focused mentoring” supports new teachers to “teach their specific curriculum and content and inducts them into the profession of teaching” and allows them to “deepen their understanding of the ideas students find confusing and reflect on how to adjust their instructional strategies” (Mundry, 2005, p. 10). This process also encourages the experienced teacher to acknowledge, deepen and expand upon their knowledge and practice through a deliberate process of action and reflection (Mundry, 2005). This is so important because McCann and Radford (1993, p. 25) propose that “the accumulation of tacit knowledge about teaching, the wisdom of years, is rarely made explicit by teachers, to themselves or to colleagues”. Rudduck (1987, p. 130, cited in Sellars & Francis, 1995, p. 29) supports this and notes that experienced teachers tend to become immersed in a “world of routine practice” and this reduces their capacity to “contemplate alternative courses of action”. Indeed, as Mundry (2005, p. 10) concisely states, “since teaching matters so much to student learning, veteran teachers, too, must continue to deepen their knowledge and skills throughout their careers”. Content focused mentoring between mathematics specialist teachers and generalist teachers gives some scope to filling the void existing in the middle school mathematics classroom.

To broaden the idea of joint reflection further, Turnuklu and Yesildere (2007) incorporate the role of the academic researcher into the school environment. The idea is to encourage and provide support for teachers to “learn how to inquire systematically into their pedagogical practice” (Battista, 2001, p. 30). However, the literature acknowledges that teachers often have low self-esteem and doubt their ability to effectively integrate joint reflection into their everyday tasks (Battista, 2001; Goodell, 2000; Hiebert & Stigler, 2004; Turnuklu & Yesildere, 2007). The culture of teaching places teachers in the “role of knowers and tellers working in isolation” and teachers are not accustomed to viewing themselves as “learners” (McCann & Radford, 1993, p. 39). Some teachers are reluctant to participate in discussions, and for some it is another time-consuming demand, contributing to work overload. Encouraging teachers to take risks and avoid the type of “teacher talk” which Brookfield (1995, p. 141) describes as a “swapping of mutually reinforcing prejudices, an experience of groupthink” appears to be critical starting point. Furthermore, Hargreaves (1992) suggests that teachers need to move beyond the contrived collegiality prevalent in subject department meetings. Indeed, the formation of teaching “teams” in the middle phase of schooling has “challenged the work histories and traditions of many experienced and not-so-experienced teachers” and has been one of the factors manifesting a “dip” in the implementation of the middle school reform transition phases (Main, 2007, p. 12). Thus, Turnuklu and Yesildere (2007, p. 151) go so far as to say that joint reflection is so demanding that “it can be realised only sporadically, may be only within a research context”.

On the other hand, Hiebert and Stigler, (2004, p. 3) insist that “teaching can only change the way culture changes: gradually, steadily, over time as small changes are made in the daily routines of teaching”. The processes involved in the weekly and daily routines of teachers: lesson planning, implementing, assessing and reflection were examined by Hiebert and Stigler (2004). What they discovered was that in order for change to occur, each phase of the teaching routine needs to be deliberately slowed down to allow for more cautious self-examination. Three suggestions were offered by Hiebert and Stigler, (2004) on how teachers can shift the stagnating culture. First was the reallocation of time for “studying and improving teaching in a systematic and continuing way” (Hiebert and Stigler, 2004, p. 3). Second was that teachers need to be provided with “vivid examples that illustrate alternative ways of teaching” (Hiebert and Stigler, 2004, p. 3). For example, the authors suggest that examining videos of teaching that show “exemplary practice” and “everyday teaching, with its missed opportunities” should be essential experiences for teachers (Hiebert & Stigler, 2004, p. 3). Third, they suggest that teachers need to learn how to critically analyse student work and draw conclusions from such analyses so that significant changes to teaching practices can be made. Hiebert and Stigler, (2004, p. 4) acknowledge that the suggestions they propose for “changing the culture of teaching to enable targeted changes in teaching practice assume many of the features recommended numerous times in the professional development literature”. However, to supplement this, they endorse that educators need to slow down, be retrospective and transform the familiar procedures to yield the desired impact on the fatigued culture of teaching.

2.14 Conclusion

The literature highlights the need to move to a multi-dimensional approach in the second phase of middle school reform. Certainly, developing academic rigour in the middle school mathematics classroom is vital in the provision of the next generation of students who have the mathematical proficiency to choose careers requiring higher-level mathematics skills. The research literature reveals the limited availability of suitably qualified teachers to teach mathematics in the middle school and its impact on students’ successful progression to higher-level mathematics courses.   A greater focus on core mathematical reasoning and effective pedagogical techniques that scaffold on students’ prior knowledge is identified within the literature as crucial to ameliorating the status of mathematics in the middle school.

This literature review examined the concept of pedagogical content knowledge with the intent of using it to empower teachers to become life long learners through self-perpetuated professional development. Strategies such as content focused mentoring and reflection are examples used to exemplify how teachers can intersect and transform their knowledge in teaching, content and curriculum to create specialised mathematics knowledge that can be used effectively in the classroom and curriculum leadership. Clearly the research literature encourages the next stage of middle schooling to build upon the conceptual framework of the initial reform through the consideration of new contexts. One such context involves emboldening teachers to continually reactivate their pedagogical goals to enhance their practice and bring academic rigour to the mathematics classroom. This academic rigour is essential for students to be mathematically powerful, individually and collectively.

Chapter 3

Methodology and Research Design

Qualitative inquiry cultivates the most useful of all human capacities – the capacity to learn from others.

Halcolm's Evaluation Laws

3.1 Introduction

This chapter discusses the methodology and methods of this qualitative case study. Both methodology and methods are defined since they are taken to mean different things within this research. The methodology refers to the theoretical framework and epistemological assumptions that underpin this study. Method refers to the specific techniques of the case study and hence describes the use of observations and interviews to facilitate data collection. While the methodology and methods are specifically defined, the two discussions are blended within this chapter since the methods "are consistent with the logic embodied in the methodology" (Bogdan & Biklen, 2007, p. 35). Furthermore, the methodology and methods are entwined with the discussion on analytical techniques into an axis which focuses on understanding in context.

3.2 Qualitative case study research: methodological lens

The aim of this research was to understand how the pedagogical content knowledge (PCK) (Ball, et al., 2007) of the mathematics teacher steers the intellectual engagement of students into doing rigorous mathematics in the middle school classroom. This epistemological problem was approached by attempting to better understand how the teacher and students are mutually engaged in a "learning community" (Wegner, 1998, p. 214). The mathematical interactions within the middle school classroom are viewed within this study as contributing to the unfolding and reforming of students' mathematical identities and thus their mathematical dispositions. How students are guided by the teacher to work and think mathematically to construct their mathematical knowledge is the focus of this research. Intellectually engaging, quality mathematical interactions in the middle school classroom have the potential to empower students into being efficacious and proficient doers of mathematics with productive mathematical dispositions.

A social constructivist view to teaching and learning is a critical feature within this research framework. This is a salient feature in the current context of mathematics education within Australia. The research literature emphasises the importance of students having the "opportunity to 'create' mathematics concepts and link them to existing concepts for themselves" (DEEWR, 2008, p. 27). From this perspective, neither the teacher, nor the students, should be the focus of the classroom learning community; but rather, this important position is taken by the "Great Thing": which in the mathematics classroom is the mathematics concept (Palmer, 1998, cited in Neuenschwander, 2000, p. 94). Effective teaching may be characterised in the following way:

> Good teachers replicate the process of knowing by introducing students into this community centred on the Great Thing. Bringing the students into the ongoing conversation occurs through many pedagogies, traditional and experimental…the "grace of the Great Thing" provides the "plumb line" by which experts and novices alike are measured. As they confront this power beyond themselves, teachers recall the passions that originally drew them to the subject, and students are exposed to a world larger than their own experience and egos.
> (Palmer, 1998 cited in Neuenschwander, 2000, p. 94)

Indeed, effective teaching that promotes the emancipation of students in the mathematics classroom so that they may "stand outside the teacher's authority on forms of knowledge…to discover and own it for oneself" (Hopkins, 2008, p. 2) foreshadowed the commitment of this research.

A qualitative approach was chosen for this case study to gain a holistic understanding of the epistemological problem. Certainly in the domain of gaining insight into “learning as social participation” (Wegner, 1998. p. 4) within the middle school mathematics classroom the “interpretivist” (Howe, 2003, p. 2) epistemological view appeared appropriate. This qualitative stance considered the “value” side of knowledge which included “irrationality, politics, ends, interests, subjectivity and power” (Howe, 2003, p. 2). Merriam (1998, p. 4) distinguishes interpretive research in education as gaining knowledge by “understanding the process or experience” within schools settings. It is this “value-ladeness” that Howe (2003, p.2) suggests is “especially salient in social and educational research, whose vocabulary is rooted in the description of social practices and whose aim is to evaluate and improve such practices”. In this sense, the contextual distinction of this qualitative case study was taken as an opportunity to “establish an empathetic understanding” (Stake, 1995, p. 39) of the middle school mathematics classroom. The microscopic view of this research facilitated the rich description of a single school context and might establish parameters that could be used in further research.

This research attempted to better understand how the social and mathematical norms of the middle school classroom contributed to a student’s mathematical disposition, a key strand of students’ mathematical proficiency, as defined by Kilpatrick, et al. (2001). A student’s mathematical disposition and the development of a belief that they are doers of mathematics is the “strongest predictor of mathematics performance, stronger than general mental ability, and also stronger than intrinsic motivation” (Stevens, Olivariz Jr & Hamman, 2006, cited in DEEWR, 2008, p. 50).

The social constructivist view to learning regards “intellectual autonomy” as a “student’s way of participating in the practices of a classroom community” (diSessa & Cobb, 2004, p. 94). This study placed an emphasis on understanding the goal of mathematics education that is focused on developing students' mathematical proficiency, by contemplating how their intellectual and social autonomy might cultivate their productive dispositions. From this social constructivist epistemology, the formation of students' mathematical dispositions is contingent on and "formed in norm-sanctioned social encounters" (Howe, 2003, p. 5). Thus, this research was undertaken from the "tranformationist" (Howe, 2003, p. 5) view which implies that facts and values may both have to be used in a constructivist epistemology. Indeed, there are "forms of normalisation that are good and the practice of education should promote" them (Howe, 2003, p. 5). One perspective of norms of practice that involve facts and values in the mathematics classroom are described by the complementary activities involved in the *mathematical norms* and *socio-mathematical norms* (Cobb & McClain, 2001).

3.2.1 Theoretical framework

This case study used an interpretive framework from the work of Cobb and colleagues (Cobb & McClain, 2001; Cobb, Stephan, McClain, Gravemeijer, 2001; diSessa & Cobb, 2004; McClain, 2002). This framework (Table 2, p. 61) involves the psychological and social perspectives of classroom norms. The mathematical norms within the mathematics classroom are the mathematical reasonings and interpretations from which procedures, tools and facts of mathematics can be cultivated The socio-mathematical norms are described as including: "what counts as a different mathematical solution, a sophisticated mathematical solution, an efficient mathematical solution, and an acceptable mathematical explanation and justification" (McClain, 2002, p. 218) within the mathematics classroom learning community. Moreover, the socio-mathematical norms include "the usual practices, organisational procedures and modes of communication that impact on approaches to learning, types of responses valued, views about legitimacy of knowledge produced, and responsibilities of individual learners" (DEEWR, 2008, p. 28). The classroom social norms do not have a mathematical focus. However, they are included since they are a general norm within classroom learning communities and contribute to students' mathematical habitus (Zevenbergen, 2004). Indeed using the interpretivist framework of the social and psychological perspective within the classroom learning community acknowledges that habitus is the “embodiment of culture and provides a lens through which the world is interpreted” (Zevenbergen, 2004, p. 202).

3.2.2 The reflexivity of the social and psychological perspectives

The columns in Table 2 (p. 61) offer two perspectives from which the analyses of the norms are developed. As discussed by McClain (2002, p. 218), the social perspective column offers “three aspects of the classroom microculture” that are useful in the analysis of the classroom as a learning community. The psychological perspective column suggests that the formation of students’ identities is influenced by the social interactions within the classroom. The arrows within this framework indicate that the social and psychological perspectives are viewed “as reflexively related in that one does not exist without the other” (McClain, 2002, p. 218). The psychological and social perspectives recognise that ‘learning as participation’ has “broad implications for what it takes to understand and support learning” (Wegner, 1998, p. 7). Viewing the reflexivity between the two perspectives in this interpretivist framework recognises that students’ habitus “predisposes” but does not determine their “thoughts, actions and behaviours” (Zevengergen, 2004, p. 202).

Analysing the psychological and social perspectives of the norms of the classroom explicitly acknowledges that “learning is an issue of engaging in and contributing to the practices” of the classroom microculture (Wegner, 1998, p. 7). Furthermore “it means that learning is an issue of refinement of knowledge and practice” (Wegner, 1998, p. 7). The social perspective frames an individual student’s or teacher’s thinking as an “act of participation” within the “normative activities” of the classroom (Cobb et al., 2001, p. 119). On the other hand, the psychological view focuses on the individual’s own interpretations and adjustments to thinking as they adjust their particular ways of participating in the learning community and thus develop as autonomous doers of mathematics. However, the social perspective and psychological perspective are reflexive since individual learning is affected by, and contributes to, the ongoing regeneration of classroom norms. Taken together, the two perspectives are coordinated to treat the construction of mathematical knowledge as a synthesised process.

Table 2: Interpretive Framework of the classroom norms (McClain, 2002, p. 219)

SOCIAL PERSPECTIVE		PSYCHOLOGICAL PERSPECTIVE
Classroom social norms	⇔	Beliefs about own role, others' role and the general nature of mathematical activity
Socio-mathematical norms	⇔	Mathematical beliefs and values (self-efficacy…feel mathematically autonomous, a doer of maths)
Classroom mathematical norms (classroom practice)	⇔	Mathematical interpretations and reasoning

Maintaining the classroom as a learning community brings with it inherent tensions for the teacher since they need to "be constantly judging the nature and quality of the students' contributions against the mathematical agenda to ensure that the issues under discussion offer means of supporting students' development" (McClain, 2002, p. 218). Using the socio-mathematical norms acknowledges that the mathematics does not stand separately from the learning community. In supporting the idea of students becoming doers of mathematics, the conjecture (Cobb et al., 2001) is that classroom practices should encourage the active engagement of students into judging, justifying and arguing about the quality of, and differences between, mathematical solution methods. From this perspective, an active learning community is formed rather than a learning space that is controlled by the "authority of the teacher or the textbook" (Cobb et al., 2001, p. 124). Thus, the socio-mathematical norms have been used as a focus point in this research to investigate how the teacher and students are mutually engaged in quality mathematical interactions.

This case study converged particularly on analysing and understanding the social and psychological perspectives of the socio-mathematical norms. The "psychological correlates of the socio-mathematical norms are taken to be the specifically mathematical beliefs and values that constitute what might be termed student's mathematical dispositions" (McClain, 2002, p. 222). Also, the socio-mathematical norms recast the idea of intellectual autonomy as "a characteristic of a student's way of participating in the practices of a classroom community" (diSessa & Cobb, 2004, p. 94). The mutual engagement that is the hallmark of effective learning communities (Wegner, 1998) is embedded in the construct of socio-mathematical norms in the following way:

> The link between the growth of intellectual autonomy and development of classrooms in which mathematics is realised as a form of inquiry is readily apparent given that in such classrooms, the teacher and students together constitute a community of validators.
> (diSessa & Cobb, 2004, p. 94)

This focus was chosen since the aim of the research was to gain a greater awareness of the *understandings in action* for the students and the teacher as they contribute to the establishment and re-establishment of socio-mathematical norms. For the teacher this potentially augments their PCK and for the students this contributes to the productivity of their mathematical dispositions to continue with doing mathematics at a higher level.

3.2.3 Defining the social constructivist view in the middle school mathematics classroom

A transformationist perspective would suggest that the socio-mathematical norms influence the "normalising" processes that set up a student's view and formation of "self" (Howe, 2003, p. 74). The classroom normalising processes, where students and teachers negotiate and judge what constitutes valid mathematical contributions and the specific "mathematical beliefs and values they entail, capture and specify much of what is implied by the notion of mathematical disposition, a major focus of reform recommendations" (diSessa & Cobb, 2004, p. 94). Hence, when teachers use their PCK to initiate and negotiate the socio-mathematical norms, they are concurrently inviting students to be doers of mathematics. The distinction that needs to be made, however, is that this normalising process within the mathematics classroom does not imply that students are passive recipients of mathematical ideas. Rather, they are participants within a community of learning (Wegner, 1998) that encourages intellectual autonomy. Indeed, socio-mathematical norms are "joint accomplishments" and "students develop their personal classroom identities as they contribute to (or oppose) the ongoing regeneration of the normative identity as a doer of mathematics" (diSessa & Cobb, 2004, p. 97). A socio- constructivist view to learning promotes that knowledge is "actively constructed" (Howe, 2003, p. 6) by the students.

For the purposes of the establishment of positive socio-mathematical norms within the middle school mathematics classroom, this research assumes a post-Kantian position to constructivism and refers to "Wittgenstein's philosophy of mathematics" (Howe, 2003, p. 85). Wittgenstein's "naturalised epistemological constructivism eschews a given in the traditional sense" (Howe, 2003, p.85). That is, there is an assumption of shared meanings of mathematical ideas and procedures, around which socio-mathematical norms are moulded. It is in accepting and understanding the "constitutive meanings" (Howe, 2003, p. 92) or mathematical norms that promotes participation within the classroom. For example, the definition of an irrational number as a number that cannot be expressed as a ratio of two integers is a fact, around which other mathematical conceptions and reasonings can be built; it is a given. Thus, while it is important that students are involved in the social construction of their knowledge, there are also facts in mathematics around which concepts are shaped. In this way we acknowledge that we stand on the shoulders of giants (Newton) in shaping our mathematical knowledge.

This assumption does contrast in some ways with the research into socio-mathematical norms of early primary school classrooms by Cobb et al. (2001). In their research they take the "classroom community rather than the discipline as their point of reference" since they see the "practice to be an emergent phenomenon rather than an already established way of reasoning and communicating into which students are to be inducted" (Cobb et al., 2001, p. 120). This case study of a grade 9 mathematics classroom took the view that the "historically developed practices" (Cobb et al., 2001, p. 120) within mathematics do contribute to the normative practices. Furthermore, the horizon knowledge of the effective mathematics teacher in the grade 9 classroom would promote the concept that certain foundational mathematical concepts, procedures and processes need to be well established in students' minds if they are going to pursue mathematics successfully in the senior years. This idea is supported in the *Shape of the Australian Curriculum: Mathematics* (ACARA, 2009, p. 8) document which suggests that the grade 9 mathematics classroom learning community should build upon the foundational mathematics that has been established in the earlier years of schooling so that more abstract ideas can be developed. The challenge for the teacher is to engage students in genuine ways within the learning community so that they can achieve mature and compressed understandings of mathematical concepts and procedures. Thus, while the socio-mathematical norms are distinct processes in classroom learning communities, this case study resonates with the view that certain processes and procedures are essential to pursuing higher level mathematics. The socio-mathematical norms should not be confined to distinct ways of working mathematically, since discussing different ways to do the mathematics is essential. However, at some point as justifications and judgments of mathematical solutions become more sophisticated, the classroom learning community might acknowledge the efficiency and sophistication available within some of the traditional ways of working.

3.2.4 The power of pedagogy

In terms of the role of the teacher, Howe (2003, p. 93) suggests that constructivist pedagogy is "broader in scope than constructivist learning" and characterises it as "embracing a constructivist learning theory, but mixing ostensibly constructivist and non-constructivist teaching techniques as appropriate". This is especially pertinent in the middle school mathematics classroom. That is, students who are aiming to gain a conceptual understanding of mathematical ideas, may first need to understand basic mathematical procedures and concepts (the vertical mathematics of the RME framework); and these procedures may be considered as *facts* that sometimes require non-constructivist teaching techniques. It is scaffolding upon these facts and procedures of mathematics that the teacher's pedagogical content knowledge (Ball, et al., 2007) may become especially critical in steering quality mathematical interactions in the middle school classroom. Thus, examining mathematical interactions and socio-mathematical norms requires research that considers how facts also shape the subjective nature of classroom interactions. This qualitative approach was used to gain a better understanding of what quality socio-mathematical norms might look like in the middle school mathematics classroom and how this contributed to students' mathematical dispositions. Furthermore, qualitative research and the investigation of the "social arrangements" of the mathematics classroom, also acknowledges "oppressed groups" or individuals and aims to "criticise existing conditions to suggest the direction that transformations should take" (Howe, 2003, p. 77).

The constructivist view may be defined through the works of Piaget and Vygotsky (Howe, 2003, p. 93), with the basic premise that learning begins with student conceptions and preconceptions and that knowledge construction evolves in a scaffolding process through social interaction with others. Essentially though, the learner constructs "their own understanding from the inside" (Howe, 2003, p. 93). Wegner (1998, p. 4) argues that a focus on "learning as social participation" is essential to understanding a constructivist epistemology since our social participation "shapes not only what we do, but also who we are and how we interpret what we do" (p. 4). To use Wegner's (1998, p. 72) term, "community of practice" within the mathematics classroom, suggests that teachers and students are mutually engaged in a joint process of knowledge construction in which mathematical reasoning allows for the negotiation and redefining of significant mathematical concepts and procedures. Wegner suggests that negotiating within a community of practice establishes a "mutual accountability among those involved" (1998, p. 81).

In the practices of the mathematics classroom it is within the negotiation of mathematical ideas that the socio-mathematical norms of the classroom are established. The teacher within the classroom assists (and guides when necessary) the negotiation of mathematical concepts, so that students become accountable for their mathematical ideas by explaining and justifying their ways of working and ways of thinking. Therefore, Wegner's (1998, p. 81) regime of mutual accountability needs to be defined within the mathematics classroom as being mediated by the pedagogical knowledge and practices of the teacher and this places the teacher as having more power within this community of practice. This is a salient point in the aim of 'understanding' in this research. As mentioned earlier, Howe (2003, p. 74) suggests "education is – or ought to be" a normalising process, and "in contrast to the relatively passive role that is frequently implied by the metaphor of the teacher as a facilitator…analyses of socio-mathematical norms contribute to the development of an empowering vision of the teacher's role in the mathematics classroom" (diSessa & Cobb, 2004, p. 98). Wegner (1998, p. 80) also suggests that power within communities of practice can be used to inspire, help, support, enlighten, unshackle and empower. From this viewpoint, teachers' pedagogical content knowledge may be considered as a source of benevolent power in the establishment of positive socio-mathematical norms. Indeed, this case study regards pedagogy as an "opportunistic" act within the mathematics classroom since "what matters is the interaction of the planned and the emergent – that is, the ability of teaching and learning to interact so as to become structuring resources for each other" (Wegner, 1998, p. 267). Moreover, as mentioned earlier, the power in an effective learning community is focused on the "Great Thing" or the mathematical idea (Palmer, 1998, cited in Neuenschwander, 2000, p. 94). Indeed, students can have authorship of the mathematical ideas, by becoming increasingly intellectually autonomous in building their understanding of the mathematics they are doing.

Another important perspective in establishing effective socio-mathematical norms within learning communities involves Vygotsky's (1978) *Zone of Proximal Development*. This is described as the "distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined by problem solving under adult guidance or in collaboration with more capable peers" (Vygotsky, 1978, cited in the DEEWR, 2008, p. 28). This idea urges teachers to extend students beyond what they can do individually into doing more within the social support systems of the classroom. This may be conceptualised as scaffolding or participating in a community of practice, where students are "inducted into more disciplined and rigorous modes of thinking that involve exploration, speculation, conjecture, gathering evidence, and providing proof (Goos, 1999, p. 6). The Zone of Proximal Development suggests that the problem solving activity initially solved in cooperation with others can eventually become internalised and then confidently pursued and used independently by the student. Effective teachers use their pedagogical content knowledge to place the classroom practice slightly beyond what the student can do independently. As suggested by Vygotsky "there are highly complex dynamic relations between developmental and learning processes that cannot be encompassed by an unchanging hypothetical formulation" (1978, p. 88). For teachers this idea suggests that adjusting pedagogical practice according to the idiosyncrasies of how students learn is a key component in negotiating worthwhile learning trajectories within the classroom interactions. For the students, their input into negotiating mathematical ideas has the potential to initiate and empower them to become intellectually autonomous doers of mathematics, with productive mathematical dispositions. It is from this viewpoint that students' mathematical proficiency may be developed effectively.

The perspective offered by Vygotsky's Zone of Proximal Development reinforces the notion that both the sociological and psychological perspectives are useful in analysing the socio-mathematical norms of the mathematics classroom. The sociological perspective suggests that the "mutual engagement" in negotiating mathematical ideas is a "privileged locus for the acquisition of knowledge" (Wegner, 1998, p. 214). The psychological perspective acknowledges that a constructivist view to knowledge creation involves individual reflection. These two perspectives regard "mathematical learning as both a process of active individual construction and a process of enculturation into the mathematical practices of wider society" (McClain, 2002, p. 218). Moreover, the foundation of this study recognises that the psychological and sociological perspectives are in a reciprocal relationship within the community of practice of the classroom.

3.3 Rationale for qualitative case study research

Qualitative case study methods provide an ideal opportunity to gain an emic understanding of the uniqueness of people and programs within educational settings and may be employed to give meaning to the diverse situations within this setting (Bassey, 1999; Merriam, 1998; Stake, 1995; Tellis, 1997). The emic perspective endeavours to capture a sagacious insight into the participant's point of view. Case studies can involve both qualitative and quantitative methods for data collection. Choosing a qualitative perspective for this case study research stemmed from a requisite focus on "insight, discovery, and interpretation rather than hypothesis testing" (Merriam, 1998, p. 29). In this sense, the aim was to uncover some of the emic issues (Stake, 1995) of the teachers and students who were involved in the learning community of a grade 9 middle school mathematics classroom. The emic perspective provided an ideal opportunity to add the richness of the empathetic viewpoint when interpreting the data.

3.3.1 Features of qualitative case study research

A feature of qualitative case study methods is that the researcher is the "primary" or "human instrument" mediating the collection of data and analysis (Merriam, 1998, p. 7). This is considered beneficial since the researcher can: respond to and adapt data collection techniques to suit the context; consider the micro elements and broader implications of context; expand upon interpretations through discerning observations of nonverbal details; summarise and elucidate as the research story unfolds, and make decisions about the relevancy of investigating irregularities (Merriam, 1998). Essentially qualitative case study research aims to understand how components of a phenomenon fit together. The focus is on quality meanings and rich descriptions which are "inductively derived from the data" (Merriam, 1998, p.8). The researcher attempts to consolidate the meanings of the participants' experiences by filtering it through their own perceptions to enrich understandings of particular contexts. Denzin (2001, in Miles & Huberman 2002, p. 350) discusses the importance of the researchers having a "sociological imagination" so that they may think "reflectively…critically, historically,... comparatively, and biographically". Thus the human research instrument with a sociological imagination "self-consciously make their own experience part of the research" (Denzin, 2001, in Miles & Huberman 2002, p. 350). Therefore, I did not attempt to reject what I knew from my teaching experience. However, I was aware of how I used that knowledge so that it was used constructively to give meaning to the research context. Framing questions from the how rather than the why certainly helped in that respect. Indeed as the human instrument in this research I had to develop my ability for "reflective intelligence" (Bolam, McMahon, Stoll, Thomas, Wallace, Greenwood, 2005, p. 20).

This case study was used to understand education in action and the research was concerned with adding to the “thinking and discourse of educators” (Stenhouse, 1985, cited in Bassey, 1999, p. 28). The investment was in understanding the “process rather than outcomes, in context rather than specific variable, in discovery rather than confirmation” (Merriam, 1998, p. 19). The “interpretation in context” (Cronback, 1975, cited in Merriam, 1998, p. 29) is a distinctive feature of case study designs and it is particularly suited to research that cannot and does not strive to separate diverse phenomenological variables from their context. The research literature defines the characteristics of qualitative case studies as being “particularistic, descriptive and heuristic” (Merriam, 1998, p. 29). Merriam (1998, p. 29-30) uses the research literature on qualitative case study to describe these characteristics.

The particularistic feature defines that case studies focus on a specific bounded situation and concentrates “attention on the way particular groups of people confront specific problems, taking a holistic view of the situation” (Shaw, 1972, cited in Merriam, 1998, p.29). Olson (1982) illuminates the nature of the particularistic view by suggesting that it can: “suggest to the reader what to do or what not to do in a similar situation” and that although it is examining only one particular instance, it may assist the clarification a broader problem (cited in Merriam, 1998, p. 30). This notion is assisted by the descriptive feature of case study methods. That is, the end result of a case study is a rich or thick description that can “include as many variables as possible and portray their interaction” to “illuminate the complexities of a situation” (Merriam, 1998, p. 30). The thick description may also be described as a “subjective description” (Stake, 1995, p. 42) that does not purport to claim formal truths. Instead, the thick description strives to stimulate thought and reflection from the reader. In this way “previously unknown relationship and variables can be expected to emerge from case studies leading to a rethinking of the phenomenon being studied” (Stake, 1981, cited in Merriam, 1998, p. 30). The heuristic feature aims to give readers an insight into a phenomenological issue so that it can be interpreted in a way that adds to their experience and understanding. Thus, the descriptive nature of qualitative research contributes to the heuristic feature of case studies.

Patton (1990) distinguishes heuristic inquiry from the phenomenological approach by discussing how the investigation "brings to the fore the personal experience and insights of the researcher" (Patton, 1990 p. 71). In this sense, heuristic enquiry asks, "What is my experience of this phenomenon and the essential experience of others who also experience this phenomenon intensely?" (Patton, 1990, p. 71). The tacit dimension of heuristic inquiry resonated as particularly important from the outset of this research since the aim was to articulate the inner essence of the context of the middle school. Certainly I recognised the importance of the "sense of connectedness" between myself as the human research instrument and the participants so that I could "elucidate the nature, meaning, and essence" of the middle school mathematics experience (Patton, 1990, p. 72). In summary heuristic inquiry:

- emphasises "connectedness and relationship"
- leads to "depictions of essential meanings and portrayal of intrigue and personal significance that imbue the search to know"
- concludes with a "creative synthesis that includes the researcher's intuition and tacit understandings"
- encourages that the research participants remain visible throughout the research experience.

(Douglas & Moustakas, 1984, cited in Patton, 1990, p. 73).

The various attempts to define case studies have resulted in debates that go beyond the qualitative/quantitative divide. The debates continue to contribute to the ongoing interpretations relevant to improving case study research. However, Merriam's (1998, p. 27) conclusion that the case needs to be viewed as a bounded entity so that "I can fence in what I am going to study" was the point of departure in conducting this study. Stake (1995, p. 3) defines a case study in this sense as "instrumental" since it aims to understand a particular bounded system. The literature on case study methods suggests that “defining and bounding cases is the catalyst that brings together theory, methodology, and analysis” (Huberman & Miles, 2002, p. 332). Representing and refining this case study design by considering a funnel (Bogdan & Biklen, 2007) certainly helped in the research process. The initial research question of

- How do middle school mathematics teachers empower students to be proficient doers and users of mathematics?

is formulated in the design of this case study in the following way:

Middle School Mathematics: intellectual rigour for mathematical proficiency

Two grade 9 mathematics classrooms at a single school

Interpreting the classroom interactions: social and psychological perspectives of the socio-mathematical norms

Emic perspectives

The following section describes the characteristics of this case study within the bounded system of the middles school mathematics classroom.

3.3.2 Research design for this case study

The teacher’s PCK arises in the research literature as an avenue to support mathematics learning for students. The National Numeracy Review Report

(DEEWR, 2008, p. 73) recommends that “professional development that focuses on teachers understanding how children’s mathematical understandings develop can be used to build teacher pedagogical content knowledge”. This report also suggests that a definition of quality pedagogy is difficult to quantify and measure, since what works “in one context, may not work in another” (DEEWR, 2008, p. 30). Therefore, one way to get an insight into what quality teaching might look like and how this is influenced, is to study the classroom micro-culture within a particular school.

This case study was fenced within the confines of grade 9 mathematics at one Queensland school: Amethyst College. The qualitative study attempted to gain an emic understanding of how the teacher and students were engaged in a learning community. In particular, the interactions of this learning community were filtered through the lens of the socio-mathematical norms. In this way the case study became more tightly bound since the observations were from the mathematical perspective of the community of practice. Even though the case study was bounded, it wasn’t “locked into [a] rigid design” so that new “paths of discovery” could emerge (Patton, 1990, p. 41).

The aim of this case study was to describe and analyse the complexities inherent within the mathematical interactions of the classroom that influence the intellectual autonomy of students as doers of mathematics. This qualitative research acknowledged the uniqueness of a mathematics classroom by using a “thick description” (Stake, 1995, p. 30) of how teacher’s pedagogical content knowledge and students’ mathematical proficiency evolved within the socio-mathematical norms of the classroom. The rich descriptions within this qualitative research were “used to convey what the researcher has learned about…the context, the players involved, and the activities of interest” (Merriam, 1998, p. 8). The realities of the classroom context were highlighted for the reader by using classroom snapshots and direct quotations from the participants. This further enhanced the emic understanding of how students and teachers were mutually engaged in doing mathematics.

The emphasis of this case study was on the holistic view. It attempted to study what is "actually taken for granted" in an "attempt to gain entry into the conceptual world of the informants in order to understand how and what meaning they construct around events" (Bogdan & Biklen, 2007, p. 26). The events were the mathematical processes of justifying, judging and discussing mathematical solutions and procedures within the mathematics classroom. The holistic perspective recognised that the interactions that occurred in the mathematics classroom might be interpreted in multiple ways and the meanings we attach to these experiences are what "constitutes reality" (Bogdan & Biklen, 2007, p. 26). Thus, this phenomenological research focused on the concept of: "What is the structure and essence of experience" of the socio-mathematical norms for the students and the teacher? (Patton, 1990, p. 69).

How the teacher's PCK contributed to the socio-mathematical norms within the mathematics classroom was a focus point. For example, the study conducted by McClain (2002, cited in diSessa & Cobb, 2004) established that significant mathematical ideas are initiated and mediated by the teacher's pedagogical decisions. The socio-mathematical norms are an avenue to understanding courses of action that nurture the engagement of students into the mathematical processes that cultivate the formation of their intellectual autonomy and mathematical dispositions. DiSessa and Cobb (2004, p. 98) discuss how socio-mathematical norms can "operationalise the idea of the normative identity as a doer of mathematics that student are, in effect, invited to develop in particular classrooms". However, trying to gain an insight into: how students develop their sense of being doers of mathematics; and how teachers can find the right tools to negotiate the various paths presented by the socio-mathematical norms within the middle school mathematics classroom is the challenge. The observation and identification of regularities and patterns in the classroom socio-mathematical norms and how the teachers PCK and students' habitus contributed to these norms were the interpretive structure supporting the process of understanding in this case study.

At the time that this study was undertaken, Queensland teachers used the curriculum assessment and reporting framework document: Essential Learnings by the end of Year 9 (Education Queensland, 2007). A focus of the Essential Learnings framework is that students should: "build upon their existing understanding of mathematical concepts…relate mathematics to real-life and purely mathematical situations"; and "understand that mathematics is a way of thinking" (Education Queensland, 2008, p.1). The Ways of Working part of the framework suggest students should be able to:

- analyse situations to identify the key mathematical features and conditions, strategies and procedures that may be relevant in the generation of a solution
- pose and refine questions to confirm or alter thinking and develop hypotheses and predictions
- plan and conduct activities and investigations, using valid strategies and procedures to solve problems
- select and use mental and written computations, estimations, representations and technologies to generate solutions and to check for reasonableness of the solution
- use mathematical interpretations and conclusions to generalise reasoning and make inferences
- evaluate their own thinking and reasoning, considering their application of mathematical ideas, the efficiency of their procedures and opportunities to transfer results into new learning
- communicate thinking, and justify and evaluate reasoning and generalisations, mathematical language, representations and technologies
- reflect and identify the contribution of mathematics to their own and other people's lives

- reflect on learning, apply new understandings and justify future applications.

(Education Queensland, Essential Learning by the end of year 9, p. 2)

For this study these ways of working were seen as embedded within the socio-mathematical norms of the classroom. However, the task was to give some insight into how these ways of working can be used to establish and negotiate the socio-mathematical norms that encourage students to mathematise. That is, how were these ways of working visible in the grade 9 mathematics classroom and how did they contribute to students establishing their ways of thinking and therefore their mathematical dispositions. In particular, the intersection of the ways of working and socio-mathematical norms were investigated in the following ways:

- how were the mathematical ways of working negotiated in the classroom?
- how were teachers and students mutually engaged in making decisions about accepting and justifying ways of working?

From a holistic, emic perspective the socio-mathematical norms were used to understand how the teacher and students could become mutually engaged in these ways of working and how they then individually accepted responsibility for their learning within the classroom and individually.

In essence, this research was bounded by the overlap of several core factors within different frameworks from Education Queensland, namely, the core factors of: teachers' knowledge and pedagogy from the document Numeracy: Lifelong confidence with mathematics (Education Queensland, 2007) and the ways of working from the Essential Learnings by the end of year 9 document (Education Queensland, 2007). Furthermore, this overlap was investigated from the social and psychological perspectives of the socio-mathematical norms. A "significant piece in the jigsaw of understanding" (Bassey, 1999, p. 89) of this research was to explore gaps and connections between teachers' PCK, students' mathematical dispositions and the classroom learning community.

This idea is summarised in the cognitive map in Figure 1 below:

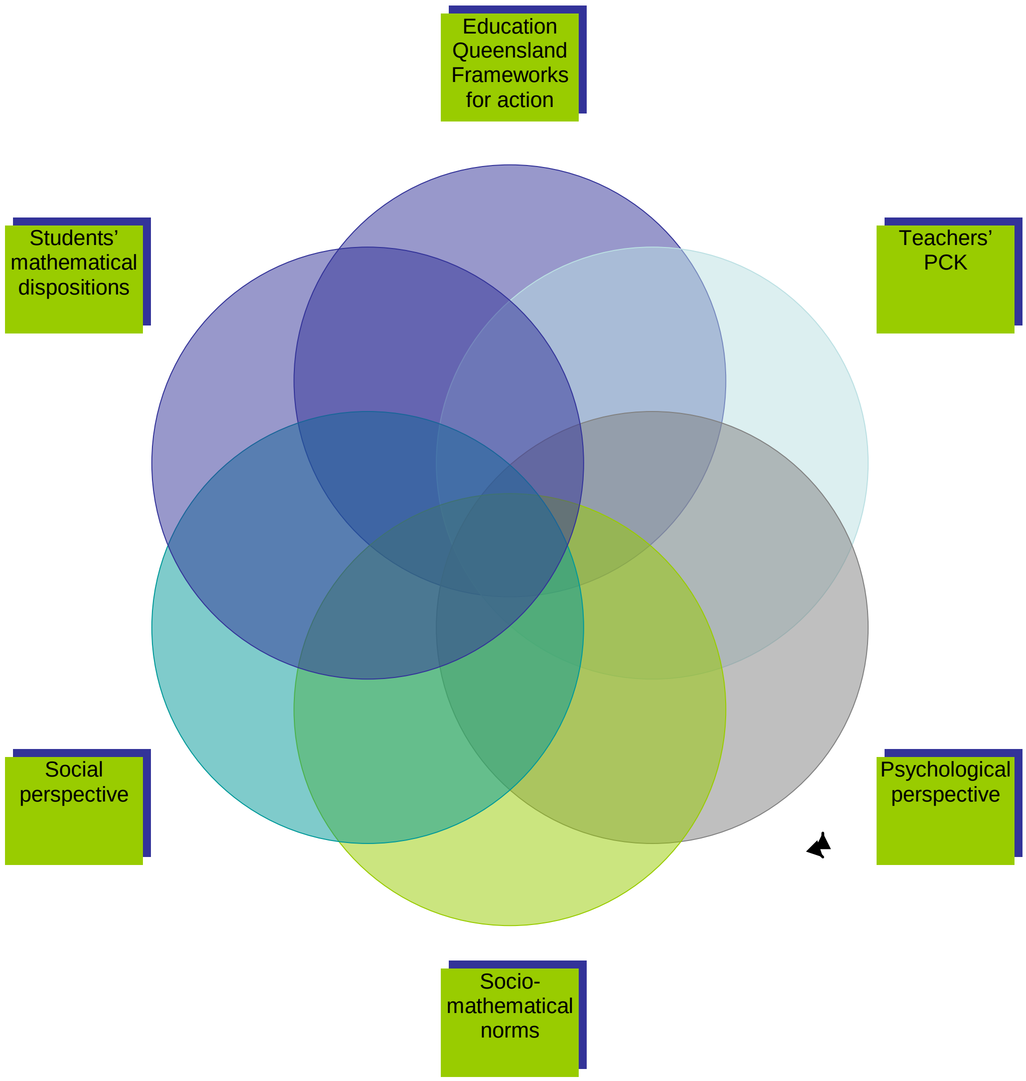


Figure 1: Cognitive Map of the Constituents of the Learning Community

3.4 Research setting: Amethyst College

Amethyst College was established in the mid 1980s as a co-educational, non-government, catholic high school (grade 8 to grade 12) in North Queensland. Grade 8 and grade 9 comprise the middle school. The college is located in a region that has undergone significant land development and expansion over the last twenty years. The total enrolment from year 8 through to year 12 in 2009 hovered around 900 students: 52% girls, 48% boys. The student attendance rate in 2009 was 94% (My School, 2010). The school employed 78 teaching staff and 35 non teaching staff.

Amethyst College has a hierarchical structure for school administration and management, and there is also a School Board and a Parents and Friends' Association. The staff hierarchy follows a traditional pyramid structure: the principal and assistant principals are at the vertex, subject coordinators and year coordinators in the middle management layer, and teaching staff and non-teaching staff making up the lower layer of the pyramid structure.

3.4.1 Social climate of Amethyst College

Amethyst College services a diverse range of economic and social groups. The School *Index of Community Socio-Educational Advantage* (ICSEA) value is 966 (My School, 2010). The ICSEA is a measure that has been specifically developed for the My School website and is described in the following way:

> The variables that make up ICSEA include socio-economic characteristics of the small areas where students live, as well as whether a school is in a regional or remote area, and the proportion of Indigenous students enrolled at the school…the average ICSEA value is 1000…most schools have an ICSEA score between 900 and 1100.
> (My School, 2010)

Student background for Amethyst College is rated by the ICSEA according to Table 3:

Table 3: Index of Community Socio-Educational Advantage for Amethyst College (My School, 2010)

Index of Community Socio-Educational Advantage (ICSEA)			
School ICSEA value: 966			
Bottom Quarter	Middle Quarters		Top Quarter
54%	25%	17%	4%

In 2009, the grade 9 NAPLAN numeracy average for Amethyst College was 581 (My School, 2010), statistically similar schools' average was 571, with the Australian schools' average being 589 (My School, 2010). According to these results, Amethyst College's results on the grade 9 NAPLAN numeracy test were close to the other relevant averages. Similarly, the school literacy averages for NAPLAN were close to other statistically relevant averages across Australia. (My School, 2010)

Amethyst College describes the social climate of the school in the following way:

> Each individual at the College is given the opportunity to pursue academic excellence and personal fulfilment in a culturally rich and caring Catholic environment. Effort and achievement are applauded.
> High expectations and standards held by caring staff help nurture students into maturity. Pastoral care permeates all aspects of College life. [Amethyst College] strives to enable its students to face the future with confidence. A comprehensive program of camps, retreats and work

experience assist students to develop into mature social, emotional and spiritual adults.
Each student is under the care of a Home Form teacher and Year Co-ordinator and the program is co-ordinated by the Deputy Principal – Pastoral. The Assistant Principal for Middle School gives extra assistance to students in their formative years during Years 8 and 9.
(Amethyst College Annual Report, 2009)

3.4.2 School organisation

Amethyst College has a timetable which cycles fortnightly. All year levels follow the same timetable. The school day starts at 8.30am with a fifteen minute homeform. During this time, the attendance roll is marked, the school notices are read to students and general administrative and pastoral issues are discussed. There are two breaks during the day, a morning tea break of 30 minutes at 11.00am and a lunch break of 45 minutes commencing at 1pm. The school day finishes at 3.15pm, with the exception of Thursday. Every Thursday, the students finish at 1.55pm. Staff use this student free time for such things as: collaborative planning; subject and year level meetings; and professional development activities.

