Chapter 7

Complexity and self-organisation
Acknowledgement of Contributions

N.H.S. Oliver – normal supervisory contributions
7.1. **Introduction**

Most of this thesis concerned the study and interpretation of the dynamical evolution of different geological systems. Research questions were focused on the mineral potential of the Lawn Hill Region and on the origin and timing of rare geological events such as the genesis of Century and the Lawn Hill Megabreccia event. Answers to these questions were all derived from recognised patterns, explored qualitatively and quantitatively with the aid of computational tools.

In this conclusive chapter, new theoretical models (chaos, complexity, fractals, attractors, self-organisation, etc.) are combined with information derived from previous sections, to better define the origin of some of these “exclusive patterns”. Introduced theoretical concepts are then used to propose a theoretical discussion on the genesis of Pb-Zn mineral deposits.

7.2. **Complexity and self-organisation**

Earlier chapters focused on different scales and perspectives of the Lawn Hill Region, allowing the identification of several interrelationships within, and among, presented models. Each “universe” (e.g. a geological terrain, a mineral deposit, a deformed structure, a textural style) contains in fact other sub-universes that are often characterised by complex interrelationships different from the larger scale counterpart and with their own emergent properties (Flake, 1998). A system also contains both
stochastic and deterministic components, and can display similar organisation among different scales (self-similarity). In this regard, Flake (1998) distinguishes three main groups of rules that are useful to consider when dealing with complex systems: (1) Collection, Multiplicity and Parallelism; (2) Iteration, Recursion, and Feedback; (3) Adaptation, Learning and Evolution.

7.2.1. Collection, Multiplicity and Parallelism

Complex systems with emergent properties are often highly *Parallel Collections* of similar units. A natural example could be a colony of bees or ants that are numerous groups of equivalent individuals (agents) although they accomplish specialised tasks. *Multiplicity* considered as system duplication in space can be also seen in geological systems. A mineral deposit often comprises intricate, irregular networks of veins that formed ideally following the same ore genetic process. The spatial distribution of mineral grades could be seen as an example of specialisation, because the same type of brine (e.g. within a Pb-Zn system) can interact differently, via feedback relationships with the environment producing different paragenetic styles (e.g. Sedex, MVT and others).
7.2.2. Iteration, Feedback, and Recursion

*Iteration* is similar to multiplicity except that it develops in time rather than in space. A SEDEX-type deposit (e.g. Large et al., 1998; Schardt et al., 2006) is composed of numerous laminae and bands derived from sequential deposition of mixtures of sediment and sulphides. Also the periodicity of hydrothermal pulses can be thought as an example of iteration. In this context, a *Feedback* relationship controls the system evolution in time. The feedback is represented by the interaction of a system with its environment. For instance, if the system is a brine precipitating mineralisation the outcome of this process will be strongly controlled by the environment parameters and by how they evolve in time, because the system can also actively modify the environment. The morphology of a reaction front created by brines diffusing in a host rock will be then strongly dependent on the type of the occurring feedback relationship (e.g. Ciobanu and Cook, 2004). *Recursion* represents more the path that arises from the combination of iteration and feedback relationships. The recursion of systems produces energetically convenient structures (self-similar) or fractal objects in nature. The term *fractal* was firstly introduced by Benoit Mandelbrot (Mandelbrot, 1983; Flake, 1998; Barnett et al., 2005) who used it to describe the geometry of non-Euclidean objects. Fractals are self-similar on multiple scales (e.g. a fern leaf looks like a fern tree) and their dimension is fractional, Fig. 7.1.
Fig. 7.1 Mandelbrot-set and relative enlargement views of smaller details on the edge of the Cardioid. Each enlargement contains more detail and is composed of self-similar patterns that are slightly rotated copies of their neighbourhood. From Mandelbrot (1983).
7.2.3. Adaptation, Learning and Evolution

Adaptation can be seen as the combination of Multiplicity and Iteration. This can be considered equivalent to natural selection in a biological system, in which only the fittest individuals survive within multiple groups of agents and they reproduce (iteration) leading to adaptation of the whole collection. However, the concept can be extended to a geological context using adaptation to qualify how a mineral deposit evolves in response to changes in pressure and temperature or other thermodynamic variables. Reciprocal feedback type relationships in space (multiplicity) and time (recursion) control these modifications. In summary, a system is usually governed by simple rules repeated in time and space and its evolution is strongly influenced by feedback type relationships. These simple rules are, for instance, represented by fractals.

