JCU ePrints

This file is part of the following reference:

Young, Ian Robert (1983) *The response of waves to an opposing wind*. PhD thesis, James Cook University.

Access to this file is available from:

http://eprints.jcu.edu.au/12010



1. INTRODUCTION

The obvious causal relationship between wind and surface gravity waves is evident to even the most casual observer. Nonetheless, the science of air-sea interaction remains in its infancy. The first real impetus towards a rational understanding of the processes responsible for the generation of waves was the 1956 review by Ursell (129), whose much quoted opening sentence succinctly summarises the state of the art at that time: "Wind blowing over a water surface generates waves in the water by physical processes which cannot be regarded as known".

In the twenty-seven years which have followed, air-sea interaction has become one of the most intensely studied (both theoretically and experimentally) areas of fluid mechanics. The theoretical predictions of Miles (78, 79, 80, 81, 82) and Phillips (89) have been investigated in the field and the laboratory on numerous occasions. Although these measurements have often proved contradictory, to theory and to each other, the most recent (117) indicate that the combined Miles-Phillips theory is a reasonable model for the energy transfer from the atmosphere to the ocean. In evaluating such experimental results, the considerable difficulties of the task should be realized. As pointed out by Phillips (94): "It is little wonder, then, that the results show considerable scatter - the fact that there is any consistency at all is a tribute to the experimental skill of those involved". This work, together with theories of wave-wave interactions (38, 39, 40), bottom dissipation (65, 43, 110) and a specification of the saturation spectrum (90, 45, 61) have provided a sufficiently reliable specification of the source terms of the Radiative Transfer Equation to obtain reliable numerical wave predictions (120, 136).

The considerable effort which has been applied to the problem of airsea interaction has been largely fueled by man's increasing development of, and reliance upon, the ocean and its resources. The energy crisis of recent years has increased the economic viability of many offshore oil and gas deposits. The development of these resources, together with other offshore mining activities, port facilities and coastal protection works, are all dependent upon accurate wave prediction models.

One aspect of air-sea interaction which has had scant attention is the response of waves to an opposing wind. Such situations are not especially frequent but, in situations where the wind field varies rapidly either temporally or spatially, as in tropical cyclones or in the vicinity of strong frontal activity, they are not uncommon. King and Shemdin (59) measured the directional properties of waves in a number of U.S. hurricanes, finding that many waves propagate as remotely generated swell into areas where they experience adverse winds. Stewart and Teague (124) have measured the rate of decay of waves in an opposing wind, following the passage of a front. Measurements of the actual energy transfer at the air-water interface in opposing winds (26, 117) are available from experiments specifically designed to measure wind wave growth, but these data sets are far too brief to draw any meaningful conclusions.

The object of this research is a laboratory study of opposing air flow over mechanically-generated water waves. A special-purpose wind-wave flume was designed and constructed, in which mechanically-generated water waves propagate against the wind. Measurements of both normal and shear stresses above the water surface have been made with the aid of a wave follower, from which the energy transfer at the air-water interface has been inferred.

2. ENERGY TRANSFER AT THE AIR-WATER INTERFACE

2.1 THE RADIATIVE TRANSFER EQUATION

An examination of wave records from any wind sea will immediately reveal the apparent confusion of the sea surface. The most suitable representation of the sea surface is in terms of a statistical model, current practice utilizing the Gaussian random wave model in which the complex sea state is described in terms of the variance spectral density F(k) of the surface gravity waves in directional wave number space. At each position and time, F represents the superposition of free linear wave components of all wave numbers and from all directions. There are equivalent variance spectral representations in directional frequency space, both in terms of cyclic frequency f and angular frequency $\omega = 2\pi f$, but the F(k) representation leads to the more natural description of spectral evolution.

Assuming F to be a slowly-varying function of position and time, it follows from differential calculus that wave energy conservation may be written as (41)

$\frac{dF(k_x, k_y; x, y, t)}{dt} = \frac{\partial F}{\partial t}$	(temporal accumulation)
$+ \frac{\partial F}{\partial x} \frac{dx}{dt} + \frac{\partial F}{\partial y} \frac{dy}{dt}$	(propagation)
$+\frac{\partial F}{\partial k_{x}} - \frac{dk_{x}}{dt} + \frac{\partial F}{\partial k_{y}} - \frac{dk_{y}}{\partial t}$	(refraction/shoaling)
= $Q(k_x, k_y; x, y, t)$	(source/sink) (2.1)

The terms x and y are orthogonal co-ordinate directions, k_x is the component of the wave number vector in the x direction and k is the component in the y direction. Reading down the page the terms of Equation 2.1 represent the local or temporal accumulation, propagation and combined refraction and shoaling. The right hand side of the equation, indicated by the source

function Q, represents the net transfer of energy to or from or within the spectrum at the wave number \underline{k} due to all interaction processes which affect the component \underline{k} . This equation, known as the Radiative Transfer Equation, formally summarises all the various physical processes that contribute to the evolution of the directional spectrum.

In addition, the kinematics of wave propagation are described by ray theory (66). The wave number \underline{k} and the angular wave frequency $\omega(k,d)$ are also assumed to be slowly varying functions of position and time, related by the conservation of crests equation

$$\frac{\partial \underline{k}}{\partial t} + \nabla \omega = 0 \tag{2.2}$$

and the dispersion relationship, which for linear surface gravity waves in the absence of currents is

$$\omega^2 = gk \tanh kd \tag{2.3}$$

Equation 2.3 describes a time invariant but space dependent medium through the depth $d(\underline{x})$. Equations 2.2 and 2.3 can be manipulated to yield Eulerian time rate of change equations for both k and ω , namely

$$\frac{\mathrm{d}\mathbf{k}}{\mathrm{d}\mathbf{t}} = -\frac{\partial\omega}{\partial\mathrm{d}} \quad \nabla\mathrm{d} \tag{2.4}$$

$$\frac{d\omega}{dt} = 0 \tag{2.5}$$

which have the same characteristic curve

$$\frac{\mathrm{d}x}{\mathrm{d}t} = C_{g} \tag{2.6}$$

where $\underline{C}_{g} = \nabla_{k}\omega$ is the group velocity. Equations 2.4 to 2.6 are the characteristic equations defining the wave orthogonals, again in the absence of currents. Diffraction, involving wave energy transfer normal to wave orthogonals, is a boundary value problem and cannot be accommodated by the present approach, which describes an initial value problem.

Considering together Equations 2.1, 2.4, 2.5 and 2.6, gives

$$\frac{dF}{dt} = Q \tag{2.7}$$

along wave rays defined by Equations 2.4 to 2.6. This relationship was first established in the context of gravity waves by Longuet-Higgins (70) for Q = 0 (no generation, decay or interaction) in which case $F(\underline{k})$ is conserved along the wave rays. Generally however, generation, dissipation and interaction are non-zero and Equation 2.7 describes the evolution of the directional wave number spectrum relative to waves moving along wave orthogonals at the group velocity C_g. In deep water the right hand side of Equation 2.4 is identically zero, C_g is constant for each frequency and the wave rays are straight lines, but this is not generally the case in shallow water where the bathymetry progressively reduces C_g after a small increase and bends or refracts the rays.

Conventional engineering practice considers the directional frequency spectrum $E(f,\theta)$ rather than the directional wave number spectrum $F(k_x,k_y)$, in recognition of Equation 2.5 which specifies the angular frequency ω as a constant along wave rays. These alternative spectral representatives are related through

$$\int_{-\infty}^{\infty} F(k_{x},k_{y}) dk_{x} dk_{y} = \int_{0}^{\infty} F(f,\theta) df d\theta = \sigma^{2}$$
(2.8)

where σ^2 is the variance of the surface gravity waves. Evaluating Equation ².8 yields

$$F(k_{x},k_{y}; x,y,t) = \frac{CC}{2\pi\omega} E(f,\theta; x,y,t)$$
(2.9)

where $C = \omega/k$ is the phase speed. Substituting Equation 2.9 into Equation 2.1 gives

$$\frac{\partial}{\partial t} (CC_{g}E) + C_{g} \cos\theta \frac{\partial}{\partial x} (CC_{g}E) + C_{g} \sin\theta \frac{\partial}{\partial y} (CC_{g}E) + \frac{C_{g}}{c} [\sin\theta \frac{\partial C}{\partial x} - \cos\theta \frac{\partial C}{\partial y}] \frac{\partial}{\partial \theta} (CC_{g}E) = CC_{g}S \qquad (2.10)$$

where $S(f,\theta; x,y,t)$ becomes the forcing (source) term. Likewise Equation 2.7 becomes

$$\frac{d}{dt} (CC_g E) = CC_g S$$
(2.11)

the product CC E being the new action variable which is conserved along wave rays for S = 0. The characteristic equations defining the wave rays now become

$$\frac{dx}{dt} = C \cos\theta \tag{2.12a}$$

$$\frac{dy}{dt} = C \sin\theta \tag{2.12b}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{k} \frac{\partial \omega}{\partial d} \left[\sin \theta \frac{\partial d}{\partial x} - \cos \theta \frac{\partial d}{\partial y} \right]$$
(2.12c)

The well-known Munk and Arthur (85) equations describe the same ray paths with wave fronts propagating at speed C, whereas wave energy propagates at speed C. In deep water, both C and C are independent of depth and constant for each frequency, the wave rays become straight lines and Equation 2.10 reduces to the familiar form

$$\frac{\partial E}{\partial t} + C_g \cos\theta \frac{\partial E}{\partial x} + C_g \sin\theta \frac{\partial E}{\partial y} = S$$
(2.13)

utilized in deep water wave prediction by Gelci et al (35), Pierson, Tick and Baer (96), Barnett (3), Ewing (30) and Cardone, Pierson and Ward (17).