Semester 2, 2009 commenced on July 13th and consisted of: Term 3 (10 weeks); 2 weeks spring vacation; and Term 4 (8 weeks) followed by the summer vacation. Term 3 was a particularly busy term at Amethyst College, with a lot of interruptions to the regular timetable. The year planner showed that in Term 3, close to one week of classroom learning time was lost over the course of the term due to: 1 student free day (Week 1), 1 show holiday (Week 1), 1½ days for the Athletics Carnival (Week 3), 1 Feast Day (Week 6).

The grade 9 mathematics classes have 9 lessons of 45 minutes within the fortnightly cycle. Within these 9 lessons, there are 3 double lessons of 90 minutes and 3 single lessons of 45 minutes. Therefore, over a 2 week cycle students and mathematics teachers are timetabled to be in their mathematics classrooms for 6 hours and 45 minutes.

3.4.3 Middle school mathematics

The middle school (grade 8 and 9) at Amethyst College follows the same timetable as the entire school. All mathematics classes from grade 8 through to grade 10 are streamed into advanced, ordinary and general. Students entering into grade 8 are streamed immediately, according to the results from: their grade 7 NAPLAN test; and a PATMaths test (ACER, 1998) that they sit for during their grade 7 into grade 8 orientation day held at the college.

Students who commence grade 8 at Amethyst College are aware that they are in advanced, ordinary or general mathematics, since mathematics appears on their timetable as: 8 MAA, 8 MAO, or 8 MAG. The grade 8 mathematics program is the same for all students regardless of how they are streamed. That is, students in advanced, ordinary and general mathematics all follow the same program, use the same text book and do the same assessment. The difference arises in the comparison of class sizes, since the general mathematics classes are the smallest in size.

The grade 9 mathematics program is the same for advanced mathematics and ordinary mathematics. However, the assessment items are different. Depending on students' progress and results, they are able to shift from one tier of streaming to another throughout their journey in middle school mathematics. Such a move is made via consultations between the assistant principal curriculum, the mathematics coordinator, the classroom teacher, the student and parents. Significantly, timetabling constraints may limit the opportunity for a student to shift from one stream of mathematics to another. By the end of grade 9 the movement between the three streams of mathematics decreases to a minimum.

The grade 10 mathematics program and assessment are different across the three streams of mathematics. Year 10 advanced mathematics is a prerequisite for year 11 mathematics B and mathematics C and is preferable for senior physics and chemistry.

3.4.4 Participants from Amethyst College

I approached the principal of Amethyst College at the end of 2008 to ask if I could conduct my research at the school in Semester 2, 2009. After receiving approval from the principal, I approached the mathematics coordinator (Naomi) and asked if I could observe two year 9 mathematics classes and their teachers. Naomi said that I could observe her and her year 9 advanced mathematics (9MAA) class and Dan another mathematics teacher volunteered to be observed in his year 9 ordinary mathematics (9MAO) Class.

After agreeing to participate, Naomi made this remark, referring to research in education:

> You can tell me how I should do things, but I will still do what I think.

Initially I was surprised by this comment, but then I realised that Naomi was self-assured and candid, which is exactly what I was looking for. Similarly, Dan's character was such that he wouldn't need to put on a contrived performance while I was observing him in the classroom. I did explain to Naomi and Dan that the intention of my research was to observe the interactions between the different members of the classroom learning community. Essentially, I felt that I had found two teachers who would allow me to immerse myself within their environment to gain the desired sagacious insight into the real world of doing mathematics in two middle school classrooms during 2009.

I also got the impression from both Naomi and Dan that while they were happy to help me, they were "not optimistic that the research" was going to be "of benefit to them" (Stake, 1995, p. 58). However, this did not prove problematic since I didn't feel the need for my research work to be lauded. Furthermore, as suggested by Patton (1990, p. 354) the research process is "first and foremost" about gathering data not trying to "change people". In fact, these teachers were doing me a favour and I tried to minimise the imposition on them without compromising or subtracting value from the data collection (Lankshear & Knobel, 2004). In this way, I felt that an atmosphere of mutual respect developed between us and this ultimately benefited the entire research process (for me). What's more I did get the impression from Naomi, that she wasn't totally removed from education research, more that she appeared to be overwhelmed by the many aspects of her role and that perhaps education research was a luxury that she didn't have a lot of time for.

3.4.5 Teacher participants

Naomi had 16 years experience teaching mathematics in secondary school, as well as experience in teaching senior physics. Her qualifications are:

> Bachelor of Science (Physics, Mathematics)
> Postgraduate Diploma of Education (Physics, Mathematics)

Naomi started teaching at Amethyst College in 2003 and became the mathematics coordinator in 2004. In 2009 Naomi taught 4 classes: 9 advanced mathematics; 11 mathematics B; 11 mathematics C; and 12 mathematics B (see timetable in Appendix 1)***.*** Naomi had been on the mathematics C Panel for over 10 years. Naomi wrote the current mathematics programs at Amethyst College for grade 8 through to grade 10, and was in the process of writing the new senior mathematics programs in 2009.

Dan had 38 years experience in teaching mathematics from primary through to senior secondary school. I asked Dan to write down his teaching qualifications and he wrote:

Certificate of Teaching: Primary, obtained in 1970 in a 2 year course.
My mathematics teaching experience is:
1971-1985 Primary school (mostly Years 6 and 7)
1986 – 2009 Secondary school (mostly Years 8, 9 and 10, some Year 11 and 12 Mathematics A, Trade and Business Mathematics, Pre-vocational Mathematics)

In 2009 Dan had a year 10 homeform class, taught 5 classes and had a year 8 sport (see timetable in Appendix 2). The 5 classes he taught were: 8 advanced mathematics; 2 classes of 9 ordinary mathematics; 10 ordinary mathematics; and 12 English.

3.4.6 Student participants

The year 9 students in the two mathematics classes were aged between 13 and 15 years. The majority of these students went to primary school at one of the three nearby catholic feeder primary schools.

The year 9MAA class (Naomi's class) had 23 students: 17 girls; 6 boys.
The year 9MAO class (Dan's Class) had 21 students: 9 girls; 12 boys.

3.4.7 Ethics Approval and Constraints

The conduct of this qualitative case study research was aligned with ensuring that the interests and welfare of the participants were respected. The initial conceptualisation of the research methods and methodology didn't present any foreseeable issues that would compromise my duty of care obligations (Lankshear & Knobel, 2004). However, I was mindful of potential risks to teachers and students, particularly when: conducting interviews; choosing which students to speak to at different times during the semester; when to speak to them and where. Flexibility on my part in relation to these courses of action was an avenue that minimised my intrusion. Essentially, I felt that minimising my intrusion was one way of respecting the goodwill of the participants.

Guidance was sort from the JCU ethical framework and research literature on ethical dilemmas (Lankshear & Knobel, 2004; Patton, 1990). Ethics approval was obtained from JCU, Ethics Approval Number H3358 and from Catholic Education Services, Cairns. I also have current Queensland teacher registration and a Blue Card for child related employment. Furthermore, I followed the necessary protocols set down by the school procedures for visitors to the school. Naomi and Dan and all of the students in their grade 9 classes were given an information sheet and an informed consent form (see Appendix 3).

The use of pseudonyms in this research offered the traditional criteria of protection to the participants in terms of privacy, confidentiality and anonymity. However, as suggested by Lankshear and Knobel (2004, p. 110), “assuring confidentiality and anonymity is actually quite difficult to put into practice as some schools are readily identifiable because they are…easily recognised in the region”. Both Naomi and Dan would be easily identified in this school. On the other hand, the use of pseudonyms did give the students a higher degree of anonymity than the teachers. With this in mind, I did need to maintain my duty of care when writing and reporting my research findings. In particular, I attempted to critique situations and contexts and how they melded in a reciprocal relationship with people and actions. That is, as discussed in the methodology, this research attempted to better understand and report on the reflexive nature of the psychological and socio-cultural contexts of the middle school mathematics classroom. Hence, I don’t feel uneasy about participants reading this research report, especially because I would invite and encourage them to critique my interpretations. After all “research is best practised as a ‘two-way’ street” (Lankshear & Knobel, 2004, p. 112).

3.5 Data collection strategies

3.5.1 Key questions guiding data collection

It has been established that this research regarded the socio-mathematical norms of the classroom as a focus point that influences students' mathematical dispositions. This research attempted to understand the socio-mathematical norms from the social and psychological perspectives. The reflexive relationship between the social and psychological perspectives implies that "neither perspective exists without the other, in that each perspective constitutes the background against which mathematical activity is interpreted from the other perspective" (Cobb et al., 2001, p. 121). Therefore, data collection within this case needed to consider:

- How the classroom interactions (social perspective) transpired and how the teacher and students individually contributed to these interactions (psychological perspective).

Two main research tools were used for data collection: classroom observations and interviews. These are displayed in Table 4:

Table 4: Research tools

Socio-mathematical norms	Social perspective	⇔	Psychological perspective
Teacher PCK	Observations	⇔	Interviews
Students' mathematical dispositions.	Observations	⇔	Interviews

The data collection strategies for this case study evolved from the following key questions:

- how did the teachers use their pedagogical tools to steer the classroom learning community?
- how was knowledge constructed by students and how was this knowledge legitimised within the norms of the classroom?
- how did the learning community contribute to the ongoing development of students' intellectual autonomy and mathematical disposition?
- how did students become involved in mathematising?
- how did the socio-mathematical norms of mathematical argumentation and mathematical difference function in the classroom learning communities?

3.5.2 Classroom observations

Classroom observations examined how the teacher and students participated in the negotiation of socio-mathematical, mathematical and social norms within the classroom learning community. In particular, the observations focused on the processes of how mathematical solutions methods were:

- justified
- accepted
- and legitimised

in the classroom learning community. Therefore, the observations provided the opportunity to investigate how members of the classroom learning community were mutually engaged in the learning process.

The natural observations of the classroom setting represented a direct source for understanding the emic issues of the socio-mathematical norms. Research observations within this study were methodical in that they were:

- planned and deliberate

- recorded systematically after each lesson
- continually reflected upon to continue with data collection.

(Merriam, 1998, p. 95)

As the human research instrument, this planned and deliberate action required that observations were written descriptively and systematically (Merriam, 1998). Furthermore, observations were funnelled through the ongoing reflection and analysis of observed data. In this way, the patterns that emerged were pursued in subsequent observations (Lankshear & Knobel, 2004, p. 220). In particular, the following prompts were initially used to focus the observations of the classroom setting and to they were used to create the Observation Form (see Appendix 4):

- Where do the teacher and students place themselves within classroom discussions?
- How do the classroom interactions emerge; who has the greater input?
- How do the different members of the classroom community act within the discussions?
    - Who speaks to whom?
    - Who listens?
    - Who is silent?
    - What are some of the non-verbal cues?
    - What doesn't happen (that perhaps should have)?

      (Merriam, 1998, p. 98)
- What are the norms of the classroom and how are they legitimised?
- What types of responses are valued and why?
- What are the responsibilities of the students/teacher?
- How were the responsibilities determined?
- How does the mathematical concept become evident and legitimised?
- How are students given opportunities to make links with previous concepts/ideas?

- How does the teacher positively reinforce student's mathematical participation?
- Opportunities for risk taking in mathematising?
- Evidence of PCK (Ball et al., 2007), how is it adapted?
- How was it evident that teaching and learning were structuring resources for each other? (Wegner, 1998, p. 267).

The field notes that were taken were highly descriptive so that they allowed me to "return to that observation later during analysis" and to ultimately "permit the reader of the study findings to experience the activity observed" (Patton, 1990, p. 239)

It is important to note that observations for this case study were from a semi-structured, observer as participant perspective (Lankshear & Knobel, 2004; Merriam, 1998). This meant that the research activity was evident to the classroom community, yet researcher participation was ancillary to the role of human research instrument. There were "informal interviews and conversations…interwoven with observation" within the data collection activities (Merriam, 1998, p. 94). In this way the researcher's role was to "observe and interact closely enough with members to establish an insider's identity without participating in those activities constituting the core of group membership" (Adler & Adler, 1994 cited in Merriam, 1998, p. 101). This allowed for the psychological aspects of the socio-mathematical norms to be more directly observed as the teacher and students "interpreted and responded to each other's actions" (Cobb et al., 2001, p. 121).

Observations were made from two main perspectives: things that happened and things that didn't happen. The capacity of the observations in this study were optimised by adopting Patton's (1990, p. 236) suggestion to restate "the observation about what did not occur… [as] the opposite of what did occur" since "that restatement…[would] attract attention in a way that the initial observation might not". This type of restatement was considered to be an important facet in the collection and analysis of data. This was vital to analysing the decision making processes regarding what constitutes an efficient or a different mathematical solution. Essentially, this led to the question: how was responsibility and closure brought to the decision making process when establishing classroom socio-mathematical norms?

The patterns and informal conversations from the observations were used within this study to carefully examine the social and psychological perspectives of the socio-mathematical norms. Furthermore, the “ongoing interpretive role” (Stake, 1995, p. 43) of these observations acknowledged that the socio-mathematical norms were generated and re-generated through the social and individual aspects of classroom interactions. Moreover, these observations allowed for the emic issues to emerge, which could be used as “reference points for subsequent interviews” (Merriam, 1998, p. 96).

The observations facilitated the process of purposeful sampling (Merriam, 1998; Lankshear & Knobel, 2004). Purposeful sampling is used as a strategy by researchers who “use their judgment to choose participants for the specific quality they bring to the study” (Lankshear & Knobel, 2004, p. 148). This is also referred to as “opportunistic sampling” since it “takes advantage of new opportunities during actual data collection” (Patton, 1990, p. 179). This strategy was a strength of this qualitative research since it allowed me to be more open and able to follow useful lines of enquiry. In gaining insights for the rich description of this case study, purposeful sampling was a logical and efficient method to assist data collection because some decisions regarding data collection could not be made in advance. That is, to understand the psychological and social perspectives of the socio-mathematical norms, students who were willing to explain how they viewed themselves within the classroom learning community were chosen through observations and informal conversations. These participants were chosen for subsequent interviews since they were “information-rich” sources, willing to add to the rich description of this case study (Merriam, 1998, p. 61). Using the criteria of willingness and ability to explain their point of view reflected the purpose of this case study. These criteria contributed to the funnelling feature of qualitative research and increased the feasibility of data collection.

3.5.3 Interviews

The observations of classroom interactions together with the purposeful sampling of participants revealed key student informants. These students were subsequently interviewed since they were "particularly helpful, insightful, and articulate in providing data" (Bogdan & Biklen, 2007, p. 273). The main purpose of the interviews was to find out what was "in and on someone else's mind" (Patton, 1990, cited in Merriam, 1998, p. 71). It is important to acknowledge that the interview data cannot be used as though "they are a direct representation of some definitive 'truth' as expressed by the respondent" (Lankshear & Knobel, 2004, p. 199). The interview data are "contrived", "partial and incomplete" since there is always going to be "some conscious shaping of the verbal exchanges" between the researcher and the interviewees (Fetterman, 1998, cited in Lankshear & Knobel, 2004, p. 199). Still, within this case study the interviews were a valuable opportunity to gain an insight into the insider's perspective of doing mathematics in the middle school classroom.

The interviews were recorded by using notes and an audio-recording. The notes concentrated on what was being said by the interviewee. The audio-recording facilitated investigation into how data were spoken; for example "interviewee's intonation…hesitations" and "self-corrections" (Lankshear & Knobel, 2004, p. 199). Also, as the human research instrument I found that listening to the interview tapes allowed reflection on how I used my voice in the interview, and how I could improve upon my "empathetic neutrality" (Tuettemann, 2003, p. 20). Empathy within the interviews "communicates interest in and caring about people". Similarly, neutrality was important since it meant "being non-judgmental about what" the participants said during the interview" (Tuettemann, 2003, p. 20). In this way, the interview procedure acknowledged the significance of understanding the meanings of participants' responses.

The interviews were timed so that it suited both parties. From the researcher's perspective, it was important to have sufficient time immediately after the interview to fully record and interpret the interview using the notes and audio-recording (Stake, 1995). For both parties it was also important to keep the interviews short and focused on specific questions to clarify the psychological perspectives of the students and the teachers. The interviews took place in the mathematics classroom after the lessons that went into lunch breaks.

A hallmark of effective interviews is asking good questions. Good questions within qualitative research are used to stimulate the participants into telling their story (Merriam, 1998). One of the purposes of this case study was to obtain a rich description of the classroom learning community. The interview questions were the vehicle used to investigate the "multiple views" (Stake, 1995, p. 64) within the middle school mathematics classroom. The interviews provided valuable insights into how participants viewed themselves in the learning community and it highlighted the emic issues of the participants. Some of the questions were structured in advance so that they could elicit more than yes and no answers. Furthermore, organising good questions beforehand avoided the use of leading questions and multiple questions, since leading questions reveal researcher bias and assumptions that may synthetically change the respondent's point of view. Multiple questions needed to be re-structured into a series of questions so the participants had a chance to answer each question fully (Merriam, 1998). Certainly, research questions needed to be realistically reviewed in order to optimise the opportunities available within the interview timeframe. Returning to the aim of gaining an insight into the insider's understanding of what constituted doing and learning mathematics and what influenced this in the middle school context assisted in this respect.

Within this study, the aim of the questions was to understand the students' and teachers' interpretations of the middle school context and what influenced their dispositions. Thus, the interpretive questions were used to check my understanding of classroom interactions and they also provided an opportunity for "more information, opinions, and feelings to be revealed" (Merriam, 1998, p. 78). The interviews were semi-structured and open ended to allow for the emic perspective of the participants to emerge. In this way, the interview was moulded around the responses of the interviewee and facilitated the elaboration of important issues as they emerged (Lankshear & Knobel, 2004, p. 199). The classroom observations of the learning community were also used to shape the interview questions. That is, the interviews were used to ask students and teachers questions in an effort to understand their interpretations of previously observed classroom interactions (Merriam, 1998).

For the students the interviews concentrated on understanding how they viewed themselves as doers of mathematics and things that might have been of concern to them. A questionnaire (Appendix 5) was given to students in the week before the interviews so that I had time to look at their responses in order to gather further insight before framing possible questions for individual students. The formal interviews with both Naomi and Dan were open-ended so that their concerns about what was influencing their classroom learning communities could be raised. Both Naomi and Dan had through the course of my classroom visits discussed their concerns with me regarding different issues specific to their contexts through informal conversations. The formal interviews with both of them allowed me to gain a deeper insight into these issues. The specific words the key informants chose to use in their responses within the interview conversations were considered important in understanding the respective psychological perspectives of the socio-cultural norms (Lankshear & Knobel, 2004). The semi-structured format used for the interviews within this case study allowed for the elucidation and clarification of the "emerging worldview of the respondent" (Merriam, 1998, p. 74).

Halcolm's *Evaluation Interviewing Beatitudes* (cited in Patton, 1990, p. 359) were a useful source which I continually reflected upon during this research:

> Evaluators, listen. Do you know that you shall be evaluated by your questions?
> To ask is to seek entry into another's world. Therefore, ask respectfully and with sincerity. Do not waste questions on trivia and tricks, for the value of the answering gift you receive will be a reflection of the value of your question.
> Blessed are the skilled questioners, for they shall be given mountains of words to ascend.
> Blessed are the wise questioners, for they shall unlock hidden corridors of knowledge.
> Blessed are the listening questioners, for they shall gain perspective.

3.6 Data Analysis

3.6.1 Using the vitality of the socio-cultural perspective

A strength of this qualitative case study research was that data analysis was given consideration from the initial conceptualisation of the research problem through to the final stages. The iterative process of data collection and analysis was a critical element of this research. Ideas were built upon within data collection and analysis to achieve a thick description of the emic perspective of the participants.

The framework for analysis follow a process involving four distinct stages that can be identified in both Becker's (1958) and Glaser and Strauss's (1967) approaches (cited in Hopkins, 2008, p. 130). The four stages of data analysis that were used in this research are summarised as follows:

- The initial categories were generated from an analysis of the research literature and the methodology. A case study was selected for data collection and analysis.
- The categories were validated within this case study through data collection and analysis
- The categories were interpreted and integrated with emic themes through ongoing data collection and analysis
- The presentation of findings through a thick description.

The division between qualitative and quantitative techniques becomes even more clearly defined during data analysis. That is, qualitative research focuses on interpreting instances, pulling them apart and then putting them back together to gain an insight. On the other hand, quantitative research concentrates on an aggregate of instances for meanings to emerge (Stake, 1995). Quantitative research looks at using the repetition of instances to support hypotheses whereas qualitative research concentrates on interpretations of individual instances to allow hypotheses to evolve. This reiterates that qualitative research is focused on understanding the variables and dynamics in context.

Data analysis for this qualitative research followed what Merriam (1998) described as an ethnographic analysis and sought to reach across multiple sources of data. The task of analysis involved finding a way “through a forest of data, theory, observation and distortion” (Fetterman, 1998, p. 92). Like Cobb et al. (2001), this study used the pre-existing categories of the social and psychological perspective of the social, mathematical and socio-mathematical norms to organise and analyse data. Gaining an insight into students’ mathematical dispositions, the socio-mathematical norms, and the consideration of how mathematical solutions were legitimised and justified within the grade 9 mathematics classroom were considered to be the “template” for analysis (Stake, 1995, p. 7). However, it is important to acknowledge that case study research is not general qualitative research. The focus is on using the case to understand the socio-cultural patterns and relationships within the grade 9 mathematics classroom. Therefore, the case itself should determine the analytical strategies best suited to gaining insight and understanding.

The aim of the data analysis was to continually sift through the multiple sources of data in search of patterns and correspondence. The focus of the case converged on understanding how the teacher and students interacted when working and discussing mathematical solutions. The search was for consistencies and patterns in the ways of working mathematically within the classroom. The social and psychological perspectives of socio-mathematical norms were taken as the general view or defining categories. However, these categories did not limit opportunity for “conceptual creativity” (Tuettemann, 2003, p.11). That is, patterns emerging unexpectedly from the data were considered a crucial component in understanding the emic issues of this particular case. Therefore, in contrast to quantitative analysis where the focus is on generating “frequency counts” of items in categories, this qualitative analysis aimed at producing a rich description by looking for patterns within and across the categories (Lankshear & Knobel, 2004, p. 270).

The pre-existing categories within the socio-mathematical norms may appear to contradict the openness of qualitative research. However, this case study considered these categories as a salient starting point compatible with the purposes of this research. Furthermore, the decision to use these categories provided the funnel which minimised the potential of collecting data that was diffuse and incongruent with research aims. The emic issues of the participants remain the key concern, since the research attempted to understand the psychological and socio-cultural perspectives of the participants within the case study. The initial categories used for data analysis in this case study did not intend to limit the potential of new possibilities or relationships between data. From the outset this research acknowledged that “if a researcher knew all the relevant variables and relationships in data ahead of time, there would be no need to do a qualitative study" (Corbin and Strauss, 2008, p. 57).

The cognitive map of the learning community (Figure 1, p. 77) showed how the constituents of the learning community were initially conceptualised for this research. This cognitive map transpired through the analysis of the research literature. Reviewing the literature revealed an overlap between the socio-mathematical norms, the productivity of the teachers' PCK, the productivity of student mathematical dispositions and the willingness of students to pursue higher level mathematics. What the data collection and analysis in this case study attempted to do was inductively investigate and compare these elements within the grade 9 mathematics learning community at Amethyst College. The interpretivist approach to data analysis was used in the sense that analysis went beyond what was observed into understanding the educational processes from the social and psychological perspectives within the grade 9 mathematics classroom (Merriam, 1998). Moreover, as discussed within the theoretical framework, analysis involved looking for connections between how the social and psychological perspectives continually regenerated each other and how this contributed to the socio-mathematical norms of the classroom (Cobb et al., 2001).

The actual process of data analysis involved focusing on particular emic themes that emerged from the data collection. Data was collected by keeping "two parallel records, one labelled social and the other psychological" (Cobb et al., 2001, p. 128). Thus, learning from research experiences of Cobb et al. (2001), this strategy was used so that the two perspectives weren't confused. The analysis involved cross referencing across the perspectives so that important inferences could be made. What's more the emic issues of the participants meant that both a micro and macro view of the classroom learning community needed to be used to analyse the data. In this way the two perspectives were coordinated in describing how the socio-mathematical norms of the classroom were generated and regenerated in this particular case.

3.6.2 The strength in the triangulation of data

In architecture, a triangle is considered to be the most rigid shape. The triangle is the strongest geometric shape and is used to bring strength to a structure. So the term triangulation when used metaphorically calls to mind strength (Patton, 1990).

This case study used the triangulation of data to bring strength to the analysis process. Triangulation allows the researcher to “compare information sources to test the quality of the information (and the person sharing it)…to put the whole situation in perspective” (Fetterman, 1998, p. 93). The triangulation approach involved pattern matching analysis across the three sources of data: documentation; observations; and interviews. These three methods, “increased confidence in the interpretations” within the case study (Tellis, 1997, p. 2). Validating the data by continually checking the interpretations across the three sources of evidence was an essential part of the research process. The triangulation of data within this case is represented in Figure 2:

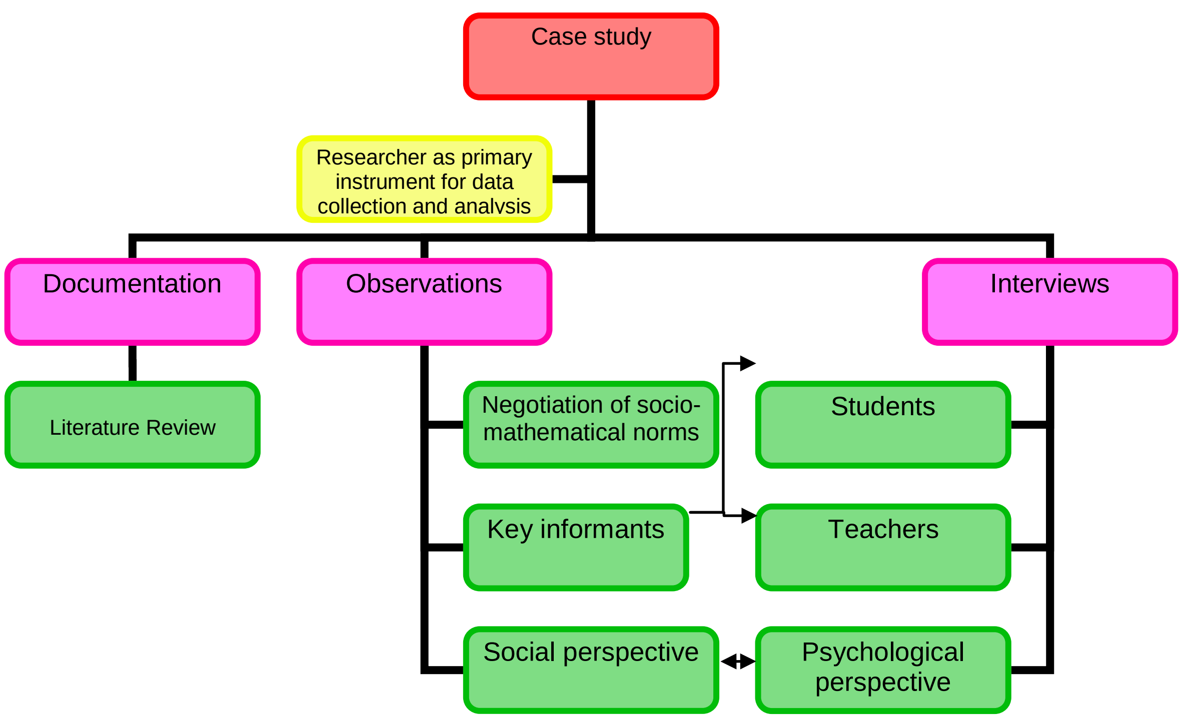


Figure 2: Triangulation of data

The processes within the observations and interviews laid foundations for each other since they re-generated and re-defined opportunities for quality data to be collected and ultimately analysed. This is particularly significant within this study since the processes involved in data collection reflect the reflexive processes involved between the social and psychological perspectives of the socio-mathematical norms.

3.7 Strengths and limitations of qualitative case study research

It is important to acknowledge that this case study brings with it the “centrality of interpretation”, of qualitative research and a “subjective description” where I am ultimately offering my “personal view” (Stake, 1995, p. 42). This appears as a limitation within a quantitative paradigm. However, the thick description of this case study does not profess to “map and conquer the world, but to sophisticate the beholding of it” in order to stimulate “further reflection, optimising readers’ opportunity to learn” (Stake, 1995, p. 42). The appeal of case study research is that it may “reveal how theoretical abstractions relate to common sense perceptions of everyday life” (Walker, 1985, p. 57).

As the researcher I acknowledged the limitations of being the primary instrument for data collection and analysis. Merriam (1998) discusses several characteristics and limitations about the human instrument as investigator and several of them I recognised as being particularly relevant to me.

To bring strength to the qualitative research method I adopted a neutral stance with regard to the findings in the case study. However, attaining a neutral stance requires that qualitative researchers use techniques that acknowledge personal bias. These techniques include the systematic methods of data collection and the triangulation of data (Patton, 1990, p. 56). However, it is also important that the researcher uses their life experiences and insights as an asset within the research setting. Thus it is important to reach a balance, between empathy and neutrality within the research setting. In order to achieve this balance it was important to remember that empathy refers to the stance towards the participants as people, and neutrality refers to the non-judgmental stance taken when collecting and analysing the data (Patton, 1990).

The qualitative research characteristic of “tolerance for ambiguity” was one area that required continual intelligent reflection for me, since traditionally, as a classroom teacher I have worked in “structured situations” with “set procedures” and “protocols” (Merriam, 1998, p. 20). As suggested by Stake (1995, p. 21), qualitative case study research needs to allow for issues to “emerge, grow and die” by “remaining open to the nuances of increasing complexity”. From the outset of this research I have certainly needed to establish routines, yet have seen the benefit in settling into an acceptance that flexibility is essential and that while decisions about how data are to be collected should be made, they (and I) needed to be circumspect and resilient to the idea of change.

On the other hand, having taught in middle and senior school does induce a certain degree of “sensitivity” and intuition about the “covert and overt agendas” that exist within these contexts (Merriam, 1998, p. 21). In this way an empathetic rapport was established within an “atmosphere of trust” (Merriam, 1998, p.23) when conducting the research. At this point it is important to note that communication was a key factor in empathising, listening, establishing a rapport and asking quality questions. That is, having teaching experience within the research setting does not automatically qualify me as being totally empathetic or intuitive. The very nature of this research acknowledges that different contexts produce unique experiences and perspectives. Therefore, I was conscious that I needed the vital skill of listening carefully and patiently.

The limitation that is often propounded about case study research involves generalisation issues. This relates to the terms, reliability and validity, and is discussed by Walker (1985, p. 57) as sometimes being framed through questions such as:

- How can you justify studying only one instance?
- Even if it is justifiable theoretically, what use can be made of the study by those who have to take action?

These criticisms may be answered by reiterating that the research is worthwhile if it "captures the attention" of the audience so that the reader may ask themselves "what is there in this study that I can apply to my own situation, and what clearly does not apply?" (Walker, 1985, p. 57). From this viewpoint, case studies also invite a socio-constructivist interpretation of the research, which supports the emancipatory intent of qualitative research. That is, this case study was undertaken within a "democratic mode" which suggests "a shift in power, a move away from researchers' concerns, descriptions and problems towards practitioners' concerns, descriptions and problems" (Walker, 1985, p. 57). Underpinning this research is the endeavour to seek understanding about some of what influences the power in the potential of pedagogy in the middle school mathematics classroom and how this becomes reflected in the classroom learning community and ultimately in students' mathematical proficiency. Indeed, this research resonates with Palmers' (1998, p. 95) view:

> To teach is to create a space in which the community of truth is practiced…The hallmark of the community of truth is not psychological intimacy or political civility or pragmatic accountability, though it does not exclude these virtues. This model of community reaches deeper, into ontology and epistemology…into assumptions about the nature of reality and how we know it - on which all education is built. The hallmark of the community of truth is in its claim that reality is a web of communal relationships, and we can know reality only by being in community with it.

Chapter 4
The microculture of grade 9 MAA

I am always ready to learn although I do not always like being taught.
Winston Churchill

4.1 Introduction

A first impression of Naomi and her grade 9 advanced mathematics class was that they were exuberant and energised. They seemed to share a purpose of talking about and doing mathematics. Naomi seemed to be confident in her pedagogical content knowledge and in what she felt students should know. During the lessons most students wanted to be involved in what was going on, they were alert and interested. Several students, Kelly, Cale and Shay (see Figure 3 below) dominated the classroom discussions. However, most students in the class would participate in the mathematical discussions at different times.

Naomi's mathematics classroom routine was steeped in a traditional instructional setting. The classroom routines followed the school mathematics program that focused on doing chapters out of the textbook to cover a certain amount of content in a set amount of time. It could be anticipated that there would be little opportunity for the classroom learning community to be mutually engaged in the exciting processes of mathematisation. However, these enthusiastic students didn't sit passively within these routines. Instead, they became actively engaged at different times. It is the potential within this active engagement, despite the traditional instructional setting, that is of particular interest within this study.

The remainder of this chapter examines this classroom and its productive capacity as a learning community. The intention of this interpretation in context is to delve into the microculture of the classroom. Different lesson observations, interviews and informal discussions are used to investigate the socio-cultural and psychological correlates of the norms within this classroom learning community.

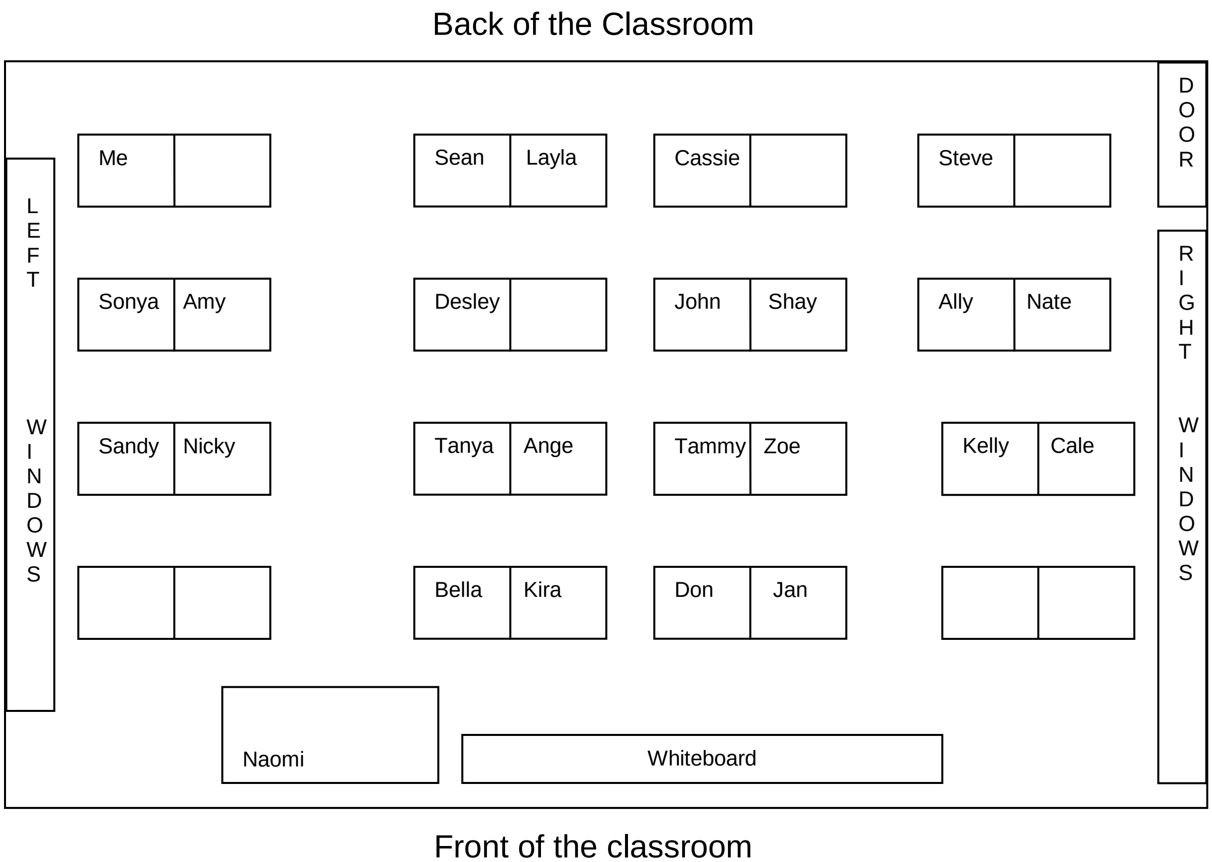


Figure 3: 9MAA Classroom layout

4.2 The social norms of 9MAA

The social, socio-mathematical and mathematical norms of this mathematics classroom were interpreted to understand if and how students develop positive mathematical dispositions. The socio-mathematical norms are specific to how mathematical interactions evolve and these "appear to influence the learning opportunities for both the students and the teacher" (Yackel & Cobb, 1996, p. 461). Therefore, in the context of understanding students' mathematical dispositions, the processes which allow the socio-mathematical norm of mathematical difference to continually evolve became a focal point. Investigating how the learning community "attempts to make sense of explanations given by others…compare others' solutions to their own and make judgements about similarities and differences" (Yackel & Cobb, 1996, p. 466) is part of the process involved in the ongoing re-establishment of socio-mathematical norms. Lesson snapshots are presented to interpret the classroom norms. How Naomi chose to use her role and how students became involved in the learning processes are explored.

4.2.1 Mathematical argumentation

The following lesson snapshot introduces this learning community. Of particular interest is how members interact during a classroom discussion on significant figures in mathematics.

Lesson snapshot 1

The lesson was on significant figures and began with a review of homework. Naomi had asked students to investigate significant figures using the textbook, or the internet and then complete questions in the textbook. Students had problems with some of the questions and they asked Naomi to work through the questions on the board. Many students were calling out and asking questions.

> Shay: Is it just rounding?
> Layla: What's the point of doing that?
> Kelly: Can you do a harder one Miss?

Students were trying to use their prior knowledge of rounding in the unfamiliar context of significant digits. This scaffolding on prior knowledge appeared to be a classroom mathematical norm and was an important tool for the students and Naomi. Naomi and the students worked together to come up with a set of rules that could be used to determine how many significant figures there are in a number. So while this class was involved in the usual classroom routines of reviewing homework, students were mathematising since they were mutually engaged in coming up with a set of rules that appeared to work for them.

Establishing how the number of significant figures could be determined was interactively constituted by the members of the classroom as evidenced by the following exchanges.

> Shay: No, you need to do it from the first non-zero digit.
> Naomi: Thank you that's a rule I didn't remember.

Amy argued a point with Naomi:

> Amy: But you said…, but I think…
> Naomi: Let's check the rule.

As it turned out, the class agreed that Amy was right.

> Amy: Yahoo, yes I get it.

Naomi would make comments as the class worked together through the problems:

> Naomi: I am using what Kelly said.
> Naomi: Kelly has refined our understanding.

It soon became evident that arguing about mathematical possibilities was encouraged within this classroom. The students and Naomi appeared to be a "community of validators" mutually engaged in realising mathematics as a "form of enquiry" (diSessa & Cobb, 2004, p. 94). It is within such a learning community that students have the opportunity to develop a productive mathematical disposition (McClain, 2002).

4.2.2 Community of validators

While solving problems on significant figures, a discussion transpired about how and where significant figures were important in real life. Students were constantly asking questions and seemed to be involved in judging, justifying and arguing about solutions. The classroom looked like a community of validators, where the students and Naomi "interactively" constituted what was an "acceptable mathematical reason" (Yackel & Cobb, 1996, p. 468). Students constantly sought clarification of their understanding from Naomi in particular, but also from one another. At one point, after Naomi worked through a problem on the board, several students called out that the answer in the back of the book was different from the answer obtained on the board. A chorus of students replied:

> The back of the book is wrong.

That chorus of students said this with authority; they weren't saying this because they didn't care and wanted to move on to the next question. At that point in the lesson an active learning community legitimised the answer. Those students felt that they had the mathematical power to override the authority of the textbook. Naomi, together with the class, checked the question against the normative rules on significant figures (that they had created during the lesson) and they all seemed to agree that their answer was correct. Furthermore, a recurring theme that Naomi weaved into this lesson was that questions on significant figures needed to have a context so that they could be understood.

> Naomi: We use the rules, but we have to think about how it works in real life. We practise the rules so that we can use them in real life.

Naomi reinforced her statement by discussing the injection of drugs by ambulance officers or medical practitioners. Students had suggested that the 0 on the end of the number was useless and therefore should not be counted as significant. Naomi used the example of the **0** in 0.256**0** to clarify the importance of the 0:

> If I go 0.256 and 1/10 000 then the drug would be useless or if I injected 0.255…therefore the 0 is a significant figure on the end of 0.2560.

Naomi also talked about significant figures in the context of physics and chemistry and the need for a normative way of determining how many digits are considered to be significant within a scientific measurement. Even so, Naomi attempted to engage students into becoming proficient in the use of significant figures by encouraging them to understand both the rules and the limitations that those rules might bring. So Naomi's implicit judgements throughout the interactions took a leading role in the classroom interactions. In this lesson snapshot, Naomi used her knowledge to transform the textbook questions into something that students might attach some meaning to. However, the students also participated in this transformation since they were trying to understand and reason through the notion of significant figures. In terms of the mathematisation cycle framework Naomi's pedagogical content knowledge encouraged this classroom learning community to "make sense of" significant figures in terms of what might be a "real solution" and importantly "identify the limitations of the solution" (OECD, 2003, p. 38).

When participating in the negotiation of the socio-mathematical norms, Naomi's role became especially powerful when she encouraged students to be reflective in their mathematical activity. That is, students were encouraged to look beyond the rules within significant figures into considering the processes involved. This is an example of how the teacher "plays a central role in establishing the mathematical quality of the classroom environment" (Yackel & Cobb, 1996, p. 475) which contrasts with the "relatively passive role that is frequently implied by the metaphor of the teacher as a facilitator" (Howe, 2003, p. 74).