7.2.4. Fractals, attractors and self-organisation

Recent theories, derived from experimental mathematics (e.g. the Mandelbrot-set in the 80s), offer a description of this latter concept defined symbolically by a power law relationship of the following type:

\[ N_i \approx r_i^{-D} \]  

(7.1),
where $N_i$ is the number of objects of size $r_i$ and $D$ is the fractal dimension, a scaling exponent introduced by Mandelbrot (1967; 1983). The equation (7.1) known as fractal scaling relation seems to hold for many natural systems (Turcotte and Rundle, 2002), providing a mathematical explanation for at least some of the patterning observed in nature. Probably the most striking geological example of a complex system reducible to a fractal object is the spatial distribution of faults in the lithosphere (e.g. Sornette, 1991; 1999). Fractals and self-similarity are related to the concept of self-organisation. The idea of an Earth’s crust self-organised to minimise the energy in response to plate tectonic motion could be well explained by classic thermodynamics, but the use of self-organised-criticality links this re-equilibration process to the formation of observable and non-observable (e.g. chemical) patterns (e.g. Grasso and Sornette, 1998). In other words, natural systems in certain conditions tend to reorder “spontaneously” forming fractals. This system behaviour was firstly recognised in classical experiments in chemistry such as the Belousov-Zhabotinski reaction or the Bénard cell. In the latter an external constraint is imposed to the system causing its reorganisation in convective cells (Nicolis, 1995; Haken, 2004). These experiments led to the idea that in a certain limiting condition a system breaks its symmetry spontaneously (Bifurcation, Fig. 7.2), and demonstrated intuitively that disorder can be chaotic (deterministic). The deterministic component can potentially be mapped also when dealing with more complex cases (e.g. it might hold also for the patterning of faults in the crust). For example, there is interest in constructing more refined models based on self-organisation that could predict the future location of earthquakes (e.g. Sornette, 1999).
However, the validity of these models has not yet been demonstrated convincingly, probably because of the incompleteness of information in the geological applications. The problem is also apparent in other fields such as Economics, where a significant wealth of data exists (e.g. the variability of time series for the stock market), and the application of chaos and self-organisation seems to be predictive only for a limited extent (on a short-time scale).

Fig. 7. 2 Bifurcation diagram representing a change from a single state of equilibrium to two stable phases. Continuous line represents stable conditions, dashed line represents an unstable condition.

The initial considerations made regarding deterministic and stochastic processes (Chapter 2) helped their distinction, but here chaos and self-organisation provide more richness to this “binary” subdivision. This makes it difficult to define the proportion of a stochastic component in a system. In other words, measuring the randomness of a system does not provide indications relative to its statistical nature, because a deterministic process can generate indistinguishable random distributions as well. In this regard this concept is explained referring to the example of the *Logistic map* (a univariate system):
\[ x_{t+1} = 4rx_t(1-x_t) \]  \hspace{1cm} (7.2),

where \( r \) is a parameter that can be set to reflect the reproduction rate in time \( t \) of a generic population, and four is a numeric constant used to constrain the interval of variation of \( x_t \) between \([0, 1]\). Flake (1998) illustrates several examples of how the logistic function behaves for specific values of \( r \) (Fig. 7.3). The interesting point is that this deterministic relationship assumes a chaotic (nonlinear) variation for a value \( (r = 1) \) that approximates a random statistical distribution although yet fully deterministic.