The formulation of the problem is completed by the specification of the source term S. Consistent with linear wave theory, the source term is considered as the summation of a number of separate influences which transfer energy to, from or within the wave field:

$$S(f,\theta) = \sum_{i} S_{i}(f,\theta)$$
(2.14)

Three classes of source terms can be identified, namely atmospheric input, non-linear wave-wave interactions and dissipation (wave breaking or white capping, bottom friction, percolation, bottom motion), i.e.

$$S(f,\theta) = S_A + S_N + S_D$$
(2.15)

These terms will be discussed later in this chapter.

The Radiative Transfer Equation describes an Eulerian convective transport problem, for which analytical solutions are only available under very special conditions. In general it is necessary to resort to discrete numerical solutions. The numerical alternatives have been reviewed by Young and Sobey (136), and by Sobey (118) in another context.

2.2 THEORIES OF AIR-WATER ENERGY EXCHANGE

2.2.1 Potential Theory

The potential theory for gravity waves on a density discontinuity between two fluids was first investigated by Stokes (125). The theory, for the case of a uniform mean velocity U in the upper fluid has been developed in many texts. The development below follows Lamb (65).

The flow in both fluids is considered to be irrotational, incompressible and inviscid. The velocity potential ϕ is introduced so that

$$u = -\frac{\partial \phi}{\partial x}$$
(2.16a)

$$w = -\frac{\partial \phi}{\partial z}$$
(2.16b)

where x and z are the horizontal and vertical coordinates and u and w, horizontal and vertical velocities respectively. The water surface is assumed to be sinusoidal such that

$$n = \operatorname{aexp}\left[i(\omega t - kx)\right] \tag{2.17}$$

where a is the wave amplitude and only the real part of Equation 2.17 is physically realistic. The problem is in essence one of small oscillations about a state of steady motion, the velocity potential being defined as

$$\phi = -\mathbf{U}\mathbf{x} + \phi' \tag{2.18}$$

where by hypothesis φ^{*} is small and given by

$$\phi' = B \exp \left[-kz + i \left(\omega t - kx \right) \right]$$
(2.19)

where B is a constant. Substituting Equations 2.16 into the mass conservation equation yields the Laplace equation

$$\frac{\partial^2 \phi}{\partial \mathbf{x}^2} + \frac{\partial^2 \phi}{\partial \mathbf{y}^2} = 0 \tag{2.20}$$

and the kinematic free surface boundary condition is, to first order in η_{J} ,

$$\begin{bmatrix} \frac{\partial n}{\partial t} + \underline{U} \frac{\partial n}{\partial x} = \frac{\partial \phi'}{\partial z} \end{bmatrix}$$

$$z = 0$$
(2.21)

Substituting Equations 2.17 and 2.19 into 2.21 gives

$$B = i a(C-U)$$
 (2.22)

where $\mathbb{C}=\omega/k$ is the phase speed of the waves. From Equation 2.22, the velocity potential becomes

$$\phi = -\mathbf{U}\mathbf{x} + i\mathbf{a}(\mathbf{C}-\mathbf{U}) \exp\left[-\mathbf{k}\mathbf{z} + i(\mathbf{\omega}\mathbf{t}-\mathbf{k}\mathbf{x})\right]$$
(2.23)

The air pressure is obtained from Bernoulli's equation for unsteady irrotational motion:

$$\frac{p}{\rho_{a}} = \frac{\partial \phi'}{\partial t} - \frac{1}{2} \left\{ \left(U - \frac{\partial \phi'}{\partial x} \right)^{2} + \left(\frac{\partial \phi'}{\partial z} \right)^{2} \right\} - gz + \dots$$

$$= \frac{\partial \phi'}{\partial t} - U \frac{\partial \phi'}{\partial x} - gz + \dots, \qquad (2.24)$$

the terms omitted being either of the second order or not relevant in the present context. Equations 2.23 and 2.24 give

$$\frac{p}{\rho_{a}g} = -a e^{-kz} \left[1 - U/C\right]^{2} \exp\left[i(\omega t - kx)\right] - z$$

and since the pressure is a real quantity

$$\frac{p}{\rho_{a}g} = -a e^{-kz} [1 - U/C]^{2} \cos(\omega t - kx) - z$$
 (2.25)

this result being illustrated in Figure 2.1. The horizontal and vertical velocity components, u and w, can be found from Equations 2.23 and 2.16 as

$$u/C = U/C - ka[1 - U/C]e^{-kz} \cos(\omega t - kx)$$
 (2.26)

and w/C = ka[1 - U/C]
$$e^{-kz}$$
 cos($\omega t - kx + \pi/2$) (2.27)

which are illustrated in Figure 2.2.

The pressure predicted by Equation 2.25 is only the wave-induced component, any background static pressure being neglected. The negative sign indicates that this wave-induced pressure is 180° out of phase with the water surface. In addition, the pressure can be quite large for an opposing wind (U/C negative) due to the term in square brackets. The magnitude of the pressure will also decay quite rapidly with increasing kz, due to the e^{-kz} term. Equations 2.26 and 2.27 predict that the magnitudes of the u and w wave-induced velocities are equal. The phases of u and w are dependent upon the magnitude of U/C. If U/C is less than one, u is in anti-phase with the water surface and w leads the water surface by 90°. Again the velocities decrease in magnitude with increasing kz, at the same rate as the pressure but the increase in the magnitude of the velocities with U/C is slower than for the pressure.

As will be shown later (Section 2.6) potential theory has proved unsuccessful in predicting the pressure or velocity fields above gravity waves in a following wind. Does it have any relevance in opposing winds? The major assumptions of the theory are that the flow is irrotational, incompressible and inviscid. The assumption that air is an incompressible fluid is made in many areas of fluid mechanics and at the velocities being considered (< 10 ms⁻¹) seems quite reasonable. The assumptions of irrotational flow and an inviscid fluid are related. Irrotationality requires that the vorticity be zero. That is, for two dimensional flow

$$\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} = 0 \tag{2.28}$$

Rotation may be caused by viscous forces, but a rotational solution also exists for a perfect fluid, and irrotational flows exist in a viscous fluid (67) and hence the existence of viscosity is not a sufficient condition to assure rotational flow. Whether a motion is physically rotational or irrotational however is evident from a consideration of friction effects. Near a fixed boundary, along which particle velocities are zero, and hence, a velocity gradient exists, particles on adjacent paths will have significantly different velocities (see Figure 2.3). A line joining at one time two particles on the same path will rotate much less than that of a line joining two particles on adjacent paths. The difference in direction of the friction forces acting on the opposite sides of the particle, causes a torque resulting in a net rotation.

In general, the motion can be assumed to be almost irrotational when the velocity gradients are small, when streamlines converge rapidly, and when the velocity distribution depends on the shape of the boundaries and not their roughness. Thus, within the boundary layer the flow must be rotational. A flow may be considered almost irrotational only if the boundary layer is of little importance, i.e. relatively thin. In the present problem interest is concentrated within the surface boundary layer and it would be expected that the predictions of potential flow theory would be in error. The degree of error, however, will depend upon just what effect the boundary layer has on the wave-induced pressure and velocities.

2.2.2 The Kelvin-Helmholtz Instability

The first theory to consider the possibility of an instability on a density discontinuity between two fluids moving relative to each other was developed by Helmholtz (47) and later extended to include the effects of surface tension by Lord Kelvin (56). This solution is known as the

Kelvin-Helmholtz Instability" and was the earliest attempt at predicting the growth of sea waves.

Both fluids are considered to be inviscid with an air boundary layer of negligible thickness compared with the wave height. The velocity U_a of the air is uniform, ρ_a is the air density and ρ_w is the water density; surface tension T is retained in the analysis. The water surface elevation is assumed to be of the form given by Equation 2.17. Solving for the phase speed C gives

$$C = \frac{\rho_{a} U_{a}}{(\rho_{a} + \rho_{w})} \pm \left\{ \frac{g}{k} \frac{(\rho_{w} - \rho_{a})}{(\rho_{w} + \rho_{a})} + \frac{T k}{(\rho_{w} + \rho_{a})} - \frac{\rho_{w} \rho_{a} U^{2}}{(\rho_{w} + \rho_{a})^{2}} \right\}$$
(2.29)

where C is, in general, complex. In the absence of mean flow, U_a , and neglecting surface tension, this result reduces to the expression for the interfacial wave speed

$$c_{I}^{2} = \frac{g}{k} \frac{(\rho_{w} - \rho_{a})}{(\rho_{w} + \rho_{a})}$$
(2.30)

With U_a non-zero however, the disturbances move relative to the weighted mean air flow speed $\overline{U} = \rho_a U_a / (\rho_a + \rho_w)$ with speed ± S, where

$$S^{2} = C_{I}^{2} - \frac{\rho_{a} \rho_{w} U^{2}}{(\rho_{a} + \rho_{w})^{2}} , \qquad (2.31)$$

provided $S^2 > 0$. If $S^2 < 0$, C is complex and the disturbances grow exponentially but remain stationary with respect to \overline{U} . Thus the flow is stable or unstable according to whether $S^2 > 0$ or $S^2 < 0$. When $S^2 = 0$, the transitional case, the flow is marginally or neutrally stable.