Lesson snapshot 1 on significant figures is an example of how the normalising processes within this classroom learning community were developed. Students' participation in making judgments about theirs and others' mathematical ideas and explanations contributed to developing their "mathematical disposition" (diSessa & Cobb, 2004, p. 94). This mathematical disposition undergoes "ongoing regeneration" (diSessa & Cobb, 2004, p. 97). Students had the opportunity to participate actively or vicariously within this classroom learning community. In this way they could become engaged in the process of knowledge construction within the socio-mathematical norms. Similarly, the socio-mathematical norms also contributed to the regeneration of Naomi's pedagogical identity and her pedagogical content knowledge. The importance of Naomi's role was evident in this classroom discussion. The next lesson snapshot illustrates how several students interacted independently of Naomi.

4.2.3 Students mutually engaged in assignment work

Students worked on one assignment each term. Most of the assignment was completed by students in their own time. However, several lessons were spent where students could work together. It was during such a lesson that I observed John and Nate talking about their mathematical solutions to a question on the assignment.

John and Nate were two students in Naomi's class who weren't loud participants during the classroom discussions. They both seemed to follow the lesson and mathematical negotiations closely. Even though John wasn't dominant or loud in the discussions, his role was significant and I noted that many students would go to John for help or to compare answers. Students said to me that they found John's way of explaining things very helpful. John appeared to generate his "own personally meaningful ways of solving problems" (Yackel & Cobb, 1996, p. 469) and his articulations of mathematical ideas were viewed as valuable by other students. A feature of John's participation in the classroom learning community was that he seemed to understand the processes involved in interpreting other students' explanations and this made his contributions especially valuable. John was one of several students in this classroom who were “intellectually autonomous in mathematics” (Yackel & Cobb, 1996, p. 462).

On the occasion when students were working on their assignment in class I observed Nate walk over to John to compare one of his answers. Their answers for a particular question, which involved an equilateral triangle and using Pythagoras’ theorem, did not agree. John grabbed his calculator and said:

> I’ll check my answer.

Both boys were very focused yet polite in how they spoke to each other; neither boy accepted or denied that he might be wrong. John started doing his calculation and talked about using Pythagoras’ Theorem and how he had to halve the base of the equilateral triangle. So John immediately started justifying and explaining the mathematical processes. Earlier in the lesson, I had seen John checking Pythagoras’ theorem in his text book. Nate watched and listened as John talked through his answer. All of a sudden Nate said:

> Oh, I see now.

Nate had worked out his error (he hadn’t halved the base of the equilateral triangle so that he could use Pythagoras’ theorem). Neither boy made a big deal about the error; Nate went back to his desk to fix his error. John didn’t say anything and continued working on his assignment.

The discussion between John and Nate is an example of a mutually beneficial interaction. Both boys were actively engaged in judging and justifying their solutions. Naomi encouraged students to compare answers and this is an example of a socio-mathematical norm that invited John and Nate into being doers of mathematics. As suggested by Schoenfeld (2007, p. 70) “students pick up their beliefs about the nature of mathematics from their experiences in the mathematics classroom". Both boys owned the mathematics in the assignment question and their interaction allowed them to reflect on the mathematics that they chose to use. Of course it also involved the psychological perspective, since both boys felt comfortable with the interchange of ideas. Significantly, neither boy brought with himself a knowledge gap that suppressed the mathematical discussion. Their mathematical dispositions appeared to allow them to make "good use of" their mathematical knowledge and employ effective "problem solving strategies" (Schoenfeld, 2007, p. 71). In this way, John and Nate demonstrated how they were building their mathematical proficiency in terms of the five interconnecting strands: conceptual understanding; procedural fluency; strategic competence, adaptive reasoning and productive disposition (Kilpatrick, et al., 2001).

4.2.4 Mathematical Difference

A key feature of mathematical proficiency is that students develop and attach their own meaning to mathematical concepts and ideas. This may be promoted when students discuss different ways of solving problems. It is in encouraging discussions on how solutions are "mathematically different" that students may "develop personally meaningful solutions that they can explain and justify" (Yackel & Cobb, 1996, p. 462). The role of the teacher is to "develop an enquiry form of practice" (Yackel & Cobb, 1996, p. 462). At times, Naomi attempted to develop this and would encourage students to use different methods and justifications to arrive at their answer. The students in Naomi's class would often offer their different solutions without being asked. Acknowledging and discussing different solutions was a socio-mathematical norm that was "interactively constituted" in this classroom (Yackel & Cobb, 1996, p. 462). There were several students in this class who modelled their mathematical autonomy during mathematical discussions. The following lesson snapshot illustrates how the classroom community discussed the questions on their assignment.

Lesson snapshot 2

The lesson began with students asking Naomi if they could hand in their rough drafts for her to mark. Naomi suggested that they compare their answers with someone else. A big discussion ensued about plagiarism.

> Naomi: Talk it through [referring to the assignment], teach each other, but you don’t copy from someone else.
>
> A student asked: Can your parents help?
>
> Naomi: Yes, you can get help from your parents, tutor, google, as long as you are learning, But don’t get them to do it for you.
>
> Layla suggested that in mathematics: We all end up with the same answer.

Naomi acknowledged that the answers might be the same but the justification would be different.

> Naomi: You must show your working out.

When the classroom discussion turned to a particular question and a student talked about their solution Shay called out

> I did mine differently
>
> Naomi: Excellent. If another technique makes more sense to you, that is how you should do it.

Naomi encouraged all students to use their own ways of working, thinking and reasoning mathematically.

4.2.5 Efficient and sophisticated solutions

What became evident in this classroom was that even though students offered different solution methods and justifications, not a lot of time was spent on discussing how and why solutions were mathematically different. That is, the classroom didn't discuss what counts in making a method more efficient or elegant. It is in the elaboration of how and why solutions are mathematically different that Cobb et al. (2001) suggest make the socio-mathematical norm of mathematical difference potentially powerful. The following lesson snapshot is an example of how the power within discussions of mathematical difference often remained latent.

Lesson snapshot 3

The class discussed homework on the concept of average speed.
Students were asking about the homework question that required them to find speeds that would give an average speed between 50 and 60km/hr. Naomi talked about the idea of how to find an average or mean: that you add two numbers together and divide by two.

Naomi started with a simpler idea and asked students to find two numbers that have a mean of 50.

Students offered different pairs of numbers that add to 100.

Naomi then returned to the homework question and asked students to find an average speed between 50 and 60.

Unprompted, Cale asked and answered his own question:

> How would I justify my solution? Would I say that the two numbers lie between 100 and 120?

Cale often asked about how he justified his answers since his last test results were low due to a lack of justification in his solutions. It was an area that he had to ‘work on’.

> Naomi: That’s right, exactly right, very clever I hadn’t thought of it that way.

The discussion about this question and Cale’s idea stopped short and Naomi moved onto the next question.

In terms of pedagogy Naomi was modelling the Polya strategy where the problem solver needs to:

- think to use the strategy,
- generate a relevant and appropriate easier related problem,
- solve the related problem, and
- figure out how to exploit the solution or method to solve the original problem.

(Schoenfeld, 2007b, p. 66)

Such strategies are used by “accomplished mathematicians” and “high school students can learn to master such strategies” (Schoenfeld, 2007a, p. 66) and therefore should be encouraged in the classroom. However, students should have the opportunity to reflect on this process and to develop their metacogntion. Students monitoring of their own mathematics processes may be developed within the socio-mathematical norm of establishing mathematical difference.

However, the power in the socio-mathematical norm of mathematical difference was lost in this interaction because the different solution methods and their efficiency weren’t compared and discussed in depth. For example: What was clever about Cale’s solution? What if there were more than two speeds? Is there a different way of thinking about it? What if we used different numbers? Naomi commented that Cale's solution was clever. However, the students were left to independently decide what was clever about it. Eliciting a discussion about what was clever about Cale's explanation would have turned the interaction into a potentially meaningful mathematical sense-making opportunity. Certainly with hindsight, there could be a number of different ways that the interaction could have evolved into a richer and more meaningful learning experience for a broader portion of the classroom learning community. Given Naomi's pedagogical content knowledge and the intellectual autonomy of many of the students in this class, the interaction could have become mathematically powerful in terms of socio-mathematical norms.

4.2.6 Classroom mathematical norms

Naomi's classroom practice often converged on encouraging students to set their work out and justify their work using sophisticated and efficient mathematical syntax. This norm is illustrated in the following lesson snapshot:

Lesson snapshot 4

Naomi had put Sean's setting out for solving an equation on the board:

$3a – 2 =7$

$2+7=9$

$9/3=3$

$a=3$

> Naomi: I am not trying to embarrass you [Sean], but I am concerned about people's setting out. This is not a good way to set your work out. It is good thinking, but not for setting work out. When the equations get complicated, this won't work. You need to set it out; each line needs to follow on from the previous line.

Again Naomi commented that Sean's thinking was good. However, Naomi didn't invite the class to share in the elaboration of what was good about the thinking. Rather, the discussion became about how to set the work out.

> *Naomi: You must follow this setting out*

$$3a – 2 =7$$
$$3a -2 + 2 = 7 + 2 \quad \leftarrow A$$
$$3a = 9$$
$$3a/3 = 9/3 \quad \leftarrow B$$
$$a = 3$$

Naomi commented that steps A and B do not have to be included.

> Kelly: You know how you said we don't need to include steps A and B, can we still include it? Do we get marked down in communication if we don't include it?
>
> Naomi: It depends who you are and where you are at. If you have to practise it, you should still include it until you have got it right in your own mind.

So Naomi used her horizon knowledge to inject what she believed was important for students to proceed successfully with solving more complicated equations: a structured approach to mathematics. This is an example of how Naomi used her pedagogical content knowledge to encourage students to use sophisticated and efficient procedures for solving problems. While Kelly seemed to be concerned about getting marks on the end of term test, Naomi encouraged her to understand what she was doing, but only in terms of the procedures involved. The mathematical classroom norm was that using the correct mathematical syntax to solve these equations is an important tool within mathematics.

The distinction that needs to be made here is that these procedures and tools of mathematics are part of the general social mathematical norms of this classroom. Naomi and the students focused on the norms involving the expectation that mathematical solutions are justified and set out efficiently. This norm is encouraged within the Ways of Working framework of Education Queensland (2007, p.1) that suggest students should:

- plan and conduct activities and investigations, using valid strategies and procedures to solve problems
- select and use mental and written computations, estimations, representations and technologies to generate solutions and to check for reasonableness of the solution
- communicate thinking, and justify and evaluate reasoning and generalisations, mathematical language, representations and technologies

Furthermore, Naomi's horizon knowledge from her experience as senior school mathematics teacher placed this setting out and structure as being very important. However, the importance of mathematical communication within mathematical justification appeared to override the possibilities available in the socio-mathematical norm of mathematical difference. That is, little or no emphasis was placed on creating taken as shared ways of deciding what counts as a different way of thinking about the mathematics involved.

I observed that a lot of time and effort was spent during lessons on students understanding the correct setting out and the need for well communicated justification. However, I saw limited evidence of students analysing solutions to determine what counts as an "acceptable mathematical explanation" (Yackel & Cobb, 1996, p. 461). Similarly the mathematical classroom norms saw students offering different solutions, but not the socio-mathematical norm of students having the opportunity to understand "what constitutes mathematical difference" (Yackel &

Cobb, 1996, p. 461). Therefore, the classroom mathematical norms were overpowering the socio-mathematical norms within this learning community.

Encouraging more productive, inquiry based socio-mathematical norms in the classroom would attend to the analysis components of Educational Queensland's (2007, p. 1) Ways of Working. The crucial socio-mathematical norms that include the opportunity for sustained enquiry into the different solution methods offered by members of the classroom learning community were lacking from the classroom interactions I observed. Hence, the process of "active individual construction" (von Glaserfeld, 1984, in Yackel & Cobb, 1996, p. 460) that is an essential component of mathematical learning became rather superficial in the classroom interactions. Unless students are involved in the sense making processes within the socio-mathematical norms of the classroom microculture, then "working mathematics problems involves rather meaningless operations on symbols" (Schoenfeld, 2007, p.70). As a result doing mathematics might be considered an unthinking routine to be performed at a certain time. This seemed to be happening for some students in this classroom. The following lesson snapshot illustrates that some students might be wedged into viewing mathematics as an unthinking routine.

Lesson snapshot 5

Naomi's horizon knowledge appeared to feature prominently throughout her interactions with her students. She often referred to how a mathematical idea or concept becomes especially important later on in physics, chemistry or senior mathematics. Naomi encouraged students to see the usefulness of what they were doing and how it is going to be applied as they progress with choosing and using mathematics.

For example, during a lesson on speed Naomi suggested to the class:

> Can you use v for speed? Because later on in physics, s stands for distance, so we don't want to get confused.

While this is important, it again sees Naomi focusing her horizon knowledge at the level of mathematical communication.

However, Naomi also encouraged her students to understand and consolidate basic concepts and procedures of mathematics since these are a prerequisite to continuing with mathematics. For example during the lesson on speed Naomi asked:

> Can we convert km/h to m/s?
> Kelly: Yes
> Tammy: We did this in science, but I didn't understand.

As this particular lesson progressed students' misconceptions were highlighted. Embedded within Naomi's specialised content knowledge, was her knowledge of content and students. Several students had misconceptions about converting from hours to minutes to seconds or vice versa. The classroom discussion on these conversions vividly illustrated a duopoly of misconceptions that the students held. These involved misconceptions with: converting from fractions to decimals; and understanding how time is measured. Some students didn't appear to attach any meaning to the numbers. During the lesson, the class was working on the following problem together:

> d=1520km
> v=?
> t= 18 hours 40 min.
>
> Naomi: What are we going to do with the 40 minutes?
> A student suggested: Round it up

Naomi didn't dismiss this idea. The class talked about it and decided that it was better not to, and that precision was important here. So Naomi was inviting students to participate in evaluating the plausibility of a claim, which is part of the ways of working of the Essential Learnings.

> Cale: 40 minutes is 40/60 = 2/3 = 0.6 recurring

Naomi wrote this on the board as Cale said it, but didn't comment on whether it was correct. The class talked amongst themselves, to the person next to them and I heard students saying:

> Do you get that?

Students asked Cameron how he knew it was 2/3. However, Naomi jumped in here and offered the explanation. It could be that Naomi had decided to intervene because time was short.

After Naomi's explanation, the class *seemed* satisfied about how to change 18 hours and 40 minutes to 18.6 (recurring) hours. Naomi was about to proceed when Shay asked:

> Why can't it be 18.4?

Shay was thinking of 40 minutes as 40/100, and this misconception seemed to be firmly implanted in her mind. The mathematics didn't appear to be making sense and perhaps she was attempting to use "meaningless operations on symbols" (Schoenfeld, 2007a, p. 70).

Naomi explained that it is not in decimals, it is not per 100, it is per 60 minutes, 40 minutes out of one hour. Naomi returned to encouraging students to set their work out, illustrating the setting out on the board as follows:

v=d/t
=1520km/18.6 hours
=81.42857143 km/hr
=570/7 (using the calculator to convert decimals to fractions)

However, as Naomi was writing the setting out on the board a misconception arose, regarding fractions. Students were confused about what 570/7 was. One student called out:

It is 81 and something over 7
Layla: Is it 81 4/7?

Naomi asked the class. They disagreed. Layla told everyone that she got the **4** from the **4** in the decimal number 81.**4**2857143. Layla might have been guessing or there could have been a profound reason for this. However, she didn't explain why she chose to say this. The missing socio-mathematical norm of explaining how solutions are mathematically different is evident here.

John offered a suggestion of how to find the numerator in the fraction. Certainly, the social classroom norm of participation and mutual engagement was well established since students offered ideas without being asked. Naomi acted as the scribe as John explained:

81x7=567,
so there is 3 left over…from 570…
so the answer is 81and 3/7

Shay: I don't get why you did 81x7=567.

Here, Naomi intervened, using her specialised content knowledge and knowledge of students to talk about fractions. Shay participated and appeared to evaluate her own thinking and reasoning by saying:

Shay: I found it, I get it now.

Naomi's specialised content knowledge and knowledge of students and their misconceptions, featured as important tools as the classroom learning community discussed their interpretations of the mathematics being done during the lesson. Naomi appeared to be aware of how her pedagogical content knowledge was a catalyst in the learning processes involved in the classroom learning community. Now, while Naomi didn't overpower the mathematical discussion, she did tend to intervene during discussions, especially to explain basic mathematical knowledge that *should* already be known by the student. This could be because she thought that by transmitting her knowledge, students would understand the basic ideas more quickly. Perhaps in this way, Naomi contributed to the creation of a social norm in the classroom that knowledge is transmitted rather than individually created within a culture of participation.

Interestingly, near the end of this lesson when students were working individually on problems, Shay who had already brought up her misconception on converting seconds into hours called out a question:

> Is 26 minutes 0.26 hours?

Naomi had already talked through this misconception with Shay. However, Shay made the same error as soon as she started a new question requiring her to make a time conversion. Shay wasn't the only person who was struggling to understand since Sandy also called out in a frustrated voice:

> I'm still stuck on understanding how to change 26 minutes into hours.

Shay ended up getting help from John who sat next to her; she looked at his working out and asked him questions. John helped her. Naomi moved over to Sandy to talk to her individually about changing from minutes into hours.

So students' misconceptions didn't appear to be easily remedied even with effective pedagogical content knowledge. That is, Naomi was aware of how to deal with misconceptions by using her knowledge of content and students. However, students continued to make the same errors. This could be an example of how the social norms of this classroom that emphasised structure and transmitting knowledge through the demonstration of different solution methods remained inadequate. On the other hand, negotiating socio-mathematical norms that explicitly encourage students to be involved in explaining the mathematical differences between solutions so that they can attach meaning to their mathematics might be an avenue that would allow Shay to move into making sense of the mathematics she was doing. Shay is a motivated student who persists with helping herself. However, looking at John's solution might not help her to make sense of the mathematics. This might explain why she continued to make errors as she chose and used mathematical ideas.

So even though Shay pushed herself to understand, the classroom microculture focused as it was on efficient setting out and structure didn't contribute to a more sophisticated view of mathematics. Essentially socio-mathematical norms encourage students to be involved in processes involving mathematical suspicion of different solutions. In this way, students might participate in making critical decisions regarding the sophistication and efficiency of an explanation, beyond correct mathematical syntax such as setting out. Hence, understanding and learning mathematics involves being involved in a culture of mathematising. Even though Naomi and her students appeared to comprise a community of validators who do mathematise, they seemed to be skimming the surface of sense making in mathematics.

Staying at the surface of making sense of the mathematics was illustrated also in the following lesson snapshot. The interactions do seem to lose the mutual engagement that is evident in some of the other lesson snapshots. This loss of mutual engagement became clear as students struggled with their misconceptions.

That is, the classroom practice became about fixing the deficiencies as quickly as possible.

Lesson snapshot 7

During one particular lesson the class was working on linear relationships and solving linear equations. Naomi asked the students to work through the *Prep Zone* page. Prep Zone is the first page in the new chapter of the textbook. It is a review section for students and aims to guide them in their thinking for the coming chapter. Specific questions are used to identify students' misconceptions. Essentially the questions in Prep Zone are revision from work done in grade 8.

Most students in the class started working through the problems on their own; several students didn't know where to start, for example:

> Cassie: I have no idea what I am doing.
>
> Amy: How do you divide by a negative?
>
> Shay: I don't get the working out, I can write the answers.

Naomi wrote the working out on the board for a particular question:

> r/7=6
> r/7 x 7 = 6x7
> r = 42
>
> Shay: Oh the 7s just wipe each other out.
>
> Naomi: Seems to me that we aren't understanding the steps.

By saying that the 7s just wipe each other out, Naomi assumed that Shay didn't understand the underlying mathematics. Naomi's assumption may be correct, but Shay didn't explain her thinking.

Naomi would have liked to work from the assumption that students have already understood how to balance and solve equations in grade 8. Ideally she wanted to move on to setting out and solving linear equations efficiently. But some students were having trouble attaching meaning to the mathematics they were doing. Naomi's focus on understanding the steps didn't appear to be helping them.

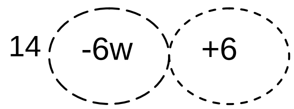
Students had constant questions about negative numbers and struggled with subtracting negatives. Over half the class lacked flexibility in subtracting or adding negatives. They hadn't made sense of the use of negative numbers. The students continued to work through the questions, in pairs or individually. They called out questions and Naomi wrote the solutions on the board. Naomi spoke about the difficulties students had when working with negative numbers during our interview:

> Multiplying and dividing by directed numbers, working with number, difference between minus and negative. I ask them every year but they can't tell me why they can't transit between minus and negatives [i.e. students need to write plus and negative as being distinct from subtracting]…I don't know the technique they are taught in primary school. This is what I want to know [pause] what is their way of thinking?

So Naomi was aware that she needed to delve into students' ways of thinking. But during the lessons I didn't observe her inquiring into how students think. Rather, Naomi encouraged students to understand that 2x – 5 was the same as 2x + -5. Her focus was on the syntax of the mathematics. Naomi offered sound methods to help students perform the procedures, and encouraged students to use the rules of directed numbers.

Naomi reminded students once again that the sign stays with the numbers.

Naomi wrote on the board that 14 – 6w+ 6 was the same as



A classroom norm that I noticed was that Naomi answered individual questions out loud for the class to hear, so that other students could listen if they had a similar misconception or problem. However, students didn't seem to be mutually engaged at this point. When Naomi went through the steps and suggested methods, the students became more like passive recipients in the interactions.

Amy asked Naomi to do part 2 (d):

Expand and simplify:

3a + 1 + 6 (2a+7)

Amy was unsure about the order of operations, a recurring issue for her. I had taken particular notice since Amy seemed to be a proficient mathematics student who often came up with creative insights into a problem.

Amy thought that perhaps they should add the 1 + 6 first.

> Naomi: Kelly is going to run me through the steps.
> Kelly stated out loud:
> 3a+1 + 6 x 2a + 6x7
> Naomi explained the expanding steps.

Kelly:
3a + 1 + 12a + 42 and then I went
15a + 43

Naomi: Kelly is collecting like terms in her head. Is that OK, Amy?

Amy nodded her head. It is difficult to determine though if Amy had made sense of the mathematics involved. Amy didn't participate in the interaction, but rather sat and listened. She didn't argue the point, nor did she explain why she thought the 1 and the 6 should be added first. Later in the lesson Naomi talked to Amy and suggested that while she might feel uncomfortable with it now, "it will come with practice". It could be that one of the classroom norms was that if you felt confident in the answer, you could become involved in the interaction, but if not, then listening was the best option. Consequently, Amy would need to go away and practise solving equations, and then she might become involved in the interactions. In fact Naomi then told the class

You should know this; you need to fill in the gaps.

Thus, another classroom norm was that mathematical misconceptions are something that aren't welcome in the discussion. Furthermore, the students might interpret that they were isolated in their endeavours to fix their deficiencies. Certainly, Naomi was focused on getting through the mathematics. The students' misconceptions were challenging her pedagogy since they did not seem to be easily remedied within the transmission model of knowledge construction.

At this point it is important to consider Naomi's and the grade 9 students' beliefs about doing mathematics in this classroom. The participants' beliefs are used to triangulate the interpretation made of the classroom microculture so far. Furthermore, consideration is given to other factors that may be influencing the classroom microculture.

4.3 Naomi’s beliefs about teaching and learning mathematics

It became clear during the lesson observations that Naomi had mathematical content knowledge and pedagogical content knowledge as discussed in the domains by Ball et al. (2007). Naomi was aware of her knowledge and suggested that

> You want teachers in the room who could inspire the kids. Look at what I had to know this morning. I needed to understand vast gaps in their knowledge to guide them in their understanding.

Furthermore Naomi enjoyed being in the classroom

> I stay in teaching because it is so interactional, and I like interacting with the students.

However, “a basic assumption of interactionism is that culture and social processes are integral” (Yackel & Cobb, 1996, p. 458). Both my lesson observations and Naomi’s comments suggest that Naomi appeared to view classroom interactions in terms of how she used her knowledge to guide and transmit knowledge to her students. Naomi also placed great importance on the knowledge, skills and procedures that are required for students to progress into higher level mathematics so that these seem to be taken “as the main performance of the culture” (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458). By placing her pedagogical content knowledge within the limitations of how she might best transmit knowledge, Naomi may be undermining the potential of her pedagogy. That is, Naomi could also use her pedagogical content knowledge to develop the processes involved in the socio-mathematical norms of mathematical difference. Hence, the prospect of encouraging productive dispositions within a culture of doing mathematics has the opportunity to evolve.

4.3.1 The value of structure

Naomi often spoke about the importance of structure during our interview. Good structure was a social norm that Naomi valued. I asked Naomi why she always started her lessons by asking students to rule up their page:

> They [students] need to have structure. They are coming from primary school and they don't know how to use a text, or how to take good notes, and to then revise from their notes. I assume they don't know how to use a text book and I assume they don't know how to revise from their notes. Note book skills; taking good notes, look through their work books to revise. I don't know if that type of study /revision is done in primary school.

As the interview progressed Naomi brought up once again the importance of structure, this time referring to students setting their work out:

> It [structure/setting out] seems to be usurped by creativity being more important [in primary school], and while I think creativity is important I don't think it is more important. If you don't give them a basic structure, they have nothing to build a creative mind on.

Naomi's point is important and supported within the literature, since the knowledge, skills and procedures in maths are the "bricks for the building" (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458). However, the bricks need mortar and "the design for the house of mathematising is processed on another level" (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458). It is this processing on another level that appeared to be overshadowed in the classroom. This could be, as Naomi suggested, due to students' knowledge gaps. That is, the processes involved in mathematising become skewed and protracted as students continually grappled with their knowledge gaps. However, it might also be perpetuated by a narrow focus on how knowledge is constructed. That is, the absence of an awareness of socio-mathematical norms that encourage students to reflect upon the when, how and why of choosing and using mathematics at particular times might be part of the reason for recurring knowledge gaps. In this way, the procedures and tools of mathematics are viewed as part of a process within mathematical sense making instead of the key performance. Interpreting the mathematical tools in action is the art of mathematisation.

Naomi did seem to be very concerned with the bricks of the building and talked about how she felt she needed to "constantly plug holes". Her focus on how knowledge might be better transmitted by the teacher was reinforced when she organised a professional development afternoon:

> I have two primary school teachers coming from Floating Hill to present *first steps* I think it is called, where they are presenting what primary school teachers do when they are teaching multiplication, division, subtraction, addition, to kids that are struggling. Because we aren't primary school trained and we have no idea sometimes of how to help a kid come from grade 2 level to grade 8 level or from grade 6 level up to grade 8 level, we're trained from grade 8 up, we can go down a bit from there, I have difficulty on how to teach them stuff, I'm not primary trained.

It is well established that a teacher's pedagogical content knowledge is extremely important. However, how the teachers use their pedagogical content knowledge to engage students into meaningful mathematical learning involves a culture of participation rather than a model of knowledge transmission (Yackel & Cobb, 1996).

4.3.2 Structure and streaming

Naomi's focus was on having better strategies to skill up students. One of the recurring themes from this data was that Naomi felt that structure could help students build their mathematical knowledge. What's more, she felt that mathematical structure such as correct setting out and justification provided a solid basis from which new mathematical ideas could evolve. As the mathematics coordinator at Amethyst College, Naomi sought to improve the structure also from the macro perspective of middle school mathematics by streaming students into advanced, ordinary and general. Naomi spoke of this structure:

> We've got structure, and we've done our curriculum to cater to that structure, so that we reduced the amount of work for the ordinary maths classes. The advanced maths classes try to do just about everything in the text, but we haven't really achieved that. Ordinary maths, we've cut out stuff that we don't think is necessary: harder stuff. The general course is a different, alternative course.

I noticed during the lessons that students struggled with algebraic manipulations that they had done within the structured environment of a streamed grade 8 mathematics class. Here we have a situation in which a structured environment does not automatically effectuate what is coveted: that students understand the mathematics that they have done in the past so that it can be used as the stepping stone to new mathematical concepts and ideas. In this way the proficiency and productiveness of students' mathematical repertoire appeared to have a tenuous existence.

Naomi spoke about streaming in terms of how much content needed to be covered. Once again the social mathematical norm of teaching and learning mathematics was considered in terms of a body of knowledge that needed to be transmitted to students.

4.3.3 Knowledge gaps

It seemed as though the knowledge gaps (such as those with fractions, directed numbers and time conversions) were stagnating the quality of the interactions. That is, Naomi wanted to use her pedagogical content knowledge to scaffold upon students' understanding so that students could "build upon their existing understanding of mathematical situations" as in the Essential Learnings (Education Queensland, 2007, p.1). Naomi talked about her frustration in our interview:

> I don't think I am working at the pace I should be for advanced. I feel as though I am filling in all these gaps, plugging all these holes along the way, so that by the time they finish with me and begin grade 10, perhaps the teachers can move more quickly.

Once again, Naomi referred to her pedagogical content knowledge in terms of how her knowledge could be given to the students. The mathematical proficiency of students was assumed to evolve from the knowledge and skills of mathematics that are presented in the classroom. However, these form the "procedural surface only" (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458). The lesson snapshots indicated that students' misconceptions were not easily remedied even with Naomi's pedagogical content knowledge. Perhaps Naomi's knowledge of content and students is only one part of the glue to repair the gaps. What might need to be considered is learning through participation so that students are involved in thinking about mathematics in terms of "when to do, what to do and how to do it" (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458). Thus, the strength of the glue lies within how pedagogy can be used to improve the culture of participation. In this sense, mathematical proficiency when considered from the perspective of the five interconnecting strands (Kilpatrick, et al., 2001) places the negotiation of mathematical ideas in a classroom learning community driven by an inquiry tradition as a meaningful and productive way to learn mathematics.

A challenge to Naomi's pedagogy was spending enough time on misconceptions while also getting through the advanced mathematics program. During our interview Naomi discussed how mathematics teachers in the middle school were telling her that there was too much work to cover within the Grade 9 curriculum:

> I wrote the curriculum for 8, 9, 10 and 11 and 12 and I taught grade 8 for two years because I wanted to make sure that that curriculum was running properly, in that structure. That was the first two years that we implemented the general through to advanced flexible grouping into grade 8, so we stream them in grade 8. So I wanted to know that was correct and working to my satisfaction. It is, I think. All the maths teachers think it is working. Some people say, 'Oh you stream. You label your students, blah blah'. No, no we don't. They can move.
>
> I wanted to teach year 9 because I hadn't taught it and people were telling me there is too much in the curriculum and we can't get through it, and I wanted to see for myself. So there is a lot in it and I don't know what I can pare back from it. We have a NAPLAN test which tells us that we should have seen all of this by this time, so I'm kind of like yes, we have a lot in there but it is difficult to pare back from it because everything is on the NAPLAN test and it is aimed very high. We could sacrifice a whole topic I suppose, and then [Amethyst] is bad at I don't know say, indices or something like that, but I don't think that is a great decision.

So Naomi saw herself caught between a triad of problems: a significant number of students without a strong mathematical foundation; excessive content in the curriculum; and the quality control measures of the NAPLAN test. Naomi didn't openly admit to feeling the pressure of Amethyst College performing well on the numeracy section of the NAPLAN test. However, the complexity of the issue was clear since Naomi used the NAPLAN test as the reason why it was difficult to cut back on the content within the curriculum. In this way Naomi might be thinking of mathematical proficiency as mathematical content rather than content and action combined (ACARA, 2009).

Consequently, students were being rushed though a shallow curriculum so that they could cover a greater breadth of content. As a result, the depth of students' mathematical comprehension became undermined. Certainly, using a transmission model of learning and teaching expedites the learning process, but any learning appeared to be short lived. Moreover, it might have added to the *holes* in students' mathematical foundations. In this way, students' knowledge appeared to cycle through a spiral of remembering and forgetting. Thus, the distinction needs to be made that the holes might not be within the mathematical knowledge of the students. Instead, the holes might be within the processes of understanding how and when to choose and use the knowledge. That is, knowledge "will not be of much help, if the learner is unable to flexibly relate and transform the necessary elements of knowing into his/her actual situation" (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458).

Students in grade 9 at Amethyst College did the QCAT during semester 2. The QCAT did bring to light a deficiency in the flexibility and functionality of students' mathematical repertoire, particularly in modelling and problem solving. Naomi suggested that the grade 9s really "failed" the QCAT. What's more, Naomi felt that the QCAT didn't reflect the ability of the students at Amethyst College since results were markedly different from those achieved on school based assessment.

Indeed the QCAT results might be a reflection of the quality of the learning that occurred in the classroom. For example, the students in grade 9 did poorly on the QCAT, which did involve a question using Pythagoras' theorem, a topic students had done in Semester 1. However, many students couldn't make the connections with the question using Pythagoras' theorem on the QCAT. This could be an indication of the inactiveness and weakness of their knowledge of Pythagoras' theorem. Moreover, the "depth of settling is nothing else than the connectedness to lived-through reality and its persistence can be guaranteed by the strength of these relations" (Freudenthal, 1973, p. 77). Moreover, it could be that the classroom norms made "the pieces of mathematics…foreign substances in students' lived through reality…children can learn all you want…it is another fact that they can forget it just as completely" (Freudenthal, 1973, p. 77). Therefore, testing students' ability to apply what they have just learnt (as in the end of term test) may be irrelevant since it does not tell "how deeply the taught material has settled and how long it remains active" (Freudenthal, 1973, p. 77). This perhaps reiterates the idea that the problem might not be about knowledge gaps, but rather about students being able to participate in identifying when and how knowledge is useful.

Another important factor to consider is the breadth of the grade 9 mathematics curriculum and the influence of the NAPLAN test. That is, the pressure of high stakes testing and doing sufficient content before the end of term test may also provoke teachers into feeling pressured to teach knowledge and skills. In this way "other aspects of mathematical proficiency tend to be given short shrift…thus, assessment shapes what students [and teachers] attend to, and what they learn" (Schoenfeld, 2007, p. 72). On the other hand, the QCAT appeared to focus on students choosing and using their mathematics to justify answers to real life problems. It could be that students performed badly on the QCAT because it was testing something different from what they thought they were expected to know.

4.3.4 Naomi's beliefs about education research

Naomi does appear to be a reflective practitioner. But I also noticed at times that she may have felt isolated in her endeavours and would often be on the defensive regarding different issues. During our interview Naomi's comments tended from *we* initially to ***I*** statements as the interview progressed. Naomi appeared reticent in talking about the other mathematics teachers within the school in case it appeared that she might not be appreciative of their efforts. Her sense of collegiality provided a platform from which she discussed her distrust of research in education:

> Good teachers are driven away because they are driven insane with all the rubbish that is handed down from useless research that has no idea about real, practical issues that teachers face in the classroom: when they are teaching seven classes every day. No purpose to some research.
>
> It seems as though a lot of the changes to the curriculum over the past ten years are based on ideologies that have no practical basis. For example: outcomes ‘we will change our curriculum so that every time a student achieves we will record an outcome’. If you’d asked any teacher at all whether that would be possible, it is ridiculous, it was a stupid idea. Sure we can do that if we employ the same number of teachers who are currently teaching and let those teachers do that kind of researching/recording. Then that will be done: one teacher teaching and the other teacher can assess that is all they do.

The various education reforms that have “swamped” teachers (Handal & Herrington 2003, p. 63) over the last decade have compounded Naomi’s role as the mathematics coordinator and resulted in job intensification, seemingly without improved learning outcomes for students. Handal and Herrington (2003, p. 63) acknowledged teachers’ scepticism toward policy–orientated change and discussed “outcomes-based education” as being “poorly defined in operational terms and without positive gains in students’ learning”. Naomi sees herself removed from research in education since she doesn’t feel it is meaningful and worthwhile within her context. Furthermore, Naomi views herself at the receiving end of the education reforms rather than participating in the processes of how to improve mathematics education. It could be that the continuous reforms in education have resulted in her succumbing “to pedagogies of resentment that are driven by a logic of deficit views of students” (Prosser, 2006, p. 13). Naomi’s overarching concerns about students’ misconceptions and her reliance on structures such as streaming as the cure all may suggest that she is struggling with her “pedagogical identity” (Prosser, 2006, p. 13) since she is searching for solutions outside her classroom practice.

4.4 Students’ beliefs about doing mathematics

The aim of the informal discussions with several students was to gain an insight into how they viewed themselves within the norms of their mathematics classroom. Furthermore, the analysis contemplates the student’s mathematical proficiency, in terms of the five interconnecting strands (Kilpatrick, et al., 2001) as discussed in the literature review. Students completed a questionnaire (Appendix 5) and I used their responses as a guide for our discussion. This discussion was open so that different issues could emerge.

4.4.1 John’s beliefs about mathematics

On the questionnaire that I had given to students, John wrote that he found maths class “amusing”, so I asked John why this was so.

John had a broad grin on his face when he responded:

> Oh, it’s the class’s attitude and how the class acts. Like other students have one small error, but when I explain it to them they can do the whole page of work.

John seemed to be very comfortable in his mathematical ability and also commented that:

> I do forget some maths sometimes and do need to be reminded how to do some things.

John felt confident that he understood mathematics very well, but he also wasn’t overly hard on himself about forgetting things. He was in a situation where his mathematical understanding and confidence were nurtured by the other students in the class asking him for help.

Also within the questionnaire John wrote what worried him:

> Exams that exploit the few weaknesses in maths; they should be more general and less questions that give 8 marks.

John went on to explain:

> On the test in term 1 or 2, there was one section with two questions, worth 8 marks and 6 marks. It exploited my weakness and I really don't see myself as a weak maths student. It was a question on Pythagoras, with two circles inside a rectangle.

John was disappointed that he couldn't show how good he was at mathematics and Pythagoras' theorem in the exam. Essentially John criticised the exam's structure for not "capturing" his "mathematical proficiency" (Schoenfeld, 2007a, p. 72). What's more, because students "take tests as models of what they are to know" (Schoenfeld, 2007a, p. 72), John is critical of two questions that appear to have too much weight attached to them.

I then asked John if he liked doing the assignments.

> John: Yes, I like the assignments, better than other subjects because they are more interactive, like measuring things like height.

So John enjoyed interacting with the mathematics and in having the opportunity to choose and use mathematics. He appeared to be developing his mathematical proficiency since he seemed to be aware of the need to turn mathematical content into mathematical action (ACARA, 2009). Overall John appeared to have a productive mathematical disposition and high degree of intellectual autonomy. John’s disposition may be able to thrive within this classroom, since the classroom practice places value on those who can contribute correct solutions and ideas to the classroom learning community.

4.4.2 Sandy’s beliefs about mathematics

I had taken particular notice of Sandy since her motivation and interest in maths decreased as the semester progressed. Her confidence levels seemed to drop and she didn’t seem to be experiencing a lot of success. She was not interested in continuing with advanced mathematics, and planned to go to ordinary mathematics in grade 10. Sandy was very open about her dislike of mathematics. She appeared to be overwhelmed and was finding everything difficult and felt that she needed help with everything.

> Sandy: Maths is boring and hard
>
> I: When wasn’t it boring and hard?
>
> Sandy: It was easy and fun in primary school where we played games. I went to primary school in England. I found maths easy in grade 8 because it is like grade 7 in England. That’s why they put me in advanced maths [at Amethyst College last year]. Maths is boring and hard this year and I want to move to ordinary maths next year. I just couldn’t do that water test last week [Sandy is referring to the QCAT]

Sandy told me that she does feel relaxed in class:

> I can just sit here while she [Naomi] talks, and I just copy stuff down. I think that is what everyone does.

Sandy might have fitted into the classroom practice of sharing knowledge in grade 8 because she felt she had knowledge to contribute and could therefore participate. However, as the mathematics became more challenging in grade 9, Sandy might have felt ineffectual in the interactions and therefore came to see mathematics as boring. As mentioned, the norms in Naomi's class meant that those students who knew the right answers would participate in the interactions, and those who didn't needed to listen to the more knowledgeable others. Consequently, Sandy became further removed from the classroom learning community since her knowledge gaps became an overpowering source of disengagement. What's more, Sandy felt that all of the students were as disengaged as she was, even though this wasn't necessarily the case. Moving to ordinary mathematics appeared to be a viable option to Sandy perhaps because she thought she might be able to participate more effectively.

In this way participating in the classroom interactions depended on Sandy's having the mathematical proficiency to action the mathematics. However, the classroom practice which was focused on transmitting knowledge didn't encourage students to become engaged in the processes involved in choosing and using mathematics. Consequently, Sandy's mathematical proficiency did not appear to be developing in a productive way within this classroom. Sandy's mathematical disposition appeared to become less productive over the course of the semester as the mathematics became more rigorous.

4.4.3 Shay's beliefs about mathematics

Shay was a highly motivated and focused student who had a willingness to persist. She constantly asked questions and also sought help from other students in the class. Shay believed that learning was her responsibility, perhaps to the point where she seemed to be hard on herself. On her questionnaire Shay wrote:

> Maths is a very hard subject, but I know if I listen more in class, I can do it very well.

Shay continually referred to maths as being hard.

> Maths is hard for me. It is either a yes or no and it is the hardest subject for me. I do find maths interesting, but I don't think I'm good at it. If I don't study I will fail. In other subjects I can get by, but not in maths.

The discussion continued in an informal way and Shay commented that:

> I'm not scared to ask questions, you have to ask questions. I put myself with people who don't annoy me. John is smart so I sit next to him, because he is smart and doesn't distract me. In term 2, I sat with friends and got low marks, my marks have improved since I moved. When I sit with friends, we chat and I get distracted. I did better on the last test and I can feel myself achieving.

I spoke to Shay about her questionnaire and I asked her to explain how she stopped herself from being discouraged when she found the maths difficult.

> Shay: Well, if you give up you won't get anywhere. If you can't do a question, you leave it and try the next one and it may help you work out what you couldn't do in the one before it.

This was an interesting comment since in one of the classroom observations I noted that Naomi said, "What I like very much about students in this class is that if you can't do Q1 part b you still go on and try part c, d, e, etc; you keep trying to see if you can work it out for yourself". It is an example of the reflexivity of the socio-cultural and psychological aspects of the classroom learning community.

Certainly, a recurring social norm of the classroom is that students will succeed if they keep practising and trying. Therefore, Shay's persistence saw her fitting comfortably into the classroom practice. What's more, Shay suggested that she needed to ask questions so that she could get answers from other members of the classroom learning community, another opinion that was reinforced by the classroom norm which cast knowledge construction within a transmission model. However, this may not be developing Shay's intellectual autonomy or a productive mathematical disposition. That is, Shay's mathematical proficiency appeared to be hovering around conceptual understanding and procedural fluency at the expense of incorporating strategic competence, adaptive reasoning and a productive disposition (Kilpatrick, et al., 2001). This may be inducing a vulnerability to Shay's conceptual understanding and procedural fluency. Instantiating this conclusion are Shay's recurring misconceptions that are evident in the lesson snapshots.

4.4.4 Tammy's beliefs about mathematics

Tammy was a quiet student in class, although she was willing to participate in the classroom discussions. She did work well with other students and seemed happy and relaxed in class.

> Tammy: Maths is just the same thing over and over. It's not fun. It's only fun when you sit next to your friends. Today was fun because I drew pretty patterns.

I asked Tammy what were some of the new things she learnt last term.

> I learnt how to expand [an equation]. It makes things easier. I couldn't do it and then one day I was doing maths in English and it just worked out. I like algebra. I'm good at it right from the start. I could do it and because I wasn't behind it was more enjoyable.