Other examples include in higher dimensional space the Henan map (a bivariate system) or the Lorenz attractor (a multivariate systems). These mathematical experiments based on iterative mapping provide the formal demonstration of how a simple linear process can evolve into nonlinear chaos. A good approach to handle the more complex cases (in multivariate examples) is to represent their variation in a phase space, which is a representation of the evolution of the considered system variables in time, as illustrated in Fig. 7.4, for the Lorenz attractor. An attractor is a representation of the whole time trajectory of the system in phase space (Nicolis, 1995). Usually natural systems are dissipative therefore they can be approximated to quasi periodic motion (Barnett et al., 2005). In such cases the attractor often has an irregular shape and is therefore defined as a Strange Attractor. Phase space representations are useful because the attractor may have a fractal dimension suggesting that the system evolves following specific iterative rules. Problem envisioning in phase space allows then to
identify at least the possible deterministic component of a system outlining the existence of a chaotic motion. However, pattern formation and consequent ordering of a certain structure can arise from either a stochastic or a deterministic process (i.e. existence of stochastic and deterministic fractals). The interpretation of the significance of an attractor in the more complex examples is also aggravated by a remaining component of random noise (Chatterjee and Yilmaz, 1992). Identification and iterative mapping of patterns or bifurcation is possible, but the system selection of relative states of equilibrium remains governed by chance (e.g. coin tossing in Chapter 2; Haken, 2004). Despite the limitations, if a component of order is recognised in a system and a deterministic model is suitable to map its evolution, then prediction is possible in the simplest cases (few variables) or at least in a short-timeframe (e.g. the successful prediction of the path of hurricanes on the order of days; Flake, 1998). Beside predictability the identification of a fractal component in a system can be used to better understand the origin of certain patterns, and how the system converges to, or diverges from a certain condition.
Fig. 7.3 Plots of logistic function for different values of \(r\). (a) Logistic map with \(r=7/10\). The time series stabilised to a constant value \(x(t)\). (b) Logistic map with \(r=8/10\). The time series evolves into a periodic two limits cycle. (c) Case similar to b, with \(r=88/10\), except that the period has four limits per cycle. (d) Logistic map with \(r=1\). This latter example has a chaotic time series approximating noise (adapted from Flake, 1998).

### 7.3. Practical applications to the Lawn Hill Region

In this final section we discuss the application of introduced concepts on self-organisation and chaos in complex systems. The objective is to use some of the results obtained in previous chapters to propose a theoretical discussion that focuses on the genesis of Pb-Zn mineral deposits.
Fig. 7.4 Perspective views of the Lorenz attractor in phase space. (a) Stereoscopic diagram of the strange attractor. To view simply cross your eyes until the image overlaps then refocus. (b) Sectional view in $XY$-plane. (c) Sectional view in $YZ$-plane (adapted from Flake, 1998).
7.3.1. The fractal nature of faulting and mineralisation

Assuming that self-organisation criticality is a valid model to describe the deformation of the crust (Grasso and Sornette, 1998), by analogy it could be extended to the fault patterns observed in the Lawn Hill Region. The multi-scale analysis (Chapters 3, 4, 5) has outlined a component of self-similarity among structures that would favour this interpretation (i.e. observed self-similarity in the fault patterns at regional to deposit scale). The advantage of working with a geological system is that often system transformations are partial allowing evaluation of multiple states of evolution. The information gained is sufficient to understand the variables that effectively lead to macroscopic fractal geometries. Other techniques can also be helpful in defining non-morphological fractal properties of the system, because self-organised patterns can also form during chemical reactions (Ciobanu and Cook, 2004). In the Lawn Hill case a component of structural self-similarity among faults has been documented at different spatial and temporal scales (Chapter 3 and Chapter 6). Self-similar ordering is also evident in the base metals that have close spatial association to faults as demonstrated in Chapter 3 (correlation between prospects/deposits and faults), but is also valid for a component of base metals at Century (as illustrated by the 3D reconstructions of Chapter 4 and 5). These patterns suggest that similar fractal functions controlled the spatial and temporal organisation of faulting and fracturing and vein-style mineralisation at Century as well as in veins/lodes and prospects surrounding the giant
ore deposit. The following equations could be defined to describe the spatial association between faulting and mineralisation:

\[ \nabla M = \nabla P_s \] (7.3),

for a system in equilibrium. The (7.3) becomes in a time series equal to

\[ \nabla \dot{M} = \nabla \dot{P}_s \] (7.4).