As a consequence of surface tension, water waves have a minimum velocity C_m (see Lamb (65), Section 267) and there is a range of wind speeds which do not generate waves. Substituting C_m into Equation 2.29 yields approximately 6.5 ms⁻¹ as the minimum wind speed at which this mechanism can be effective. Since naturally occurring wind generated waves are initiated at much lower speeds and in addition the assumption of a very thin interfacial boundary layer is unrealistic, this mechanism is not thought to be very effective at the wind speeds and wavelengths commonly observed at sea.

2.2.3 Jeffreys' Sheltering Theory

Jeffreys (52) assumed that the energy transfer between the wind and the water was exclusively form drag, associated with flow separation. It was assumed that flow separation occurred on the leeward side of the wave crests with re-attachment somewhere further down on the leeward slopes of the wave. Tangential stresses were completely neglected and the normal pressures were assumed to be solely responsible for wave growth. In a progressive wave train, the rate of working (or energy flux) by the atmosoheric pressure distribution is (94)

$$\frac{\partial E}{\partial t} = -p \frac{\partial n}{\partial t}$$
(2.32)

Thus, it is the component of pressure in quadrature with the water surface which does the work. Due to the type of flow separation assumed by Jeffreys, this component of pressure is positive and hence the energy flux represented by Equation 2.32 will also be positive. Based upon dimensional arguments, Jeffreys assumed that the pressure can be represented by

$$p = S \rho_a (U_{\infty} - C)^2 \frac{\partial \eta}{\partial x}$$
(2.33)

where the constant of proportionality, S, is called the sheltering coefficient. Considering Equations 2.32 and 2.33 for $U_m/C > 1$ yields

$$\frac{\partial E}{\partial t} = \frac{1}{2} S \rho_a (U_{\infty} - C)^2 k^2 a^2 C \qquad (2.34)$$

and, as shown by Kinsman (60), for U_/C < 1

$$\frac{\partial E}{\partial t} = -\frac{1}{2} S \rho_a (U_{\infty} - C)^2 k^2 a^2 C$$
 (2.35)

which indicates that the waves are doing work on the wind. Equations 2.34 and 2.35 predict exponential rates of wave energy growth and decay respectively.

The sheltering coefficient S was calculated by determining the rate of energy loss due to molecular viscosity and the least wind U_{min} that can maintain waves against this loss, and then comparing this with observed least winds which seemed just capable of generating waves. The calculated value of S varies as $(U_{min})^3$. The choice of U_{min} is thus critical; Jeffreys chose 1.1 m/s, giving an S of 0.3. Experiments of air flow over solid wave models (122), however, appeared to give values of the pressure difference much smaller than Jeffreys required to account for the observed rate of wave growth and the 'sheltering hypothesis' fell into disrepute. It is now recognised that these experiments were almost irrelevant to the problem of wave generation, and more recent research by Banner and Melville (2) indicates that air flow separation only occurs with the onset of wave breaking. The type of air flow separation mechanism postulated by Jeffreys cannot be responsible for wave growth.

2.2.4 The Miles-Phillips Theory

As mentioned in the preceding sections, the central question in wind wave generation concerns the distribution of stress on the water surface under the action of wind. The surface stress can be resolved into normal stresses and shear stresses. In addition, these stresses can be of two kinds: those produced by turbulent eddies in the wind and those produced by the air flow over the wavy water surface. These two forms of surface stress were considered independently by Phillips (89) and Miles (78). In the following review, rather than follow their original derivations, the approach of Phillips (94) is adopted.

Water surface normal stresses σ and shear stresses τ may be written as

$$\sigma = \overline{\sigma} + \widetilde{\sigma} + \sigma' \tag{2.36}$$

and
$$\tau = \tau + \tilde{\tau} + \tau'$$
 (2.37)

respectively, where $\overline{\sigma}$ and $\overline{\tau}$ are the time-averaged values representing the mean flow, $\tilde{\sigma}$ and $\tilde{\tau}$ are the wave-induced stresses caused by air flow over the wavy surface and σ' and τ' represent the atmospheric turbulence. The

various components of Equations 2.36 and 2.37 are easily separable in Fourier space. If the wave field near wave number <u>k</u> is defined by $dA(\underline{k}) \exp[i\underline{k} \cdot (\underline{x} - \underline{Ct})]$, the induced stress components near the same wave number can be represented by

$$d\tilde{\sigma}(\underline{k}) = (v_1 + i\mu_1) \rho_w c^2 k dA(\underline{k})$$
(2.38)

and
$$d\tilde{T}(\underline{k}) = (v_2 + i\mu_2) \rho_{\mu} C^2 k dA(\underline{k})$$
 (2.39)

where v_1 , v_2 , μ_1 and μ_2 are coupling coefficients. The total surface stress is the sum of the directly induced variation, Equations 2.38 and 2.39, together with the random contributions from the atmospheric turbulence. The wave-induced components provide a selective energy input to the wave component at wave number \underline{k} , whilst the turbulence provides a contribution over a wide spectral range.

It has been shown by Longuet-Higgins (71) that a fluctuating tangential stress applied at the free surface is dynamically equivalent to a normal pressure fluctuation of the same magnitude, lagging $\pi/2$ in phase behind the tangential stress; a tangential stress in phase with the wave elevation is equivalent to a pressure in phase with the wave slope. Hence, the effective Fourier component of pressure at the water surface is

$$p = (v + i\mu) \rho_{\nu} C^{2} k dA(k) + dp'$$
(2.40)

where $v = v_1 - \mu_2$, $\mu = \mu_1 + v_2$, $dp' = d\sigma' + i d\tau'$ and in practice the nett coupling coefficients are both small, |v|, $|\mu| << 1$. The linearised gravity wave equations for the wave number component become

$$d\ddot{A}(\underline{k},t) + N^2 dA(\underline{k},t) = -\frac{k}{\rho_w} dp'(\underline{k},t) \qquad (2.41)$$

where the complex frequency N = $\omega(1 + i\mu/2)$ and the dots imply differentiation with respect to time. Subject to quiescent initial conditions, Equation 2.41 solves (94) to

$$F(\underline{k},t) = \frac{\pi \Pi(\underline{k},\omega)}{\rho_{w}^{2} C^{2}} \left[\frac{\sinh \mu \omega t}{\mu \omega}\right]$$
(2.42)

where $\Pi(\underline{k},\omega)$ is the wave number-frequency spectrum of the turbulent atmospheric pressure at the water surface (i.e. p'). To this approximation, the wave frequency (the real part of N) remains unchanged by the coupling with the wind, whereas the imaginary part of N, which is proportional to the component of normal stress in phase with the wave slope, determines the development with time. Thus it is the normal stress in phase with the wave slope or the tangential stress in phase with the water surface which supplies energy to the moving wave.

Equation 2.42 reduces to a simpler form when the wind duration t is either small or large compared to $1/\mu\omega$. These two conditions correspond to different primary mechanisms of wave growth. For small time (t << $1/\mu\omega$), the square bracketed term in Equation 2.42 becomes t and Equation 2.42 becomes

$$F(\underline{k},t) = \frac{\pi \Pi(\underline{k},\omega)}{\rho_{W}^{2} C^{2}} t$$
(2.43)

describing an initial linear growth in the wave spectrum, as established by Phillips (89). For large time (t >> $1/\mu\omega$), Equation 2.42 becomes

$$F(\underline{k},t) = \frac{\pi \Pi(\underline{k},\omega)}{\rho_{w}^{2} C^{2}} \left[\frac{\exp(\mu \omega t)}{\mu \omega}\right]$$
(2.44)

describing an exponential rate of growth determined by the coupling coefficient μ , as originally formulated by Miles (78).

The linear growth mechanism (Phillips' mechanism) is broadly responsible for initial excitation of the sea and has an important influence on durationlimited seas. This mechanism involves a type of resonance between the free surface waves and the exciting turbulent stress fluctuations. It provides potentially a broad-band input across the complete wave number-frequency spectrum with the resonance condition directing energy to those wave components with a phase speed equal to the convection velocity of the atmospheric turbulence component. Numerical evaluation of this mechanism requires knowledge of the $\Pi(k,\omega)$ spectrum, measurements of which will be discussed in Section 2.6. In order to evaluate the exponential growth mechanism, knowledge of the dimensionless coupling coefficient μ is required.

Miles (78) made the first attempts at evaluating μ by assuming the air flow to be quasi-laminar, atmospheric turbulence being neglected except in so far as it determined the basic mean velocity profile. The basis of the solution is the inviscid Orr-Sommerfield equation, from which Miles obtained an approximate solution for μ for a shear flow with a logarithmic velocity distribution. His solution has the form

$$\mu = \frac{\rho_a}{\rho_w} \frac{\pi}{c^2} (\bar{u}_c - c)^2 e^{-kz} c \frac{(d^2\bar{u}/dz^2)}{(d\bar{u}/dz)} c$$
(2.45)

where the subscript c indicates that terms are to be evaluated at the so called critical height where the wind velocity equals the wave phase speed. For an opposing wind, U_{∞}/C is negative, there is no critical layer, $\mu = 0$ and there will be no energy transfer.