Tammy's comments reinforce the idea of the microculture of the classroom being more valuable to students who know and understand more quickly. Since Tammy felt good at algebra "right from the start", she enjoyed it and would persist with working through the ideas for herself.

Tammy turned the discussion around on her own and started talking about Naomi:

> I have no idea what she [Naomi] is saying. When I think of my own way it makes it easier than when she writes it on the board. She makes it more complicated. The arrows make it confusing [arrows refer to the distributive law]. I get my friends to explain it and I get it.

This is an example of where a student placed great value on participating in a culture of negotiating mathematical ideas with other members of the learning community.

I asked Tammy what she found frustrating about maths.

> I get frustrated when I spend so much time on the one subject, but you still get things wrong. You see what you get wrong and think you get it, but then when you do it again later you still get it wrong.
>
> Kids have to wait with their hand up for ages until Miss comes round, and then if she is helping someone else and someone near them puts their hand up, she helps them and you have to wait even longer.
>
> I: Can you ask the person next to you for help?
>
> Tammy: Yes, Zoe explains it easier, easier than the teacher sometimes, so the teacher is like the book sometimes; if the question is really hard then I need the teacher.
>
> I: What do you mean?

> Tammy: Well, if me and my friend have different answers, we have to ask the teacher to tell us which is right or wrong, because we can't always work it out on our own.

Tammy's comments reiterated the precarious nature of constructing knowledge in mathematics. Certainly, the fact that Tammy can reason about how she is thinking about the mathematics suggests that she is developing her intellectual autonomy and thus has the potential to develop a productive mathematical disposition. However, the socio-mathematical norm of mathematical difference isn't encouraged within this classroom. Thus, as suggested by Tammy, students wait passively for Naomi to tell them the right way to do things. While Naomi does tell students to work together, attaching meaning to the idea of mathematical difference is something that is "interactively constituted" by the members of the learning community (Yackel & Cobb, 1996, p. 462). These types of interactions flourish in an "inquiry form of practice" (Yackel & Cobb, 1996, p. 462). This form of practice is something which cannot be told to students, but rather is negotiated and constructed through mutual engagement and encouraged in the classroom practice. So regardless of Naomi's telling students to work together, students are locked into seeking answers from more knowledgeable others since the classroom practice values this transmission model of knowledge construction. Tammy's intellectual autonomy is evident in her responses. However, there appears to be a tension in the development of her mathematical disposition since how she would like to do mathematics conflicts with the classroom practice.

I asked Tammy to tell me about the things she worries about when it comes to maths.

> I worry about my overall result on the exam, so I rush the exam so I can finish each section, because you need to get an A, B or C in each section, I won't pass the test if I don't do well in each section, so I have to rush to get through the questions.

> I hate graphs and linear equations. It was on the exam and I had no idea what to do. It was like I had never seen it before.
>
> I: Did you spend much time on it?
>
> Tammy: No, it was rushed through at the end of term.

Tammy's frustration was clear. However, this frustration doesn't seem to be directed towards the art of doing mathematics, but rather towards how mathematics is done at school. For this student it seemed that the conditions of *mathematics in school*: listening to the teacher; waiting for the teacher to help; rushing through excessive content to perform on the exam is turning mathematics into a chore with little reward.

Tammy actually enjoyed thinking about mathematics, particularly when she had the opportunity to think about it in her own way. Tammy also seemed to enjoy interacting with her peers. However, the current conditions of mathematics in school: the expectations of what to know and how to do the exam; and the transmission model of knowledge construction seemed to be taking the enjoyment out of the subject for her**.** It seemed as though Tammy wanted to keep doing mathematics, but this was in spite of school mathematics instead of because of it. What's more Tammy's mathematical proficiency appeared to be undermined by school mathematics since there was little opportunity for Tammy to make the connections between the five strands within the concept of mathematical proficiency (Kilpatrick, et al., 2001). Thus, Tammy's mathematical proficiency might not be able to develop in the robust way that she would have liked.

4.4.5 Amy's beliefs about mathematics

Amy appeared to have confidence in her own mathematical proficiency. I observed her in the classroom as being able to reason and think through the mathematics. She relished the opportunity to feel that she was right. She did have a knowledge gap when it came to order of operations. However, she turned this around with independent consolidation before the test.

In the questionnaire Amy wrote that her biggest worry in mathematics was that:

> I can't stay concentrated enough and my grades are dropping. My grades dropped on the last exam, I might have to study for this next exam. I only studied for half an hour for the last exam.

I asked her to tell me more about this.

> Like at the end of the day I just can't be bothered; I really don't like to do big chunks of work; I don't really like to do lots and lots of work; I'm a bit lazy. Miss goes over and over it again and again for the less clever people in the class who keep asking questions, and I get it, so then I get side-tracked and I don't concentrate and when I'm supposed to start work I'm not in the mood.

Amy was highlighting that the classroom practice of the transmission model of teaching was not serving the needs of the learning community. The classroom practice wasn't expediting the repairs to students' knowledge gaps and it was leaving Amy feeling bored. The classroom interactions which placed greater value on more knowledgeable others didn't suit Amy, even though she appeared to be proficient in her mathematical knowledge. So Sandy and Amy were both bored within this classroom, yet apparently for different reasons.

I asked Amy how she thought we could improve our mathematics classes:

> I'm competitive, so like today when we were doing all that work I was racing Sean. In exams I compare myself and compete with other people.

> When I was in year 7 I was in an extension group where we did sheets with problem solving and I was always really competitive with that.

Amy used competition to relieve some of the boredom she felt in the classroom. Problematically though, it seemed as though it wasn't so much the doing of mathematics that interested Amy, but rather the opportunity for competition in school mathematics. In fact, Amy's mathematical disposition had been nourished within the competitive nature of doing school mathematics. However, her mathematical disposition may end suffering from mathematical malnutrition. Moreover, this type of mathematical disposition may not be productive in the long term, since for Amy the reward is in the competition rather than in feeling that she is a doer of mathematics. In this way Amy's mathematical proficiency becomes questionable, since she may not be seeking to action mathematical content for the sake of doing mathematics.

4.4.6 Layla's beliefs about mathematics

Layla was a student in this class who talked to me throughout my classroom visits about her dislike of mathematics and how boring she found it. She also told me a number of times during the lessons that she felt she was "dumb at maths". I asked Layla what she found so boring about mathematics.

> Layla: Because once you get something, you still have to do the whole exercise and it's repetitive and boring.

So the classroom norm of practising the mathematics seemed to be redundant to Layla's way of doing mathematics. However, Layla also continually told me that she was "dumb at maths" and I asked her to explain this:

> Because it feels like I take longer than other people to get it, I get frustrated; I don't move quickly enough; I wouldn't be able to do extension maths because I'm not quick enough.
>
> I: How is it that you are in advanced maths?
>
> Layla: Well, it seems that I'm good at it. Dad's good at it and he helps me. I was really good at maths in primary school; I got it really easily; I don't know what happened when I came here. I was in ordinary last year and they put me up into advanced even though others were doing better than me and they didn't move up.

It appeared that Layla didn't feel that she belonged in the advanced mathematics classroom and it seemed that she equated being good at mathematics with being quick at it and understanding it easily. Certainly, one of the social classroom norms equated speed at doing mathematics to being competent. Naomi would encourage students to get quick at doing their mathematics and students would race each other to get the exercises done. Naomi often told students that they needed to get quicker at doing their mathematics by practising their mathematics. However, the classroom norms of practising mathematics and doing the mathematics quickly appeared to conflict with the development of a productive mathematical disposition for Layla. Moreover, this mathematical norm might be trivialising the focus of learning in mathematics as a process of sense making.

I asked Layla how she found the QCAT that they had done during the week.

> Layla: I found that really hard; I wasn't confident at all in how I did, so I wasn't surprised when I found out I did really badly. I mean really bad [Layla's eyes widen in disbelief]. I just couldn't relate any of it to what we do in class; I don't remember doing the maths for that at all.

Layla’s comments reiterate the notion that students’ mathematical proficiency is transitory in this context. The transitory nature of students’ mathematical proficiency is sustained within this classroom practice which fleetingly attended to mathematical ideas. The value placed on mathematical agility in the classroom to do the exercises quickly appeared to produce acute yet temporary knowledge. The recurring knowledge gaps and the inability to use mathematics in flexible ways on the QCAT are two recurring themes within this data that substantiate this interpretation. As Layla suggested, she couldn’t “relate any of it to what we do in class”. As mentioned earlier, mathematical knowledge is not much help unless the user can “flexibly relate and transform the necessary elements of knowing into his/her actual situation” (Bauersfeld, 1993, p. 4 cited in Yackel & Cobb, 1996, p. 458). Layla’s experiences appear to confirm this view.

4.4.7 Concluding thoughts about students’ beliefs

There was an underlying theme that emerged when I spoke to students. Students were often stressed and worried about doing mathematics and about mathematics tests. Several students made comments similar to this:

> I forget what I have learnt, and I worry that I won’t remember it for the test. Even though I understand it, I forget how to do things that I haven’t done for a couple of months. I really have trouble with integers and negatives and what to do with them, and on the test I forgot how to divide by a fraction.

The focus of mathematics in school seemed to be on students achieving on a test. This focus and the classroom practice resulted in transient mathematical knowledge. The social mathematical norms where students “experience skills-focused instruction” meant that they tended “to master the relevant skills, but didn’t do well on tests of problem solving and conceptual understanding” (Schoenfeld, 2007b, p. 64). Moreover, students’ mathematical dispositions were not developing in productive ways. That is, students were focused “on doing a particular computation… they…never stopped to consider how wise it was to invest their time in doing so, they never reconsidered" (Schoenfeld, 2007b, p. 66). It is this continual reconsideration and mathematical suspicion within the socio-mathematical norms of mathematical difference that might encourage students to mathematise in action when required. Certainly, the data appeared to converge on the idea that missed opportunities for students to continually engage in the negotiation of mathematical difference and the focus on a transmission model of knowledge construction was weakening the prospects of sustaining a classroom culture engaged in mathematising. In turn this influenced the usefulness of students' mathematical dispositions and thus how well placed they were at choosing and using mathematics in flexible ways. Consequently students' mathematical proficiency was undermined since they had few chances to make the links between the interconnecting strands that constitute mathematical proficiency (Kilpatrick, et al., 2001).

4.5 Conclusion: the potential of active engagement

The potential of Naomi and the students within this classroom learning community seemed to be unfulfilled. The lesson snapshots showed that Naomi and the students were often mutually engaged in talking about the mathematics that they were doing despite the classroom focus on a transmission model of learning within a traditional instructional setting. But the social classroom norms appeared to exist in a dichotomy of participation involving either being *a knower* and *a teller* or *a questioner*. Therefore, the mutual engagement became about knowing or not knowing mathematics rather than about the processes involved in how someone comes to know the mathematics that they are doing. The focus on knowing or not knowing appears to have influenced students' mathematical proficiency and flexibility.

Naomi's pedagogical content knowledge is clear in the lesson snapshots. But it seems as though her pedagogical content knowledge is not being used to its full potential. Similarly, students aren't using their intellectually autonomy to develop productive mathematical dispositions. This may be because developing productive mathematical dispositions involves participation within a culture of mathematising (Schoenfeld, 2007; Yackel & Cobb, 1996). The key processes of mathematising through the negotiation of the socio-mathematical norm of mathematical difference seemed to be "constrained by the current goals, beliefs, suppositions and assumptions" of the classroom microculture (Yackel &Cobb, 1996, p. 460).

The classroom microculture is interactively constituted by the members of the learning community. However, Naomi in particular, steers the microculture with how she chooses to use her pedagogical content knowledge. Certainly, Naomi plays a "central role in establishing the mathematical quality of the classroom environment and in establishing norms for mathematical aspects of students' activity" (Yackel & Cobb, 1996, p. 475). What's more, Naomi's role within the microculture is also influenced by how mathematics is interactively constituted in the macro-perspective by Naomi and the mathematics teachers at Amethyst College.

The next chapter examines the microculture of the other classroom within this case study. The data gathered from Dan and his grade 9 ordinary mathematics class are presented and analysed using the same methodological lens as in this chapter. Furthermore, the next chapter draws attention to how another teacher uses his view of learning and teaching mathematics from a macro-perspective and how this also shapes the microculture of the learning community. These data supplement the breadth and depth of the data which will be used in the comprehensive analysis of the middle school mathematics culture at Amethyst College in Chapter 6.

Chapter 5

The microculture of grade 9MAO

> *It is important that students bring a certain ragamuffin, barefoot irreverence to their studies; they are not here to worship what is known, but to question it.*
>
> Jacob Bronowsk

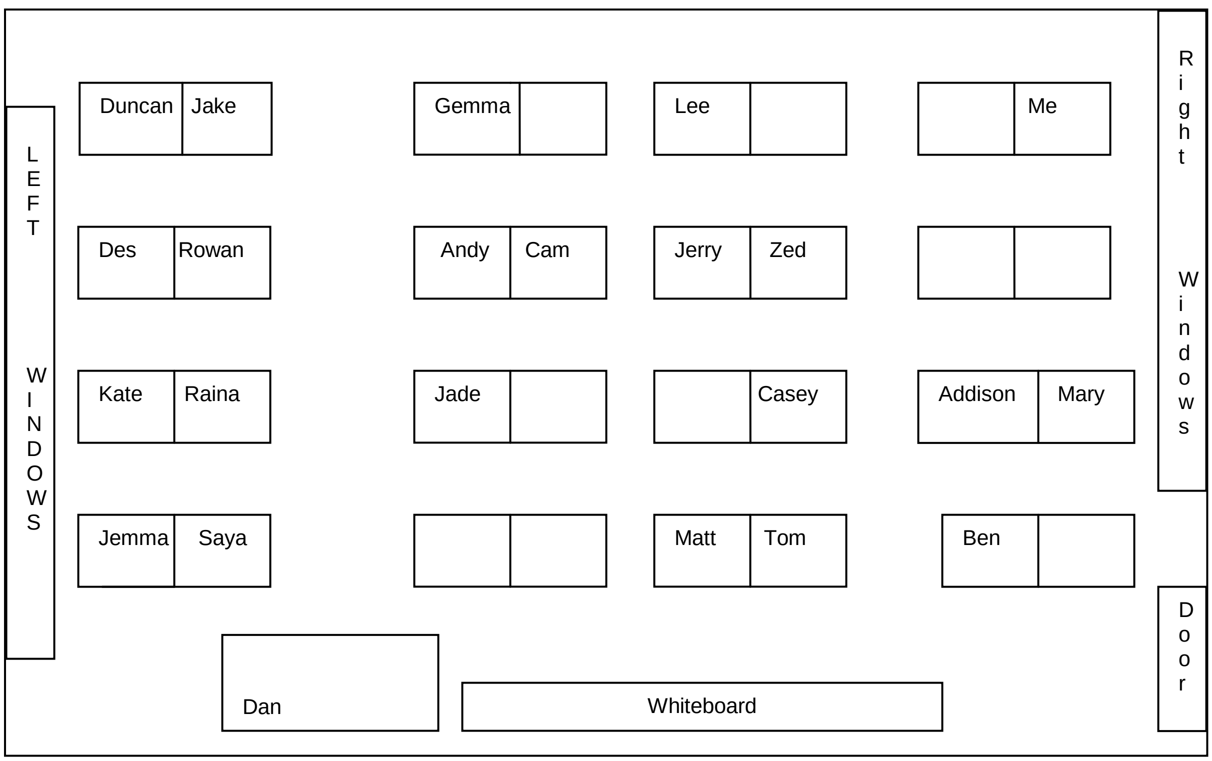
5.1 Introduction

Dan and the grade 9 students in this ordinary mathematics classroom appeared to have developed a rapport of mutual respect. Dan is a very experienced teacher who planned each of his lessons meticulously. His lessons followed the traditional pattern of a review, a presentation, followed by students doing exercises out of the textbook for consolidation (Romberg, 2001). Dan planned his lessons according to how he could best adapt the content to the perceived needs of his students at a particular point in time. In the presentation part of his lessons, Dan seemed to use his specialised content knowledge and his knowledge of content and students to sequence mathematical content to perhaps improve the accessibility of the mathematics. Dan didn't teach from the textbook and he was very selective when choosing questions out of the textbook for students to do. In this way, Dan used his own knowledge of content and teaching to structure his lessons.

Dan's mathematical authority was clear in the classroom and most of the students appeared to try to listen to Dan. However, there was an underlying listlessness in how students participated during the lessons. Even so, Dan felt that the class had really improved as the year progressed and he was pleased about their participation and how they worked together. He told me that one disruptive student (Agatha) had left the class at the end of last term. Dan felt as though her shifting had been very settling for the class. It appeared that Agatha had a lot of power in the class, and after her departure, the class dynamics changed so that the class was more settled and willing to participate in the discussion.

The remainder of this chapter investigates the norms of this classroom learning community. Also, an insight into Dan's beliefs and the students' beliefs about doing mathematics highlights the reflexivity of the social and psychological perspectives of the classroom norms. Finally, data which explore how the mathematics curriculum is structured at Amethyst College draw attention to how this might influence the microculture of the classroom learning community.

Back of Classroom



Front of classroom

Figure 4: 9MAO Classroom layout

5.2 The social norms of 9MAO

The members of this classroom learning community appeared to be going through the same routines in most of the lessons that I observed. Dan chose examples to do with the class and invited students to contribute their ideas about how to solve the questions. It seemed to be the same students participating in the discussion. A pattern of participation became apparent.

Des took the lead if the topic was on finding area or volume; Jemma took over if the discussion was on algebra and solving linear equations. Other students fell into several categories; for example, Jake, Addison, Ben, Cam and Rowan contributed their ideas on all topics; Gemma, Jerry, Kate, Raina, Duncan and Zed participated only if they were directly asked a question; Matt and Tom oscillated between being really interested or totally uninterested.

Thus a social norm of this classroom was that students participated if they felt confident that they knew the *right* answer. Ultimately though, most students in this class tended to accept Dan's statements so that "students arrive at the understanding that mathematics is, in fact, whatever the instructor wants it to be" (Milgram, 2007, p. 40). A mathematical norm had developed whereby students had learnt that "answering a mathematical problem amounts to guessing what the person stating the problem wants" (Milgram, 2007, p.40).
The following lesson snapshot illustrates this norm:

Lesson snapshot 1

Dan commenced the lesson on measurement, starting with the volume of a prism.

Dan began by asking students to share their ideas about what a prism is. Some suggestions from the class were "a tissue box" and "a coke can".

> Dan: What about the pyramids?
> Ben: A pyramid's not a prism because it goes to a point.

The discussion about prisms continued and Dan illustrated that a prism needed to maintain a constant cross section. Dan drew several different diagrams on the board and discussed different types of prisms and what's not a prism: for example a truncated cone or a tetrahedron.

Dan worked several examples which were similar to some of the questions that students had to do out of their textbooks. This was pre-planned so that students could use the examples and the setting out as a guide. He started by doing an example to find the volume of a rectangular prism.

> Dan: What is the shape of the base? What is the area of the base?
> Des: Length by width, it's a rectangle.

Students worked out the answer.

> Dan: What are the units?

The class talked about why the units were in m^3.

Dan wrote the working out for the question up on the board, the students copied it down.

Dan moved onto finding the volume of a cylinder.

> Dan: What is the radius here? What is the area?

Ben suggested that $2\pi r$ is the area of the base.

Dan talked about this misconception:

> A lot of people get confused with that.

Des: Is it πr^2?

Dan wrote the working out for the question up on the board and the students copied it down.

It was mainly the boys in the class who participated in the discussion by offering answers. I noted later during my observations that when the topic was algebra, the girls in the class took the leading role in the discussions.

Dan drew a triangular prism lengthways. Des commented that it looked like a *toblerone* (chocolate) bar, Dan and the class agreed.

Dan: How would you have to stand it so it looks like a prism?

The students talked about the need to have the triangular prism sitting on its end, so that they could see the height.

Matt: Doesn't a prism have 4 sides?
Dan: What about a cylinder?
Matt: Oh yer.

Dan: What is the area of the base, the triangle?

Lots of ideas were offered, students were trying to remember rules. One student suggested: "s+s+s" another student offered "3xs". The ideas were random and students seemed to be trying to guess the right answer for Dan.

Des: Half base times height.

Dan set out the problem on the board.

Dan also included in this question the need to change cm to mm.

Dan: How do we change 6.1cm to mm?

The class talked randomly about moving the decimal or multiplying by ten

Dan: Multiply by 10, why?

Several students responded with 1cm = 10mm

Dan constantly questioned students. However, students' explanations seemed to revolve around guessing and regurgitating rules rather than thinking about the how and why of the mathematics. Students rarely asked questions. In the lesson snapshot Matt was the only student who made an inquiry about prisms. Thus, there didn't appear to be a great deal of mutual engagement. It could be that the social norms of this classroom meant that the students participated because it was a requirement of being in there. This contrasts with the notion of mutual engagement, where the learning community has a shared purpose of thinking and reasoning about the mathematics.

5.2.1 Triadic dialogue

The triadic dialogue evident in *Lesson snapshot 1* contributed to the classroom norm of minimal mutual engagement. Triadic dialogue is described by Lemke (1990, cited in Zevenbergen, 2004, p. 206) in the following way:

> Triadic dialogue is an activity structure whose greatest virtue is that it gives the teachers almost total control of the classroom dialogue and social interactions. It leads to brief answers from students…It is a form that is overused in most classrooms because of a mistaken belief that it encourages maximum student participation. The level of participation is illusory, high in quantity, low in quality.

So while Dan might view his students as participating in the classroom, it might not have developed their mathematical proficiency. However, on the surface the students were exposed to mathematical knowledge, skills and procedures. Dan's approach to teaching mathematics in most of the lessons I observed revolved around a model of transmitting knowledge that was sustained through triadic dialogue. Thus, the social norm of doing mathematics that seemed well established in this classroom was "through a mutual compliance with the implicit rules" (Zevenbergen, 2004, p. 209) rather than a mutual engagement and participation in a culture of mathematisation.

5.2.2 How students participated in the social norms of the classroom

Some students in the classroom seemed to be good at pretending that they were following the discussion. For example, at the start of one lesson Zed asked Dan about a question from the homework involving the volume of a cylinder. It was one of the few times that Zed asked a question. Zed was sitting towards the back of the classroom and I had a clear view of him and his exercise book. Dan talked through each step of the question, drew a diagram on the board and set up the question by writing down what was known and what they were looking for. Certainly, the mathematical thinking and procedures were effectively modelled by Dan (Ball et al., 2007; Schoenfeld, 2007a). However, the norms of the classroom dictated that students sat passively and thus the opportunity for mathematical sense making may well have been severely restricted.

As Dan went through the answer on the board, Zed drew pictures of skate board ramps. He was half listening since he offered answers when prompted by Dan, but his focus was on drawing his skate board ramp. Most of the answers offered by Zed regarding conversions from cm to m were correct. From the front of the room Zed would have looked like he was copying down the question from the board. Zed continued to draw and refine the quality of his skate board ramp for the entire 90 minute lesson. He also copied down all of the worked examples that Dan did on the board during the remainder of the lesson. It is difficult to know whether Zed understood these worked examples and whether he would ever refer to them again.

Zed seemed to be going through the routines of the classroom. He didn't disrupt anybody else and appeared to be *on task*. So Zed appeared to stay under the radar in the classroom by pretending to follow the expected social classroom norms of participating in the triadic dialogue which was conducted by Dan.

5.2.3 Students staying on task

Dan spent quite some time disciplining students and trying to get some of them to stay on task during the lesson. Towards the end of term 3 they appeared to be really tired. They seemed to have reached saturation point. During one particular 90 minute lesson, two weeks before the end of term, Dan introduced how to plot points and draw linear equations. He invited students to the board and asked some open questions:

> Can you mark any point on the grid where y=3?

Tom and Matt were doing their own thing and were not following the lesson at all. They were chatting and playing with a ruler. The lesson progressed with quite a few students participating in the discussion about different gradients of lines. Matt and Tom were scribbling in their diaries. Dan asked students to work together to draw graphs of y=-2, x=5, y=x-4, y=-2x+3. Dan had given the students graph paper to use.

> Dan: Get started, Matt
> Matt: Can't we just chill?

Matt didn't know where to start and called out:

> How do you do them again?

I walked over to see what Matt and Tom were doing and Matt said to me:

> I can't do this. This exam next week is going to be an epic failure.
>
> Tom: I have to do well, because I got zero on my assignment because I didn't hand it in. Last term I screwed up my exam because I was too lazy. Now I have to try hard to get my marks up so Mum and Dad don't yell at me.

Neither of the boys made any attempt to do the questions. After some time Dan went through the questions. As Dan went over the answers, Matt called out random numbers:

> 5, 9, 7, 82.

Matt had had enough:

> I'm over this.

As the class approached the end of term, I observed that students were overwhelmed with the amount of content in some of the lessons. During one of the 90 minute lessons on a Wednesday during term 3, Dan went through a lot of content. This was because the following week they were going to miss several lessons due to various interruptions. Dan felt that he needed to get through the work because it was in the school mathematics program, and would therefore be tested at the end of term. It is not clear how much of the work that was covered by Dan was actually understood by the students.

5.3 Mathematical participation rather than mathematical interaction

Something obvious by its absence in this classroom was the sense of mutual engagement I had seen in Naomi's classroom. The triadic dialogue and the model of transmitting knowledge meant that the norm was that students relied on Dan to tell them the right way to do the mathematics. I didn't see students talking to each other about the mathematics that they were doing. Students who did the exercises tended to work in isolation, even though they were sitting in pairs. Some of the students were consistently easily distracted; for these students it was as if any diversion was better than doing the mathematics out of the textbook.

The atmosphere in the classroom was respectful; students didn't tease one another or put others down if they made mistakes during the triadic dialogue. In fact, students didn't really appear to be overly interested in one another's mathematical ideas. However, several students put themselves down very easily. Students appeared to have a deficit view of their mathematical proficiency and this might be reflexively related to the unavailability of quality mathematical interactions in this classroom. Quality mathematical interactions in this respect refer to the negotiation of the socio-mathematical norms.

5.4 The socio-mathematical norms of the classroom

The socio-mathematical norm of engaging in disagreement about solutions was a rare occurrence in this classroom. The following lesson snapshots illustrate how the socio-mathematical norm of arguing about mathematical ideas evolved.

Lesson snapshot 2

The lesson was on the relationships between capacity, mass and volume of water. Dan wrote the following table on the board:

Volume	Capacity	Mass
$1cm^3$	1mL	1g
$1000cm^3$	1000mL =1L	1000g = 1kg
$1m^3$	1000L =1 kL	1000kg = 1t

Dan and the students talked about how to convert between each measure. Dan chose $1000cm^3$ to show the relationship between each measure, since students had covered this in their homework and he was scaffolding upon earlier work. Certainly in this case Dan was demonstrating how to make sense of the mathematics by looking at the relationships between the three measures. As Dan worked though a conversion on the board, Rowan called out:

> Sir, you got that wrong.

Dan checked what he has done, went through the table and asked the class to check what they had done; Dan was correct.

> Rowan: I'm confused, I'll just shut up.
>
> Matt: I'm lost.

The power in the socio-mathematical norm of argumentation was lost in this interaction. Students seemed to start to argue or disagree with a solution, but the interaction could not gain momentum. Perhaps the well established social norm of triadic dialogue dominated any potential for meaningful interactions. Other students didn't offer their ideas, only Dan responded to Rowan's claim. In the end, students didn't persist with their idea and they put themselves down as Rowan did.

Lesson snapshot 3

During another lesson Dan was working on solving an equation on the board and he asked the class a procedural question:

> What do I do first?

Several students called out their ideas and Dan asked why. The students immediately changed their mind about what to do first; they were unwilling to take the risk of offering an explanation. At this point Dan said in exasperation:

> Don't change your mind. When I ask why, just explain.

This example highlights that the social norm of doing mathematics as a way of guessing what Dan wanted to hear was preventing the participation from turning into a meaningful interaction. While Dan might have encouraged students to persist with stating and arguing their ideas, the students appeared to be committed to adhering to the social norm that placed them as passive recipients of mathematical knowledge. This issue was also identified by Lee and Majors (2000, in Ball, 2003, p. 35), who found that when a teacher asked "a student to explain a method he has used, he will probably think that he made an error". This may accentuate the notion that in this classroom the social norms, steered by the teacher's actions, spoke louder than words.

It could be that students did not know how to explain what they were thinking. The taken as shared way of participating in a mathematical discussion in this classroom involved students offering snippets of knowledge and then Dan clarified their ideas using mathematical skills and procedures. Dan might have asked students questions, yet the classroom practice appeared to maintain a sense of unsureness whereby students didn't have the opportunity to develop the cognitive capacity to think through the problems for themselves. Dan would use his knowledge continually to make the mathematics accessible and therefore less challenging. The important point is that these students had not developed a sense of obligation to "try to develop personally meaningful solutions that they could explain and justify" (Yackel & Cobb, 1996, p. 462).

5.4.1 Students' lived through reality: unfulfilled opportunities for mutual engagement

During my classroom observation I noticed that students seemed more willing to engage in disagreeing with an idea if they could use their real life experiences. During one lesson Dan asked the class to do an example which involved a large can of fruit juice. The exchanges were as follows:

> Dan: What's the most logical way of writing 3887.72g?
> Addison: 4000g.
> Dan: or
> Addison: 4kg.
> Addison: That's heavy [suggesting that it may be wrong]
> Dan: Well, it is 4 L. What about your 3L bottle of milk?

The class discussed the feasibility of the solution. It appeared that the real life context furnished an opportunity for them to be doers of mathematics and students felt as if they could participate. This was an area that could potentially develop the productivity of their mathematical dispositions. However, Dan's input meant that he ended up justifying the answer rather than the students' thinking and reasoning about the mathematics.

A similar discussion occurred later during the same lesson. The class was working on a question that resulted in an answer suggesting that a pool had a volume of $12m^3$. Matt called out:

> That's tiny, sir.
>
> Dan: Well, let's see.

Dan seemed to value students' views and opinions. However, capitalising on students' views might have been overshadowed by the traditionalist view of knowledge construction. Dan asked Matt to come to the front of the room and they paced out the possible dimensions of the pool. In this way, it was Dan steering Matt's thinking rather than Matt making sense of his idea and sharing it with the class. The interaction might have evolved in a more mathematically productive way for all of the students if Matt had been asked to explain his thinking. The idea of understanding how Matt and other students in the class viewed $12m^3$ might have given Dan a valuable insight into how the students think. In the end, the interaction lost the potential of mathematical power for Matt and the other students since it was Dan who gave the answer:

> Dan: Yes, if you dived in, you would hit the other side.

Consequently, Matt lost the ownership of the idea and Dan recast the interaction into being about transferring his knowledge. While the class did discuss the different dimensions of the pool, the discussion became stereotyped by Dan's knowledge rather than developing into a mutual negotiation about the mathematical ideas.

5.4.2 Socio-mathematical norm of mathematical difference

Interpreting the socio-mathematical norm of mathematical difference clarifies "the process by which teachers foster the development of intellectually autonomy" (Yackel & Cobb, 1996, p. 473). Dan would often ask students if anyone had done a mathematical problem a different way. The following lesson snapshot highlights how the social norms of the classroom impacted on the socio-mathematical norm of exploring mathematical difference.

Lesson snapshot 4

Toward the end of term 3 students were working on making different variables the subject of a formula. Both Jake and Cam powered through all of the questions and seemed to enjoy what they were doing. Dan did the following question on the board:

A = (a+b)h/2 Make b the subject.

Dan set his answer out and checked with Cam and Jake to see how they did it.

Cam: Yes, I got that.

Jake: There is another way to do it.

Dan: What's the other way to do it, Jake?

Jake: You can expand it.

Dan used Jake's idea to make b the subject of the formula, by expanding first. Dan and Jake talked about it and they agreed that it was a bit more confusing and that the first method was more efficient. Dan valued Jake's input and they were mutually engaged in making decisions about the quality of the solution process. No other students in the class contributed ideas during this particular interaction. So again, the social norm of triadic dialogue overpowered the potential available in the interaction for other students in the class since members of the classroom learning community had not developed social autonomy (Yackel & Cobb, 1996).

What is interesting is that both Cam and Jake wanted to do advanced mathematics in grade 10. It might be this desire to move to advanced mathematics that encouraged Jake to move beyond being a passive recipient into interacting to make sense of the solution process. In this way Jake chose to override the social norm of the classroom so that he could improve his understanding of the solution method and thus develop his intellectual autonomy.

5.5 Dan's pedagogy: helping or hindering students' intellectual autonomy?

The lesson snapshots support the notion that Dan had both mathematical content knowledge and pedagogical content knowledge (Ball et al., 2007). Dan seemed to be aware of how important his pedagogical content knowledge was in the classroom. During an interview with Dan I asked him what he thought made a good mathematics teacher. He answered:

> Content - you need to know your content. This is the most important. You need to be able to adjust/adapt content and methods to suit and reach as many kids as possible because they don't all suit the same style.

Dan's comments were confirmed by his actions in the classroom and how he steered the social norms of doing mathematics. Dan refined his role in the traditional sense of knowledge transmission, acknowledging that students have different ways of learning. He viewed his role as being able to adapt the mathematical content and methods to suit the students. In this way Dan chose to adapt content while still retaining the traditional routines of the classroom rather than adapting his classroom strategies to open the possibilities for students to negotiate the mathematical content. Hence, while he acknowledged that students learn in different ways, he was doing the thinking for the students, even though the development of productive mathematical dispositions involves students participating as "increasingly autonomous members of an inquiry mathematics community" (Yackel & Cobb, 1996, p. 474).

The next lesson snapshot is an example of how Dan used his PCK and careful lesson planning to create opportunities for students to experience success in the topic of solving linear equations.

Lesson snapshot 5

Dan wrote 4x – 3 =17 on the board and asked:

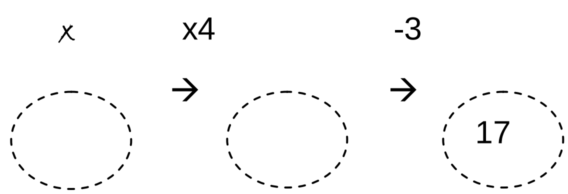
> What is my aim when I solve an equation?

Students called out randomly:

> Finding x; finding the unknown.
>
> Rowan: x equals 5
>
> Dan: Yes, but you can't always use guess and check. This is an easy equation. They're not all that easy to check.

So Dan tried to make students aware that they needed to have efficient ways to solve their equations. Essentially he was trying to establish that mathematics is a useful tool that may be used to solve equations with some efficiency and sophistication.

Dan put the following representation on the board and called it the bubble method:



Dan talked the students through the steps:

> What did you do first? What did you do next?

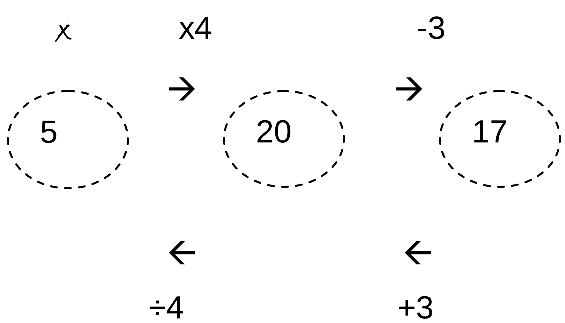
Students called out their answers.

> Dan: How do we undo this now?

A student called out: Do the opposite

Dan: Plus/minus are opposite operations so how do we undo multiply by 4?

Class answered divide by 4*.* Dan talked through the steps on the board with the class:



Dan: How do I check?

Ben: Do it the other way.

Dan: Now if you were in advanced maths you would do it like this:

4x-3=17
4x – 3 + 3=17+3
4x = 20
4x/4=20/4
x=5

Dan talked through the procedures of solving the equation.

Misconceptions arose here because students were getting confused as to whether they were doing the inverse operation. Some of the students seemed to recoil at the sight of the advanced mathematics setting out. However, Dan felt the need to mention it since some of his students wanted to move to advanced mathematics.

Dan spoke about how balancing an equation was like a set of scales, and suggested to students that they imagine that the equal sign meant that “you have to keep it balanced”. He attempted to bring students back to understanding what they were doing. Once again Dan’s traditional ideas of knowledge construction saw him using his pedagogical content knowledge to taper students’ thinking rather than to encourage a process of inquiry so that students could draw upon their own intellectual capabilities. Consequently, students had to “rely on the pronouncement of an authority to know how to act appropriately” (Yackel & Cobb, 1996, p. 474).

Throughout many of the lessons that I observed, Dan kept reminding students to do their working on their paper, and to show their mathematical working rather than use trial and error. Dan said during the lessons:

> Think, then write, then share.

However, sharing didn’t appear to fit into the social norms firmly embedded in this classroom. What’s more, if students did share, they only shared with Dan since there appeared to be little devolution in the responsibilities of constructing knowledge in this classroom.

Dan then put a new equation up on the board (specifically chosen because the -2x isn’t the first term):

> $7 - 2x = 18$

> Dan: I like to put the x at the front of the equation, what sort of 2x is this?

Students talked about it being a -2x. I heard students mumbling about how they didn't like negatives.

Dan wrote -2x + 7 =18 on the board.

As Dan went through the different examples he highlighted the misconceptions that students might bring to the classroom. He was aware that students sometimes have trouble with: negatives; what to do with denominators; fractions and the different ordering of the terms. He did this before students asked questions about it; it was part of his lesson plan. It is an important pedagogical tool to anticipate students' misconceptions (Ball, et al., 2007). However, in terms of socio-mathematical norms this may be likened to a wet blanket that puts out fires that should perhaps be allowed to burn a little first. That is, Dan took responsibility for the misconceptions rather than students thinking about their own misconceptions. Dan told his students:

> You need to think, 'What makes it easier for me to do this?'

However, the classroom practice adhering to established norms didn't encourage students to be involved in a process whereby they could think about and share what made it easier for them. Socio-mathematical norms of understanding mathematical differences would encourage students to take responsibility for understanding their misconceptions. It is in the opportunity to pull apart the mathematics for themselves and share what they have found, that students may develop productive mathematical dispositions.

Most students seemed to enjoy solving the equations during this lesson. Dan's pedagogical content knowledge certainly seemed to have helped students feel as though they could use the mathematics that he had shown them. It also gave students an opportunity to feel successful at solving linear equations. However, these might be temporary gains. The absence of the socio-mathematical norms of establishing mathematical difference and the opportunity for shared mathematical inquiry into theirs and others' thinking meant that students may not have made sense of the mathematics. While it is important that students have opportunities to experience success, if they aren't part of the processes involved in producing their success, it may not add to building their mathematical autonomy in the long term. In fact, some of the students in the class seemed to have developed a *mathematical indifference*, as exemplified by Matt's question:

> Why do we have to do this for? Did the people who make the maths up get bored and so they made this up?

5.5.1 Pedagogical content knowledge: Assignments.

The assignments in mathematics seemed to be an opportunity for students to work together and apply their knowledge. Teachers took turns in setting the assessment item. Dan set the term 4 assignment for the ordinary mathematics students. I asked Dan the following question:

> I: How do you decide what to put in the assignment?
>
> Dan: It is activity based and theory based, and the practical stuff helps kids develop the theory side, to develop an understanding of the theory for themselves and make the connections for themselves. Although a lot in my class didn't click, the kids didn't make the connections for themselves at all.

The activities in the term 4 assignment illustrate how Dan used his pedagogical content knowledge to create opportunities for students to construct their mathematical knowledge. The assignment used an array for students to build a robust understanding of the distributive law. Dan's aim was for students to:

> Think about it for themselves; so that they understand what they are doing.

However, Dan felt that many of the students in his class didn't really understand what they were doing. I observed students during one of the lessons when they were working on this assignment. Some of the students were doing the questions in any order, without really thinking about what they were doing. Several students told me that they were too "lazy" to do the assignment. However, it could be that these students were not accustomed to thinking mathematically. Their mathematical indifference didn't place them as doers of mathematics, so it wasn't going to "click" for them. The classroom norm that focused on the traditional model of knowledge construction and distinctly lacked an emphasis on inquiry meant that students hadn't developed their intellectual autonomy.

It is interesting though that Dan expected that his students would "think about it for themselves, so that they could understand what they were doing" since this wasn't a process that had developed during the classroom practice. What is also worth noting is that Dan used his pedagogical content knowledge to construct an assignment that encouraged mathematical inquiry, which contrasted with how he used his pedagogical content knowledge in the classroom. A corollary that could be proposed is that Dan thought that students would automatically develop an inquiry approach, possibly as a result of his modelling. So Dan might not see a connection between the social experiences available for mathematical knowledge construction and intellectual autonomy.

On the other hand, how Dan chose to use his pedagogical content knowledge at different times to suit different purposes might suggest that he was being influenced by the social norms that existed outside the classroom. Perhaps Dan's disconnected use of his pedagogy is a reflection of the bigger picture of mathematics education; that is, "we do not agree on a philosophy of education that can offer guidance about what should be taught and how and most importantly, for what reasons" (Ramaley, 2007, p.20).

5.6 Dan's beliefs about teaching and learning mathematics

Dan spoke to me a number of times about his dislike of the textbook:

> Dan: The exercises cover too much theory; it isn't divided up well enough; it needs to be subdivided.

Dan's pedagogical content knowledge also seemed to have provoked an apprehension about the school mathematics program for grade 9 at Amethyst College. Throughout the semester when I visited Dan's classes he talked to me about the limitations that the school mathematics program brought to his teaching. It was something that he was very anxious about.

Dan felt that the grade 9 ordinary mathematics program didn't suit the students, and that the exercises in the text book were too hard for many of them. What's more, he said that many students "don't see the relevance in doing the algebra". He acknowledged that the algebra was in the program to cater for those students who wanted to do advanced mathematics in the following year. However, a majority of students in his class did not want to do advanced mathematics. Certainly, some of the students in Dan's class didn't view algebra as relevant to their life world. This might be a symptom also of the classroom norms, since students' way of engaging in the classroom practice of doing mathematics involved relating it to their real life experiences. It could be that algebra appeared to be a foreign concept to them. Furthermore, the traditional model of knowledge construction that was the norm of this classroom didn't seem to improve the accessibility of the algebra in the long term. Moreover the habitus (Zevenbergen,

2004) that saw students view algebra as irrelevant might have been reinforced within the norms of this classroom

Arising from this, Dan was especially concerned that the ordinary mathematics students:

> Don't have an opportunity to achieve.

That is, he felt that the ratio of the results A, B, C, D and E should be the same in ordinary mathematics as they were in advanced mathematics. Dan felt that wasn't the case, since the majority of the ordinary mathematics students were getting a C or below. In Dan's 9MAO class the end of year results were: 14% of students achieved a B overall; 48% achieved a C overall and 38% achieved a D overall. The students who did achieve a B overall planned to move to advanced mathematics.

Underlying much of Dan's apprehensiveness was the fact that the school program provided only pages from the text book and nothing else. Dan felt that some teachers were teaching straight from the textbook and this limited what they were doing with their students. Dan spoke about how he had been involved in writing a number of mathematics programs. From his experience he felt that the program should be written around curriculum requirement rather than the text book as "it is done now". Dan had this to say about what should be in the mathematics program:

> A list of what kids need to know and what they need to be able to do, rather than a reference to the text book pages. Examples of applications of concepts, like the old style programs we used to write; in order for them to be prepared for the following year.