In equation 7.3 \( \nabla M \) is the spatial distribution of mineralisation (\( M \)) and \( \nabla P_s \) is the permeability structure (\( P \)). In equation 7.4 the quantities expressed are the respective rate of variation in space and time of \( M \) and \( P \). The fractal component in space of epigenetic mineralisation can be also inferred simply considering the geometrical branching of Pb-Zn mineralised veins because their morphology mimics the bifurcation diagrams of (Fig. 7.2 Nicolis, 1995; Flake, 1998) and approximate a power law size distribution (Mandelbrot, 1967). \( \nabla P_s \) is not only controlled by mechanical processes such as brittle faulting and fracturing of rocks. There is also a chemical reaction and transport component that regulates both \( \nabla P_s \) and \( \nabla M \). Further complication is also introduced by an “alternative” component of \( \nabla M \) which is independent of \( \nabla P_s \). Notice that it may be ordered as well, but its degree of ordering is governed by a different iterative (stochastic or deterministic) process. For example, in an exhalative
Pb-Zn system a purely exhalative component which forms at seawater/sediment interface would be defined by a relationship of this type:

\[ \nabla M \neq \nabla P_s \]  

(7.5),

involving possible self-organisation of mineralisation during early deposition of sulphides (e.g. the chemical oscillatory behaviour induced by pycocline fluctuations, Chapter 3). The periodicity and patterning is however unrelated to \( \nabla P_s \). However, for a stringer zone where replacement fronts form in response to lateral diffusion of Pb-Zn brines, \( \nabla P_s \) will be controlling the spatial distribution of mineralisation (eq. 7.3).

### 7.3.2. The mineral system attractor

The dynamic evolution of a mineral system (eq. 7.4) could be envisaged using a strange attractor, which reflects the quasi periodic nature of this dissipative system, meaning that each modification will contribute to a loss of energy or matter (Chatterjee and Yilmaz, 1992). In a Pb-Zn mineral deposit, sulphides can be (re)mobilised or dispersed (Chapter 5, Fig. 7.5a). There is then the possibility to deplete or enrich (upgrade) the initial base metal reservoir introducing epigenetic mineralisation. However, from the definition of a dissipative system a loss of matter/energy is required. The process is invariant working for any scale of transformation (small or large sub-systems, Fig. 7.5b). Each transformation causes a loss of matter or energy within the
considered sub-system. The process is then self-similar. A practical example that fits a Pb-Zn scenario could be the evolution of a sedimentary basin during compaction and diagenesis. To compact a basin, the expulsion of basinal fluids is required, and also dissolution of certain mineralogical species (e.g. quartz, carbonates etc.) occurs involving the transfer of solutes in relatively empty domains such as pores, cavities, fractures or faults. In this regard, the chemical and physical transformations reflect the general tendency of energy to equally distribute within all available states (Chapter 2). A small scale transformation will cause a local redistribution of matter/energy in its proximal neighbourhood, for example, a fracture filled by hydrothermal minerals. Following this consideration rather than an internal process generating local permeability enhancement, perhaps an external process involving large scale solution/precipitation reactions would be more appropriate to incorporate base metals in a dissipative system (Fig. 7.5c, d). For example, if a local transformation produces a loss of volume (e.g. hydrocarbon maturation producing low-pH solvents may dissolve silica in shales creating secondary porosity) the neighbourhood will react to equilibrate such permeability imbalance. Therefore, to introduce epigenetic mineralisation, a replacive process would be energetically convenient, because it does not require a large change of volume to introduce base metals within the system.
Fig. 7.5 Schematic diagrams representing the basic rules that constrain the mineral system evolution. (a) Possible transformations in a mineral deposit involving chemical and mechanical redistribution of sulphides. The processes are mainly Remobilisation (R), Dispersion (D) and Mobilisation (M). (b) Example of transfer of energy/matter from a small- into a larger-scale system (environment) containing the dissipative sub-system. (c) Similar case applied to a vein sealed by hydrothermal minerals. (d) Same dissipative behaviour although in space. Arrows outline the imbalance corresponding with dissipative behaviour and relative non-conservativity, expressed by the integration in a closed cycle.
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The mineral system attractor converges toward a specific condition leading the spatial distribution of mineralisation to gain a fractal component that is similar to the fractal component present in the evolving permeability structure (e.g. newly developed fractures tend to be spontaneously filled by mineralised veins). Clearly a number of exceptions apply to this example (i.e. a hydrothermal mineralising fluid must be present, veins may be barren). However, any transformation involving mineralisation either remobilised or externally introduced will work according to the following:

$$
 \left( \nabla M_{\text{syn}} + \nabla M_{\text{epi}} \right) \rightarrow \nabla P_s $$

(7.6).

Subscripts (syn, epi) refer respectively to syngenetic and epigenetic mineralisation. Notice that equation 7.6 represents (in case of Pb-Zn deposits) either a SEDEX or an MVT deposit. Overprinting affects the syngenetic component of the mineralisation where the permeability variation is more intense such as portions of the mineral deposit intersected by faults and subject to new hydrothermal pulses.

Two considerations are thus made using the theoretical models on self-organisation and accepting a fractal component for the mineral system, i.e. replacive processes are favoured, and the system tends during its evolution to converge to an epigenetic style. These have implications for the understanding of the genesis of Pb-Zn mineralisation. In a SEDEX-type ore there should be evidence of both styles of
mineralisation (dependent and independent from $\nabla P$, equations 7.3 and 7.5). Exclusively syntectonic or late-diagenetic ores represent the epigenetic end-member that would be equivalent to the final stages of the trajectory in phase space of an attractor representing the evolution of a SEDEX system (eq. 7.4), because mineralising brines get access to the host as a direct function of $\nabla P$. In agreement with intuitive interpretations provided in Chapter 5, I conclude that the genesis of shale-hosted and relatively low-grade metamorphosed or unmetamorphosed Pb-Zn deposits found in the Mount Isa Inlier might be assessed by evaluating the proportion of mineralisation associated to the permeability structure $\nabla P$, and throughout the identification of the type of self-similar ordering occurring (Fig. 7.6). Understanding and mapping the exclusive fractal distributions in a mineral deposit may provide a chance to understand how and when the mineralisation formed. In this context, mathematical models of reaction fronts that evaluate nonlinear instabilities represent the most direct application of self-organisation in porous media flow (e.g. Chadam et al., 1986; Xin et al., 1993). However, a close understanding (e.g. depending on the mineral system considered) of the parameters controlling the stability of reaction surfaces is required to establish the condition in which instability occurs (i.e. in the form of morphological fingering, scalloping etc., Fig. 7.6).
Fig. 7. Diagram summarising distinctive patterns as a function of spatial location within an exhalative system. (a) Schematic illustration showing the spatial location of different styles of mineralisation as direct function of their distance from the source of hydrothermal Pb-Zn brines. (1) Zone where the mineralisation occurs in laminae deposited with sediment at the seawater/sediment boundary. Mineralisation in this case is unrelated to the permeability structure. (2) Overlapping zone in which either syngenetic or epigenetic mineralisation is present. (3) Feeder where predominantly the sulphides are hosted in veins. (b) Evolution of exhalites as a function of their spatial location and consequent interaction with later hydrothermal fluids that can either add, remove or simply transform the base metals. (c) Relatively undisturbed laminae of sphalerite at Century. (d) Intense modification of sedimentary bedding (arrow) in a reaction front in the Ocna de Fier-Dognecea Orefield, Romania. The latter comparison is a clear example that in certain cases it is possible to use the spatial organisation of minerals and their relative morphological character to infer the timing of mineralisation.