In a series of subsequent papers (79, 80, 81, 82), Miles extended his theory to account for the effects of turbulence and viscosity. He concluded that the energy transfer is given by

$$\mathbf{F} = \mathbf{F}_{\mathbf{C}} + \mathbf{F}_{(1)} \tag{2.46}$$

where the energy transfer F_c is identical in form to the prediction of the laminar model. The second term F_ω represents the sum of a vertical integral of the mean product of the vertical velocity and the vorticity and the perturbation in the turbulent shear stress at the air-water interface. F_ω depends on the turbulent Reynolds stresses which are strictly dependent variables. Phillips (93) has made certain simplified closure assumptions, later criticised by Miles (82), and obtained a solution for F_ω . This solution is shown in Figure 2.4 as a function of u_\star/C and for a number of angles α between the wind and waves. As u_\star/C decreases, the height of the critical layer increases, the wind profile curvature decreases and

the laminar model term, F decreases. For u_*/C less than approximately 0.05 the contribution to μ from the critical layer becomes insignificant compared with the contribution from the undulatory turbulent flow near the water surface. The break in the curve reflects this change in the mechanism of generation of the induced surface pressure.

This solution has special significance for opposing winds, where $F_c = 0$ and wave growth (or decay) is determined solely by the undulating turbulent flow over the wave form. The Reynolds stress of the induced air motion extracts momentum and energy from the waves, the rate of loss of wave energy being given by the exponential coupling coefficient μ . Phillips (93) calculations show that for opposing winds

$$\mu = (1.07 \times 10^{-5} / (u_{\perp}/C) + 1.24 \times 10^{-4}) \cos \theta \qquad (2.47)$$

where θ is the angle between the wind and waves ($\theta = 180^{\circ}$ indicates wind and waves in opposition) and u_{*} is the shear velocity. Equation 2.47 is also plotted in Figure 2.4. This predicted decay rate is quite slow, and comparable with the growth rate produced by the undulatory turbulent flow in a following wind.

Subsequent investigators (25, 36) have applied more sophisticated turbulence closure models to the flow over water waves, in an attempt to assess the importance of turbulence in wave growth. As pointed out by Phillips (94), however: "Closure schemes in turbulent shear flow are still rather ad hoc and different methods, which may be reasonably satisfactory in other flows, give very different results when applied to this problem. The situation is not one in which firmly established methods lead to results that one might seek, with some confidence, to verify experimentally. On the contrary, because of sensitivity of the results to the assumptions made, the air flow over waves appears to provide an ideal context to test the theories of turbulent stress generation themselves".

2.3 WAVE-WAVE INTERACTIONS

2.3.1 Resonant Interactions

As gravity waves grow under the influence of atmospheric forcing, the individual waves steepen and the nonlinearity of the governing equations begins to have a major impact on continuing wave growth. The individual Fourier components of the spectrum are no longer independent and interactions among wave components redistribute energy within the spectrum. If the nonlinearity is weak, a perturbation analysis with the wave slope ak as the small parameter will describe the nonlinear effects as small perturbations on linear wave theory. The nonlinear effects will be small unless there is dynamic resonance of some kind among wave components.

For second order resonant interactions among a triad of deep water surface wave components, the conditions

$$k_1 = k_2 + k_3$$
 (2.48)

and
$$\omega_1 = \omega_2 + \omega_3$$
 (2.49)

where $\omega_i = g k_i$ must be satisfied simultaneously. Phillips (91) has shown that there are no nontrivial solutions to these equations, so that resonance cannot occur to this order. For third order resonant interactions among a tetrad of waves, the conditions

 $\underline{k}_{1} \pm \underline{k}_{2} \pm \underline{k}_{3} \pm \underline{k}_{4} = 0$ (2.50)

and $\omega_1 \pm \omega_2 \pm \omega_3 \pm \omega_4 = 0$

must be satisfied. For many of these sign combinations, no solutions are possible, but there do exist solution sets to

$$\underline{k}_1 + \underline{k}_2 = \underline{k}_3 + \underline{k}_4 \tag{2.52}$$

$$\omega_1 + \omega_2 = \omega_3 + \omega_4 \tag{2.53}$$

since such wave-wave interactions occur only at the third approximation, they are not simply weak, but very weak. They nonetheless give rise to a number of interesting and observable phenomena.

The effect of the resonant interactions on the entire energy spectrum was developed by Hasselmann (38, 39, 40). The resultant energy transfer is

$$Q_{N}(\underline{k}_{4}) = \int_{-\infty}^{\infty} \frac{9 \pi^{2} D_{4} \omega_{4}}{4 \rho_{W}^{2} (\omega_{1} \omega_{2} \omega_{3} \omega_{4})^{2}} \{ D_{4} \omega_{4} F_{1} F_{2} F_{3} + D_{3} \omega_{3} F_{1} F_{2} F_{4} - D_{2} \omega_{2} F_{1} F_{3} F_{4} - D_{1} \omega_{1} F_{2} F_{3} F_{4} \} \delta(\omega_{1} + \omega_{2} - \omega_{3} - \omega_{4}) \delta(\underline{k}_{1} + \underline{k}_{2} - \underline{k}_{3} - \underline{k}_{4})$$

$$d k_1 d k_2 d k_3$$
 (2.54)

where $F_j = F(k_j)$ and Equation 2.54 represents the net energy transfer to (or from) the wave number component k_4 . The terms D_j are complicated coupling coefficients which are functions of the wave numbers k_1 , k_4 and are given by Hasselmann (38, 39). The Dirac delta functions $\delta()$ suppress energy contributions from all tetrads save those which satisfy the resonance conditions, Equations 2.52 and 2.53. The cubic form of the integral arises since the interaction occurs at the third order. Equation 2.54 can be interpreted in terms of quadruple interactions between three active wave components, which determine the interaction rate, and a passive fourth component, which receives energy from the first three components but has no direct influence on the interaction. Wave-wave interactions of this type are conservative and merely redistribute the energy within the spectrum.

The actual evaluation of the energy transfer represented by Equation 2.54 is extremely complicated. As well as the computational effort involved in evaluating such a six fold integral, the determination of the coupling coefficients D is no simple task since it involves long and taxing algebraic manipulations followed by numerical computations. Due to these complexities it is not surprising that there appear to be inconsistencies in the calculations which have been performed (40, 106, 73, 32). In an attempt to overcome some of these inconsistencies and to provide an adequate data base for the development of a suitable parameterisation of the S_N term for wave prediction models, Hasselmann and Hasselmann (44) have evaluated the integral

for twenty-nine different spectral shapes. Their computations for the mean JONSWAP spectrum along with independent computations by Sell and Hasselmann (106) and Fox (32) are shown in Figure 2.5. The Hasselmann and Hasselmann and Sell and Hasselmann computations were based on the complete integral Equation 2.54 and are similar although neither are smooth. The Fox computations are based on Longuet-Higgins' (73) narrow spectrum approximation and, although, the curve is smooth it is significantly different from the other results.

Despite possible numerical problems which may be responsible for the irregular results, the extensive computations of Hasselmann and Hasselmann (44) provide data from which to determine the characteristic features of the nonlinear transfer. The transfer generally consists of positive lobes at high and low frequencies with a mid-frequency negative lobe in the general region of the spectral peak. The positive energy transfer at high wave numbers leads generally to a directional broadening of the spectrum at higher frequencies. The positive lobe at low frequencies has a narrow directional distribution and leads to a shift of the peak to lower frequencies without appreciable directional broadening. The shape of the peak has a strong influence on the relative position of the two positive lobes and the intermediate negative lobe of the nonlinear transfer. In situations where there is both swell and wind-sea present, there is little interaction between the two, provided they are sufficiently separated in frequency. Thus, when swell experiencies an opposing wind, the locally generated waves would have little effect on the swell due to this form of resonant interaction.

The significance of wave-wave interactions was first established in the JONSWAP (45) study where they were considered as the principal source term, causing a self-stabilising process responsible for the pronounced peak and steep forward face of the spectrum. Phillips (94) has questioned the dominant role assigned to wave-wave interactions in the rapid growth phase for each wave component, arguing that the observed growth of the peak and forward face of the spectrum is inconsistent with the influence Hasselmann ascribes to the S_N term at that stage. The magnitude of wavewave interactions grow with spectral density, becoming more influential at later stages of wave growth and evolution. Phillips denies wave-wave interactions a dominate role in the wind-wave prediction, but nonetheless concedes their significance.