Dan identified an issue with the school mathematics program as being one dimensional. His solution appeared to consider a second dimension of identifying what students should be able to do. A third dimension that might be worth considering is in how to develop students' conceptual understanding and intellectual autonomy. In this way the school program might use an "integrative model" that moves towards teachers developing a "shared understanding about what mathematics must be taught, how and to what end" (Ramaley, 2007, p. 19).

Dan appeared to be unhappy about the direction that middle school mathematics had taken at Amethyst College. He was concerned about how more students seemed to be in the lowest stream. That is, ten years ago there would have been only one general mathematics class (the lowest stream) of about 15 students in grade 9. Now there were three general mathematics classes in grade 9. This would account for approximately 30% of the year level. He also had this to say when I asked him what worried him about mathematics in the middle school:

> Student apathy, an unwillingness to work and do homework. It has gotten worse, it seems a widespread disease, not just in mathematics; they don't see education as important. One of my grade 8 students, who is very capable and bright and capable of doing year 9 advanced maths, said to me, 'I don't want to be in your class [because I make them work] and other teachers don't; I just want to have fun'. Another student in my grade 9 maths class said to me 'I'm glad I'm in your class because you make me work'.

So, as identified in the lesson snapshots, Dan appeared to take responsibility for making the students work in class. Certainly I observed that students in Dan's class did do their mathematics. However, the mutual rapport between Dan and his students did not have the same mathematical power as might be available if there were mutual engagement in doing the mathematics. Dan's traditional approach to teaching mathematics appeared to constrain the availability of mutual engagement. What's more, students in Dan's class appeared to have arrived with fragile mathematical dispositions, so that it might be difficult for them to feel capable of interacting with the mathematical concepts presented by a mathematical authority such as Dan. Certainly, the norms of obligatory participation rather than intellectually autonomous participation do not seem to be a source of enrichment to students' mathematical dispositions.

5.7 Students' beliefs about doing mathematics

The students in Dan's class seemed to be reticent in their participation and negotiation of the mathematical ideas. Certainly, the classroom social norms were reflexively related to how students viewed their own mathematical proficiency. While students arrived into the classroom with a mathematical habitus (Zevenbergen, 2004), the "development of individuals' reasoning and sense-making processes cannot be separated from their participation in the interactive constitution of taken-as-shared mathematical meanings" (Yackel & Cobb, 1996, p. 460). Thus, a student's participation cannot be separated from their mathematical identity or from the classroom social norms which were steered by Dan.

As I did with Naomi's class, I spoke to several students in Dan's class. The following data provided some insights into how students viewed themselves within the social norms of their mathematics class.

5.7.1 Duncan's beliefs about mathematics

Duncan was a student who generally avoided participation in class discussions. He participated if Dan asked a direct question. Duncan did distract other students in the class and Dan often asked him to move and sit by himself.

Duncan discussed how he wanted to be a musician and told me that mathematics was not going to help him later on in life. He had no desire to do mathematics because he planned to pay people like accountants to do the mathematics for him.

> Duncan: It is hard to pay attention; some maths is totally pointless. Algebra is totally pointless. Other students may have a longer attention span and may want a different career [that needs mathematics].

I asked Duncan if he was worried about anything affecting his work in mathematics at the moment and he replied:

> As long as I'm passing it's OK. I got a D earlier in the year, but it didn't worry me; worried my Mum though. I just feel slack and can't be bothered.

He also told me that mathematics classes should be more entertaining and that he felt bored. When I asked Duncan to tell me about when mathematics had been good for him, he said:

> At Floating Hill in grades 1 to 4 and a half we played games and that was fun. Then I went to Waxberry Primary and we didn't do much maths, mainly literacy.

Duncan was very open about his lack of interest in mathematics and wanted to make it clear to me how pointless it all seemed to him. Duncan didn't feel obliged to follow the classroom routines. The indifference that he felt towards mathematics was damaging his own mathematical potential and was also affecting other students due to his disruptiveness. The classroom social norms and the expectations of compliance to Dan's mathematical authority might have been aggravating Duncan's antagonism towards mathematics. Certainly, Duncan appeared to have developed a "personal classroom identity" which opposed the "ongoing regeneration of the normative identity" of doing mathematics in this classroom (diSessa & Cobb, 2004, p. 97). Overall though, Duncan's mathematical proficiency appeared to be of little concern to him.

5.7.2 Jake's beliefs about mathematics

Jake always appeared to be motivated to be involved in the mathematical discussions in the classroom; moving to advanced mathematics in grade 10 was high on his agenda. Even though Jake sat next to or in front of Duncan, he didn't allow himself to be distracted. Jake would always seem to be following Dan carefully and he made sure that he understood how to set his work out correctly by asking questions to clarify his understanding. When I asked Jake to tell me something important he had learnt in mathematics he replied:

> Setting work out better; it helps to work out the answer more easily.

Jake used words like "relaxed", "successful", "happy" and "clever" to describe how he felt in the classroom. He appeared to enjoy doing mathematics and seemed to thrive in the social norms of this classroom. For example, Jake appeared to appreciate the social mathematical norm which valued setting out the solutions since it helped him "work out the answer more easily". The classroom observations *(Lesson snapshot 4)* also suggest that Jake used the setting out as an avenue to negotiate different ways of solving the problems. Although Jake complied with the norms of this mathematics classroom, the focus on setting out the mathematics does not necessarily transmit to improving the flexibility of his mathematical repertoire. The absence of key socio-mathematical norms of mathematical argumentation and mathematical difference within this classroom learning community might result in Jake's gaining short term proficiency in this context only, instead of a productive mathematical disposition which could transfer across contexts.

5.7.3 Addison's beliefs about mathematics

Addison was contemplating whether or not she should do advanced mathematics. She spoke about a future career in architecture and this was her motivation for wanting to do advanced mathematics. Addison found that ordinary mathematics was easy and she admitted to getting bored during the lessons. However, Addison did tell me:

> I'm scared of trying advanced maths in case I fail.

Addison told me that she really enjoyed doing algebra and found it easy. She felt though that she could go more quickly than the pace set in ordinary mathematics. I often noticed that during the lesson Addison would finish her work early and she would sit and read a novel.

Addison was happy to work within her comfort zone, yet there appeared to be little challenging her within this particular classroom. She could do the work easily in ordinary mathematics. However, the classroom practice didn't seem to be improving her confidence about doing advanced mathematics. This might suggest that the normalising processes of doing mathematics in this classroom weren't advancing her to gain a productive mathematical disposition. Moreover, it seemed as though Addison's mathematical disposition was no match for the advanced mathematics juggernaut. Perhaps Addison had concerns about her own mathematical proficiency and the limited opportunities for Addison to make connections between the key strands of mathematical proficiency (Kilpatrick, et al., 2001) didn't empower her to move out of her comfort zone.

5.7.4 Des' beliefs about mathematics

Des came alive during mathematics lessons involving geometry, area and volume. It was almost as if he were two different students depending on what the mathematics topic was. Des told me about his despair at not being able to do algebra; it felt totally foreign to him:

> I just don't get all the different ways to do it.

During a lesson observation I commented to Des:

> Gee you know all of the areas of the different shapes really well.
>
> Des: Oh yes, I remember those from primary school, but I had trouble with these [he flicks back in his book and shows me some work on indices and scientific notation]. I did really badly on the exam because of these; I couldn't do them. Normally I get Bs or Cs, but last term I got Ds [and he talks further about how he can't do indices].

So the fragility of Des' mathematical disposition becomes evident. When presented with a positive comment about his mathematics ability Des immediately changed the subject to what he couldn't do and couldn't understand. He didn't really want to acknowledge his strengths in mathematics. Des felt that he couldn't do indices and didn't do well on the last test, so being good at finding the area seemed to be irrelevant to him. Perhaps, Des felt that with one or two weak links in the chain of proficiency he was destined to fail.

Des' comments about his algebraic inadequacies might lend some evidence to why Dan felt that algebra should not be a significant component of the ordinary mathematics course work. That is, the algebra seemed to be an area where students didn't feel they had an opportunity to achieve. However, the culpability of the lack of achievement seemed to be directed toward the topic of algebra rather than to the classroom practices that rely on a model of transmitting knowledge.

Des shared his disappointment about his test results:

> I thought I'd get them right, but I got them wrong; I thought I might get Cs. It makes me think I should try harder.

This comment might suggest that the classroom social norms lead students to the belief that their lack of achievement is a direct reflection of how hard they try. However, if the culture of the classroom relies on students passively receiving knowledge, then trying harder involves practising through repetition. Certainly practising the mathematical ideas is important, but gaining the flexibility to choose and use mathematics requires that students have the opportunity to make sense of the mathematics that they are doing. Des thought that he would "get them right". However, he "got them wrong" which may suggest that the practice that he did do didn't improve his ability to transform his knowledge when required on the test. Therefore, it could be that the classroom practices that do not see students mutually engaged in the socio-mathematical norms of mathematical sense making become manifested in disappointing test results where students are unable to apply the mathematics that they have learnt. In this way the mathematical proficiency of the students is reflected in their test results when they can't apply the mathematics that they have learnt and practised during the semester.

5.7.5 Rowan's beliefs about mathematics

Rowan's decision to participate in mathematics lessons depended on how he felt on the day. Sometimes he participated in the discussion, offered ideas and worked well independently. However, during other lessons he was distracted from doing mathematics and looked for ways to distract other students.

> Rowan: Maths is hard, hard to get. I've never liked maths.

Rowan does admit to me rather coyly that:

> Algebra isn't too bad.

I observed Rowan during a lesson when he was factorising an algebraic expression. This is the working out that Rowan had in his exercise book:

4a + 6ab – 14ac

2x2xa + 2x 3xaxb – 7x2xaxc

2a (2 +3b – 7x2xaxc)

Rowan was confused that there were 3 terms in the expression, and he was accustomed to working with 2 terms. Rowan did understand which terms were multiplied and how to factorise the first two terms, he just ignored the third term though, because it was out of the realm of what he had experienced, and he felt that he didn't know what to do with it. Rowan was doing what the classroom practice encouraged, and that was to apply procedures that Dan taught to solve problems. Rowan could factorise and he wanted to factorise. However, he didn't seem able to apply his understanding of factorisation in new ways. This may expose the superficiality of the mathematical sense making that is perpetuated within the norms of doing mathematics within the classroom.

Rowan wanted to be a carpenter or an electrician or a chef. He felt that the mathematics done in school wasn't really useful since:

> I know the basics, other stuff that we do I don't really need for outside.

Another consequence of the classroom practice might be that the mathematics becomes "framed by high interest in use and low interest in advancing understanding" (Ramaley 2007, p. 21). Dan's traditional approach to knowledge construction placed an emphasis on his teaching students how to use the mathematics rather than guiding the negotiation process in a culture of participation so that students developed the processes involved in understanding mathematics. The consequences might be echoed by Rowan's comments whereby he didn't see any point to the mathematics that was occurring in the classroom. Ultimately students will use their proficiency in mathematics in different ways that cannot be predicated by choosing the right mathematics to do out of a textbook. As suggested by Ramaley (2007, p. 21) "we must keep all of our students in mind and teach them authentically and honestly, being faithful to the discipline of mathematics and mindful of our students and how they are developing".

5.7.6 Ben's beliefs about doing mathematics

Ben was good at participating in the triadic dialogue with Dan. This participation appeared to lead Ben into thinking that he understood the mathematics. Ben's comment illustrates this idea:

> I understand everything and then I try to do homework and I don't understand. It's like I've got selective short term memory in maths and I'll get it and then when I go home I forget it. I really worry about remembering how to do algebra.

Ben also had trouble working independently during class time. During my classrooms visits I noticed that Ben rarely consolidated any work in his exercise book. He sat by himself and would pull apart his pens and put them back together again. So this might reinforce the notion that Ben's participation in the triadic dialogue of the classroom practice was generating superficial understandings of mathematical ideas, so that the mathematics was not accessible to him when required. Hence, opportunities for Ben to develop his intellectual autonomy and mathematical proficiency were constrained.

Therefore the classroom social norms contributed to the creation of conflicting views for Ben when he came to reflect upon his mathematical ability. That is, he liked doing mathematics, and felt that he understood it, but he seemed to be overwhelmed by the breadth of work that he needed to cover. The breadth of work that overwhelmed Ben was compounded by his superficial understandings. Moreover the breadth of the work might have proliferated for Ben due to the lack of opportunities available to make connections between mathematical concepts. What's more Ben was disappointed about his results:

> I got a C once last year. On the last exam I got D E D [he pauses and laughs quietly] …I got dead.

If we take the view that "assessments should help students figure out what they know and what they don't know" (Schoenfeld, 2007, p. 9), then Ben's synopsis suggests that he wouldn't know where to start. Surely though, a prerequisite for students to judge their test results in terms of how to improve their performance requires that they have developed a sense of intellectual autonomy. However, the personal beliefs that Ben has constructed while participating within the classroom are not advancing his mathematical autonomy. It is possible that Ben's mathematical disposition may have become immobilised within the norms of this classroom. In this way, there appears to be little chance for Ben to develop his mathematical proficiency.

5.7.7 Students' mathematical dispositions

In summation, the views that students shared about doing mathematics reiterate that the classroom norms limited the opportunities available for them to develop productive mathematical dispositions. Further to this though, assessment continually arose as an area of concern for students and for Dan. So it is worthwhile contemplating how the assessment is structured at Amethyst College. A powerful aspect of school based assessment is that it can be used to help "teachers develop a better understanding of students' mathematical understanding" to "identify content and curricular areas that need attention" (Schoenfeld, 2007a, p. 9).   With this in mind, the next section examines how the teachers at Amethyst College collaborated on the construction of the curriculum for middle school mathematics and how this might contribute to the classroom microculture.

5.8 Constructing a middle school mathematics culture

I attended a key curriculum planning meeting at the end of the school year at Amethyst College. This meeting occurred during the student free week at the end of semester 2. There were approximately 18 mathematics teachers in attendance. I sat near Dan; Naomi as the mathematics coordinator chaired the meeting.

Several of the teachers at the meeting were very agitated. They spoke about the pressure of getting marking done so that reports could be completed according to the timeline set down by the school administration. Students' reports had to be done during the last week of term (the week before the meeting), when teachers who had all junior classes still had a full teaching load (grades 11 and 12 had already finished). So teachers felt exasperated about having to mark exams and complete comprehensive reports for each of their classes, while also teaching a full load.

It seemed as though all of the teachers were able to express their views openly during the meeting. The purpose of the meeting was to discuss the structure of the curriculum for the following year.

5.8.1 Planning a curriculum around assessment

As the curriculum meeting progressed, discussion became centred on assessment in grades 8 and 9. Students had one assignment and one test per term. There was a discussion about cutting out some of the assignments. One of the grade 8 teachers (Emma, who is also a senior school mathematics teacher) had this to say:

> I think the maths assignments are very important, since the grade 8s are not accustomed to setting their work out. Their communication is shocking, it stems from primary school. I think that first assignment in grade 8 (term 1) is so important because it allows us to give them valuable feedback on setting out. They can

get a half decent mark on the assignment rather than using just the exam, especially because they are not used to setting their work out.

Another grade 8 teacher agreed and suggested that when students start high school they:

> Feel keen and grown up and so a maths assignment early on is usually well done.

So it seemed as though in this instance assessment was viewed in terms of how teachers could guide students to set their work out better and adjust themselves to the expectations of high school mathematics. Certainly for these teachers, it seemed that setting work out in mathematics was an essential tool that required attention. However, this narrow focus could mean that teachers weren't using the assessment to reveal how well students were making sense of the mathematical content and processes. Moreover, if assessments "reflect the mathematical values of their makers and users" (Schoenfeld, 2007a, p.1) then little value appeared to be placed on understanding how students were thinking and reasoning mathematically. Essentially though, setting work out in mathematics and thinking and reasoning in mathematics should not be viewed as opposing actions.

The discussion then turned again to discussing the pressure of getting marking and reporting done on time at the end of term. Naomi suggested having the test mid-term to reduce the end of semester pressure of marking and reporting. However, a teacher disagreed:

> If you don't have a test at the end, kids won't do anything. The test is your *big stick* and if you don't have it, your class will break down.

Another teacher commented:

You need assessment from each topic.

Dan then suggested that instead of all assignments being marked by the teacher, why not do activities at the end of semester, interesting tasks that would see students applying the mathematics they had done during the semester.

> Dan: Students can use self-reflection, report their finding to the class, it is completed in class, peer marking, kids present what they have done to the class. This work isn't used as part of their marks, it is not summative.
>
> Emma: I don't like that idea because students need to have the assignment marks counted because it gets them over [the line] and boosts their self-esteem when they do well.
>
> Dan: Assignment marks aren't really even valid because I have seen too many that have just been copied.

Certainly, within this subject department meeting, teachers took risks in the discussion and disagreed with one another. There wasn't a sense of contrived collegiality (Hargreaves, 1992). However, a consensus of opinion did seem to emerge so that there was a "swapping of mutually reinforcing prejudices" (Brookfield, 1995, p. 141) that appeared to be detached from a focus on improving the learning opportunities available in their classroom practice.

The teachers appeared to reach a consensus that assignments needed to be summative for students to take them seriously. Essentially, the majority of teachers didn't appear to place any value on formative assessment. However, Keith (a mathematics teacher with 30+ years experience), who was sitting at the back of the room and hadn't said much during the meeting disagreed with the consensus and instead supported Dan's idea by saying:

> The best learning is the incidental learning; it is not subjected to an exam; there is no pressure on them; they're learning through investigation and their own research. Richness is lost in education because of assignment and test constraints.

Everyone turned and listened to Keith. When he had finished, the teachers in the room turned around and the discussion immediately returned to each year level's mathematics program and about how to best set the assessments' due dates to suit the constraints of reporting. It was as if Keith and Dan hadn't spoken. It appeared that the testing and reporting deadlines steered how the mathematics was going to be delivered in the classroom. The quality of the mathematics that was occurring in the classroom appeared to be overshadowed by the assessment and reporting agenda. This may instantiate Schoenfeld's (2007a, p. 3) claim that "tests can have a strong impact on the very system they measure". This idea is reinforced in the following section which looks at how external tests are impacting on curriculum construction at Amethyst College.

Interconnecting the five strands for true mathematical proficiency (Ball, 2003) involves developing processes that focus on effective classroom practices. Using test results diagnostically to interpret the effectiveness of classroom practices may encourage the development of students' mathematical proficiency in constructive ways. A powerful aspect of school based assessment is that it can be used to help "teachers develop a better understanding of students' mathematical understanding" to "identify content and curricular areas that need attention" (Schoenfeld, 2007, p. 9). However, test results are only one tool in the diagnostic analysis of a students' mathematical proficiency. Certainly, a students' mathematical disposition cannot be assessed using a summative assessment item.

5.8.2 Decisions about curriculum content and processes

The QCAT and NAPLAN numeracy tests are external tests that were completed by the grade 9 students during the year that the data for this case study were collected. These tests also appeared to affect how the curriculum in grade 8 and 9 was structured. Naomi suggested that these external tests needed to be:

> Better embedded within the program.

Naomi brought up the fact that the grade 9 numeracy NAPLAN test evidenced that the ratio skills of the students were poor. Teachers agreed that ratio should be done at the start of the semester when students are “fresher” rather than at the end of the semester as in the current program. Teachers felt that students should be exposed to all of the topics before they do the NAPLAN test and that they needed to practise doing multiple choice questions. Dan made this comment:

> When I was supervising the NAPLAN test, 60-70% of kids had stopped work early, and some of them were kids out of my class, and we had practised the test and I had suggested that they need to check their work.

Quite a few of the teachers made comments such as:

> The work ethic of our students has dropped.

> Education is less and less valued.

> Naomi suggested: We have to do what we can with what we have.

Naomi also spoke about the QCAT and how the grade 9 students “failed”. She spoke about the emphasis of the QCAT being on modelling and problem solving and how students seemed to struggle with that. A teacher asked if the topic was known in advance. Naomi said yes, that they were told the topic in advance, and most teachers agreed that the program should allow for the QCAT topic to be done or revised before the test. The grade 9 teachers also spoke about how there was not enough time for the students to complete the test, and that:

> You could spend a week on it. It is a really good task, but there just isn't enough time for them to complete it. It would be a good end of term activity.

One teacher had this to say:

> The year 9s are tested to hell, the QCAT, the NAPLAN, these take at least 2 weeks out of our teaching time. Their results are not the same as our other assessment results; they don't reflect our students' ability.

The grade 9 teachers all agreed that the grade 9 program was "jam packed". There was a concern about fitting everything in: the core curriculum; practising for the NAPLAN test by doing the multiple choice questions; covering the topic for the QCAT and practising modelling and problem solving questions. It seemed that *success* hinged upon getting the mathematics curriculum in grade 8 and 9 to effectively converge all of the internal and external testing requirements. This view of success appeared to taper the focus of the classroom practice, and this may ultimately limit the opportunity for effective classroom norms, which are focused on making sense of the mathematics.

What does appear evident from the teachers' discussion is that curriculum construction is centred on content, perhaps at the expense of meaningful process. Any talk on process converged on practising setting out or practising problem solving type questions. The teachers saw their role as needing to better transmit the content to their students, by juggling the content around to fit into testing requirements. The emphasis appeared to be on "How much do they know?" or "What problems can they solve?" (Schoenfeld, 2007, p. 4). This emphasis was compounded by the introduction of the multiple choice NAPLAN test. As discussed by Schoenfeld (2007a, p. 12) these teachers felt that they had to "focus on skills that were related to similar items on the test". It seemed as though teachers were concerned about improving the mathematical proficiency of their students. However, their “efforts to improve maths learning” was focused “on simply intensifying efforts based on … common but unhelpful methods” (Mann, 2006, p. 236). The convergent focus taken by teachers had been influenced by the NAPLAN test so that they become further removed from thinking divergently about teaching for mathematical proficiency in terms of the five interconnecting strands (Kilpatrick, et al., 2001). As discussed by Mann (2006, p. 236), high stake tests that “seeks to eliminate variability in scoring for more efficient marking…does not test for creativity and does not reflect the nature of real-world problems”. So for these teachers, the current climate of education reform placed the NAPLAN test as a significant test of proficiency for their students. However, this seemed to further delimit the attention afforded to changing classroom practices to engage students in the processes of mathematisation.

Therefore, teachers’ attention aimed towards structuring the curriculum so that students’ mathematical proficiency may be improved for the NAPLAN test may be intensifying the problem that they are trying to remedy. As mentioned, noticeable absences from the discussion were: how the mathematics is being learnt in the classroom and how the content can be structured so that students can make connections and develop their conceptual understanding. Furthermore, teachers congregating their interpretation of assessment around content may suggest a narrow interpretation of what the process of understanding mathematics and thinking mathematically actually means.

The teachers’ “deficit views” (Prosser, 2006, p. 13) of their students can be implied from their approach of teaching content immediately before the QCAT as a means of generating success on the test. But while this suggests that teachers implicitly acknowledge the shallowness of their students’ mathematical understandings, they don’t seem to relate it to their classroom practice. Possibly, the blame for superficial mathematical understanding appears to be more easily shifted to the work ethic of their students. Moreover, it seemed as though the teachers felt that they were trying their best to find better ways of doing their job of transmitting knowledge to improve their students' proficiency and thus it became difficult for them to analyse their classroom practice in a meaningful way for their students. Examining classroom practice in a meaningful way might include interpreting how students are (or are not) invited into participating in the processes of mathematising within a culture of choosing and using mathematics (Yackel & Cobb, 1996). So it could be that what underpins a teacher's understanding of what meaningful teaching in mathematics looks like becomes reciprocated in how they view their students.

5.8.3 Streaming in mathematics

Streaming arose throughout data collection as an area that appeared to influence the microculture of the classroom. Furthermore, a number of concerns were discussed during the curriculum planning meeting. Dan had this to say:

> I think it is unfair that students are put into classes called advanced, ordinary and general in year 8, if they are all doing the same course and the same assessment. I'm not saying that they shouldn't be streamed, but why is it called advanced, ordinary and general?

Naomi suggested that it was a timetabling code that couldn't be changed. However, Naomi also acknowledged that she was not convinced that the streaming of the students going into year 8 in the following year had been done correctly. What's more, one teacher brought up that there were a number of students doing advanced mathematics who shouldn't be there. Naomi explained that streaming in grades 9 and 10 was:

> Based on September results crunched into the timetable. The results are diluted due to timetabling constraints. There are some

> students who are in ordinary maths that should be in advanced maths. Their subject selection impacts on how they can be placed.

Even so, most teachers appeared to value streaming at this school, as suggested by one teacher:

> Well it is good for the kids. You can aspire to be advanced if you are in ordinary.

Another teacher commented that students can move between classes, and Dan replied:

> But they don't!

All of this seemed to be a contradiction. Streaming in mathematics had unpredictable effects on students' mathematical dispositions, as evidenced by some of the comments that students made during our informal discussions. It is also a structure that Naomi suggested could support and improve students' progress through grade 8 and 9 mathematics. Furthermore, grade 10 advanced mathematics is a prerequisite for senior mathematics B and C. Accepting the haphazard treatment of the placement of students is inconsistent with the importance that is placed on streaming at this school. The lack of careful attention to the details of how students are streamed at Amethyst College may be reciprocated in how students value their education. Moreover it may be a reflection of how the teachers are struggling with their "pedagogical identity" (Prosser, 2006, p. 13) since they appear to be consistently searching for solutions outside their classroom practice. Furthermore, the atmosphere of the meeting suggested that they felt their efforts were not supported by the school administration. In this way, the commitment to changing their understanding of what incorporates effective teaching and learning in mathematics and the implications of this may be beyond the domain of possibility for them at that point in time.

As suggested by Naomi, “Good teachers are driven insane with all the rubbish that is handed down from useless research…that has no practical basis”. So it could be that teachers were focused on keeping up with the seemingly unfeasible expectations from the ongoing reform agendas in the middle school and timelines from the school administration rather than participating in processes that seek to bring about meaningful curriculum change. So the shallow decision making processes and the quick fix solutions around a transmission model of knowledge construction that teachers seem to rely upon in this school may be perpetuated by the reform policies that are trying to bring about revolutionary change. Certainly, a lack of attention given to the processes involved in developing a culture of participation in the mathematics classroom continually arises in the data as a significant issue. This issue is especially significant in contemporary mathematics education since it is within such “a culture of mathematising as a practice” (Bauersfeld, 1993 cited in Yackel & Cobb, 1996, p. 459) that students may develop their intellectual autonomy in mathematics. It is this intellectual autonomy that transforms the power in the potential of students’ mathematical dispositions. Hence, the macro-perspective of mathematics education may be negatively influencing how teachers feel able to flexibly use their pedagogy in productive ways for students’ learning.

5.9 Conclusion

The latest research literature in mathematics education places the development of a student’s productive mathematical disposition as an essential element of mathematical proficiency (Ball, 2003; DEEWR, 2008; OECD, 2003; Schoenfeld, 2007a; Yackel & Cobb, 1996). Therefore, ongoing reflection upon classroom practice appears to be salient starting point when contemplating how to develop processes whereby students’ mathematical dispositions may unfold in productive ways for their long term mathematical proficiency.

The next chapter expands the analysis and ultimately contemplates how students' productive mathematical dispositions may be cultivated when teachers are engaged in the processes involved in developing productive pedagogical dispositions.

Chapter 6
Analysing classroom practices

Give me a fruitful error any time, full of seeds, bursting with its own corrections. You can keep your sterile truth for yourself.

Vilfredo Pareto

6.1 Introduction

A key focus of this research was to gain a sagacious insight into the realities of mathematics education in the middle school at Amethyst College. The social and psychological perspectives of the classroom learning community were the lens through which the emic issues of the participants could be captured. Thus the analysis moves between the objective categories of these socio-cultural perspectives to gaining some meaning through subjective interpretation. This analysis attempts to go beyond interpreting the social and psychological perspectives in terms of reflexive binaries into acknowledging the variables and dynamics in context.

The socio-mathematical norms were the funnel used to understand the "structure and essence of experience" (Patton, 1990, p. 69) for the classroom learning communities at Amethyst College. This research used the idea from Yackel and Cobb (1996) that socio-mathematical norms such as judging, arguing about and justifying what constitutes a different solution or a sophisticated and efficient solution are critical for students to develop their intellectual autonomy. It is this intellectual autonomy in mathematics that promotes the development of students' productive mathematical dispositions (Yackel & Cobb, 1996). A productive mathematical disposition is one of the five interconnecting components that, when combined in effective classroom practices, supports the development of students' mathematical proficiency (Kilpatrick, et al., 2001). In this analysis, intellectual autonomy is thought of in terms of its contribution to students' productive mathematical dispositions and their mathematical proficiency. Underpinning the analysis is the premise that an effective mathematics classroom attempts to engage all members in thinking about mathematics so that they may become increasingly proficient at doing and using mathematics.

In accordance with contemporary research (Ball 2003; Kilpatrick, et al., 2001; DEEWR, 2008; Stephens, 2009) in mathematics education, this study positioned classroom practices aimed at engaging students into being intellectually autonomous and productive users of mathematics as the quintessential aim. Becoming increasingly autonomous users of mathematics optimises opportunities for students to continue with doing mathematics at a higher level. Of course, intellectual autonomy is mutually dependent on the socio-mathematical norms. Indeed, these are interactively constituted by the members of the classroom learning community, that is a "self-sustaining system of shared power that involves the knower in a community" (Bussey, 2008, p. 141). An interpretation of the interactions in the learning communities at Amethyst College was used to understand the "epistemic culture" (Cetina, 1999, cited in Bussey, 2008, p. 141); these interactions were interpreted in the two preceding chapters in terms of how responsibility and closure were brought to the decision making process in the construction of knowledge. What those interpretations about the learning communities highlighted was *what didn't happen* during the interactions between members of the Amethyst classroom learning community and why this might be the case.

The remainder of this chapter uses what didn't happen when mathematical solutions were legitimised and justified within the classroom as the analysis template (Stake, 1995). Critically though, this analysis template is applied with an empathetic view of the learning communities. The analysis attempts to distance itself from a deficit view of teachers and students; the aim is to acknowledge from a holistic perspective the possible reasons why things may not have happened. This distinction is made since the analysis is using what didn't happen in the classroom learning communities as an avenue from which possibilities for genuine transitions in classroom practices may transpire; genuine transitions are continually directed towards empowering students to develop their mathematical proficiency.

6.1.1 The role of the mathematics teacher

The role of the mathematics teacher "as a representative of the mathematics community" (Yackel & Cobb, 1996, p. 475) is underscored throughout this case study. Dan and Naomi's role is vital to establishing the quality of the epistemic culture in their classrooms. This case study has focused on the feature of productive learning communities whereby students can develop their intellectual autonomy by becoming actively involved in the negotiation of the socio-mathematical norms. In this way neither the students nor the teacher play passive roles. However, the teacher does play a critical role, since it is their pedagogical content knowledge that may open possibilities and experiences for students to view the mathematics as accessible and doable. The research literature unequivocally places profound pedagogical content knowledge as an essential prerequisite for the middle school mathematics teacher. Naomi and Dan had "deep and connected" (DEEWR, 2008, p 1) mathematical knowledge that spanned the curriculum. One outstanding feature of their pedagogical content knowledge was their horizon knowledge and a tacit awareness of students' mathematical knowledge gaps. However, the data from this study found that having high quality pedagogical content knowledge as described within the domains by Ball et al. (2007) is a necessary, but not a sufficient condition for developing students' intellectual autonomy and mathematical proficiency.

Perhaps Naomi and Dan used their pedagogical content knowledge in the best way that they thought they could, given the confines of the school curriculum and how they saw themselves within the broader educational domain. Both of these teachers worked very hard and helping their students learn mathematics was the essence of their endeavours; they cared about what they were doing. They are at the coal face of mathematics education: the classroom, which ultimately is the most influential place to be in terms of developing students' mathematical autonomy. Therefore, teachers such as Dan and Naomi who do have solid pedagogical mathematical knowledge are viewed within this study as diamonds in the coal field. However, their pedagogy was influenced by: their personal belief systems; the mathematical proficiency that the students arrived with in their classrooms and the socio-cultural domains in which they worked. As a consequence, the data showed that their pedagogical content knowledge might have lacked the necessary lustre to promote students' intellectual autonomy. This analysis considers how teachers might use their role and their pedagogical content knowledge in different ways so that students can develop their intellectual autonomy. In this way, the critical process of teachers developing their agency for change in their socio-cultural domains is underscored.

6.2 The teacher's view of mathematics

The role of the teacher is influenced by how they view mathematics teaching and learning. Naomi and Dan appeared to use their pedagogical content knowledge in a convergent way through the transmission of knowledge, to prepare students for the next year of mathematics at school. It seemed as though they viewed mathematics from the "noun view…as a …structured body of knowledge" (Schoenfeld, 2007, p. 30). In this way they used their pedagogical content knowledge in terms of how to organise the mathematics so that their students could "best apprehend it" (Schoenfeld, 2007, p. 30). Therefore, the overarching view was that teaching mathematics involved transmitting knowledge rather than encouraging participation within a culture of mathematising (Yackel & Cobb, 1996). Perhaps this view may have reduced the "repertoire of strategies" (DEEWR, 2008, p 1) available to Naomi and Dan when steering the classroom interactions. Thus, learning mathematics in both of the classrooms was "seen as mastering a predetermined body of knowledge and procedures" where the teachers presented the "subject matter in small, easily manageable pieces" and demonstrated the “correct procedure or algorithm, after which students worked individually on practice exercises” (Goos, Galbraith & Renshaw, 2004, p. 92). Indeed, Naomi and Dan’s epistemological view may have contributed to students becoming relatively passive in the receipt of mathematical knowledge. This view is far removed from students being continually engaged in arguing about the differences between solution methods to make their understanding of mathematical ideas more sophisticated.

At times during the lessons some students in Naomi’s class became actively involved in thinking and reasoning about the mathematical concepts. Certainly, in *Lesson snapshot 1* on significant figures, students appeared to be mutually engaged in reasoning through the mathematics. It was an empowering experience for the class to override the authority of the textbook. Similarly, when John and Nate talked through the ideas on the assignment they were mutually engaged and they were using their intellectual autonomy to reflect upon their mathematical solution methods. It seemed as though the students in Naomi’s class made the most of her transmission mode of teaching and continually attempted to understand the mathematical ideas. Naomi also appeared to encourage students to be involved in the mathematical discourse. However, the focus of the classroom practice was on different answers rather than different solution methods to get to the answer. What appeared to be missing was time for students to be actively engaged in the socio-mathematical norm of explaining their different ways of thinking, regardless of the correctness of the answer. It is the opportunity for students to explain their thinking that has the potential to develop their intellectual autonomy to gain ownership of the mathematical ideas.

Students in Dan’s class also attempted to participate in the classroom discussions. The triadic dialogue dominated the classroom interactions, so students’ participation was constrained. *Lesson snapshot 5* involved Dan using his pedagogical content knowledge to give students an opportunity to experience success at solving linear equations. Students were involved in this process but Dan had the authorship of the process. So in this way, while Dan had good pedagogical intentions, the process might not have empowered students to be intellectually autonomous since their obligation remained within the confines of the procedures presented by Dan. Indeed, when students in Dan's class worked on their assignments, they couldn't make the connections between the different questions and so they treated questions in the assignment as they would the exercises in their textbooks. It seemed as though a significant number of students in Dan's class had not developed the intellectual autonomy and capacity to think and reason mathematically.

So while the teachers and some of the students participated in doing and talking about mathematical content and procedures, this practice is distinct from the teacher and the students interactively constituting and negotiating mathematical ideas though the processes of mathematisation. Dan and Naomi may have verbally encouraged students to think and share what they were doing. However, the classroom practice was more focused on getting the answer using efficient procedures, taught by the mathematical authority, the teacher. This practice placed limitations on the opportunities for students to be intellectually autonomous in these classrooms. What was missing from the classroom practice was a sense of responsibility or obligation for the students to be involved in understanding the how and why solution methods were more efficient or sophisticated. Placing mathematics as an action, as in mathematisation, shares the opportunity for mathematical authority with all individuals in the learning community.

6.2.1 Viewing mathematics in a different way

Viewing mathematics as an action involves thinking about "what mathematicians do" (Schoenfeld, 2007, p. 30). The processes involved in mathematisation calls into service a repertoire of activities that are involved in solving problems. Mathematisation in this respect refers to using the vertical and horizontal mathematics (Romberg, 2001; Zulkardi, 2004). This repertoire of activities is distinct from a “repertoire of strategies” (DEEWR, 2008, p. 1) employed by the teacher since it is interactively constituted by the members of the classroom learning community. The negotiation of the socio-mathematical norms (Cobb et al., 2001) has been proposed as being an avenue through which students can develop their ability to generalise, organise and reflect upon the mathematics that they are doing. In this way students might develop their ability for abstract thought and intellectual autonomy. Thus, mathematisation may be thought of as an extended process of mathematical action which develops through a culture of participation. The evolving culture of participation is highly dependent on how the mathematics teacher identifies their role as the mathematics agent.

The culture of participation in Naomi’s and Dan’s classroom was analysed by using the norms of practice. What appeared to be missing was a focus on developing the processes involved in the socio-mathematical norms of mathematical difference and mathematical argumentation, key features in developing a culture of “mathematising as practice” (Bauersfeld, 1993, cited in Yackel & Cobb, 1996, p. 459). However, there was a distinct contrast in how students in the two classrooms became involved in the interactions that were available. Certainly, how students became involved in these classrooms reflects the reciprocating relationship between the psychological and sociological perspectives (McClain, 2002, p. 218). Moreover, the grade 9 students’ capacity to participate in the learning community appeared to be influenced not only by the current classroom practice, but also by the mathematical disposition and proficiency they had developed over prior years.

6.3 Different views of proficiency across contexts

Naomi and Dan had similar transmission teaching styles and they both followed the school mathematics program. However, the classroom norms of practice became highly dependent on the mathematical disposition and proficiency of the students. In Dan’s classroom, the triadic dialogue dominated and students continually attempted to guess the answers for Dan. Naomi's class tended to override the triadic dialogue and at different times students were more able to open up the possibilities available in the classroom and turn them into learning opportunities for themselves and others. So the difference between the two learning communities was related to the already established mathematical proficiency of the students. This difference is highlighted when we consider how students attempted to deal with their mathematical knowledge gaps.

6.3.1 Different views of knowledge gaps

As suggested in the National Numeracy Review Report (2008), it is important to gain an insight into the knowledge gaps that students appear to have in the classroom. What is interesting in the data is how students appeared to view their knowledge gaps. Students in Naomi's mathematics class appeared to struggle with their misconceptions even more so than students in Dan's mathematics class. This could be because a greater majority of students in Naomi's class sought a deeper understanding of core mathematical ideas as they became engaged in the norms that were interactively constituted. In this way the "lived culture of the classroom" presented "a challenge for students to move beyond their established competencies… to enter more fully into disciplined and scientific modes of enquiry and values" (Goos, et al., 2004, p. 97). Some students, like Shay, would continually attempt to try to overcome their knowledge gaps by finding someone who could help them. However, the lived culture was continually influenced by the expectation that accompanies being in the advanced mathematics class doing school mathematics. So the process of mathematisting in a culture of participation might have become overpowered by the mathematical artificialness of the school mathematics curriculum.

Amy is another student who acknowledged her misconceptions, took responsibility for them and appeared to master them. Amy came from the extension group in primary school and she admitted to enjoying the competition of school mathematics. Amy's habitus empowered her to reach her potential in school mathematics (Zevenbergen, 2004). However, it is difficult to determine if the classroom practice was encouraging Amy to view the "mathematics as a vehicle for sense-making" (Schoenfeld, 1989, cited in Goos, et al., 2004, p. 97). Therefore, the productivity of Amy's mathematical disposition may be called in to question. Indeed, her mathematical disposition seemed to be nourished by the competition of doing school mathematics rather than by the socio-mathematical norms that engage students into being intellectual autonomous at mathematising. Still, the mathematical disposition and proficiency that Amy had already established allowed her to participate within this classroom and continue with doing school mathematics. So under the contrived conditions perhaps Amy could be considered as being proficient. But Amy was using the competition aspect of school mathematics to overcome her boredom, which contrasts with the opportunity for intellectual autonomy that would be made available in a classroom learning community that is caught in a "creative tension" (Johnson, 1999, cited in Bussey, 2008, p. 142). The socio-mathematical norms of mathematical difference and argumentation might intellectually engage Amy to "hold opposites together" and "create an electric charge" to keep her awake (Johnson, 1999, cited in Bussey, 2008, p. 142).

On the other hand, students' misconceptions in Dan's mathematics class didn't become obvious during the classroom interactions. The triadic dialogue resulted in students continually attempting to tell Dan the answers that he wanted to hear. In this way students might not have been making sense of the mathematics during their classroom participation. However, as I walked around the room I observed that these students' misconceptions were aligned with the misconceptions in Naomi's class. The difference was that Dan's students didn't appear to take ownership of their misconceptions. Perhaps for many of these students their habitus thwarts their potential within this classroom (Zevenbergen, 2004). Dan attempted to make up for this by using his pedagogical content knowledge to proactively acknowledge misconceptions in his lesson preparation. However, the "teacher has to establish the conditions in the classroom where students become engaged in the process of enquiry and are willing to share their insights” (Goos, et al., 2004, p. 113). Dan’s transmission model of teaching and the triadic dialogue may have disempowered students and limited the productiveness of their participation in the negotiation of the mathematical ideas. Therefore, the students felt little opportunity or obligation to become involved in thinking to make sense of their misconceptions. Furthermore, as mentioned, the mathematical dispositions that the students arrived with may not have seen them well placed to even attempt to override Dan’s mathematical authority. Moreover, students in Dan’s class appeared to

> resist thinking: they live in a world were relationships are often quite fragile…they are desperate for more community, not less, so when thinking is presented to them as a way of disconnecting themselves from each other and from the world, they want nothing of it.
> (Palmer, 1993, cited in Bussey, 2008, p. 142)

6.3.2 Building the Great Thing by scaffolding on prior understanding

The mathematical community is represented by how Naomi used her pedagogical content knowledge in the classroom learning community. That is, she was “responsible for structuring the cognitive and social opportunities for students to experience mathematics in a meaningful way” (Goos, et al., 2004, p. 113). The methodological assumptions of this thesis emphasised that the classroom learning community should have the Great Thing, the mathematics concept at its core. This idea of a subject centred classroom doesn’t suggest that the mathematics be thought of from the noun view. Rather, the teacher’s task is to let the mathematics have its chance to grow in students’ minds. One way to build the capacity of the mathematical concept is when students can scaffold on their prior knowledge and understanding.

A key feature of both classrooms was that Dan and Naomi attempted to scaffold upon students' prior knowledge. Naomi and Dan used their pedagogical content knowledge to engage the zone of proximal development (Vygotsky, 1978) so that the classroom learning community solved questions together and then attempted problems individually. However, both Naomi and Dan tended to rely on "an unchanging hypothetical formulation" (Vygotsky, 1978, p.88) of how they structured their classroom routines. In this way, their pedagogical practice which steered the classroom learning community did not attend to the idiosyncratic ways that students might construct and understand mathematical knowledge. Therefore the focus on the transmission model of knowledge construction, and a focus on procedures and setting out might have prevented students from becoming involved in worthwhile learning trajectories. According to Vygotsky "making mistakes" within the process of scaffolding "is an essential part within concept formation: the child relies on their own perception to make sense of objects that appear to be unrelated" (Dahms, M., Geonnotti, K., Passalacqua, D., Shilk, J. N., Wetzel, A. & Zulkowsky, M., 2007, p. 2). A worthwhile learning trajectory where students are engaged in the socio-mathematical norms of mathematical difference to take risks might open the possibilities for students to be involved in the process of developing their intellectual autonomy for a productive mathematical disposition (Yackel & Cobb, 1996). In this way the students might become engaged in thinking about the Great Thing in their own ways.

Some of the students in Naomi's class attempted to take responsibility for their learning, by acknowledging their misconceptions. However, Naomi seemed to view the misconceptions as barriers. Naomi would suggest to students that they needed to "fill in the gaps", since the misconceptions were slowing the pace of the lessons. In this sense, the knowledge gaps prevented many of the students from being engaged in the classroom learning community. These students couldn't seem to make sense of the mathematics that they were doing since they appeared to grapple with fundamental mathematical concepts. Within the confines of the school mathematics program, Naomi didn't feel that there was time to continually go back to basics.

The students who were able to scaffold upon their prior knowledge in Naomi's class were able to move forward with the sense making process. For example, they could use their prior knowledge of foundational mathematics involving fractions and directed numbers to solve linear equations using the efficient procedures and processes suggested by Naomi. However, it was difficult for other students to become involved in the interactions since their mathematical misconceptions on basic mathematics meant that they didn't have the key basics to use as stepping stones to understanding new concepts and procedures. These students appeared to lack a fluidity and flexibility in basic mathematics so that they couldn't move into using mathematics in abstract ways.

This issue was identified by Milgram (2007, p. 48) in the following way:

> I can tell you, from personal experience with students, that it is a grim thing to watch otherwise very bright …students struggle with more advanced courses because they have to figure everything out at a basic level. What happens with such students, since they do not have total fluency with basic concepts, is that—though they can often do the work—they simply take far too long working through the most basic material, and soon find themselves too far behind to catch up. Skill and automaticity with numbers is only part of the story. Students must also bring abstraction into play. This is also very commonly an unconscious process. There are huge numbers of choices for what to emphasize and what to exclude so as to focus on the core of what matters.

This issue (Milgram, 2007) was reflected in Naomi's class. As the classroom learning community negotiated new mathematical ideas, some students would be unable to keep up, since the basic mathematical knowledge that was assumed to be available just wasn't there. So Naomi felt that she was constantly attempting to "plug holes". Her attempts at plugging holes by showing students the procedures of the basic mathematics were unsuccessful since students became so overwhelmed in trying to understand the basic ideas that they couldn't make the connection to solving more complex problems. As suggested by Kilpatrick et al. (2001, p. 135) "proficiency in mathematics is acquired over time". Certainly, the continual development of fluency, flexibility and proficiency with mathematics involves an ongoing process from the early years through to the middle years rather than something that can be taught and learnt within a short time span. Higher level mathematics depends on students developing their mathematical proficiency across the early and middle years of schooling.

Essentially many of the students whom I observed had a problem with viewing the mathematics in abstract ways. Indeed by grade 9 mathematics, the building of students' mathematical proficiency for higher level mathematics involves an expectation that students have developed the intellectual autonomy to think abstractly. It is from this that students might continue with building their understanding of the Great Thing and therefore their mathematical proficiency. Also, classroom practices across all levels of schooling develop students' mathematical dispositions. A productive mathematical disposition may develop through a process whereby students participate in effective classroom learning communities that engage in the practice of mathematisation. The process of mathematisation involves students becoming increasingly autonomous at choosing and using mathematics so that by grade 9 their capacity for mathematical abstraction is part of their mathematical repertoire. Significantly Leontiev (1981), Vygotsky's colleague wrote that children

> Cannot and need not reinvent the artefacts that have taken millennia to evolve in order to appropriate such objects into their own system of activity. The child has only to come to an understanding that is adequate for using the culturally elaborated object in the novel life circumstances he encounters.
>
> (cited John-Steiner & Mahn, 1996, p.193)

The processes of mathematisation do not assume that students need to create all of their mathematical knowledge in order for them to choose and use mathematics. We can stand on the shoulders of giants when trying to learn new things in mathematics. However, this doesn't mean that students should be told exact methods of how mathematics should be done. What the process of mathematisation encourages is that students are involved in coming to their own understanding of why the vertical mathematics is needed to do the horizontal mathematics. Coming to an understanding of how the vertical and horizontal mathematics work together in the process of mathematisation is interactively constituted in productive classroom learning communities. However, the productivity of such classrooms is steered by teachers who can use their pedagogy in effective ways to engage students in the processes of thinking and reasoning about the mathematics by using mathematics. In this way productive classroom learning communities acknowledge that the facility for mathematical abstraction is connected to students' mathematical disposition. Students need to want to be engaged in thinking about mathematics in abstract ways. Certainly, classroom practices can generate or depreciate opportunities for students to view the mathematics as accessible. Also, these classroom practices need to make the opportunities worthwhile in terms of how students can continually build their mathematical proficiency. Therefore, classroom practices which are steered by the teacher need to continually work at building all of the five components of mathematical proficiency (Kilpatrick, et al., 2001).

In the primary school context, the research literature discusses how "it is well known that early grade teachers are very concerned with making mathematics accessible to students, and believe that it is essential to make if fun" (Milgram, 2007, p. 56). Such an emphasis implies that students may create the essential mathematical knowledge if they are participating in mathematical activities that are fun and accessible. However, while such participation may engage students, it may not being developing productive mathematical dispositions for their long term mathematical proficiency. Therefore, too much concern with engaging students by making it accessible and real life may conflict with opportunities for students to gain the mathematical precision and flexibility to make more abstract connections (An, et al., 2004; Milgram, 2007). The *Shape of the Australian Curriculum: Mathematics* (ACARA, 2009, p. 8) suggests that mathematics in the middle years “needs to include a greater focus on the development of more abstract ideas” and with this is the expectation that “the foundations that have been built in the years prior, provide a solid basis” for students to draw upon “in unfamiliar sequences and combinations to solve non-routine problems and develop more complex mathematical ideas”. Certainly several students in Naomi’s class (and a majority in Dan’s class) lacked a solid foundation of basic mathematical ideas and this appeared to suppress their ability to think in abstract ways to develop their mathematical repertoire at a grade 9 level.

6.3.3 Creativity versus procedures: why the conflict?

A tension seems to arise between the idea of students being creative in the mathematics classroom and the notion of structure. That is, “many educators may believe that precision and accessibility are in direct opposition to each other” (Milgram, 2007, p. 56). Naomi raised this issue in this way:

> It [structure/setting out] seems to be usurped by creativity being more important [in primary school], and while I think creativity is important I don’t think it is more important. If you don’t give them a basic structure, they have nothing to build a creative mind on.

The conflict between creativity and precision may develop due to a narrow understanding of what mathematical creativity means. It may be assumed that creativity involves making the mathematics fun, with an *anything goes* attitude to knowledge construction in an effort to make the mathematics accessible. However, Mann (2006, p. 236) suggests that:

> Creativity in maths consists not just of finding alternative answers to problems, but also of ‘finding problems’ and being

> sensitive to deficiencies, ‘disharmonies’ and gaps in current knowledge. It means finding unrecognised links between ideas, techniques and areas of application, and breaking from old mindsets that limit intellectual exploration.

Certainly, a prerequisite for engaging students into participating in the socio-mathematical norms which would encourage a “creative tension” (Johnson, 1999, cited in Bussey, 2008, p. 142) is profound pedagogical content knowledge. However, Naomi used her pedagogical content knowledge to focus on speed, accuracy and procedures in order to prepare students for senior mathematics. The norms of practice established in Dan’s classroom involved “a student mentality of ‘waiting for the correct answer’” (Mann, 2006. p. 236). This type of classroom practice “dims curiosity and discourages creative thinkers from seeing themselves as good at or interested in mathematics” (Mann, 2006, p. 236). It could be that the knowledge gaps of the students made Dan and Naomi feel that a transmission model of teaching was their only option to get through the school mathematics program, even though this narrow focus limited even further the possibilities for students to become flexible and creative in their mathematics thinking. Thus, the classroom practices might have contributed to eroding any inclination that students might have had with continuing with mathematics at a higher level. Therefore, students might have arrived into grade 8 and 9 with an inefficient mathematical proficiency because of a primary school context that was focused on mathematics being fun and accessible. However, classroom practices in the middle school which focus on transmitting knowledge, skills and procedures may swing the proficiency pendulum to the opposite extreme. This extreme does not bring about the desired improvement to the productivity of students’ mathematical proficiency since it doesn’t acknowledge the importance of developing students’ mathematical disposition and ability for adaptive reasoning.

Indeed overcoming the tension between creativity and structure in teaching and learning mathematics involves allowing the proficiency pendulum to continually swing across all five elements (Kilpatrick, et al., 2001) of mathematical proficiency.

Classroom practices across primary and middle school contexts need to allow such a fluid movement rather than be caught at extremes or on exact descriptions of the pendulum's path. Students' mathematical proficiency depends upon the opportunities that are available in the classroom learning communities. Teachers using effective and adaptive pedagogy to engage students into using socio-mathematical norms when mathematising in classroom learning communities appears to be a worthwhile process in the development of intellectual autonomy and mathematical proficiency. Importantly, the "learning culture" that is interactively constituted in the middle school mathematics classroom learning communities is heavily dependent on the "input of all stakeholders" (Bussey, 2008, p. 141) acknowledging the interconnectedness of the five proficiency strands (Kilpatrick, et al., 2001). Furthermore, there may not be an exact path to how the proficiency pendulum should swing. Indeed Galileo's simple model of the pendulum taking a circular path was too simple. Understanding the concept of time by using the cycloid curve of a simple pendulum involved the work of Bernoulli and Huygens. However, it was the seed of knowledge from Galileo that evolved through ongoing research into an accurate way to measure time. So understanding how to teach for mathematical proficiency is an ongoing process that takes the time to build on the work of others to continually look for ways to refine the path of the proficiency pendulum.

6.3.4 Reflecting on the path of the proficiency pendulum

Mathematics in grade 8 and 9 is the last port of call that primes students with the mathematical proficiency to pursue senior mathematics and beyond. Within this preparation, teachers at Amethyst College discussed the need for students to communicate and justify their mathematical thinking and reasoning. They felt that students needed to understand how to choose, use and understand the correct mathematical syntax. In the view of the senior school mathematics teachers, it is flexibility and efficiency with mathematical syntax that enhances students' potential success in higher level mathematics. Teachers at Amethyst College were acutely aware that students arrived into the grade 8 mathematics classroom without an ability to set their work out and use the vertical mathematics to do the horizontal mathematics. This may be a reflection of “Australian curriculum documents of the last decade” showing a “reduced emphasis on computational skill and algebraic procedures, and substantial emphasis on students obtaining deep understanding of the underlying ideas and being able to use them in real contexts” (DEEWR, 2008, p. 31). However, the data from the TIMSS (1999) video study

> pointed to a less than expected performance on conceptual understanding and problem solving ability and that the greater emphasis on conceptual understanding and problem-solving skills did not appear to have given Australia the benefits from the trade off of routine skills.
> (cited in DEEWR, 2008, p.31)

There was a discussion at the curriculum meeting about how students had trouble with moving within the world of mathematical symbols and this appeared to be an obstacle to their progression in mathematics. This has also been identified in the research literature, since “for many children, mathematics is seen as a ‘foreign language’; the symbols and expressions provide a formidable barrier to understanding mathematical concepts” (DEEWR, 2008, p. 32). The senior school mathematics teachers who taught in the middle school had a tacit awareness of the importance of students becoming confident and competent at using vertical mathematics to help them apply their mathematics to solve problems. During their lessons Naomi and Dan constantly reminded students that they needed to set their work out and show their working and justification when solving problems, since the students appeared to be in a mathematical habit of only writing an answer.   In this way, it may be that Dan and Naomi were acknowledging that “mathematical communication skills are crucial for creativity to be ‘recognised, appreciated and shared’” (Mann, 2006, p. 236). What’s more, Ball (2003, p. 37) discusses the “fluent use of symbolic notation” as a “critical practice” in developing students’ mathematical proficiency. The symbolic notation that needs to be used and further developed in the middle school classroom is essential for students to justify their thinking. The idea of using mathematical syntax to construct logical justification builds upon understanding and moves understanding mathematics away from being “education reform rhetoric” into not only “knowing it” but also “knowing why it is true” (Ball, 2003, p. 37). Certainly, an emphasis on students justifying their solutions during their problem solving endeavours was a classroom norm that was encouraged within the classroom practices of both Naomi and Dan. However, “justification is a practice supported by intellectual tools and mental habits” (Ball, 2003, p. 38). Thus, students also require opportunities for authorship of the mathematical tools that they choose to use when justifying their solutions in effective classroom socio-mathematical norms, rather than accepting the authority of the teacher as to which is the most efficient method.

As suggested by Milgram (2007. p. 48), there are “verbal and nonverbal aspects to problem solving”. Activating the nonverbal areas of the brain involves practice. That is, Milgram (2007, p. 48) suggests that for students to be able to activate the “nonverbal mechanisms in their brain” when required in solving mathematical problems, they need to become “fluent with the basic operations” so that they don’t have to think about each separate step”. He likened the process to becoming proficient at playing a musical instrument. Another analogy to understanding the consequences of a lack of fluency in mathematics is in literacy and creative writing. A lack of fluency would mean that students had to consistently think about whether every word was a noun, a verb or an adjective before they wrote every sentence in their narrative. Central to facilitating the “non verbal processes of problem solving” (Milgram, 2007, p. 48) is that students must practise with the basic ideas of mathematics for example: addition, subtraction, multiplication, division, fractions, directed numbers in real life and purely mathematical contexts so that they gain the flexibility for the ideas to become automatic. In this way when the basics of mathematics become automatic, students may feel better equipped to work at an abstract level, thus improving their overall ability to be creative in solving mathematical problems.

Naomi and Dan’s focus on students practising their setting out and procedures might be because they were trying to compensate for missing elements of students’ mathematical repertoire. But the focus on setting out and procedures appeared to deflate the potential of the socio-mathematical norms in the classroom. However, a tension arises since it also appears difficult for students to engage successfully in the socio-mathematical norms of mathematical difference and argumentation at a grade 9 level unless they are efficient and competent at using the fundamental ideas of mathematics. That is, the very processes involved in mathematisation in grade 9 assume a proficiency at using basic vertical mathematics together with the horizontal mathematics. So perhaps the teachers were attempting to get students to think about the mathematics by encouraging them to justify their ideas by using symbols and procedures correctly. In this way they were attempting to develop the essential processes involved in mathematical proof. Building proficiency with justification in the middle school may be thought of as a general mathematical process that encourages students to use mathematical tools to explain “Why does this work? Is this true? How do I know? Can I convince other people that it is true?” (Ball, 2003, p. 38).

These teachers were also “highly conscious of the peculiarities of the language of formulae” (Freudenthal, 1973, p. 310). Neither Naomi nor Dan intentionally taught the procedures and steps in algebra like it was a “meaningless game”, but the algebra became meaningless to some of the students. Both teachers verbally encouraged students to think about what they were doing. However, the socio-mathematical norms which encourage students to question differences in solution methods didn’t appear to be a consistent practice in the classroom learning communities that I observed. Some of the students in Naomi’s class appeared to be able to perform on the end of term test; they were “computationally fluent” and could “pick up skills and methods very quickly” (Mann, 2006, p. 236). However, as Mann (2006, p. 236) warns this may be a stagnating learning process, since “creativity is vital when applying maths to real world problems, which often require reformulation”. The limited success that students experienced on the QCAT may be symptomatic of their mathematical inefficiency at reformulating the mathematics that they have learned in the classroom. Thinking abstractly and making connections is essential when using algebra to solve problems. Indeed students need to have opportunities to build new knowledge from the mathematical intuition that has developed over their previous experiences with mathematics. Building students' ability to think abstractly and mathematical intuition appear to be critical components in the pendulum's path to guiding students into proficiently progressing into higher level mathematics.

The data seem to expose the fact that there may be a gap in the pendulum's path to building students' mathematical proficiency. There seems to be a discontinuity in what is valued in mathematics between the primary school and the middle school context in preparation for higher level mathematics. The semiotic tools within vertical mathematics seem especially important to the senior school mathematics teachers in the middle school. The socio-cultural discourse of mathematics in the middle school at Amethyst College assumes that students should arrive adequately prepared to use mathematical symbols and tools since these "are central to the appropriation of knowledge through the representational activity by the developing individual" (John-Steiner & Mahn, 1996, p. 193). This notion is supported in the latest reform document: *Shape of the Australian Curriculum: Mathematics* (ACARA, 2009). This is because the students' mathematical "tool kit of semiotic means…which become internalized and available for independent activity are critical in supporting and transforming mental functioning" (John-Steiner & Mahn, 1996, p. 193). The apparent lack of continuity of classroom contexts that place the proficiency pendulum at extremes appears to continually diminish opportunities for students at this school to become users and doers of mathematics at a higher level.

6.4 Influences of the established school culture

It has been discussed that Dan and Naomi's focus on knowledge transmission may have dampened students' involvement in the processes of making mathematical connections to think about the Great Thing in their own way. This may have constricted the critical opportunity for students to "be sensitised to the beauty of maths, the ability to see the whole and to find harmony and relationships" (Mann, 2006, p. 236). It has been acknowledged that it is important for teachers to use their pedagogical content knowledge to attend to the idiosyncratic ways that students might construct new mathematical knowledge by being flexible in their classroom practices (Vygotsky, 1978). However, in a similar way their classroom practices may have been constricted by the socio-cultural context in which they work.

6.4.1 Streaming in the middle school

Streaming in mathematics was not a preliminary concern of this research, but it emerged as a key theme that influenced the epistemic culture at Amethyst College. Naomi discussed during our interview how she introduced advanced, ordinary and general mathematics at the start of grade 8, in 2007. She spoke about how these "flexible groupings" at the beginning of grade 8 were a necessary "structure". Naomi's comments clearly portrayed her belief that the process and structures of streaming would be the "panacea to underachievement" (Boaler, 2004, p.197). Naomi felt that the teachers at the school supported streaming and this appeared to be enough for her to be "unconcerned that research has failed to demonstrate any links between setting [streaming] and high achievement" (Boaler, 2004, p. 197). Naomi spoke about streaming in terms of the amount of content covered by grade 9 students, and that the advanced class tried to do the "whole text book". The focus of the school mathematics program was on the amount of work covered through a transmission model of knowledge construction and this was how streaming was structured. So again, the nucleus of the classroom practice viewed mathematics as a noun through the transmission of a set amount of knowledge and procedures. Streaming reinforces the notion that teaching school mathematics is a one way process, but this does not promote sustainability (Bussey, 2008).

Naomi and her staff appeared to have the flexibility (and power) to make an educational decision from the macro perspective and they chose to go with streaming. The research literature acknowledges that streaming is widely accepted by teachers (DEEWR, 2008). The "hierarchy of learning approach" adopted at Amethyst College aligns with the belief that "having students clustered around their 'natural abilities'" allows teachers "to construct learning activities that match the perceived ability of the students" (DEEWR, 2008, p. 48). Naomi discussed the natural ability of students and how:

> Some of them just develop a little slower… then they hit mid year 9 and all of a sudden maths becomes easy to them …because the neurons have suddenly grown and all the rest of it…and there's nothing that anyone is doing about that.

Thus it is important to analyse how the curriculum is structured to fit in with the teachers' view of how students learn. Dan performed this analysis at the curriculum planning meeting when he highlighted that the school mathematics program does not fit in with their perspectives on ability and how students learn. So Dan was trying to initiate a discussion about how the grade 8 curriculum did not cater to the perceived ability levels of the students in the streaming hierarchy. Naomi responded that it was a timetabling issue. At this point it appeared that many of the teachers had tunnel vision and didn't attempt to acknowledge the wider implications of streaming. What's more, it was clear that Naomi and the teachers were aware of other situations where students were being streamed incorrectly. So even though the mathematics teachers were conscious that streaming at the school was flawed on a number of different levels, streaming remained. Essentially, streaming in grade 8 didn't appear to fit in with teachers' hierarchical view of how students learn mathematics. Moreover, what continually appeared to be neglected in any discussion was consideration directed towards improving the processes of how students might become better engaged in doing and using mathematics.

Amethyst College is part of an “educational market place” and it does compete with other schools for its students (Boaler, 2004, p. 196). Given the lack of congruency between streaming and the school mathematics program in grade 8, it could be that streaming is used in this school to create an “image” that is “popular with parents that schools generally want to attract” (Boaler, 2004, p. 196). This view was supported by a senior school mathematics teacher at Amethyst College who suggested that streaming was good for the kids since they can “aspire to be advanced”. What’s more, the introduction of the NAPLAN test and the potential of the leagues tables “force schools to pay more attention to potential high achievers” and there is a widespread belief that streaming “enhances achievement for high ability students” (Boaler, 2004, p. 196). In fact Clarke and Clarke (2008) report that “the research evidence is clear that generally any benefits which accrue from ability grouping are only to very high achievers, with a negative impact on average and low-attaining students” (cited in Stephens, 2009, p. 39). Coincidentally perhaps, streaming in grade 8 at Amethyst College began in 2007, the year before the NAPLAN test was introduced.

Naomi and the mathematics teachers have the ability and the pedagogical content knowledge to choose how to structure how students learn mathematics at their school. Research evidence consistently supports the view that “the two factors that are most strongly associated with growth in student achievement in mathematics…are opportunity to learn…and the degree of curricular homogeneity” (DEEWR, 2008, p. 49). Even so, the power of the streaming mindset prevails, perhaps in part due to the not so invisible hand of the market as well as an attempt to group students according to their perceived ability level to make the transmission of knowledge easier. At this point it is important to acknowledge and analyse the micro perspective of streaming and consideration must be given to how students view themselves within this setting.

6.4.2 The rate of learning mathematics: maths/second

During my discussions with students from both Naomi and Dan's classes I noticed that most students brought up the issue of pacing in the lessons. Across both classes there were conflicting opinions about the tempo of the lessons. For example in Naomi's class Amy talked about getting bored during the lesson since:

> Miss goes over and over it again and again for the less clever people in the class who keep asking questions, and I get it so then I get side tracked and I don't concentrate and when I'm supposed to start work, I'm not in the mood.

On the other hand, Layla felt too much pressure from having to go at a certain pace:

> …it feels like I take longer than other people to get it; I get frustrated; I don't move quick enough; I wouldn't be able to do extension maths because I'm not quick enough.

The need for students to get quicker was important to Naomi since she said during our interview:

> My advanced class, they're good, they get it, but they can't work quickly; they're not fast in their thinking; then some kids are just brilliant.

During the lesson Naomi would say things like:

> Remember you are not just trying to get it right…you want to get them fast

The social norms of the classroom resulted in equating proficiency in mathematics to how fast the mathematics was done. The rate at which the mathematics was done (the maths/second) was a significant measure of success in the classroom. Nate would often ask John, “What question are you up to?” even though they collaborated well together when working on their investigation. The maths/second created a competitive atmosphere in the classroom which made some students uncomfortable depending on their mathematical habitus (Zevenbergen, 2004). Problematically, such classroom practices diffuse opportunities for the creativity required to develop the processes involved in problem solving so that students are involved in mathematical action for effective mathematical proficiency. “An over-emphasis by the teacher on speed, accuracy and following algorithmic rules dims curiosity and discourages creative thinkers from seeing themselves as good at or interested in maths” (Mann, 2006, p. 236).

So Naomi may be trying to get students to gain the proficiency and automaticity at doing the basic mathematics by practising the skills and procedures. Perhaps Naomi believes that teaching from the noun view of mathematics in the middle school is this best way to prepare students to choose and use mathematics in senior mathematics. But this noun view might be at the expense of mathematical abstraction (Milgram, 2007), just as too much of a focus on creativity and accessibility might be. Essentially a narrow approach to teaching and learning mathematics which appears to view mathematics as innate causes the “underlying lacuna in school math…that students playing with manipulatives, will find all of mathematics already hiding in their memories…and they will automatically know it when they need it” (Milgram, 2007, p. 46). Therefore, even though different classroom contexts (across the primary school and the middle school) appear to place learning mathematics at extremes (students creating their own knowledge by doing fun mathematics versus students practising the skills and procedures of mathematics), there appears to be a common assumption that students will come to understand how to choose and use mathematics on their own. However, it seems that students become flexible at adjusting their mathematical repertoire and disposition to suit the focus of particular classroom contexts, often at the expense of their mathematical proficiency. Classroom norms which teach students to view mathematics as a noun do not position students as proficient doers of mathematics ready to engage in doing mathematics.

There was also a range of opinions about the pace in Dan's ordinary mathematics class. Ben wanted to "do algebra slowly and not rush into it", whereas Addison felt that she could go more quickly and this could have been one of the reasons she thought she might move to advanced mathematics. These data expose the fact that Amethyst College appears to base its streaming on what Boaler (2004, p. 215) refers to as a "fallacy" since:

> Students of a similar 'ability' assessed via some test of performance, will not necessarily work at the same pace, respond in the same way to pressure of have similar preferences for ways of working. Grouping students according to ability and then teaching towards an imaginary model student, who works in a certain way at a certain pace, will almost certainly disadvantage students who deviate from the ideal model.
> (Boaler, 2004, p.215)

The data from Amethyst College suggest that students have idiosyncratic responses to the pace that streaming generates. Indeed it seems that "students who were most able to adapt to the demands of their [streamed group] were most advantaged or least disadvantaged by" streaming (Boaler, 2004, p. 215). So the maths/second done by the student overrides the importance of their ability to think, reason, choose, use and thus justify mathematics appropriately. Layla appears to be a capable mathematics student, but the pace of the classroom setting may cause her to move to ordinary mathematics in grade 10.

Certainly, streaming and the classroom practices that are focused on the noun view of learning mathematics are continually placing students as passive recipients of mathematical knowledge. Furthermore, what may be implied from the classroom mathematical norms that are steered by Naomi and Dan is that coming to be an effective user of mathematics is an innate process. Indeed, Naomi and Dan's classroom practices that are intended to prepare students for higher level mathematics may in fact be preventing students from developing a mathematical disposition to want to continue with doing higher level mathematics. Also, there is an overarching concern about the consequence of streaming in terms of educational sustainability (Boaler, 2004; Bussey, 2008). This is particularly so at Amethyst College, since advanced mathematics in grade 10 is a prerequisite for senior mathematics B or C (and highly preferable for physics and chemistry). Therefore, the one way process of streaming in this context may be constraining the mathematical horizon of students, as are the classroom norms and practices that place mathematics learning as a one way process.

6.4.3 The scope of the curriculum: expecting students to sprint a marathon

One of the reasons that Naomi felt she needed to rush her students in advanced mathematics was because of the amount of content in the curriculum. Naomi discussed how she based her streaming on how much of the textbook was covered. She was aware that there was too much in the curriculum but didn't know how to "pare back from it". Naomi discussed the pressure of the NAPLAN test requiring that students "should have seen all of this by this time". The curriculum planning meeting highlighted that teachers felt that students needed to be exposed to certain topics before the NAPLAN test and the QCAT. What's more the horizon knowledge of the senior school mathematics teachers elevated their awareness of the importance of preparing their students for senior mathematics. Certainly, this preparation revolved around students becoming flexible at using mathematical content to do the mathematical skills and procedures.

There were several issues that Naomi and the mathematics staff discussed at the curriculum planning meeting as important when structuring the mathematics program: preparing for end of term testing; covering core topics for the NAPLAN test; practising modelling and problem solving questions for the QCAT; and preparing for senior mathematics. Thus they generated the "crowded curriculum syndrome" at Amethyst College which "provided little space for connecting, generalizing and conjecturing" (Stephens, 2009, p. 26). What's more, the "primary focus on 'doing', as opposed to inquiry tends to generate passive learning and poor learning habits" (Stephens, 2009, p. 26). Still the cluttered curriculum dominated how the mathematics was taught in the classroom. What's more, all of the curriculum requirements had to be facilitated through a transmission model that fitted in with a structure which was based on streaming and the text book. Dan was clearly apprehensive about basing the school mathematics program on the text book. Therefore, the potential effectiveness of Dan's (and Naomi's) pedagogical content knowledge was diluted within (among other things) the confines of the school mathematics program.

The contact time available in the mathematics classroom also mitigated the potential for pedagogical patience. The timetable was structured so that the grade 9 mathematics students had a contact time of about 3 hours and 20 minutes every week. In reality though, students were in the mathematics classroom for much less time than this. Term 3 at Amethyst College was particularly fragmented which meant that students would miss several mathematics lessons during the week. I often observed Naomi and Dan trying to get through a certain amount of content because they were going to be missing several mathematics lessons due to reasons beyond their control. So the teachers at Amethyst College were faced with the long standing problem of "trying to teach more in less time" (Willis, 1990 cited in DEEWR, 2008, p. 18) Moreover:

> It may be suggested that certain aspects of traditional mathematics should be de-emphasised to allow new content or processes to come in. But …if a particular procedure or fact is to be tested it has to learned. De-emphasising simply

> means it has to be learned in less time and …students ‘learn’ a lot badly, in the name of ‘getting through the course”.
> (Willis, 1990 cited in DEEWR, 2008, p. 18)

So it could be that Naomi used the textbook as a support to try to attend to as many as possible of the requirements and restrictions that were pervading the middle school context. The unfortunate point is that Naomi and her mathematics staff were not using their pedagogical content knowledge to think and reason from a macro perspective about how to engage their students into being flexible and sophisticated doers of mathematics. Dan acknowledged the problems, but he too seemed unable to shift from the traditional instructional methods of the mathematics classroom (Romberg, 2001). Instead the teachers’ pedagogical content knowledge was used to find mutual concessions between the text book and engaging students into doing problem solving. Essentially, the conservative approach from the macro perspective perpetuated issues such as passive learning in the classroom. This passive learning resulted in a significant number of students who couldn’t apply the mathematics they had learnt when attempting to solve problems on the QCAT.

6.4.4 External testing percolating within curriculum structure

The NAPLAN test with its focus on multiple choice answers ignores what the middle school mathematics teachers are attempting to accomplish: that students use appropriate setting out in mathematics to explain and justify their thinking and reasoning. Furthermore, if classroom practice becomes overly concerned with the NAPLAN test this may further depreciate any ideas that encourage efforts towards cultivating a culture of mathematical creativity as defined by Mann (2006). Interestingly though, the literacy part of the NAPLAN test has three parts which test language, reading and writing, and this includes opportunities for short responses, rather than multiple choice answers alone. The numeracy test has only two parts: calculator and non-calculator, with no opportunity for students to show how they can use their mathematical tool kit to communicate their mathematical thinking to an audience. This lends some support to the idea that policy in education reforms such as the National Curriculum becomes superficial rhetoric that doesn't bring about meaningful changes to classroom practice to ultimately improve students' mathematical proficiency. Of course, this also depends on the teachers' agency at interpreting policy. Being objective observers and interpreters of the education system so that they are willing and able to be engaged in a creative tension when they implement the curriculum may be an important part of the process. Students' mathematical proficiency thus depends on teachers using their agency for change to engage in effective dialogue about policy reforms.

On the other hand, the QCAT tests how students can use their vertical mathematics to do the horizontal mathematics. That is, it examines how well students can apply the mathematics they know in novel ways. At the curriculum meeting, there was a positive consensus regarding the quality of questions on the QCAT since it required students to:

> Interpret information or solve practical problems, apply their knowledge appropriately in contexts where they will have to use mathematical reasoning processes, choose mathematics that makes sense in the circumstances, make assumptions, resolve ambiguity and judge what is reasonable in the context.
> (DEEWR, 2008, p. 11)

Naomi suggested that the grade 9s "failed" the QCAT. Interestingly, Dan thought that the tasks were good but that students didn't have enough time to do the test. So the expectation that students do mathematics at a certain pace is an expectation also reflected in the external tests. In this way the pacing idea that permeates streaming fits into the time limits set by testing (internal and external).

Naomi felt that the QCAT didn't reflect the ability of the students at Amethyst College. But also, it could be that Naomi felt somewhat rebellious about its intrusion. Teachers were expected to mark the QCAT and this would have contributed to their work overload. However, Naomi and the teachers did acknowledge that the QCAT did bring to light a deficiency in the flexibility and functionality of students' mathematical repertoire, particularly in modelling and problem solving. So the QCAT did play a diagnostic role that teachers could use to inform their curriculum planning. At their curriculum meeting, teachers discussed how they could squeeze more modelling and problem solving questions into their current school mathematics program. Yet this appears to be a reactive response that relies on a unilateral approach rather than a true diagnostic analysis that is attuned with the complexities that are inherent to the practice of productive pedagogy.

Students need time to understand and participate in the processes of flexibly using the vertical mathematics to apply it to the horizontal mathematics. As suggested by Freudenthal the "greatest pedagogic virtue is patience" (1973, p. 413). Yet this pedagogic patience appeared to be a luxury that the teachers at this school could not afford. Moreover, as mentioned, the teachers tended to attribute the failure of the students to students' innate abilities rather than attributing it to the nature of the classroom practices. So the question becomes about whether teachers are willing to reflect upon their pedagogical practices and the broader educational system for the benefit of students' learning. In this way teachers might continually reconsider what they are trying to achieve in the classroom and how their school curriculum can be structured accordingly. Moreover, teachers might benefit from opportunities to move away from viewing their profession as being tightly bound and sealed into enriching their life world with the reciprocal action that real learning entails. In this sense, classroom practices become about how students can develop their productive mathematical dispositions so that they want to become part of the process of developing their mathematical proficiency.

6.4.5 Assessment in the middle school: the "big stick"

> The test is your *big stick* and if you don't have it, your class will break down.
> (Middle School mathematics teacher at Amethyst College)

School based assessment arose as an emic issue in this research. A teacher used the metaphor of the test being their *big stick*, suggesting that the mathematics test was their means of exerting power and influence over their classroom. What's more, this insinuates that students don't think the mathematics is worth doing unless it is on the test. However, school mathematics perpetuates this view and

> it is not too far-fetched to doubt whether mathematics as a discipline of mind is adequately represented by those mathematical subjects that are the readiest to be examined. I need not substantiate this doubt; at school every discipline is in danger of degenerating into the instruction of examinable matter.
> (Freudenthal, 1973, p. 83)

Indeed the classroom learning communities that I observed appeared to operate in sync with testing concerns. Doug and Naomi were compelled to cover a certain amount of content before the test. Any potential of the socio-mathematical norms being developed by students in Naomi's classroom became undermined by a need to cover enough work before the mathematics test. The students in both of the mathematics classes provided interesting perspectives about the mathematics test. An important difference between the views of the students from Naomi's and Dan's classes was identified.

When students from Naomi's class talked about the mathematics test, they analysed the test and could articulate what they thought was wrong with it. These students could separate their mathematical knowledge and ability from the construct of testing. For example, Tammy spoke about the time constraints of testing. She was aware that the test had to be done in a certain way so that she could achieve the results she desired:

> I worry about my overall result on the exam, so I rush the exam so I can finish each section, because you need to get an A, B or C in each section; I won't pass the test if I don't do well in each section, so I have to rush to get through the questions.

John was the most insightful since he was annoyed that the exams "exploit" the "few weaknesses" that he had in mathematics. Importantly though, John told me that he didn't view himself as a weak mathematics student, and he wasn't about to let the test make him appear weak. He also criticised the marking criteria of the test. Amy also acknowledged that she had to do some more revision and study since her "grades dropped on the last exam".

Significantly the students in Naomi's class understood that the mathematics test was a means to an end for them and that they had to prepare appropriately. In this way the students seemed to think about using the test as a tool to help them progress in mathematics. Therefore, they were thinking about how to best fit into the expectations of school mathematics. They were aware that this placed them as better mathematics students. So while this might have been empowering to students in terms of school mathematics, it might not improve the productivity of their mathematical dispositions and therefore the sustainability of their mathematical proficiency. Their time and efforts would have been better spent inquiring into how to choose and use mathematics to actually do mathematics.

On the other hand, students in Dan's class saw themselves as failures when it came to the mathematics test. This was perhaps best articulated by Ben who said "on the last exam I got D E D …I got dead". Des was equally hard on himself since he thought he understood the mathematics problems, but then he "got them wrong". Des took total responsibility for failing the test and suggested that he should just "try harder". However, I wonder how Des can try harder if his mathematical self-efficacy is subdued by the mathematics test. As suggested by Bandura:

> The most effective way of creating a strong sense of efficacy is through mastery experiences. Successes build a robust belief in one's personal efficacy. Failures undermine it, especially if failures occur before a sense of efficacy is firmly established.
> (Bandura, 1994, p.75).

The students in Dan's class didn't appear to have developed a robust view of their mathematical self-efficacy. A "belief in…ones own efficacy" is used in the descriptor for a productive mathematical disposition (Kilpatrick, et al., 2001, p. 117) and therefore influences the process of students developing their mathematical proficiency. D'Ambrosio (1998, p. 6) suggests that "tests can cause a regression in the flow of the biological and psychological tempo of an individual". As a result, unlike the students in Naomi's class, the students in Dan's class couldn't separate their results on the mathematics test from their mathematical ability. Some of the students in Naomi's class felt mathematically powerful and could withstand the vacillations that testing brought to their self-efficacy. The mathematical vulnerability of the students in Dan's class became more exposed in the shadows of the test. Dan was aware of this and was most concerned that his students didn't have "an opportunity to achieve". In this way Dan was thinking about D'Ambrosio's challenge for

> mathematics educators to stop and think a bit about the low results on tests and examinations being a consequence of the kind of test or examination that is being given rather than a lack of capacity on the part of the student.
> (D'Ambrosio, 1998, p. 47)

Rather than tests being used as a *big stick* they could be used to provide diagnostic information. However, at Amethyst College the focus appeared to be on summative assessment rather than formative assessment. This diminished opportunities for testing to be used diagnostically. I make this assertion because at the curriculum meeting at the end of term the assessment and reporting protocols appeared to dominate how the mathematics curriculum was structured. In turn the classroom practice revolved around the mathematics curriculum. There was more of a concern about how teachers could fit in the content of the curriculum before the test, and whether or not teachers could mark the tests to fit into the reporting timeline set down by the school administration. Essentially the quality of the mathematics education occurring in the classroom and the impact on students' mathematical proficiency appeared to be continually overlooked.

The tests and assignments were structured using three criteria: communication; knowledge and procedures; modelling and problem solving. These criteria did allow for the strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence and adaptive reasoning to be assessed (Kilpatrick, et al., 2001). Contrary to what D'Ambrosio (1998, p. 47) suggests, the tests and assignments at Amethyst College were not "removed from any space where creativity might be used". In fact, students did have the opportunity to apply their knowledge and appropriately select mathematical tools to justify their thinking in the tests and assignments. The actual school based assessment instruments were not problematic in terms of assessing mathematical proficiency. However, how the requirements of the test were interpreted by teachers and students appeared to militate against the productive classroom practices necessary to develop positive long term outcomes for students' mathematical proficiency. As suggested by Gardner (1991, p. 254) "one can have the best assessment imaginable, but unless the accompanying curriculum is of quality, the assessment has no use".

Naomi and Dan emphasised the correct way to set the work out and they concurrently verbally encouraged students to make connections and understand the relationships in mathematics. The school based assessment supported this emphasis. However, the social mathematical norms generated in the classroom practices which focused on a transmission model of learning mathematics didn't encourage students to really "appreciate connections between mathematical ideas and to understand the mathematics behind the problems" (DEEWR, 2008, p. 40). Therefore, while Dan and Naomi had the pedagogical content knowledge to engage students, the cluttered curriculum, the constraints of testing and reporting timelines made some of their lessons appear to have the "syndrome of shallow teaching" (DEEWR, 2008, p. 40). The proficiency strand which involves the development of students' productive mathematical dispositions was continually given short shrift.

So it does appear that the aim of "getting through the course" (DEEWR, 2008, p. 40) did override the quality of the interactions in the classroom. The questions on the test and assignments did fit in with Queensland curriculum requirements. Certainly, students did have an opportunity to "demonstrate the use of a range of solution strategies, techniques and tools" (DEEWR, 2008, p. 40). However, the conditions created within the mathematics classroom in trying to cover the content before the test, reduced the opportunities for students to gain real mathematical proficiency. This aligns with Stephen's assertion that "as a school subject mathematics has tended to be dominated by transmission pedagogies which position learners as passive receivers of knowledge" (2009, p. 4). In short, a focus on the mathematics tests seemed to propagate self-destruction on both the macro and micro perspectives.

The assignments were certainly an opportunity for students to engage and collaborate in some rich tasks, as John and Nate did. In this way the assignment was a "worthwhile learning experience" in itself (Burkhard, 2007, p. 79). However, once again, it depended on the mathematical disposition of the students as to whether or not they would take up the challenge. Also, Dan questioned the validity of assignment marks since he had “seen too many that have just been copied”. In this way the desired richness of the assessment task was lost when students copied it so that they could get a good mark. Another teacher (Emma) appeared to be relatively comfortable with this since it “gets them over [the line] and boosts their self-esteem”. However, this is a simplistic view since “unrealistic boosts in efficacy are quickly disconfirmed by disappointing results of one’s efforts” (Bandura, 1994, p. 75), as would occur on the end of term test for many of the students. So the assignments and tests may be “constricting activities” that undermine “motivation” since “disbelief in one’s capabilities creates its own behavioral validation” (Bandura, 1994, p. 75). Dan and Keith appeared to be aware of this and Keith summarised this pertinently:

> The best learning is the incidental learning; it is not subjected to an exam; there is no pressure on them; they’re learning through investigation and their own research. Richness is lost in education because of assignment and test constraints.

In this way, Keith and Dan deemed that students cannot fully appreciate mathematics and develop their mathematical thinking when they are wedged into trying to perform on regular, summative assessment items. However, the majority of the teachers in the mathematics department remained steadfast with using the numerous summative assessment items as both the nucleus and the binding force to dominate their pedagogy and their classroom learning communities. This reiterates that teachers appear to be using “pedagogies of resentment that are driven by a logic of deficit views” of their students (Prosser, 2006, p. 13). What’s more, it could be that the teachers have a deficit view of their own pedagogy. This is perhaps why they needed to have regular structured summative assessment, since that put more of an emphasis on students taking responsibility for their learning (or lack of learning). The atmosphere of accountability of tests that pervades the middle school context appears to be compromising opportunities for real learning in the mathematics classroom.

A focus on assessment in school mathematics damages students' mathematical dispositions since attention is on performance rather than learning (Kilpatrick et al., 2001). Indeed this attitude may be well ingrained by the time students reach the middle school since "preschoolers generally enter school with a learning orientation, but already by first grade a sizable minority react to criticism of their performance by inferring that they are not smart" (Kilpatrick et al., 2001, p. 171). Students in Naomi's class (for example, Layla and Sandy) spoke about moving to ordinary mathematics, perhaps because their mathematical dispositions saw them view their mathematical ability as fixed and they didn't feel smart enough to be in advanced mathematics. Certainly the processes involved in the ongoing development of students' mathematical dispositions cannot be separated from the socio-cultural norms generated by streaming and assessment at Amethyst College. Problematically though, students appeared to be showing "less interest in putting themselves in challenging situations that result in them (at least initially) performing poorly" (Kilpatrick et al., 2001, p. 171). Indeed, Addison's mathematical disposition saw her preferring to remain in Dan's ordinary mathematics class since she was thinking about her performance at doing mathematics rather than actually learning mathematics.