2.3.2 Long Wave-Short Wave Interaction

It has long been realized that short surface gravity waves should have enhanced amplitudes at the crests of long waves, due to the compression of the short waves by the orbital velocity of the long waves, to the working of the long wave rate of strain against the radiation stress of the short waves and to the increased ratio of potential to kinetic energy for the short waves near long wave crests (34). Phillips (93) has shown that energy density is a maximum,

$$E_{\max} = \bar{E} \left[1 + a_{\ell} k_{\ell} \left(1 + \frac{1}{2} \cos^2 \theta \right) \right] , \qquad (2.55)$$

at the crests of the long waves and a minimum,

$$E_{\min} = \bar{E} \left[1 - a_{\ell} k_{\ell} \left(1 + \frac{1}{2} \cos^2 \theta \right) \right] , \qquad (2.56)$$

in the troughs. The slope of the long waves is $a_{\ell} k_{\ell}$, θ is the angle between the two wave trains and \vec{E} is the average energy density of the small waves. The energy density of short waves at the crest is therefore partly 'borrowed' from the long waves; if no losses occur this is 'repaid' as the short waves move through the trough. The energy exchange is oscillatory and there is no nett flux between components. The enhanced energy of the small waves at the long wave crests may result in preferential splashing or breaking in this region.

Phillips (92) argued that the energy of the small waves had been partially acquired from the long waves and the dissipation of the short waves would damp the long waves. In contrast, Longuet-Higgins (72) reasoned that as the short waves dissipate, they also give up their momentum to the long waves; they exert a stress which is in phase with the orbital velocity of the long waves and which should lead to their growth (or decay if the waves are in opposite directions). This process is analogous to a maser, which is a device for coherent amplification or generation of electromagnetic waves. Hence this long wave growth process has been called the maser mechanism. Longuet-Higgins showed that, provided the short waves were continuously regenerated by the wind, the input of energy to the long waves due to this maser-type mechanism would greatly exceed the long wave damping proposed by Phillips (92). Hasselmann (42), however, showed that the energy input to the long waves due to the maser mechanism is almost exactly cancelled by a potential energy transfer, the residual being just the original damping term of Phillips (92).

More recently it has been shown by Garrett and Smith (34) that long wave growth can result if short wave generation (rather than dissipation) is correlated with the orbital velocity of the long waves. Since wave growth due to atmospheric input is essentially exponential and since the small waves are larger at the long wave crests, it is reasonable to assume that the atmospheric input to the small waves would be greatest at the long wave crests. Thus long waves could grow by this mechanism. Using the same arguments, the larger waves will be damped by this process if the two wave trains are in opposite directions. Garrett and Smith (34) conclude that the rate of energy transfer to the long waves is given by

$$S_{N\ell} = a_{\ell} k_{\ell} C_{\ell} \left\langle -k_{\ell} R_{S} \sin\phi + \alpha_{S} \cos\phi \right\rangle$$
(2.57)

where a_{ℓ} , k_{ℓ} and C_{ℓ} are the long wave amplitude, wave number and phase speed, respectively, R_{s} is the radiation stress of the short waves, α_{s} is the rate of transfer of momentum to the short waves by the wind and ϕ is the phase angle of the long waves. The angle brackets denote that the expression is phase averaged over the long waves. If the direction of the short waves relative to the long waves is reversed, the first term in Equation 2.57 remains unchanged, whereas the second term changes sign. The first term is the radiation stress term of Phillips (92) and is quite small in comparison with the second term. If the long and short waves are in the same direction the long waves will gain energy from the short waves, whereas the long waves will be damped by the short waves if they are in opposite directions. From Equation 2.57, at most a fraction $a_{\ell} k_{\ell}$ of the wind stress can go into long wave momentum and consequently, the effect of the small waves would not be large.

2.4 WAVE-CURRENT INTERACTION

When a wave train encounters a current, the surface velocity varies and the excess momentum flux results in an interchange of energy between the waves and the current. For deep water waves superimposed upon a steady

current U, the conservation of crests equation (Equation 2.2) gives (94)

$$\omega_{o} = \omega + kU\cos\theta \qquad (2.58)$$

where $\omega_0^2 = gk_0$ in the absence of a current and θ is the angle between the wave train and the current. This can be written as

$$C_{0} k = k (U \cos \theta + C)$$
(2.59)

or
$$\frac{C^2}{C_o^2} = \frac{\kappa_o}{k} = \frac{C}{C_o} + \frac{U\cos\theta}{C}$$
 (2.60)

which is a quadratic in C/C, solving to

$$\frac{C}{C_{o}} = \frac{1}{2} + \frac{1}{2} \left(1 + \frac{4U\cos\theta}{C_{o}}\right)^{\frac{1}{2}}$$
(2.61)

When $U\cos\theta = -\frac{1}{4}C_{o}$, the second term in Equation 2.61 vanishes, the convection velocity C is equal and opposite to the local group velocity of the waves and the wave energy can no longer be propagated against the stream.

Longuet-Higgins and Stewart (76) have shown that

$$\frac{E}{E_{o}} = \frac{C_{o}^{2}}{C(C + 2U\cos\theta)}$$
(2.62)

This result is plotted in Figure 2.6 which shows that if Ucos0 is positive the wave energy will be reduced but in an adverse current, the energy flux of the wave motion increases because of the work done by the radiation stress. Phillips (94) has extended this result for an adverse current to include wave breaking or white capping, where the waves are initially at their saturation limit with no superimposed current. The wave energy is initially $E_0(\omega_0) d\omega_0$ for a frequency band of width $d\omega_0$ centred at frequency ω_0 . At the point where the adverse current is greatest the frequency of this band is ω and the wave energy is $E_1(\omega) d\omega$. Where the current is a maximum, the waves will remain saturated and much of the energy input of Equation 2.62 will be lost. As the current speed decreases to zero, the frequency of the band returns to ω_0 , the wave energy is reduced by the expansion to $E_2(\omega_0) d\omega_0$, and no energy is lost by wave breaking. The ratio of the transmitted to the incident spectral densities is given by

$$\frac{E_2(\omega)}{E_0(\omega)} = \left(\frac{\omega}{\omega}\right)^7$$
(2.63)

where

$$\frac{\omega_{o}}{\omega} = \frac{1}{2} \left[1 + \left(1 - \frac{4 \left| \upsilon \right| \omega_{o}}{g} \right)^{\frac{1}{2}} \right]$$

Components with frequencies greater than g/4|U| are not transmitted at all, and the attenuation is very strong unless $\omega_{O}^{<< g/4}|U|$. Similar results can also be obtained for water of finite depth where the additional effects of boundary shear stress are present. Iwasaki and Sato (51) have shown that wave decay can be quite considerable in an opposing current although their analysis does not include the influence of wave motion on turbulence in the boundary layer, which their experiments indicate may be important.

2.5 WAVE DISSIPATION

2.5.1 The Saturation Range

The growth of waves under the action of wind cannot continue indefinitely. Except possibly near the peak of the spectrum, the wave-wave interactions cannot move energy sufficiently quickly to balance the atmospheric input. The waves become steeper and eventually dissipate their energy through wave breaking. The intermittent but widespread appearance of "white caps" in a growing sea is visible evidence of wave breaking and the associated energy dissipation. Such large scale gravitational breaking is a transient process, initiated when crests of the wave field run together or where a wave propagates into an area where the local energy density is high or when short waves riding over the crests of longer waves acquire excess energy as a result of radiation stress. The exact criteria for the onset of breaking in deep water remains the subject of some controversy. In addition, typical sea states comprise a random collection of irregular wave forms and the criteria may be continually changing. Taylor (128) has shown that the limiting condition for standing waves is that the local downward acceleration of the fluid near the crest is equal

to the gravitational acceleration. In contrast, in a steady, two-dimensional progressive wave, Stokes' (125) limiting form with a sharp crest is attained when this acceleration is only $\frac{1}{2}g$. It appears also that wave breaking may be even more extensive than the occurrence of white caps suggests, as smaller scale processes contribute to wave energy dissipation without the intense air entrainment and trails of foam that accompany larger scale gravitational breaking. Processes such as micro-scale breaking enhanced by the surface drift layer and the formation of parasitic capillaries are identified by Phillips (94). The surface drift layer induced by the wind stress can initiate wave breaking at a much earlier stage in wave growth than large scale gravitational breaking and without visible evidence, as the instability at the crest has insufficient vigour to entrain more than an occasional bubble of air. Phillips has shown that the surface drift contributes significantly to wave breaking only for frequencies $\omega > 2g/u_{\star}$, where u, is the shear velocity, so that it is important in the earlier stages of wave growth. The efficiency of this wind-drift model has, however, been questioned by Wright (142). Despite the lack of both quantitative measurements and a satisfactory theory for white capping, it is generally believed that white capping is the principal dissipative mechanism balancing the generating processes at more mature sea states. It has been suggested by Phillips (94) that this lack of knowledge is no great hindrance since the actual mechanism of white capping may be of secondary importance to the determination of the saturation or equilibrium spectrum.

It is reasonable to assume that in an actively generated wave field, the properties of the spectrum at high frequencies (saturation range) must be determined by the physical parameters that govern the stability and the limiting configuration of the wave crests. In deep water, these include g the gravitational acceleration, u_{\star} the friction velocity and f the wave frequency. Provided the effects of surface tension and molecular viscosity are neglected the functional form of the saturation spectrum in deep water can be found from dimensional considerations as

$$E_{\infty}(f) = \alpha g^{2} (2\pi)^{-4} f^{-5} f(\omega u_{*}/g)$$
(2.64)

where α is the Phillips coefficient. If $\omega > 2g/u_{\star}$ the function f can be neglected and Equation 2.64 reduces to

$$E_{m}(f) = \alpha g^{2} (2\pi)^{-4} f^{-5}$$
(2.65)

which was originally derived by Phillips (90). The α term was originally assumed to be a constant but measurements indicate a possible systematic dependence upon the peak frequency or alternatively the wave age. Hasselmann et al (46) have used a collection of data from a number of sources and presented this dependency as

$$\alpha = 0.0363 \hat{f}_{p}^{0.87}$$
(2.66)

or
$$\alpha = 0.0013 \hat{E}_{0}^{-0.29}$$
 (2.67)

where $\hat{f}_p = f_p U_{10} / g_r$, $\hat{E}_o = E_o g^2 / U_{10}^4$, f_p is the frequency of the spectral peak and E_o the total energy within the spectrum.