If teachers used assessment diagnostically to realign their pedagogy they might develop a process that focuses on the learning needs of their students. In this way, the teacher uses testing as a "meaningful activity…to check the influence of the teaching process, at least in order to know how to improve it" (Freudenthal, 1973, p. 84) This might "help students to become far more confident when dealing with intellectually challenging experiences" (DEEWR, 2008, p. 40), rather than leaving them grapple with how they are supposed to fill in the gaps and try harder. Another possibility to reduce the ubiquitous mathematical fatigue induced by chronic testing is for teachers to stop, listen and watch students. In fact, Freudenthal (1973, p. 84) suggests that "it is more informative to observe a student during his mathematical activity than to grade his papers". Naomi did suggest that she wanted to know:

> What is their way of thinking?

In this sense Naomi might have been acknowledging that

> knowledge of children's thinking is a powerful tool that enables the teacher to transform this knowledge and use it to change instruction. One major way to improve mathematics instruction and learning is to help teachers to understand the mathematical thought processes of their students.
> (Fennema et al. 1996, p. 432 cited in Stephens, 2009, p. 50)

The critical element, however, is that teachers use their pedagogical content knowledge in productive ways to not only acknowledge how students think, but to flexibly adapt their classroom practices accordingly to continually engage students into becoming intellectually autonomous. The distinction that needs to be made is that intellectual autonomy for a productive mathematical disposition and mathematical proficiency involves students reflecting on their own mathematical thinking. This thinking is directed towards how they learn and do mathematics rather than using their intellectual autonomy to become better at doing school mathematics.

Teachers thinking and reflecting about their practice is essential. Teachers are a critical link in the educational chain. If Dan and Naomi are going to be able to use the resource of their pedagogical content knowledge in a productive way for their students' learning, then they need to be "engaged in some form of transformative reflective practice that creates the inner space to debug their conditioned partialistic responses to relationships" that exist in their life world (Bussey, 2008, p. 144). Firstly this may involve Dan and Naomi viewing their pedagogical content knowledge in new ways so that they can move away from consistently teaching mathematics from the noun view. This move involves classroom practices that engage students into becoming proficient at doing mathematics by considering the five elements of mathematical proficiency through the processes of mathematisation in a culture of participation. Then Dan and Naomi may start viewing the curriculum in new ways so that they can be reflective and proactive in their interpretation of curriculum reforms. Looking at the big picture of what they are trying to achieve might bring about meaningful changes to their classroom practices to improve students' mathematical proficiency. Moreover, Dan and Naomi have to want to change how they view the use of their pedagogical content knowledge to transform their classroom practice.

6.5 Conclusion

The aim of the analysis was to use the psychological and social perspectives as a lens to explore the mathematical interactions and the norms of the classroom learning community. Additionally, the emic issues that emerged were used as key concerns within this analysis. Connections made between the emic constructs and the social and psychological perspectives can be viewed as contributing to making a system of doing mathematics at Amethyst College.

Amethyst College has ways of doing mathematics. These ways of doing mathematics may be viewed from the macro and micro systems. The reflexivity of these systems evolved though the complex interactions between the social and the psychological perspectives. What this analysis shows is that there are no if then statements within these systems. For example, in the micro system of the classroom, the psychological tools of the individual are not invented in isolation (John-Steiner & Mahn, 1996). Certainly, how they are mediated by the individual within the learning community is idiosyncratic. Therefore, while the norms of the classroom may open up possibilities for the individual to access the mathematics, actively or vicariously, the individual may also deny or feel denied access.

Building upon this idea, this analysis highlights that the pedagogical content knowledge of the teacher, while essential, does not automatically guarantee that the mathematics is going to be more accessible to the student in the short or long term. Both Naomi and Dan have outstanding potential in their pedagogical content knowledge. Naomi's pedagogical content knowledge appeared to be more effective in her classroom. However, the analysis interpreted this as a reflection of the mathematical dispositions of the students in her class. Quite a few of the students in Naomi's class were good at doing school mathematics, yet my analysis suggests that many of them didn't feel that they were users and doers of mathematics. On the other hand, Dan's PCK was constricted by the textbook and the school curriculum and this didn't suit his class. However, Dan did have the ability to extend his pedagogical content knowledge beyond the micro perspective and use it to critique the macro perspective. Dan analysed the streaming in the middle school, the text book and the structure of the curriculum. He tried to initiate a discourse about the structures of middle school mathematics and how it affected the psychological perspectives of the students. This discourse didn't evolve in a meaningful way though since Naomi and the staff predominantly felt that these structures supported how school mathematics needed to be done. This might suggest that these teachers do need to make the essential transition from a "calculating to a listening mind" (Miller, 1999, cited in Bussey, 2008, p. 144).

Naomi and Dan view the structure of the mathematics curriculum in different ways. Certainly, teachers' views are influenced by more than just their pedagogical content knowledge and their view of how students become mathematically proficient. That is:

> their careers - their hopes and dreams, their opportunities and aspirations, or the frustrations of these things - are also important for teachers' commitment, enthusiasm and morale. So too are relationships with their colleagues - either as supportive communities who work together in pursuit of common goals and continuous improvement, or as

> individuals working in isolation, with the insecurities that sometimes brings.
> (Hargreaves, 1999 cited in Bolam, et al., 2005, p. 23)

Naomi is the mathematics coordinator at the school and feels the pressure of the school's performance in mathematics; her decisions (supported by the majority of her staff) are reactive. What's more, Naomi feels under attack from the numerous and incongruous policy changes to the curriculum, so the structure of streaming in mathematics at Amethyst College may be a way of maintaining some control. Problematically though, this pressure became manifested in students' mathematical dispositions. Naomi's role placed her in a vulnerable position. However, vulnerability can be used in positive ways also since it is a "core aspect of our humanity…it breaks down the barriers of difference and allows for a space to emerge in which we can, in shared vulnerability, identify with other" (Butler, 2004 cited in Bussey, 2008, p. 140). Getting to the point of relating to vulnerability in this way, however, is an ongoing process that does require a professional culture that is willing to sustain such a shift.

On the other hand, Dan is a classroom teacher and he follows the protocols of the school mathematics program. He continually identified flaws in the school curriculum. Even so, Dan conformed to the traditional model of transmitting knowledge to fit into the school mathematics program. So while Dan and Naomi appeared to view the structure of their school mathematics curriculum in different ways, their classroom practices remained static. Perhaps their basic assumptions and belief systems about how students learn mathematics via a transmission model of knowledge construction prevented meaningful change within this context.

A core aim of this research was to gain an empathetic insight into this context. One insight is that teachers and students do not independently shape their level of proficiency. Whether they are students hoping to be proficient at mathematics, or as a teacher of mathematics, they are buffeted by the socio-mathematical and socio-cultural norms that prevail in their contexts. They are all at their mercy. They can be proficient only to the extent that these contextual influences conspire to make it happen. What's more their dispositions affect how they interpret their place in their context.

It appears that students had little opportunity to engage in the processes of mathematical inquiry. This affected the productivity of their mathematical disposition and this was interpreted as a disempowering process. Similarly, teachers did not appear to be engaging in a process of inquiry into their classroom practices. The classroom practices were influenced by the socio-cultural norms that saw these teachers act as passive recipients of reform agendas, also a disempowering process. So in this context the students and the teachers were engaging with mathematical knowledge in terms of what should be known, rather than continually thinking about new ways to understand how it is we come to know. Effort was directed towards quick solutions rather than taking the time to think more deeply about the problems.

Opportunities for students and teachers to continually refine and build new ways of thinking about knowledge and understandings are central to effective learning communities that promote mathematical proficiency. However, mathematical intuition to think abstractly develops over time and some of the students appeared to lack the flexibility with, or the disposition to use, foundational mathematical concepts. Teachers' tacit knowledge may be used to continually think about how to engage students into the processes of knowledge construction. The teachers at this school did have the foundations of essential pedagogical content knowledge. However, their professional culture didn't seem to encourage them to use the agency of their pedagogical content knowledge in flexible ways both in the classroom and for the classroom. What appeared to be missing from the classroom context and the teachers' professional context was reflective dialogue. Importantly, these types of conversations serve the thinking processes to formulate problems in new ways. As suggested by Einstein, "The formulation of a problem is often more essential than its solution, which may be merely a matter of mathematical or experimental skill”. These key ideas are used to position possibilities for authentic transitions in the next chapter.

Chapter 7

Re-shaping Australia's proficiency footprint

Common sense is not so common.

Voltaire

7.1 Introduction

This case study highlights some concerns about how Australia's mathematical proficiency footprint is being shaped. The proficiency footprint is currently shaped by shallow mathematical understandings resulting from a convergent, noun view to teaching and learning mathematics. The norms of practice of the mathematics classrooms at Amethyst College position mathematical proficiency (Kilpatrick, et al., 2001) as one dimensional through a focus on mathematical content and procedure. The essential strands of strategic competence, adaptive reasoning and productive disposition were not interwoven in the classroom practices. Re-casting the classroom norms by highlighting the doing and using of mathematic to acknowledge the interconnectedness of the five mathematical proficiency strands as a complex whole are tendered in this chapter as a way to re-configure Australia's proficiency footprint.

The data analysis reveals that a classroom microculture that promotes spaces for students to continually develop their mathematical proficiency is not independently moulded by the middle school mathematics teacher. The macro issues of the socio-cultural context in which teachers attempt to position their practice are also limiting opportunities for productive change. The extent to which teachers can change their classroom practice, to view mathematical proficiency in a different way depends upon their epistemological beliefs as well as the contextual influences. The educational rhetoric that accompanies policy orientated change and the implications of the current era of accountability are limiting possibilities for the desired evolution in the classroom microculture. Such an evolution would see classroom practices promote the intellectual autonomy of students so that they can be successful in learning mathematics for their progress at school and beyond.

The data analysis also showed the cognitive shortcomings in students' capacity to think abstractly and intuitively in these middle school mathematics classrooms. However, there is an urgency to move away from a deficit view of students since the data also highlight how this promotes the "pedagogies of resentment" (Prosser, 2006, p. 13) which both bring about and sustain the deficits that exist in students' mathematical proficiency. While these deficits may indeed underpin the concerns about Australia's proficiency footprint, the logic that focuses on reactive short term solutions is perpetuating the conditions. Changing the contours of Australia's proficiency footprint involves transforming the socio-cultural contexts that are maintaining deficit views of teachers and students.

7.1.1 Re-formulating the problem

This qualitative case study investigated education in action with a key objective of adding to the discourse in the area of pedagogy and students' mathematical proficiency. Opportunities for students to engage in quality mathematical experiences in the middle school sculpt their mathematical progress. This research has attempted to gain an insight into the goal of equipping Australia's students with the mathematical proficiency and mathematical disposition to pursue higher level mathematics. The methodology and the methods of this case study brought a socio-cultural perspective to learning and teaching. Effective classroom learning communities that promote mutually engaging mathematical interactions and opportunities to mathematise were the salient features. The socio-mathematical norms of mathematical difference and mathematical argumentation were the critical lenses used to interpret opportunities for students to develop their intellectual autonomy.

The implications of the data analysis are that if teachers and students remain as passive participants in their respective socio-cultural contexts rather than contributing as mutually engaged participants then the status quo shaping Australia's proficiency footprint will remain. The current footprint is shaped by shallow understanding, inadequate proficiencies and ineffectual mathematical dispositions. The recommendations, which are focused on processes and actions by teachers and students, are aimed at re-formulating problems to develop the intellectual and social sustainability required for a proficiency footprint better able to support Australia, nationally and globally.

The key intention of the recommendations is to initiate possibilities of how to change the norms of practice. The first recommendation considers ways in which placing mathematics as a verb to build students intellectual and active engagement in the classroom learning community can build students' cognitive capacity for abstract thought. In the second, consideration is directed towards how the socio-cultural contexts may be reconstructed through new practices to bring about new forms of agency for teachers and therefore also for students. So the recommendations reformulate the original problem of

- How do middle school mathematics teachers empower students to be proficient doers and users of mathematics?

into contemplating how students and teachers can build their capacity to think and participate critically and reflectively in their respective socio-cultural contexts. How teachers and students do this depends upon the psychological and the socio-cultural perspectives since these perspectives contribute to the creation of the norms of practice.

This research showed that the teacher was integral to establishing effective classroom norms, so students are depending on the teacher. The data showed that these teachers provided quality knowledge, but there was limited intellectual engagement for students during the interactions in the classroom learning communities. Furthermore, the data revealed that teachers were constrained by professional norms of practice. It appeared that the current socio-cultural context of mathematics education saw teachers "struggle for the soul of professionalism" (Day, Flores & Viana, 2007, p. 251) and similarly students were struggling to build their mathematical souls.

The opportunity for teachers and students to be mutually engaged in redefining the norms of their learning communities potentially builds their change agency to think critically and participate in the shaping of productive dispositions. The recommendations in this chapter continually reflect upon how teachers might change from feeling and thinking that they are gifting mathematical knowledge to their students into thinking about how they might help students feel at home and proficient in the world of mathematics. Such a change involves teachers thinking about what they are doing now and how this will shape the mathematical proficiency footprint of future generations.

7.1.2 Implications of proficiency: intellectual and social sustainability

The first recommendation is focused on how we can use mathematisation in a culture of participation to build students' ability for abstract thought. This surfaced from data analysis highlighting that students in the upper middle school should have an already established way to think intuitively and flexibly about foundational mathematical ideas. This type of intellectual autonomy is a valuable resource for students as they build their thinking and reasoning capacity throughout their educational journey from the middle school into higher level mathematics and beyond. This recommendation is focused on the development of intellectual sustainability.

This first recommendation acknowledges the importance of all five strands of mathematical proficiency (Kilpatrick, et al., 2001). However, while this recommendation may be considered to be a synthesis of ideas that might add to the knowledge base in mathematics education research, it is likely to remain inert unless teachers become engaged in thinking critically about the suggestions. The potential power in any recommendation depends on whether teachers are willing to think about how it applies to their context and whether it could or should become a norm of practice. Indeed, the recommendation isn't written with the intention of its remaining a noun. However, the implications of the data analysis induce reservations about its becoming a useful verb in the current professional culture in which teachers work. Therefore, the second recommendation acknowledges the complexities involved in teachers becoming part of a professional learning community that covets new and better ways to engage students in effective classroom learning communities. This may be thought of as the development of professional sustainability.

Thinking about how to action social and intellectual sustainability in new ways has the potential to re-shape Australia's proficiency footprint. The respective socio-cultural contexts that influence the dispositions of the teacher and the students are interdependent. The classroom learning community depends upon the agency of the pedagogical practices of the teacher, which in turn depend upon the professional learning community. The mathematical disposition of students and their agency depends upon the classroom learning communities that they have experienced while on their educational journey. The task of social and intellectual sustainability is a process involving actions that re-formulate questions. It involves creating the space within the socio-cultural domains of teachers and students for new practices and new forms of agency. In essence, this space promotes the opportunity for new actions by the teacher and students who have traditionally participated as passive recipients in their socio-cultural contexts. In this way, re-shaping the proficiency footprint involves teachers and students who are empowered by socio-cultural norms to be the change makers.

Before moving into the recommendations, the next section considers several implications from the data analysis that resulted in a common sense view of mathematics that disempowered members of the Amethyst College learning community, resulting in the atmosphere of indifference driven by deficit views. Indeed this common sense view, driven by policy, programs and deficit views appeared to place mathematical proficiency as a noun rather than acknowledging the complex, yet essential processes involved in mathematical action.

7.2 Implications of school mathematics at Amethyst College

A key implication of school mathematics at Amethyst College was that the teachers and the students appeared to be struggling with their productive capacity to think critically in their socio-cultural domains. At the forefront of their struggles was the fact that they lacked an agency to participate effectively in their learning communities. That is, in the microculture of the classroom, students appeared to wait to receive mathematical knowledge, then they practised the mathematics and attempted to remember it for the test. I implied in my analysis that this may be a reflection of the classroom norms that are established, in part, through the school mathematics curriculum, deficits views of students and the teachers' epistemological views. Indeed, the common sense about doing mathematics developed by the students became "a perspective that was less about mathematics than about how to cope in the mathematics classroom" (Kilpatrick, 2007, p. 1). The teachers' efforts that were directed toward students performing on assessment maintained this common sense view. Certainly, "common sense is both an individual possession and a social construction. It helps us learn, do, and teach mathematics, and it also can hinder all those processes" (Kilpatrick, 2007, p. 1). Students developing productive mathematical dispositions and effective long term mathematical proficiency appeared to be obscured by the common sense view of doing school mathematics at Amethyst College. Moreover, the common sense view of doing mathematics was not helping students feel at home and proficient in the world of mathematics.

The structure of the mathematics curriculum at Amethyst College seemed to involve drawing a line of best fit through the mathematical content and skills required for the internal and external tests. Consequently, at best, the learning landscape produced from the classroom practices placed the mathematical proficiency of students in terms of "conceptual understanding" and "procedural fluency" (Kilpatrick, et al., 2001, p. 116), a noun view. In some ways it seemed as though the "strategic competence", "adaptive reasoning" and the "productive disposition" (Kilpatrick, et al., 2001, p. 116) of the students were expected to naturally evolve from students practising the mathematics out of their textbook, rather than through a creative culture of participation in mathematisation. The school based assessment did test students' mathematical comprehension, procedural fluency, strategic ability to formulate and solve mathematical problems and their capacity for mathematical justification (NRC 2001, p. 5, cited in Ball, 2003, p. 9). However, the teachers appeared to be reactive in their interpretations of students' performance on internal and external tests and searched for quick fix solutions using a transmission model of knowledge construction. The students didn't appear to feel mathematically powerful and the teachers didn't appear to be using their pedagogical content knowledge in productive ways for students' intellectual autonomy. In fact, the essence of teaching and learning mathematics at Amethyst College became "mechanical, manipulative and lifeless", perhaps because the "fears" of teachers mingled and multiplied with the "fears inside…students" (Palmer, 2006, p. 2). Essentially, teachers and students appeared to have developed perfunctory identities that saw them going through the motions to adhere to the expectations of their respective socio-cultural domains.

7.2.1 Pedagogical identity: redundant by reform?

The teachers at Amethyst College appeared to be suffering from reform exhaustion. It did appear that the bombardment over the last decade within this school of "an unrelenting plethora of changes" meant that these teachers were finding it "hard to maintain energy, enthusiasm and, ultimately, willingness for change" (Bolam, et al., 2005, p. 25). During data collection, teachers were attempting to maintain attention towards Education Queensland's curriculum assessment and reporting document: Essential Learnings by the end of year 9 (Education Queensland, 2007), while also being concerned about their students performing on the NAPLAN test for the leagues tables that would ultimately be produced. The lack lustre results on the QCAT may have also contributed to "low teacher morale and feelings of impotence" (Bolam, et al., 2005, p. 25). What's more these teachers felt at odds with the administration of the school due to the unrealistic demands of reporting. The professional culture at the school appeared fatigued and undermined. Thus, as suggested by MacDonald (2003, p. 139):

> Despite the extensive knowledge and experience that curricularists seemingly have with respect to implementing meaningful curriculum change, the goals and processes of change are narrowly proscribed by existing structures, resources and traditions, with the result that schools always fall short of meeting the needs of young people and their communities.

Naomi and the mathematics staff appeared to rely on unproductive "pedagogies of resentment" (Prosser, 2006, p. 13) that focused on an unchanging hypothesis of how students learn mathematics. Skemp (1978, in Handal & Herrington, 2003, p. 63) suggested that "traditional mathematics teaching is easier than attempting more progressive approaches as innovations bring additional burdens to teachers, despite the merits and advances that each innovation might potentially bring". Certainly, the change agency of teachers appeared to be saturated in the current climate of "policy orientated change" (Bolam, et al., 2004, p. 25). In this way, the teachers may be asking themselves the same questions that some students asked in Dan's class, "What do we have to do this for"? Hence, the continual reforms that have attempted to bring about revolutionary changes to classroom practice appeared to have had the opposite effect in this school. This may be because "curriculum change in the last several decades relied on the simplistic assumption that teachers will, machine-like, alter their behaviours because they were simply told what was good for them and for their students" (Grant, Hiebert, & Weame, 1994, in Handal & Herrington, 2003, p. 62). Naomi suggested that continual reforms in education were driving teachers "insane" and "away" from the profession. It is acknowledged in the research literature that "stress and burn-out" makes teachers "less willing to engage in discussion with colleagues" and increases the "likelihood of individuals leaving the profession" (Bolam et al., 2005, p. 25). In this way, teachers may have become resistant to thinking critically about change since they were not part of the process; the expectation on them was to dispense a product.

The teacher is central to interpreting curriculum reform policies in order to bring about meaningful change in their school context. A critical missing element of reform movements in education in the middle school context might have been a failure to acknowledge "teachers' pedagogical content knowledge and beliefs as well as the contexts in which" they teach (Knapp & Peterson, 1995 in Handal & Herrington, 2003, p. 62). Consequently, this is reflected in the actions of the teachers at this school that may have resulted in their acting as "semi-professionals and recipients of reform policies rather than the change-makers" (Collay, 2006, p. 2). Their socio-cultural context perpetuated the noun view to curriculum implementation. This was relayed to teaching and learning mathematics so that the noun view became a classroom norm.

7.2.2 Drawing parallels between the macro and the micro perspectives

The focus of the mathematics curriculum at Amethyst College appeared to be on assessment rather than on how to create and sustain (through ongoing action and reflection) quality mathematical experiences for students in the middle school classroom. The classroom practice was focused on mathematics from the noun view. Contemporary research literature in mathematics education research urges that students should be part of the processes involved in choosing and using mathematics within a learning community (Cobb et al., 2001; Hopkins, 2008; DEEWR, 2008). That is, rather than a transmission model, the mathematics is interactively constituted by members within a culture of participation. The focus of the classroom community is on thinking and reasoning about mathematical concepts through the divergent use of mathematical processes. The socio-mathematical norms of arguing about different ways of formulating the mathematical problem are one avenue for students to develop their intellectual autonomy and a productive mathematical disposition (Yackel & Cobb, 1996). The mathematics serves the classroom conversations and the classroom conversations serve the thinking and learning processes of mathematics.

Perhaps this idea of socio-mathematical norms could be re-cast in the notion of a socio-pedagogical norm. This norm of practice would see teachers become engaged in conversations which are served by re-formulating the use of their pedagogical content knowledge. Central to this dialogue is structuring a curriculum that caters for classroom practices that encourage students to develop effective mathematical proficiency. Such a curriculum recognises that there are five strands of mathematical proficiency (Kilpatrick, et al., 2001) rather than a synthetic mathematical proficiency for the purpose of the test. This socio-pedagogical norm acknowledges that intellectual autonomy and a productive pedagogical disposition through teachers' active participation in a professional learning community is essential. The productivity of teachers' pedagogical proficiency converges on improving the productive and creative capacity of the classroom learning community.

There didn't appear to be a lot of gaps in Naomi's or Dan's pedagogical content knowledge in terms of the PCK domains (Ball et al., 2007). However, what was missing from their classroom practices were the processes of engaging students in a creative culture of mathematisation. Dan and Naomi remained within the relative safety of transmitting knowledge so that they could get through the content. Perhaps the creative culture of mathematisation was too great a risk for these teachers in the current era of teacher accountability and performance. Taking risks takes time and energy directed towards looking for different paths. However, the job intensification that resulted from these teachers having to keep up with the latest reform initiative and its implications may have clouded their pedagogical decisions and ability to use their pedagogical and mathematical knowledge in different ways. Similarly students had little time to take the risk to pursue different mathematical paths to understanding the mathematics they were doing. In this sense, the expectation for teachers and students to build their change agency involves a disposition to take risks. This disposition is connected to their socio-cultural contexts.

A proactive way to think about the parallels between the classroom learning community and the professional learning community is the idea of proficiency. A key aim of sustainability in mathematics education is for students to develop their mathematical proficiency according to the five interconnecting strands (Kilpatrick, et al., 2001, p. 116). Facilitating the processes of students developing their mathematical proficiency may involve teachers thinking about their proficiency regarding the teaching of mathematics. Kilpatrick, et al. (2001, p. 380) suggest that just as mathematical proficiency involves five interconnecting strands, so too does teaching for mathematical proficiency. Proficiency for teaching mathematics is defined by (Kilpatrick, et al., 2001, p. 380) in the following way:

- conceptual understanding of the core knowledge required in the practice of teaching;
- fluency in carrying out basic instructional routines;
- strategic competence in planning effective instruction and solving problems that arise during instruction;
- adaptive reasoning in justifying and explaining one's instructional practices and in reflecting on those practices in order to improve them; and a

- productive disposition toward mathematics, teaching, learning, and the improvement of practice.

Dan and Naomi appeared to have the key strands of: conceptual understanding and fluency in carrying out basic instructional routines and the strategic competence to solve problems that came up during their lessons. Dan also seemed to spend an extensive amount of time planning his lessons. However, this planning was shaped by his traditional views on knowledge construction. So what might require attention is their adaptive reasoning to justify, explain and reflect on their classroom practice so that they are not relying on an unchanging formula on how to teach mathematics. Furthermore, their productive disposition appears as an imperative requiring attention. Of course, the strands for the "proficient teaching of mathematics" (Kilpatrick, et al., 2001, p. 380) are interlaced. However, what the data highlighted is that the proficiency for teaching mathematics is also linked to the professional learning community in which teachers work, just as the mathematical proficiency of students is inextricably connected to their classroom learning community and the norms socially constructed within them.

Dan, Naomi and the teachers at Amethyst College have the pedagogical content knowledge to help students become mathematically proficient and comfortable in the world of mathematics. However, their pedagogical beliefs and the spaces currently available within their socio-cultural context appear to be constraining their ability to try different approaches to create new classroom norms of practice.

7.2.3 The latent power of pedagogical content knowledge

The recommendations that are made recognise that Amethyst College is straining to sustain students with robust mathematical thinking and reasoning skills. As discussed by Dan, there were more and more students streamed into general mathematics at the beginning of grade 8. Quite a few of the students in the classrooms I observed appeared to lack a basic understanding of foundational mathematical concepts. The analysis chapter discussed the use of streaming as a reactive response, with limited benefits for students in terms of their mathematical proficiency. What's more, attending to the pressure of their students performing on the external tests saw teachers focusing their efforts on "content coverage and pacing rather than teaching for understanding" (Handal & Herrington, 2003, p. 63). The curriculum planning meeting highlighted that the teachers were not directing their attention to changing their classroom practices to improve the learning opportunities for their students.

Scaffolding upon the latent power of their pedagogical content knowledge may be a viable pedestal where the teachers at Amethyst College could position themselves in order to change their view of curriculum construction from performance centred to learning centred. A first step towards this vantage point is that teachers shed the "deficit view" (Prosser, 2006, p. 13) they have of their students and their profession so that they might develop into an effective professional learning community. Bolam, et al. (2005, p. 2) described an effective professional learning community as "having the capacity to promote and sustain the learning of all professionals in the school community with the collective purpose of enhancing pupil learning". In this way, teachers' attention and effort is directed towards a holistic view of mathematics education to strengthen their students' mathematical disposition by engaging them in rich mathematical learning opportunities. This contrasts with the repressive processes of reward and punishment that are furnished by a motivation for students to perform well on tests to move to a higher mathematics stream.

An important starting point for the staff at Amethyst College may be to shift towards thinking about mathematics education and education reform in different ways so that their focus continually returns to creating positive learning outcomes for their students. Hargreaves (2003) suggests that:

> Professional learning communities demand that teachers develop grown-up norms in a grown-up profession – where difference, debate and disagreement are viewed as the foundation stones of improvement.
> (cited in Bolam, et al., 2005, p. 9).

Taking the path to an improved vantage point is an ongoing process that recognises the limitations of the modernist traditions of the education system but proceeds, anyway. Learning new ways to critically evaluate and view the socio-cultural norms involves time and space. There is a lot of blame being bandied around in education circles about why we are in a crisis in mathematics education. University mathematics faculties blame secondary schools for not preparing students well enough (Belward, et al., 2007). Senior schools blame middle schools (Ridd, 2004, cited in McPhan, et al., 2008) and middle schools blame primary schools, as evidenced within the data of this case study. This type of blame game does not develop the process of creating "grown-up norms in a grown-up profession" (Hargreaves, 2003, cited in Bolam, et al., 2005, p. 9) since it doesn't appear to be improving student outcomes. The blame game could turn into an opportunity for improving the quality of the mathematics education in schools if all stakeholders became mindful of a common sense goal: improving students' mathematical proficiency.

The recommendations in the remainder of this chapter acknowledge the difficulties and complexities of the social context in which the teachers at Amethyst College work. However, improving students' mathematical dispositions and their proficiency is a clear incentive. Certainly, improving outcomes for students' mathematical proficiency cannot be separated from how teachers view their profession and how they are viewed within the broader educational domain. Imperatively, the recommendations acknowledge several critical starting points so that teachers might use their role to be leaders in the mathematics education community (Yackel & Cobb, 1996) for the benefit of the students in their classrooms.

The recommendations focus on how to re-configure the mathematical proficiency footprint as intellectually and socially sustainable. The remainder of this chapter considers two core issues which are interrelated:

Recommendations to foster intellectual sustainability:
How can we develop students' ability to think abstractly about mathematics?
How can teachers build their reflective intelligence?

Recommendations to foster social sustainability:
How can teachers build effective professional learning communities to improve the effectiveness of the classroom learning communities for students' mathematical proficiency?

7.3 Developing students' ability for abstract thought

Re-shaping Australia's mathematical proficiency footprint involves students having the opportunity to build their capacity to think and reason effectively in the middle school mathematics classroom. The data from this case study showed that some students struggled to think abstractly and flexibly about basic mathematical ideas which obscured their path to doing algebra at a grade 9 level. The classroom norms at Amethyst College which were focused on the skills and procedures of mathematics might have been used by Naomi and Dan to "plug holes" and "skill up" students. However, the productivity of the learning community and students' mathematical proficiency appeared to be continually compromised. If we are to realistically improve the contours of Australia's proficiency footprint, students' ability for abstract thought requires attention beyond a bandaid approach.

7.3.1 Horizon knowledge: understanding how to develop abstract thought

Naomi and Dan had acute mathematical horizon knowledge of the content and efficient procedures that students required in their mathematical tool kit to continue with higher level courses. This pedagogical horizon knowledge was focused on what students should be able to do in terms of content, skills and procedures, perhaps at the expense of students developing their intellectual autonomy to think and reason mathematically. Perhaps one aim for all teachers of mathematics from primary school through to secondary school is to have the pedagogical horizon knowledge to not only know what students should know and be able to do, but to understand how students come to be proficient at thinking about mathematics in abstract ways. The analysis chapter discussed how both thinking and reasoning for conceptual development and practising the skills and procedures of mathematics may be viewed as complementary activities to develop students' proficiency at mathematising. Indeed pitting the use of procedures against conceptual development limits opportunities for students to organise the mathematics in their own minds in order to mathematise. Freudenthal suggested that algorithms:

> provide the technical means of fathoming greater conceptual depth…it is not fair to confront algorithmic and conceptual mathematics with one another as though one is the lofty tower from which you may look down on the other.
> (1973, p. 44).

Developing the mathematical intuition required at a grade 9 level requires a holistic understanding of how classroom practices can facilitate opportunities for students to learn how to mathematise. This does not place instruction at "extreme positions that students learn on one hand, solely by internalizing what the teacher or book says or on the other hand, solely by inventing mathematics on their own" (Kilpatrick, et al., 2001, p. 11).

The discussion turns now to focus on how students might come to understand and use directed numbers in flexible and abstract ways. Students in Naomi and Dan's classrooms appeared to struggle with using positive and negative numbers when attempting to solve linear equations. Naomi spoke of her frustration with students' lack of flexibility with negative numbers, and attempted to show students a way of performing operations with negative numbers. Dan attempted to get students to think about negative numbers in terms of temperature. All of this was going on while the actual aim of the lesson was for students to start using algebra to solve linear equations in efficient ways. Consequently, the process became codified since students were learning to follow and practise procedures so that they could perform on the end of term test. Clearly, the process of students developing their proficiency at mathematising with directed numbers involves a holistic approach from primary school through to the middle school. Indeed, students have the exciting world of analytical geometry, calculus and complex numbers to look forward to in the senior school.

7.3.2 Directed number: a holistic perspective

When students arrived into middle school mathematics at Amethyst College without an ability to think about directed numbers in abstract ways, the process seemed to become that students learned to at least use them. So students practised using the rules of directed numbers, by remembering that $q - 5$ is the same as $q + (-5)$, or that two negatives multiply to give a positive. Of course, students then became concerned about whether or not they needed to use brackets and they were confused about performing simple operations involving directed numbers. The focus of students engaging in abstract thought was diffused since they were trying to remember the rules and procedures of mathematics. So the process of using mathematics became disproportionately difficult rather than potentially rewarding, and this is not a good contour for a proficiency footprint.

The processes involved in abstract thinking for directed numbers involve not only attention to understanding negative numbers, but also understanding the processes of operations on numbers in flexible ways. There are no efficient quick fixes to abstract thinking. It is an ongoing process that develops idiosyncratically in students' minds and it takes as long as it takes. Effective classroom practices create the space for students to want to become involved in thinking about understanding directed numbers. This process may not fit into the artificialness of "traditional sequential steps" (Lowe, 2006 cited in Bussey, 2008, p. 141) of school mathematics that sees the requirement that Naomi referred to when discussing pressure from the NAPLAN test as students "should know this by this time". Therefore, the development of students' abstract thought depends upon teachers developing supportive networks across the professional learning communities of both primary and secondary schools. In this sense, there becomes a shared understanding or a shared horizon of how to develop students' ability for abstract thought. What the data of this case study showed is that secondary school mathematics teachers are depending on primary school teachers. Such a dependency cannot be left to chance (or to policy) if we are going to improve the shape of Australia's proficiency footprint. Professional and effective dialogue between teachers across primary and secondary school is critical if we are to authentically improve the mathematical proficiency of students.

Using negative numbers may not appear to be important in the life world of a primary school student. Indeed Freudenthal (1983) suggests that "negative numbers did not really become important until they appeared to be indispensable for the permanence of expressions, equations, formulae in 'analytic geometry'" (cited in Kilpatrick, et al., 2001, p. 111). Relating negative numbers directly to students' life world in terms of temperature or profit and loss does not automatically develop students' ability for abstract thought. Indeed, Dehaene (1997) discussed how "nature has only endowed us an intuitive picture for positive integers" therefore "teachers have to help students piece together novel models for new intuitive understandings" of directed numbers (cited in Tang, 2003, p. 5). What may be important is for teachers to use their pedagogy to facilitate opportunities for students to be mutually engaged in talking and thinking about using directed numbers in different ways using different directions and different paths.

7.3.3 Negatives and positives: directions for thought

Flexibility in understanding number begins before children start school. How flexible students are in their understanding of the notion of *negative* may be related to how the adults in their lives talk about it. For example, a parent might say “I’m taking $2.00 out of your pocket money this week because you didn’t clean your room”, rather than “I’ve added -$2.00 to your pocket money this week because you didn’t clean your room”. So in everyday talk we might describe negatives in simple ways, a debt is what we owe rather than a negative deposit into the bank account.

It becomes essential therefore that teachers use the everyday understanding that students bring to the classroom to build their flexibility to think and take risks and talk about negatives in creative ways. In this sense, the effort is directed toward helping students build their own mental picture of what negative and positives are. What may be useful is the “pedagogical use of metaphors” (Tang, 2003, p. 5) to develop a flexible understanding of the abstract idea of directed numbers. For example, a negative might be referred to as a *not* going and a positive as an *am* going. Students in a middle school mathematics classroom (grade 5 – grade 7) could engage in a discussion to make sense of the following ideas:

I *am not* going to town: means *not* going
I *not am* going to town: means *not* going
I *am am* going to town: means am going
I *am not not* going to town: means *am* going

A similar analogy is to let a positive be *good* and a negative be *bad:*

good things happening to *good* people: a *good* thing *good* things happening to *bad* people: a *bad* thing *bad* things happening to *good* people: a *bad* thing *bad* things happening to *bad* people: a *good* thing

These directions for thought involve students playing with the ideas in their own heads so that they can build mathematical analogies, and it recognises that students can think for themselves. The potential within such a discussion depends upon the socio-mathematical norms of the classroom learning community. Not only is it the type of activities that a teacher chooses to use with the classroom that is important, but also how the teacher and the students work together to actively construct new understandings in mathematics. This process builds students' mathematical intuition so they may come to use mathematics in abstract ways and begin to understand that $-(-5) = 5$. Importantly, this also implies that students understand: that the equals sign means equivalence; where 5 and -5 are located on a number line; and why brackets may be sometimes useful to represent and communicate the mathematical idea. This reiterates the necessity for a holistic approach to concept development by constantly looking at the relationships that exist in and between mathematical concepts. Teaching that $-(-5) = 5$ is a rule in grade 8 and grade 9 mathematics is something that can be avoided if students develop an understanding of the relationships between number and operations over time by continually looking for patterns. Students explaining and justifying their own ways of thinking about directed numbers is an essential part of this process.

7.3.4 Examples of mathematising with directed numbers in the lower middle school

Using a metaphor within a game to mathematise with directed numbers

Another metaphor that could be used to develop the idea of directed numbers is by using a game and a story telling approach. The mathematics is served through the story and the game. The following idea may be adapted, depending on the popular culture that exists in the classroom context. Indeed, the potential within such a game is greater if students' interest is captured from the outset. The following story could be read to a classroom (grade 5 – grade 7) and then the game used in the process of understanding directed numbers (adapted from Tang, 2003, p. 8).

The Potion in the Magic Cauldron

The Kingdom is doomed. The Emperor has lost his/her magic powers, unless he/she drinks the magic potion.

The temperature of the magic potion in the cauldron needs to be between 20°C and 25°C so that the Emperor can drink it and have his/her magic powers returned. There are two forces battling to control the temperature of the magic potion. The good Panda King has to battle the bad Serpent King. The Panda King tries to get the temperature to between 20°C and 25°C, and the Serpent King tries to keep the temperature away from that range.

The class divides into two groups: Panda King and Serpent King (or the game could be played in pairs). Each group is given two dice: a red die is hot (+ numbers) and a blue die is cold (- numbers). At the beginning the temperature of the potion is set, for example 36°C, but this can change. It can also be set as a negative temperature. Each group has its turn to throw the dice. However, each group must alternate between throwing the blue and the red die and then decide whether to add or subtract the heat or cold. That is, when the red die is thrown, students decide how to use the number on the die to add heat or take heat. Similarly when they throw the blue die they can either add cold or take cold. The group needs to make the decision. Each throw of the die and how the number is used: either addition or subtraction must be written down. Each group has a maximum of six throws of the dice (this can change, depending on the starting temperature). The Panda King wins if the temperature ends up between 20°C and 25°C, or else the Serpent King wins and takes over the kingdom

So for example, if the temperature is 36°C, when the Panda King group throws a 6 on the blue die, they would want to add the cold so it would be +(-6), to lower the temperature.

If the Serpent King group throws a 6 on the blue die, they would want to take away the cold so it would be – (-6), to raise the temperature.

Using the mathematisation cycle framework (OECD, 2003), this learning opportunity would proceed so that the metaphor allows students to view the problem in their own minds. Students make the decisions about whether or not they need to add or subtract heat or cold and should be encouraged to use the symbols of mathematics to record what they are doing. The mathematical syntax and justification is an efficient way to represent how the game proceeds. It allows students to check what they have done and encourages them to look for patterns in the mathematics. In this way the vertical mathematics is used while playing the game. The students are continually reformulating the problem, and the teachers' role is to remain rather passive in these interactions. The students are involved in keeping one another in check when adding or subtracting.

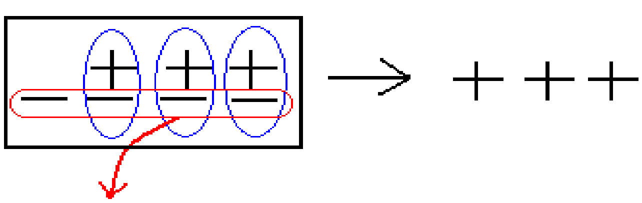
The Serpent King group would end up with a series of mathematical operations that continually attempt to increase the temperature of the magic potion, except when the temperature goes below 20°C; in which case the Serpent King would want to reduce the temperature. By writing down the vertical mathematics, students can start to view the patterns in the operations on directed numbers. For example if they threw a 6 on the blue die first, the operation could be written as: 35 – (-6), this adds heat, so the temperature would go up, so this could be written as 35 + 6. Students will make mistakes while playing the game and these may or may not be identified by themselves or by other group members. It is important that students have the opportunity to make and learn from mistakes as they continually amend their mental picture of directed numbers.

Therefore, students are using the metaphor to think mathemtically and importantly they are making the decisions. The decisions are made by the group members and so the authority is with the group. Indeed it is a chance for students to use the socio-mathematical norms to discuss *what would happen if* questions. What's more, students are using the game to develop an understanding of the vertical mathematics of the rules and operations of directed numbers, in particular the idea that two negatives multiply to give a positive.

Using number tiles to develop an understanding of adding and subtracting negative numbers

As suggested by Romberg (2001), classroom experiences need to help students develop their thinking, reasoning and understanding over time. So to develop the idea of adding and subtracting negative numbers involves different pedagogical techniques. While metaphors and games are useful tools to develop an understanding of operations with negative numbers, there should also be diversity in approach. Significantly though, different approaches are actions involving students structuring their own understanding of adding and subtracting negative numbers rather than set structures or techniques that attempt to define the ways in which students should think about directed numbers. Using the + or – signs (sometimes called integer tiles) to represent integers allows students to manipulate and group integers in order to think about the mathematical operations in different ways. Students will make idiosyncratic connections as different approaches are used to build their understanding.

Students tend to have difficulty with the abstract concept of how to subtract -1 - 4, so this will be used as an example.   The diagram below shows how -1 could be represented by integer tiles if we wanted to subtract -4. In this way students learn to represent the integers to make the mathematics work for them. The blue ovals show how the + and – can be grouped as a null set. Therefore if we take away 4 negatives, then 3 positives remain. Significantly, gaining flexibility in how to represent the integers opens up opportunities for students to develop their understanding of the rule that subtracting a negative is equivalent to adding a positive.



-1 – (-4) = 3

(http://en.wikiversity.org/wiki/File:-1--4tiles.png)

This approach may involve direct teaching, by the teacher or by the students within the classroom learning community. However, this doesn't need to imply that students sit as passive recipients. If the classroom learning community has already established socio-mathematical norms in a culture of participation, then students would become mutually engaged as they argue about the different ways to use the integer tiles. The integer tiles may be thought of as a tool that may help some students make important connections between the different ways of thinking about directed numbers. Students might see a connection between the positive and negative integer tiles and the blue and red dice from the magic cauldron game. However, the effectiveness of any approach is co-dependent on the effectiveness of the classroom learning community. What's more, this type of activity acknowledges that practising and using mathematics in different ways is part of the process for students to develop their ability for abstract thought (Milgram, 2007).