Further consideration of the saturation spectrum by Kitaigorodskii et al (62) has shown that Equation 2.65 must be modified to

$$E_{\infty}(f) = \alpha g^2 (2\pi)^{-4} f^{-5} \Phi(kd)$$
 (2.68)

in transitional and shallow water where

$$\Phi(kd) = \frac{\tanh^2 kd}{1 + 2kd/\sinh 2kd}$$
(2.69)

For deep water kd is large, Φ approaches unity and Equation 2.68 reverts to Equation 2.65. For small kd, Φ approaches $\frac{1}{2}$ kd = $\omega^2 d/2g$ whence Equation 2.68 becomes

$$E_{\infty}(f) = \frac{\alpha g d}{8\pi^2} f^{-3}$$
 (2.70)

a result which is supported by field observations in shallow water. Thus Equation 2.70 implies a significant broadening of the saturation spectrum in shallow water.

The entire concept of the saturation spectrum relies upon the presence of an active wind-generated sea. To what limit an individual swell component can grow is not well understood. In addition, the effect of opposing winds on the saturation limit has not been investigated. If such winds alter the spectral energy distribution in some manner, they may indirectly effect the saturation level.

2.5.2 Bottom Dissipation

In addition to the change in the saturation spectrum in water of finite depth, there are a number of additional shallow water dissipation mechanisms that have been identified, in particular bed friction, percolation, bottom motion and bottom scattering. Bottom friction is the most widely explored of all bottom-interaction processes. Analysis adopts the quadratic friction law $\underline{\tau} = -C_{f} \underline{u} |\underline{u}| / \rho_{w}$, where $\underline{\tau}$ is the shear stress at the bottom, u is the velocity at the edge of the bottom boundary layer and C_{f} is a friction factor. The friction factor is not constant but dependent on the flow Reynolds Number and the bottom roughness height, in a similar manner to the Darcy-Weisbach friction factor for turbulent pipe flow. Jonsson (53) has presented a wave friction factor diagram based on experimental results in the style of the Moody diagram, which shows C_{f} varying through two orders of magnitude from 0.005 to 0.5. Hasselmann and Collins (43) have applied linear wave theory, assuming that bed friction - wave interactions are weak in the mean and determined the source term representing wave dissipation to turbulent bottom friction as

$$S_{Df}(f,\theta) = \frac{C_{f}gk^{2}}{\omega^{2}\cosh kd} E(f,\theta) \left[\bar{u} + \cos^{2}(\theta-\gamma)(\bar{u_{1}}^{2}/u) + \sin^{2}(\theta-\gamma)(\bar{u_{2}}^{2}/u)\right]$$
(2.71)

where the over bars indicate time-averaged values, γ is the angle of the orthogonal co-ordinate system at the bed, u_1 is the velocity component in the principal direction of all velocities and u_2 is the perpendicular component of velocity.

For cohesionless bed materials, the oscillatory pressures induced at the bed by surface gravity waves in turn induce oscillatory flow into and out of the porous bed. Wave energy is dissipated by this bottom percolation mechanism, represented by Shemdin et al (110) as the source term

$$S_{Dp}(f,\theta) = -E(f,\theta) \ k \ \sqrt{\alpha\beta} \ \frac{\tanh \sqrt{\frac{\alpha}{\beta}} \ kd_1}{\cosh^2 kd}$$
(2.72)

but based on an isotropic analysis by Putnam (102). The horizontal and vertical coefficients of permeability are α and β respectively and d_1 is the thickness of the porous bed.

Where the bed material is cohesive, the bed material itself may respond to, and extract energy from, the wave field in a visco-elastic manner. This mechanism is believed to be responsible for significant swell decay observed off the Mississippi River delta in the Gulf of Mexico (31), where early results indicate the mechanism may be highly nonlinear. A strong dependence on wave amplitude is observed but only a weak dependence on wave frequency. An independent analytical study by Hsiao and Shemdin (48) represents the bottom motion source term as

$$S_{\text{Db}}(f,\theta) = -2k C E(f,\theta) \qquad (2.73)$$

where k_i is an attenuation coefficient dependent on frequency, water depth and on the physical properties and depth of the mud layer in a complicated but defined manner. This result is not consistent with the Forristall et al field measurements but the field program is continuing and may result in a more satisfactory representation of the bottom motion term.

A further potential shallow water dissipation mechanism is bottom scattering, identified by Hasselmann (38) and Long (69). The wave propagation medium is described by the dispersion relationship, and variations in depth change the propagation properties of the medium. The more gradual trends in the bottom topography, at length scales much greater than the surface gravity wave lengths, are accommodated by refraction but the more rapid local variations are equivalent to physical inhomogenieties in the medium, resulting in a directional redistribution of energy, even to those modes propagating in the reverse direction. Hence, this mechanism can give rise to waves propagating into the wind. An analytical expression for the source term has been developed but it depends on the spectrum of bottom displacements which is, of course, not readily available. Measurements of bottom irregularities in the JONSWAP area by Richter et al (103) suggest that bottom scattering is inadequate to account for swell decay in the area as was proposed by Long, but this does not mean that the mechanism may not be important in other areas.

2.6 MEASUREMENTS OF AIR-SEA INTERACTION

2.6.1 Wave Growth

Equation 2.42 shows that the rate of wave growth is dependent upon the wave number-frequency spectrum of turbulent atmospheric pressure $\Pi(\underline{k},\omega)$ and the exponential growth rate coefficient μ . The turbulent component of atmospheric pressure in the atmospheric boundary layer has been the subject of investigation by Priestly (101) and Elliott (29). Priestly investigated downwind and crosswind correlations of pressure along a land boundary; Elliott, downwind, crosswind and vertical cross-spectra of pressure within several metres of a land (and water) boundary. Their results are quite consistent, indicating that the pressure is typically isotropic and decays at $k^{-\nu}$ where $\nu = 3$. More recent measurements by Snyder et al (117), however, indicate that ν is closer to 2.

The first successful attempts at determining the exponential rate of wave growth due to atmospheric forcing were conducted by Snyder and Cox (116). They measured the actual growth of waves and assumed that all this growth was caused by atmospheric input, neglecting other influences such as wave-wave interactions. They concluded that the exponential growth rate parameter, μ , is given by

$$\mu = \gamma \frac{\rho_a}{\rho_w} \quad (U_{10} / C - 1) \tag{2.74}$$

where γ is a constant equal to 1. Barnett and Wilkerson (4) obtained a similar result by using an air-born radar to measure the wave growth.

A more direct and hence reliable technique for determining the energy flux from the atmosphere to the waves is to measure the induced stress at the water surface, but the problems encountered are formidable. The recording instrumentation must be as close as possible to the water surface and ideally should be in a frame of reference oscillating with the water surface; it can easily be contaminated with spray and even swamped. In addition, the aim is to measure a rather small phase difference from 180° in a pressure signal whose magnitude is quite small (of order $\rho_{\rm a}$ g a).

The first measurements of this type were made by Longuet-Higgins et al (74) using a large flat buoy, but were rather unsuccessful as no significant phase difference from 180° was found. Later laboratory experiments by Shemdin and Hsu (111) and Shemdin (108) over mechanically generated water waves and Kendall (57) over flexible wavy walls were more successful. These results are summarised in Figures 2.7 and 2.8. Dobson (27), using a very small buoy in the shallow water of Burrard Inlet, vancouver, made an extensive series of observations and found μ values consistent with those of Snyder and Cox (116); this perhaps indicates that Dobson's values are too high. Elliott (29) used a stationary probe at the same location and obtained $\gamma \approx 0.2$. Elliott however used U₅ as his reference velocity rather than U_{10} as in Equation 2.74. In addition, Elliott found that the pressure and water surface were 180° out of phase for zero wind conditions. In the presence of a wind however, he observed that the pressure lagged the water surface by 120° to 140°. In a similar set of experiments conducted in the Bight of Abaco, Bahamas, Snyder (115) found a still lower value of γ equal to 0.1. In order to reconcile the difference between the results of Dobson, Elliott and Snyder a combined experiment was conducted (117). This experiment showed that the results of Snyder were low by a factor of two because of the frequency response of his instrument and provided an extensive data set indicating γ lies between 0.2 to 0.3. Such a range is consistent with both the Elliott and the corrected Snyder values.

The experiments mentioned above have all concentrated on measuring the wave-induced normal stress on the water surface. As shown earlier (Section 2.2.4), the surface shear stress can also transfer energy to the waves. The turbulent Reynolds stress has been measured in a number of laboratory experiments (64, 127, 134, 21, 50). Although these experiments were generally designed to measure the structure of turbulence above waves for turbulent closure models, they also indicate that the energy transfer due to shear stress is quite small.

2.6.2 Opposing Winds

To date, no experiments have been specifically designed to measure the rate of decay for waves moving slower than the wind or in opposition to the wind (U/C < 1), but a number of experiments do shed some light on

the problem. In one of the experiments conducted by Dobson (26, 141), a well defined group of waves (presumably ship waves) were observed on an otherwise calm sea in the presence of a very low wind velocity. The direction of propagation of the waves relative to the wind was not recorded but Dobson inferred that they were propagating into the wind. The magnitude of the wave-induced pressure agreed well with the predictions of potential theory but the phase of the pressure signal led the water surface elevation by 165° compared to 180° predicted by potential theory. This data indicates that the waves were decaying at a rate slightly less than Dobson's growth rate. Since the growth rates measured by Dobson have been shown not to be consistent with later data, this result should be regarded with some skepticism. The data of Snyder (115) also included some upwind travelling waves. His analysis indicates that for -2 < U/C < 1 the waves are damped, whereas for U/C < -2 they are amplified. Similar waves are found in the data of Snyder et al, but they conclude that there is no appreciable shift from 180° in the p- η phase relationship and hence the waves neither decay nor grow due to the action of normal stress. Stewart and Teague (124) used a land based radar system to measure wave activity after the passage of a strong front in the Gulf of Mexico, finding that the rate of attenuation of incoming waves going against the wind was nearly independent of wind speed and quite small. The ratio of wave decay rates to wave growth rates, averaged over all their observations, was 0.15. Thus the wave growth rates were seven times greater than decay rates for identical wind conditions. Their growth and decay rates are shown against U/C in Figure 2.9. Additional radar measurements of waves propagating against the wind have also been made by Crombie et al. (140).

2.7 STRESS AT THE WATER SURFACE

As shown in Section 2.2.4, the flux of energy from the atmosphere to the ocean can be found from the stress at the water surface. In particular, the energy flux to a wave of frequency ω can be determined from the oscillating surface stresses at this same frequency ω . Invariably, the instrumentation used to measure the stress is confined to operate in an x,z cartesian coordinate system and hence measures the stress on the horizontal x,y plane. The water surface is oscillatory and since it is the stress exerted on the water surface which is required, it is necessary to transform the stress measurements in the x,y plane to an orthogonal curvilinear system in the water surface.

Consider a wavy water surface

$$\eta = a \cos(kx - \omega t) \tag{2.75}$$

which has a vertical stress σ_z and a horizontal stress τ_{zx} applied at the surface. The stress vector can be resolved into stresses normal and tangential to the water surface as shown in Figure 2.10. Firstly, considering the vertical stress σ_z , the normal and tangential stresses become

$$\sigma = -\sigma_{z} \cos \alpha \tag{2.76}$$

and
$$T = \sigma_z \sin \alpha$$
 (2.77)

respectively, where α is the angle the water surface makes with the horizontal. Based upon a limiting wave slope of ak = $\pi/7$ before breaking occurs, it is reasonable to assume that

 $\sin \alpha \approx \tan \alpha$

and $\cos \alpha \approx 1$

With these simplifications, Equations 2.76 and 2.77 become

 $\sigma = -\sigma_{z}$ (2.78)

and $\tau = \sigma_z \tan \alpha = \sigma_z \frac{\partial \eta}{\partial x}$ (2.79)

where from Equation 2.75

$$\frac{\partial \eta}{\partial x} = ak \cos(kx - \omega t + \pi/2)$$
 (2.80)

Further progress requires a specification of how the vertical stress σ_z varies with time. Two cases will be considered; the first where σ_z is constant and the second where it is oscillatory with a frequency ω .

For $\sigma_z = \overline{\sigma}_z$ = constant, Equations 2.78 and 2.79 become

$$\sigma = -\bar{\sigma}_{z} \tag{2.81}$$

and
$$\tau = \sigma_{z} ak \cos(kx - \omega t + \pi/2)$$
 (2.82)

If σ_z is given by

$$\sigma_{z} = b \cos(kx - \omega t + \phi)$$
(2.83)

where b is the amplitude of the stress fluctuation and ϕ the phase shift relative to the water surface, Equations 2.78 and 2.79 become

$$\sigma = -b \cos(kx - \omega t + \phi)$$
(2.84)

and
$$\tau = -\frac{abk}{2} \left[\cos(2kx - 2\omega t + \phi - \pi/2) + \cos(\phi + \pi/2) \right]$$
 (2.85)

Therefore when resolved into components normal and tangential to the water surface, a constant vertical stress yields a constant normal stress and a sinusoidal tangential stress of frequency ω , leading the water surface by $\pi/2$. A sinusoidal vertical stress, however, gives rise to a sinusoidal normal stress of frequency ω and phase ϕ and a shear stress with two components, one constant and the other sinusoidal with frequency 2ω .

The horizontal stress T_{xx} can be resolved in a similar manner to give

$$\sigma = \tau_{zx} \tan \alpha = \tau_{zx} \frac{\partial \eta}{\partial x}$$
(2.86)

$$\tau = \tau_{ZX}$$
(2.87)

Again if $\tau_{zx} = \overline{\tau}_{zx}$ = constant, Equations 2.86 and 2.87 become

$$\sigma = \overline{\tau}_{zx} \operatorname{ak} \cos(kx - \omega t + \pi/2)$$
(2.88)

and
$$\tau = \overline{\tau}$$
 (2.89)

Similarly if τ is sinusoidal and given by zx

$$\tau_{zx} = b \cos(kx - \omega t + \phi), \qquad (2.90)$$

Equations 2.86 and 2.87 yield

$$\sigma = -\frac{abk}{2} \left[\cos(2kx - 2\omega t + \phi - \pi/2) + \cos(\phi + \pi/2) \right]$$
(2.91)

and $\tau = b \cos(kx - \omega t + \phi)$ (2.92)

Therefore a constant horizontal stress gives a constant shear stress and a sinusoidal normal stress of frequency ω , leading the water surface by $\pi/2$. A sinusoidal horizontal stress gives a sinusoidal tangential stress at frequency ω and phase ϕ and a normal stress with two components, one constant and the other sinusoidal at frequency 2ω .

Using the relationships developed above, it is possible to resolve any stress vector measured in the x,z cartesian system into components in the orthogonal curvilinear system in the water surface. Pressure terms are, by definition, normal to the water surface but measured Reynolds and viscous stresses will of necessity be recorded in the x,z cartesian system. To determine the form of these stresses, it is necessary to consider the conservation equations of mass and momentum. In tensor form, the conservation of momentum equation is

$$\rho \left[\frac{\partial u_{i}}{\partial t} + u_{j} \frac{\partial u_{i}}{\partial x_{j}} \right] = \rho g_{j} + \frac{\partial \sigma_{ij}}{\partial x_{j}}$$
(2.93)

where $\rho = \rho_a$ is the air density, u_i is the instantaneous velocity in the x_i direction and σ_{ij} is the instantaneous stress tensor. When i = j, the stress tensor σ_{ii} represents a normal stress which, upon neglecting viscous and turbulent velocity effects, equals the negative of the static pressure (i.e. $\sigma_{ij} = -p$). Further, the pressure can be expressed as

$$p = \bar{p} + \tilde{p} + p' \qquad (2.94)$$

where \bar{p} is the mean pressure, \tilde{p} the wave-induced pressure with frequency ω and p' the uncorrelated turbulent residual. By introducing the conservation of mass equation

$$\frac{\partial u_j}{\partial x_j} = 0$$
 (2.95)

Equation 2.93 can be expressed in conservation form as

$$\rho \left[\frac{\partial u_{i}}{\partial t} + \frac{\partial}{\partial x_{j}} (u_{i}u_{j}) \right] = \rho g_{i} + \frac{\partial \sigma_{ij}}{\partial x_{j}}$$
(2.96)

If the velocities are also written as

$$u_{i} = \bar{u}_{i} + \tilde{u}_{i} + u_{i}'$$
 (2.97)

where the terms have the same meaning as in Equation 2.94, Equation 2.96 becomes

$$\rho \left[\frac{\partial}{\partial t} \left(\bar{u}_{i} + \tilde{u}_{i} + u_{i}' \right) + \frac{\partial}{\partial x_{j}} \left(\bar{u}_{i}\bar{u}_{j} + \bar{u}_{i}\tilde{u}_{j} + \bar{u}_{i}u_{j}' + \tilde{u}_{i}\bar{u}_{j} + \tilde{u}_{i}\tilde{u}_{j} + \tilde{u}_{i}\tilde{u}_{j} + \tilde{u}_{i}\tilde{u}_{j} + \tilde{u}_{i}\tilde{u}_{j} + \tilde{u}_{i}\tilde{u}_{j} + \tilde{u}_{i}\tilde{u}_{j} \right) \right] = \rho g_{j} + \frac{\partial \sigma_{ij}}{\partial x_{j}}$$

$$(2.98)$$

Time-averaging Equation 2.98 and rearranging yields

$$\rho \left[\frac{\partial \bar{u}_{i}}{\partial t} + \frac{\partial}{\partial x_{j}} (\bar{u}_{i} \bar{u}_{j}) \right] = \rho g_{i} + \frac{\partial}{\partial x_{j}} (\sigma_{ij} - \overline{\tilde{u}_{i} \tilde{u}_{j}} - \overline{u_{i} ' u_{j} '})$$
(2.99).

which has the same form as Equation 2.96. The additional 'stress' terms which appear on the right hand side of Equation 2.99 are the well known Reynolds stresses. In conventional turbulence analysis the wave-induced component \tilde{u}_i in Equation 2.97 is not considered as a separate component, in which case

$$u_{i}'' = \tilde{u}_{i} + u_{i}'$$
 (2.100)
where u_i " is the fluctuating component of velocity. Using this terminology, the Reynolds stress terms become

$$\overline{u_{i}^{"}u_{j}^{"}} = \overline{\widetilde{u}_{i}\widetilde{u}_{j}} + \overline{u_{i}^{'}u_{j}^{'}}$$
(2.101)

where $-\rho \overline{u_i u_j}$ is the Reynolds stress; the component $-\rho \overline{u_i u_j}$ is the waveinduced Reynolds stress and the component $-\rho \overline{u_i u_j}$ is the turbulent Reynolds stress. Cross terms of the form $\overline{u_i u_j}$ on the right hand side of Equation 2.101 are identically zero by definition.