Using algorithms to develop flexibility with directed numbers and place value

Algorithms are also a useful mathematical tool that can become an effective mathematical action if students have the opportunity to explore their usefulness. To extend and cement students' thinking and understanding about place value and negative numbers we can use algorithms. The teacher can set the conditions for these learning opportunities for the students, not by teaching the procedure but by encouraging the classroom learning community to explore different paths to get to a solution. Students can make idiosyncratic connections if the classroom norms encourage this practice. For example, in a grade 6/7 classroom students could be encouraged to use the algorithm and mental arithmetic to subtract two numbers 52 - 19 in different ways. Some examples students may come up with:

By using borrowing:

$$\begin{array}{rl} & 5\ {}^{1}2 \\ - & {}^{1}\underline{1\ 9} \\ = & 33 \end{array}$$

Or students could do this:

```
     52
-    19
=  4(-7)
= 40 + -7
= 40 -7
= 33
```

Certainly, mental arithmetic might be more efficient in this situation. Interestingly though, if some students were to use mental arithmetic they might think "52 take away 20", but then they might get stuck on whether they take away an extra one or add an extra one because they get caught up with the operations of mathematics, rather than thinking intuitively. That is, some students might say 52 – 19 = 31, because they have used the following logic:

52 – 19

Think: 52 – 20 = 32

Think: 20 is one more than 19, so, take another one away to get a final answer of 31 instead of thinking that because we have taken away 1 too many we need to add another 1 on.

This mental arithmetic could be written simply as:

52 – (20 – 1)

= 52 – 20 + 1 (using the distributive law)

=33

So all of these processes are intended to give student the opportunity use different representations to convince themselves about how the mathematics works to help them feel at home in the world of mathematics. It is also worthwhile for students to have options available to check their working using a different approach. While this appears to rely on a traditional approach of using algorithms, it can be a powerful learning opportunity for students since they are thinking about the different ways to use the tools, symbols, operations and procedures of mathematics, rather than being told how to use them. Of course, the potential with any learning opportunity depends on how the classroom learning community interacts. Certainly if the socio-mathematical norms of mathematical difference and argumentation were well established in a classroom, then we might expect that the discussion on the use of algorithms and mental arithmetic could evolve in a meaningful way. Indeed, students need the space and the chance to think about the efficiency of the mathematical tools that are available to them if they are going to be able to think critically about choosing and using mathematics when required.

Significantly, the learning focus is on the process rather than the answer. The discussion about how the mathematics is done is important since it stimulates thinking and reasoning about different ways of doing the mathematics. All of this involves an avoidance of students having to remember into students thinking and reasoning about what they are doing to develop their abstract thinking. Also they come to view the different tools of mathematics as a way of being efficient at doing mathematics. As suggested by Freundenthal (1973, p. 79)

> Though maybe 90% of all mathematics a mathematician learned in a lifetime can finally be ruled out as redundant, it was indispensable, because it was its role and destiny to be replaced by better mathematics...what matters is not that the mathematics one learned is not forgotten, but that it has been, and still is, operative.

Using a number line to understand directed numbers

There are many different representations and ways for students to develop an understanding of the rules of negative numbers. Helping students to develop their own mental picture is critical. Using the number line to represent positive and negative integers is an important mathematical practice since it is a core framework of analytical geometry. Directed numbers become useful in the Cartesian plane for graphing (with x and y axes), so the number line becomes even more useful for students if it is drawn (and thought about) both horizontally and vertically from the early years of schooling rather than only horizontally. This is an example of how “sometimes the way in which a concept is first learned creates obstacles to learning it in a more abstract way” (Kilpatrick et al., 2001, p. 75).

Also, how we talk about negative numbers in everyday life creates obstacles in the process of developing an understanding of how to use directed numbers in mathematics. Thus, discussing taking away money as a negative deposit becomes an important learning process so that students can understand how to use positive and negative numbers in the opposite direction. In this sense “overcoming obstacles” in our common sense ways of talking “seems to be a necessary part of the learning process” (Kilpatrick, et al., 2001, p. 75) that needs to be acknowledged by effective classroom practices.

So the idea of the socio-mathematical norms that encourages discussion about a different way to do the mathematics continually acknowledges that students should have the critical opportunity to develop their intellectual autonomy for abstract thought. Since

> abstraction is what makes mathematics work…if you concentrate too closely on too limited an application of a mathematical idea, you rob the mathematician of his [or her] most important tools: analogy, generality, and simplicity. (Stewart, 1989 cited in Kilpatrick, et al., 2001, p. 111)

In this sense, teachers’ pedagogical repertoire is directed towards how they can help students become better at taking risks by encouraging them to reformulate problems. Therefore, rather than teachers gifting mathematical knowledge to students, the classroom norms encourage students to take risks to build their mathematical intuition so that they have faith in their own mathematical ability. What’s more, these classroom norms view mistakes as learning opportunities since mistakes are part of the risk taking process.

The different approaches that have been proposed to develop students' intuitive understanding of directed numbers are part of an ongoing process that acknowledges the importance of classroom socio-mathematical norms that encourage students to build their intellectual autonomy. Certainly, the rules for operating with negatives and positives are set; we can find them in a textbook. However, how students come to understand these rules are not. The ideas I have discussed are not proposed as the best way to teach students. There is no best way to teach all students since "the elements of the mental machine…and their working does not translate directly into a prescription for educational practice" (Kilpatrick, et al., 2001, p. 24). Pedagogical content knowledge for teaching mathematics is essential and well defined by Ball, et al. (2007). However, the decisions made by teachers about how to best use their pedagogical content knowledge is not a precise process since "a teacher cannot know exactly what approach will work with a particular student or class" (Kilpatrick, et al., 2001, p. 24).

The aim of the discussion on teaching and learning directed numbers is not intended as a single-minded practical solution. Helping students feel at home in the world of mathematics is more than just good technique. The intention is for practical solutions and teaching techniques to be one part of the in-depth collegial discussions within a professional learning community. The implications of this case study analysis advocate that new paths and connections are required to improve the classroom practices that are shaping the proficiency footprint. Risk taking involves more than practical solutions, it involves teachers feeling able and willing to continually return to critical pedagogical and collegial conversations about how to facilitate effective classroom learning communities to build students' mathematical proficiency. It is this process that contributes to a strong sense of pedagogical identity that infuses the work of good teachers (Palmer, 2006).

> Good teachers join self, subject and students in the fabric of life…they manifest in their own lives, and evoke in their

> students, a ‘capacity for connectedness’… They are able to weave a complex web of connections between themselves, their subjects, and their students, so that students can learn to weave a world for themselves. The methods used by these weavers vary widely: lectures, Socratic dialogues, laboratory experiments, collaborative problem-solving, creative chaos. (Palmer, 2006, p. 6)

Working in an effective professional learning community that uses reasoned arguments when sharing practice and knowledge across the different levels of schooling, promotes a confidence that we can work towards the goal of improving students’ mathematical proficiency.

7.4 Learning communities making meaningful connections

The data highlighted that many of the norms of the professional learning community became reflected in the classroom learning community at Amethyst College. An effective learning community encourages members to make meaningful connections, not through fixed methods or best approaches, but through mutual engagement in mathematical interactions. Mutual engagement involves different approaches that allow members to take diverse paths and risks that suit them at that point in time. In this way, the learning community doesn’t set up opportunities for success or defeat, but rather opportunities for members to overcome their own feelings of inadequacy or supremacy into being involved in good conversations. Good conversations acknowledge that learning and knowing are communal activities that “require many eyes and ears, many observations and experiences” and a “continual cycle of discussion, disagreement, and consensus over what has been seen and what it all means” (Palmer, 2006, p. 15). Teachers and students need the time and space to do this. Developing the soul of effective professional learning communities becomes reflected in the essence of the effective classroom learning community.

7.4.1. Effective professional learning communities

The idea of effective professional learning communities has become discussed more extensively in the research literature since the 1990s (Wong, 2010). Certainly, a definite meaning of what a professional learning community looks like and how it evolves depends upon the educational culture or system in which the teachers work. An ideal characteristic of an effective professional learning community is that it becomes "self-perpetuating and able to reshape its own values and norms" (Wong, 2010, p. 2). In their executive report which summarises their main findings from the *Creating and Sustaining Effective Professional Learning Communities* project, Bolam, et al. (2005, p. 145) suggest that:

> Effective professional learning communities fully exhibit eight key characteristics: shared values and vision; collective responsibility for pupils' learning; collaboration focused on learning; individual and collective professional learning; reflective professional enquiry; openness, networks and partnerships; inclusive membership; mutual trust, respect and support.

Within Queensland, "supporting the establishment and sustainment of the professional learning communities, in the context of curriculum change, is no easy task" (Lamb & Spry, 2009, p. 308). What's more, the literature rhetoric about curriculum change recommending "a partnership approach that is both 'top-down' and 'bottom-up'" is discussed by Lamb and Spry (2009, p. 308). It is acknowledged that "despite good intentions, more often than not, this partnership approach to curriculum reform represents pseudo-participation and quasi-democracy" (Lamb & Spry, 2009, p. 308). Recognising such complexities is a first step in developing an effective professional learning community at Amethyst

College that can work towards the key characteristics as described by Bolam, et al. (2005).

Certainly, the numeracy leadership of teachers at Amethyst College appears to be overwhelmed by work overload and reform exhaustion, to the detriment of students' mathematical proficiency. So in this school there may be an "important place for top-down initiatives directed towards assisting the process of developing professional learning communities" (Kruse & Seashore Louis, 2007 cited in Lamb & Spry, 2009, p. 308). Most importantly perhaps, the professional learning community can support the establishment of the ground rules that keep teachers from defeating themselves (Palmer, 2006). Essentially though, the focus of the professional learning community is "Who are the young people" at Amethyst College and "What, where and how do they learn?" (MacDonald, 2003, p. 147). Creating an effective professional learning community means moving beyond the linear structures that "obfuscate difference" (MacDonald, 2003, p. 147) into teachers engaging in professional arguments with the broader educational domain so that reform movements become more relevant (at least in the teachers' minds) to improving students' learning. This process in itself is a chance for teachers to build productive pedagogical dispositions and their numeracy leadership. The common objective of teachers' professional action is ultimately for the benefit of students' learning (Wong, 2010).

7.4.2 Structuring a culture of participation in the professional learning community

Naomi and the mathematics teachers at Amethyst College did appear to value structure. However, effective professional learning communities do not view structure as "linear, static and hierarchical" but rather as "circular, interactive and dynamic" (Palmer, 2006, p. 15). A key structure that differentiates a professional *learning* community from a professional community is the notion of "double – loop learning" (Argyris & Schon, 1978 in Wong, 2010, p. 2) indicated by the dashed line between the "Goals" and "Process" in Figure 5:

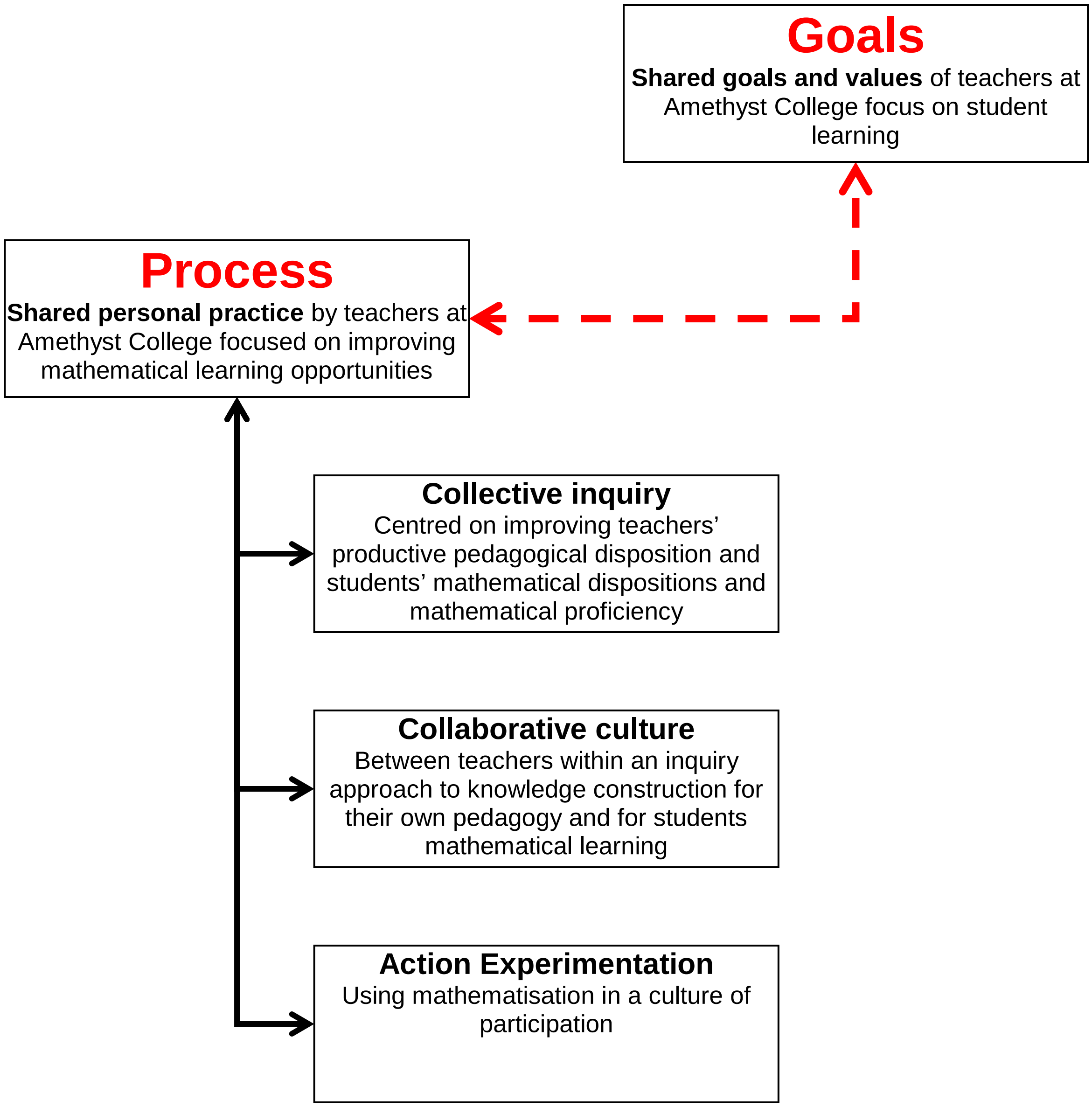


Figure 5: Elements of a professional learning community for Amethyst College (adapted from Wong, 2010, p. 2)

The reflexivity between the goals and process is especially relevant to Amethyst College since it involves teachers detecting and correcting errors in their "existing knowledge and values" in an effort to modify the "underlying norms, policies and objectives” that exist in their context (Wong, 2010, p. 2). Essentially, for Amethyst College to move from what may be called a “weak professional learning community” into a “strong professional learning community” (Wong, 2010, p. 3) requires a “reculturing” (Wong, 2010, p. 2). A critical component is that the teachers at Amethyst College work in a culture of participation and inquiry through the ongoing process of action and reflection.

This case study highlighted that an unmitigated emphasis on a traditional, transmission model for knowledge construction and a performance orientation to classroom practice may be thought of as a critical fault in the “existing knowledge and values” (Wong, 2010, p. 2) that weakened the professional learning community at Amethyst College. Similarly, pivoting the construction of the middle school mathematics curriculum around testing and reporting requirements and a reliance on the structures of streaming as the “panacea to underachievement” (Boaler, 2004, p.197) weakened the classroom learning community. Essentially it seems as though “the value of community needs to be disentangled from instrumental values of improving measurable student outcomes (e.g. achievement)…because community is really about the quality of day–to–day life in schools” (Bolam, et al., 2005, p. 11). One imperative is that teachers use their intellectual autonomy within their professional learning community to reflect upon the limitations of how the curriculum is structured as well as their classroom practice. What’s more, there needs to be time and space for teachers to put their reflections into action so that they might bring about meaningful change to classroom practices. This does require a reculturing since the traditional “image of teaching” is not about creating space for the students (or the teacher), but to “fill up” spaces in students’ mind for them (Palmer, 2006, p. 8). Indeed, creating learning spaces where students can move feely in their own process of coming to understand mathematics, involves teachers having the space to practise this in their own professional learning communities.

The ideas that I have presented in Figure 5 stem from my own reflections and interpretation of the culture of how mathematics is done at Amethyst College and I acknowledge that there are limitations to this since I am an outsider. My views may be vastly different from how a teacher at the school might reflect on the weaknesses of their professional learning community. This difference in opinion is something that should be encouraged since developing into a mature and strong professional learning community involves using disagreement and reasoning as a professional inertia to improve learning outcomes for students. Moving towards shared goals and values involves professional conversations about improving students' mathematical proficiency. Professional conversations entail a willingness to consider different perspectives. However, the perspectives need to be focused on improving the processes of how students come to think, learn and understand mathematics.

Efforts directed towards improving the mathematical proficiency of students require that we continually think about how the micro and macro perspectives of mathematics education are linked. Figure 6 (below) suggests that we should place students' mathematical proficiency at the centre of our target. However, the mathematical proficiency of the students is dependent on the idea that the four levels in the diagram are bound together through a process of action and reflection in an effective professional learning community. Furthermore, the effective functioning of the four levels in Figure 6 also depends on the importance of building upon the development of foundational mathematical ideas from the early years through to the senior years of schooling.

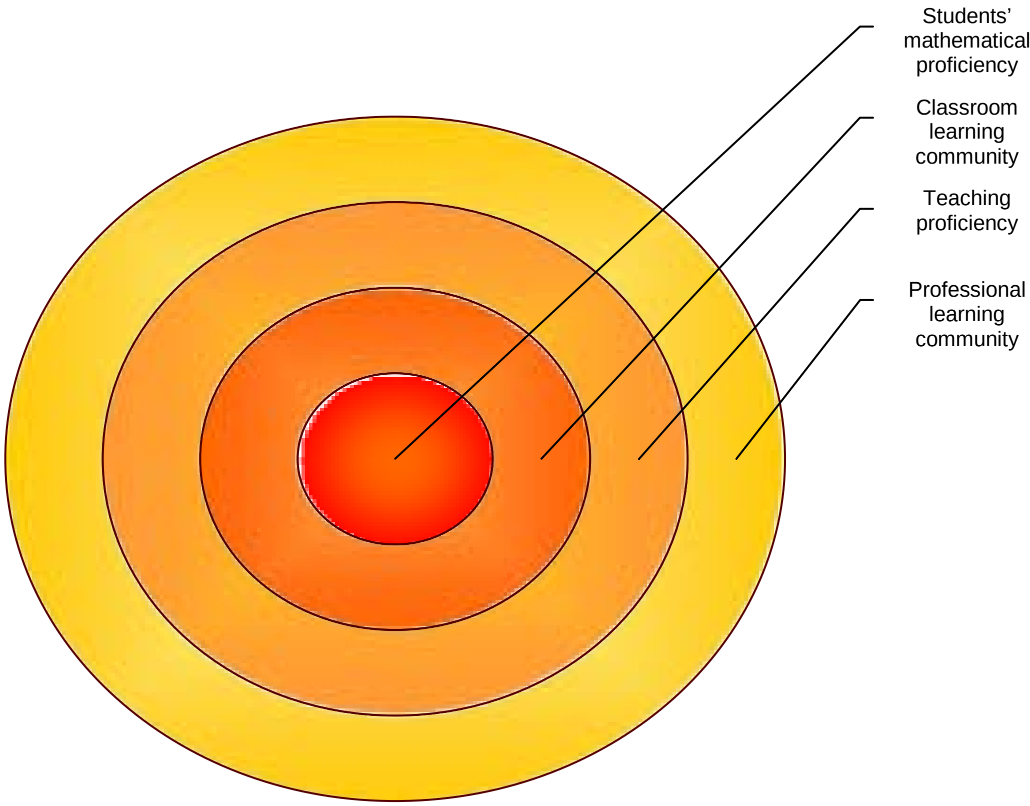


Figure 6: Target of students' mathematical proficiency.

7.4.3 The complexity of teachers' belief systems

Teachers in the middle school at Amethyst College do have a great resource in their pedagogical content knowledge. However, this resource has become inert within this mathematics department since it isn't being used in different ways to improve learning outcomes for students. This lends some insight into why the mathematics department appears to be a weak professional learning community. Hargreaves and Giles (2003) suggest that a strong professional learning community "brings together the knowledge, skills and dispositions of teachers in a school or across schools to promote shared learning and improvement" (cited in Bolam, et al., 2005, p. 21). The capacity of the professional learning community at Amethyst College may have influenced the productivity of Naomi and Dan's pedagogical content knowledge in generating consistent meaningful learning opportunities for students in their classrooms. Consideration directed towards why these teachers don't appear to be using their pedagogical content knowledge to its full potential is a starting point from which the productive capacity of this professional learning community may evolve.

First, it could be that Dan and Naomi's personal, pedagogical beliefs are that mathematics is best taught and learnt through knowledge transmission, a noun view. In turn, this influences not only how they use their pedagogical content knowledge in the classroom but also how the curriculum is structured. Naomi and Dan's pedagogy is influenced by their teaching life story. That is, teachers' systems of beliefs about how knowledge is constructed "represents implicit assumptions about curriculum, schooling, students, teaching and …act as cognitive and affective filters though which new knowledge and experience is interpreted and enacted" (Handal & Herrington, 2003, p. 59). However, Dan and Naomi's belief systems about how mathematics is taught and learnt exist in a reflexive relationship with the broader educational domain in which they work. The analysis of the data highlighted that it is a combination of their personal belief systems and the socio-cultural discourse that places them as passive recipients of education policy that contributes to rendering their pedagogical content knowledge immobile through the convergent, noun view of teaching mathematics.

Naomi and Dan's pedagogical disposition is also "shaped by standards of curriculum and assessment, and policies for hiring and promotion" (Bolam, et al., 2005, p. 25). As discussed earlier, the productiveness of teachers' pedagogical disposition appears to be critically weakened by the profusion of externally mandated policy reform initiatives. The effectiveness of the professional learning community is challenged since the "amount of policy orientated change is significant, and such change has been seen as placing demands on the learning capacity of the organisation" (Bolam, et al., 2005, p. 25). Consequently, there is a disparity between the intended curriculum, the implemented curriculum and the attained curriculum (Cuban, 1993 in Handal & Herrington, 2003). Where the intended curriculum is what is presented by curricularists, the implemented curriculum is what teachers do in the classroom and the attained curriculum is what is learnt by the students (Handal & Herrington, 2003). Therefore,

> if mathematics teachers' beliefs are not congruent with the beliefs underpinning an education reform, then the aftermath of such a mismatch can affect the degree of success of the innovation as well as the teachers' morale and willingness to implement further innovation.
> (Handal & Herrington, 2003, p. 60)

MacDonald's (2003, p. 1) metaphor of the chookhouse may be an unfortunate yet appropriate representation of how the intended curriculum intersects with the implemented curriculum:

> With no disrespect to educators or teachers in the schools, or to the curriculum theorists who informed the innovation, it seemed that…curriculum innovation... being lobbed onto schools, whereupon the principal, that is the rooster, and teachers, that is the chickens, went into a flurry of activity. However, like the modernist schooling system in which entrenched knowledge and practices often override the innovative ideal (Eisner 2000), the chookhouse quickly returned to its normal routine.

So as discussed by Fullan (1983, cited in Handal & Herrington, 2003, p. 62), "change" ends up being "cosmetic" whereby the teacher "uses new resources" or might modify "teaching practices, without accepting internally the beliefs and principles underlying the reform". Certainly, the criteria used on the tests and assignments and the questions on these assessment instruments are an example of how teachers modified their practice in cosmetic ways since their classroom practices did not align with what they were expecting their students to be able to do.

An effective professional learning community is one catalyst in promoting positive learning outcomes for students who are at the core of the educational delivery system. Part of this process is creating the space for teachers to reflect on their change agency. So perhaps the focus of the top down initiatives could be channelled towards teachers developing their capacity for change in a professional learning community. This might see a paradigm shift from teachers trying to keep up with the latest policy reform towards sharing and engaging in professional conversations. Meaningful dialogue between teachers at Amethyst College about how they can use their pedagogical content knowledge to move beyond teaching mathematical content and skills for the test into participating in the broader goals of what sustainability means in mathematics education might be a pivotal starting point in the development of their professional learning community.

It is important to make the point though, that Queensland teachers have been encouraged to work in a collaborative partnership within curriculum reform packages (Kirk & MacDonald, 2001). Teachers do have opportunities to participate in all stages of syllabus construction. However, this may have also evolved in a cosmetic way at Amethyst College. Since, the pseudo-participation in curriculum construction becomes far removed from the broader spectrum of teachers mutually interacting in the construction of the syllabus guidelines to ultimately improve learning outcomes for students. Kirk & MacDonald (2001, p. 555) discuss the difference between mutual engagement and participation of teachers in curricular reforms:

> If teachers are to be partners in the reform process and to have ownership of reforms, it may be important that they have opportunities to be agents within the recontextualising field, involved in the production of instructional discourse, as

> well as agents in the secondary field charged with receiving and delivering instructional discourse… [However,] powerful institutional forces, as well as the structure of the projects themselves, prevented teachers working as recontextualising agents.

Therefore, for teachers to participate as change agents within the "recontextualising field" (Kirk & MacDonald, 2001, p. 555), top down initiatives should perhaps be directed away from creating the curriculum reform for teachers to implement towards supporting teachers in the development of effective professional learning communities that might think critically about productive curriculum reforms. Moreover, a prerequisite for teachers to implement changes in their classroom practice for the benefit of student learning is a profound understanding of why curriculum reform is necessary.

7.4.4 Using reflective intelligence to build the capacity of a professional learning community

Creating, developing and maintaining the capacity of a professional learning community depends on the commitment of the teachers, the school administration, as well as external agents. Teachers developing their capacity for "inquiry mindedness" (Bolam, et al., 2005, p. 14) is an avenue that can build their change agency. This involves the "active deconstruction of knowledge" of how they think students learn mathematics "through reflection and analysis, and its reconstruction through action" in their classrooms (Bolam, et al., 2005, p. 14). This might then progress through to a collaborative effort with colleagues. The focus is on interactively constituting new ways of thinking about classroom practices. External agency that supports schools in this way might assist in the development of the core element of "reflective intelligence" so that the professional learning community "develops independence, the capacity to learn and apply learning more effectively over time" (Bolam, et al., 2005, p. 20). In this way, the professional learning community evolves productively through functions that can “mature into an accepted, iterative process of data collection, analysis, reflection and change” (McLaughlin and Talbert, 2001, in Bolam, et al., 2005, p. 13). However, as suggested by Little (2003), the norms of practice at Amethyst College see the “force of tradition”, the “habitual ways of thinking” and the “press to simply get on with it” resonate against the “impulse to question” (cited in Bolam, et al., 2005, p. 14).

Therefore, top down initiatives are required to promote the development of an effective professional learning community in an effort to assist teachers to:

- Have sustained opportunities to study, to experiment with and to receive helpful feedback on specific innovations.
- Have opportunities to collaborate with professional peers, both within and outside of their schools, along with access to the expertise of researchers and program developers.

(Bolam, et al., 2005, p. 15)

Some examples of practical tools that might be especially beneficial to the Amethyst College context are:

- Action research (McMahon, 1995)
- Action learning (Wallace, 1991)
- Best practice scholarships (DfEE, 2000)
- Professional development bursaries
- Sabbaticals
- Individual learning accounts

(Bolam, et al., 2005, p. 13)

Top down initiatives cannot ignore that time and funding (beyond bidding for grants) are necessary to build the space for teachers to use the practical tools to engage in individual and collective learning. What's more, the socio-cultural norms should foster an understanding that "the job of sustaining a professional learning community is never finished - it will always be ongoing" and consider this as "an optimistic view of change" (Bolam, et al., 2005, p. 142).

Importantly, building the capacity for reflective intelligence is not fixed on techniques, best practice or linear solutions, but rather on continually seeking a better understanding of how to improve the norms of practice in learning communities. Building an effective professional learning community in schools is focused on the educational agenda that continually asks questions about how to become better at making the space for students to gain the intellectual autonomy to shape their mathematical knowledge. Questions such as:

- How do we know?
- How do we learn?
- Under what conditions and with what validity?

(Palmer, 2006, p. 14)

may be used as reference points.

External support directed towards teachers engaging in professional learning communities so that they stay open and hopeful about their students is essential to shifting the deficit views that currently shape the norms of practice. However, as suggested by Watson and Fullan (1992), partnerships between teachers and external support "are not accidental; neither do they arise purely through good will or ad hoc projects...they require new structures, activities and the rethinking of the way each institution operates as well as how they might work as part of this partnership" (cited in Bolam, et al., 2005, p. 21).

Certainly, the school administration is an essential mechanism in shaping the evolving professional learning community. The mathematics teachers at

Amethyst College did seem to be at odds with their school administration team. They felt that the conditions of reporting deadlines contributed to their job overload so that the educational agenda became further detached from the focus of improving the mathematical proficiency of students. It has been acknowledged that

> for better or worse, principals set conditions for teacher community by the ways in which they manage school resources, relate to teachers and students, support or inhibit social interaction…If we are serious about building professional learning communities within and between schools then we need forms of leadership that support and nourish meaningful collaboration among teachers.
> (Bolam, et al., 2005, p. 18)

The current administration at Amethyst College did introduce an *early mark* on a Thursday afternoon. This early mark means that the students' school day on a Thursday finishes before lunch so that teachers have a time allocated (1.55pm – 3.15pm) which may be used for collaborative planning. At Amethyst College, teachers might use this time to focus on assessment and reporting since this appeared to be the emphasis of the curriculum planning meeting that I attended. Furthermore, teachers might need to use this time to plan and collaborate about how to attend to the latest reform policy in cosmetic ways, or teachers might use this time to moderate marking criteria on assessment items. It seems as though accompanying each educational reform, comes considerable administrative and bureaucratic responsibility (Bolam, et al., 2005). Therefore teachers might use this collaborative time to get jobs done rather than building their capacity to think reflectively and critically about their classroom practices.

What is interesting is that Naomi did organise a professional development afternoon (on a Thursday) where the primary school teachers from Floating Hill came and presented information about "what primary school teachers do" (Naomi).

So this professional development did involve the idea of sharing practice. However, Naomi did appear to be focused on finding better ways to transmit knowledge so that the teacher might get the student to move from a grade 2 level to a grade 8 level. Thus, regardless of what the primary school teachers from Floating Hill presented, the potential gain for teachers at Amethyst College might be bounded by their classroom practices and their traditionalist ideas about how they can teach students to learn mathematics. Therefore, while time has been allocated towards collaborative planning, collaborative work may need to be consciously directed towards innovations that improve learning outcomes for students. That is, collaborative time might not evolve beyond a contrived collegiality (Hargreaves, 1992) that is focused on the noun view of mathematics unless the socio-cultural norms promote the teachers' dispositions to engage in developing their reflective intelligence. At Amethyst College

> teacher learning seems both enabled and constrained by the ways that the teachers go about their work…the force of tradition and the lure of innovation [were] seen simultaneously and complexly at play in the teachers' everyday talk.
> (Little, 2003, cited in Bolam, et al., 2005, p. 16).

So while administrative support and external support directed towards time and funding are essential, how the time and funding is used is critical. The development of collaborative processes enabling teachers to reform their practice for the positive benefit of students' long term mathematical proficiency is a proactive direction. Naomi talked about needing teachers in the classroom who can "inspire" the students. Perhaps this can be extrapolated so that teachers collaborate to create "new practices that are inspired and energised by their dialogic encounters" (Fielding, et al., 2003 in Bolam, et al., 2005, p. 15). This idea might be reflected in the classroom learning community where the teachers and students inspire one another through classroom interactions that are focused on thinking creatively about different ways of doing and using mathematics, the Great Thing. Moreover, when the Great Thing has an opportunity to speak “for itself, teachers and students are more likely to come into a genuine learning community, a community that does not collapse on the egos of students or teachers” (Palmer, 2006, p. 12).

7.5 Conclusion: shaping the proficiency footprint

A key aim of this research was to understand how students develop productive mathematical dispositions and effective mathematical proficiency by investigating how their intellectual and social autonomy is cultivated within the classroom learning community. Sustaining classroom learning communities that cultivate the processes involved in developing the five strands of mathematical proficiency is a key challenge for the middle school mathematics teacher. Students’ developing productive mathematical dispositions in the middle school is critical to the decision making process regarding their progress with higher level mathematics. This research has highlighted the complexity of this process and this chapter has discussed two key, interrelated ideas that may foster intellectual and social sustainability.

The first recommendation calls for the development of intellectual sustainability. Specifically this refers to a holistic approach across primary school and secondary school (which depends on effective professional learning communities) to develop the essential processes of mathematical abstraction. If the middle school is taken to be the cornerstone that empowers students to pursue higher level mathematics courses, then this cornerstone uses the mortar from the primary school to build robustly defined mathematical proficiencies. Second, the recommendation calling for social sustainability involves collaborative effort directed towards developing the productiveness of professional learning communities to re-shape the norms of the classroom learning communities. These ideas call for a paradigm shift away from teachers and students being told how things should be done into building their capacity to participate by thinking critically and reflectively within their respective learning communities. While students' mathematical proficiency is placed at the nucleus of the educational agenda, teachers are the strong force binding the nucleus. The strength of this force is co-dependent on effective professional and classroom learning communities.

Central to the recommendations is that teachers continually contemplate a collective common sense goal of what effective teaching and learning in mathematics involves. Such a common sense view does not imply constant agreement, but rather intelligent reflection and argument, focused on improving the sustainability of educational practices so that students can develop effective mathematical proficiency. A common sense view recognises a balanced approach to creativity and procedure in classroom practices. Classroom practices focused on mathematics as verb through mathematisation in a culture of participation are vital for students to develop their intellectual autonomy. The idea that all students "must learn to think mathematically, and they must think mathematically to learn" (Kilpatrick, et al., 2001, p. 15) is a motivating common sense goal that could re-configure the classroom norms. Improving the quality of teaching and learning mathematics involves critical arguments about how school mathematics is done.

Underpinning the recommendations, to realign the common sense view of school mathematics is the re-shaping of classroom norms. Classroom learning communities that promote the ongoing refinement of effective socio-mathematical norms are part of this re-configuration. Socio-mathematical norms focused on mathematical difference engage students into gaining authorship of the mathematical ideas to promote their intellectual autonomy. Opportunities for intellectual autonomy shape the learning landscape for students to develop their mathematical proficiency. Classroom learning communities that promote a learning orientation to doing mathematics instead of a performance orientation are proffered as being critical to changing the deficit views that currently drive classroom practices that result in unproductive classroom norms that are detrimental to students' long term mathematical proficiency. Thus, a common sense view of doing mathematics is based on students being mutually engaged in learning better ways to choose and use mathematics.

The pedagogical content knowledge of teachers at Amethyst College is thought of in this research as an optimistic position from which effective change can proceed. Teachers can use their pedagogical content knowledge to build their agency for change. Such agency within an effective professional learning community is focused on sustaining effective classroom learning communities that help students feel proficient in the world of mathematics.

Undeniably, the shaping of students' mathematical proficiency depends on the teaching proficiency of the teacher. Teachers shape mathematical souls by the "shape of [their] knowledge in [their] modes of knowing" (Palmer, 2006, p. 14). This study discusses how an effective professional learning community creates the spaces for teachers to build their change agency, an essential component of teaching proficiency. However, this case study highlighted that:

> in our rush to reform education, we have forgotten a simple truth: reform will never be achieved by renewing appropriations, restructuring schools, rewriting curricula, and revising texts if we continue to demean and dishearten the human resource called the teacher on whom so much depends.
> (Palmer, 1998, p. 3)

Therefore, cultivating teachers' agency to remain open and hopeful about their students' mathematical proficiency depends upon support directed towards developing the vitality of a creative culture of change within professional learning communities.

If Australia's proficiency footprint is to be successfully re-shaped with mathematically proficient generations, then efforts need to be directed toward proficiency that is intellectually and socially sustainable. Social sustainability requires support aimed at developing effective professional learning communities that can effectuate meaningful changes to classroom practices. The effective functioning of learning communities depends upon the constituent members being mutually engaged in the actions of learning. For teachers, this involves action directed towards refining their pedagogical practice so that the norms of the classroom learning community model what mathematicians do. Students doing what mathematicians do through mathematisation builds their intellectual autonomy for mathematical proficiency. This type of mathematical proficiency promotes the intellectual sustainability required for students to successfully pursue higher level mathematics courses and careers. In essence, re-defining the contours of the proficiency footprint depends upon acknowledging and refining the reflexivity between social and intellectual sustainability in the middle school mathematics classroom.

Australia's global well being relies upon excellence in mathematics education. This case study emphasised that excellence in mathematics education involves teachers thinking about how they can use their pedagogical content knowledge in new ways, to establish new norms, so that students become better at thinking mathematically. As suggested by Aristotle:

> We are what we repeatedly do. "Excellence", then, is not an act
> but a habit.

Effective learning communities served by a creative culture of participation promote genuine opportunities for teachers and students to build habits of excellence. Re-shaping the proficiency footprint ultimately depends on habits that view mathematics as an action, and teachers and students who have the habits of mind to do this.

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Appendix 1

Naomi's Timetable

	Day 1 **Mon**	**Day 2** **Tues**	**Day 3** **Wed**	**Day 4** **Thurs**	**Day 5** **Fri**	**Day 6** **Mon**	**Day 7** **Tues**	**Day 8** **Wed**	**Day 9** **Thurs**	**Day 10** **Fri**
1 Homeform										
2	9MAA				11MAB	11MAC	11MAC			11MAB
3	9MAA	11MAB		12QCS	11MAB	11MAC	11MAB	11MAC	12QCS	
4		11MAB		12MAB	9MAA		11MAB			
5	11MAC		11MAC	12MAB		9MAA		11MAB	12MAB	9MAA
6	11MAC		12MAB					11MAB	11MAC	9MAA
7		9MAA			12MAB		9MAA		11MAC	12MAB
8					12MAB	12MAB	9MAA			12MAB

Appendix 2

Dan's timetable

	Day 1 Mon	**Day 2 Tues**	**Day 3 Wed**	**Day 4 Thurs**	**Day 5 Fri**	**Day 6 Mon**	**Day 7 Tues**	**Day 8 Wed**	**Day 9 Thurs**	**Day 10 Fri**
1 15min Home -form	10	10	10	10	10	10	10	10	10	10
2	12ENG		10 ASSEM	9MAOC	8SPORT			10 ASSEM	9MAOA	9MAOC
3	12ENG	9MAOC	12ENG	9MAOC	8SPORT		9MAOC		9MAOC	10MAO
4		9MAOC	12ENG	8MAAB	12ENG	10SEL	9MAOC	10MAO	10MAO	10MAO
5		9MAOA		8MAAB	10MAO	12ENG	9MAOA	9MAOC	8MAAB	12ENG
6		9MAOA	8MAAB	10MAO	10MAO		9MAOA	9MAOC		12ENG
7	9MAOA	12ENG		10MAO	8MAAB	9MAO	8SPORT			8MAAB
8	9MAOA	10MAO			8MAAB	8MAAB	8SPORT			8MAAB

Appendix 3

Sample Informed Consent Form

PRINCIPAL INVESTIGATOR	**Silvia Dimarco**
PROJECT TITLE:	**Mathematics in the Middle: Shaping the Proficiency Footprint**
SCHOOL	**Education**

I understand that the aim of this research study is to find out how the teacher and students work and learn together in the Grade 9 mathematics classroom. I consent to participate in this project, the details of which have been explained to me, and I have been provided with a written information sheet to keep.
I understand that my participation will involve being observed in the classroom, and interviews. I agree that the researcher may use the results as described in the information sheet.

I acknowledge that:

- any risks and possible effects of participating in the interview have been explained to my satisfaction;
- taking part in this study is voluntary and I am aware that I can stop taking part in it at any time without explanation or prejudice and to withdraw any unprocessed data I have provided;
- that any information I give will be kept strictly confidential and that no names will be used to identify me with this study without my approval.

(Please tick to indicate consent)

I consent to be interviewed	☐	**Yes**	☐	**No**
I consent for the interview to be audio taped	☐	**Yes**	☐	**No**
I consent to complete a questionnaire	☐	**Yes**	☐	**No**
I consent to being observed in the mathematics classroom	☐	**Yes**	☐	**No**

Student Name: *(printed)*	
Signature:	**Date:**

I consent for my child ______________________________to participate in this research study and I understand this involves classroom observations and interviews. I also understand that participation is voluntary.

Parent/ Guardian Name: *(printed)*	
Signature:	**Date:**

Information Sheet

You are invited to take part in a research project to find out how the teacher and students work and learn together in the Grade 9 mathematics classroom. The study is being conducted by Silvia Dimarco and will contribute to the research for a PhD thesis in mathematics education at James Cook University.
If you agree to be involved in the study, you may be invited to be interviewed. The interview, with your consent, will be audio-taped, and should only take approximately 15 minutes of your time. The interview will be conducted at your school. I will also be coming into your mathematics classroom at different intervals during Semester 2, 2009.

Taking part in this study is completely voluntary and you can stop taking part in the study at any time without explanation or prejudice. You may also withdraw any unprocessed data from the study.

There are no risks associated with the study. If you do feel upset or distressed in any way, please advise the researcher and you will be referred to someone who can help you e.g. the school counsellor.

Your responses and contact details will be strictly confidential. The data from the study will be used in research publications. You will not be identified in any way in these publications.

If you have any questions about the study, please contact **Silvia Dimarco or Associate Professor Mary Klein.**

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Appendix 4

Observation Form

DATE:	Teacher	Students
Placement Input		
Community Interaction Who speaks to whom? Who listens? Who is silent? Who makes positive/negative comments? What are some of the non-verbal cues? What doesn't happen? What were the responsibilities of the classroom learning community? How were responsibilities determined?		
Socio-mathematical norms What types of responses were valued in mathematical discussion? How are the mathematical solutions accepted/legitimised? How were mathematical solutions argued about?		
How are links between concepts made…and by whom?		
Evidence of risk taking by students?		
Teacher PCK…using domains CCK SCK KCS KCT		
Student mathematical disposition… during classroom interactions…how did it change?		
How was it evident that teaching and learning were structuring resources for each other in the classroom learning community?		

Appendix 5

Questionnaire

Student Beliefs

1. Answer the following questions:

a) If mathematics was a food, what kind of food would it be and why?

b) If mathematics was a colour, what colour would it be and why?

c) If mathematics was music, what kind of music would it be and why?

2. Write down the two most important things you have learnt in maths during the last Semester.

3. Write down at least one sort of problem which you have continued to find difficult.

4. What would you most like more help with?

5. How do you feel in your maths class at the moment? (Circle the words that apply to you…you may circle more than one word)

Interested	Relaxed	Worried	Successful	Confused
Happy	Bored	Rushed	Clever	

Write down one or more words of your own___________________________

6. What is your biggest worry affecting your work in maths at the moment?

____
____
____
____
____
____

7. How could we improve maths classes?

____
____
____
____
____
____

Thanks for your time. ✵

Reference:
Goos, M., Stillman, G., Vale, C. (2007). *Teaching Secondary School Mathematics.* Crows Nest, NSW: Allen & Unwin