The time-averaging step in deriving Equation 2.99 is necessary to bring the equation into the same form as Equation 2.96 and shows that the convective acceleration terms on the left hand side of the equation do not take on the properties of a stress until they are time-averaged. Fluctuating Reynolds stress components are not possible since all such terms are eliminated in the time-averaging step. All the resulting stresses are shown in Table 2.1.

Stress Group	Stress	Direction	
Pressure	q	normal	
	ğ	normal	
Viscous stresses	$2\mu \frac{\partial \overline{u}}{\partial x}$	horizontal	
	$2\mu \frac{\partial \overline{w}}{\partial z}$	vertical	
	$\mu(\frac{\partial \overline{w}}{\partial x} + \frac{\partial \overline{u}}{\partial z})$	horizontal	
Wave-induced	$-\rho_{a}$ $\overline{\tilde{u}\tilde{u}}$	horizontal	
Reynolds stresses	$-\rho_{a}$ $\overline{\widetilde{w}\widetilde{w}}$	vertical	
	$-\rho_{a}$ $\overline{\widetilde{u}\widetilde{w}}$	horizontal	
Turbulent Reynolds	-p _a <u>u'u'</u>	horizontal	
stresses	-p _a w'w'	vertical	
	-p _a <u>u'w'</u>	horizontal	

Table 2.1 Components of the x,z stress vector

In Table 2.1, $\boldsymbol{\mu}$ is the dynamic viscosity of air.

Any of the stress components in Table 2.1 which resolve to give normal or tangential stresses with a frequency ω may cause an energy flux to (or from) the waves. As shown in Section 2.2.4, this flux is proportional to the component of stress in phase with the wave slope for normal stresses or in phase with the water surface for a tangential stress. The phase relationship was represented by the coefficients μ_1 and ν_2 in Equation 2.40. These coefficients can easily be represented in terms of the measured phase angle ϕ of the stress relative to the water surface. If the surface stress is represented by

$$\tilde{\tau}$$
 or $\tilde{\sigma}$ = b cos(kx - $\omega t + \phi$) = b cos(kx - ωt) cos ϕ
= -b cos(kx - $\omega t - \pi/2$) sin ϕ (2.102)

the component in phase with the water surface is b cos φ and that in phase with the slope is -b sin φ . Thus from Equations 2.102, 2.38 and 2.39, μ_1 and ν_2 are

$$\mu_1 = \frac{-\operatorname{amp}(\tilde{\sigma}) \sin \phi}{\rho_u C^2 k a}$$
(2.103)

and
$$v_2 = \frac{\operatorname{amp}(\tilde{\tau}) \cos \phi}{\rho_w C^2 k a}$$
 (2.104)

where the amp function refers to the amplitude of the particular quantity in brackets and a is the wave amplitude. The magnitude of the phase shift determines whether the energy flux is positive (wave growth) or negative (wave decay). The signs of μ_1 and ν_2 as a function of ϕ are shown in Table 2.2.

When the stress components of Table 2.1 are resolved and the phase relationships accounted for, the final energy flux coefficients are

$$\mu_{1} = \left[-\operatorname{amp}(\tilde{p}) \sin \phi_{p\eta} - \operatorname{amp}(\partial \eta / \partial x) \rho_{a} \overline{u''u''} - \operatorname{amp}(\partial \eta / \partial x) \rho_{a} \overline{u''w''} + \operatorname{amp}(\partial \eta / \partial x) 2\mu \partial \overline{u} / \partial x + \operatorname{amp}(\partial \eta / \partial x) \mu (\partial \overline{w} / \partial x + \partial \overline{u} / \partial z)\right] / \rho_{w} C^{2} ka$$

$$v_2 = 0$$
 (2.106)

where $\varphi_{p\eta}$ is the phase difference between \tilde{p} and $\tilde{\eta}.$

φ	0°≰ ¢ < 90°	90° < φ < 180°	180° ≤ φ < 270°	270° ≤ φ < 360°
μ	-ve	-ve	+ve	+ve
ν ₂	+ve	-ve	-ve	+ve

Table 2.2. Energy flux directions as a function of surface stress - water surface phase difference

3. LABORATORY WIND-WAVE FLUME

3.1 LABORATORY MODELLING AND SIMULATION

Wind-wave flumes offer a number of advantages for the investigation of air-sea interaction phenomena. Experimental conditions may be designed to simplify, amplify or in other ways enhance the observation and study of complex processes. In the laboratory, experiments may be conducted in a deliberate, systematic fashion and, most importantly, conditions may be reproduced and measurements repeated. In the present context it is intended to investigate the response of waves to an opposing wind. Since such conditions invariably occur only with complex wind fields, the problems of investigation under field conditions are further exacerbated.

The successful use of a wind-wave flume in such a study requires that the essential features of the natural field phenomena be correctly modelled on the basis of the governing fluid mechanics equations. Cermak (19) has reviewed the subject of laboratory modelling of atmospheric boundary layers, concluding that the general requirements for geometric, dynamic and thermic similarity can be obtained directly by inspectional analysis. Appropriate time-averaged equations expressing the fundamental concepts of mass, momentum and energy conservation for motion of the atmosphere may be scaled to yield (18)

$$\frac{\partial \rho^*}{\partial t^*} + \frac{\partial (\rho^* u_i^*)}{\partial x_i^*} = 0$$
(3.1)

$$\frac{\partial U_{i}^{*}}{\partial t^{*}} + U_{j}^{*} \frac{\partial U_{i}^{*}}{\partial x_{j}^{*}} + \left[\frac{L_{o}^{\circ}}{U_{o}^{\circ}}\right] 2 \varepsilon_{ijk} \Omega_{i}^{*} U_{k}^{*}$$

$$= -\frac{\partial p^{*}}{\partial x_{i}^{*}} - \left[\frac{(\Delta T)_{o}}{T_{o}^{\circ}} \frac{L_{o}^{\circ}}{U_{o}^{2}}\right] \Delta T^{*} g^{*} \delta_{i3} + \left[\frac{v_{o}}{U_{o}^{\circ}}L_{o}\right] \frac{\partial^{2}U_{i}^{*}}{\partial x_{k}^{*} \partial x_{k}^{*}}$$

$$+ \frac{\partial \left\langle u_{i}^{*} u_{j}^{*} \right\rangle}{\partial x_{i}^{*}} \qquad (3.2)$$

$$\frac{\partial \mathbf{T}^{*}}{\partial \mathbf{t}} + \mathbf{U}_{i}^{*} \frac{\partial \mathbf{T}^{*}}{\partial \mathbf{x}_{i}^{*}} = \left[\frac{\mathbf{k}_{o}}{\mathbf{p}_{o}} \frac{\mathbf{v}_{o}}{\mathbf{p}_{o}}\right] \left[\frac{\mathbf{v}_{o}}{\mathbf{L}_{o}} \frac{\mathbf{u}_{o}}{\mathbf{u}_{o}}\right] \frac{\partial^{2} \mathbf{T}^{*}}{\partial \mathbf{x}_{k}^{*} \partial \mathbf{x}_{k}^{*}}$$
$$+ \frac{\partial \left\langle \mathbf{\theta}^{*} \mathbf{u}_{i}^{*} \right\rangle^{*}}{\partial \mathbf{x}_{i}^{*}} + \left[\frac{\mathbf{v}_{o}}{\mathbf{u}_{o}}\right] \left[\frac{\mathbf{u}_{o}^{2}}{\mathbf{c}_{\mathbf{p}o}(\Delta \mathbf{T})_{o}}\right] \phi^{*} \qquad (3.3)$$

 $\lambda^2 \pi *$

where ρ is the air density, U_i, u_i and u_i" are the ith components of mean, instantaneous and fluctuating velocity respectively, L is the upwind fetch length, Ω_{i} is the ith component of angular velocity, ϵ is the energy dissipation rate, p is the local static pressure, T is temperature, θ " is the local potential temperature fluctuation and ϕ is a viscous dissipation function. Zero subscripts in these equations refer to reference scale values, whilst asterisks indicate values which have been non-dimensionalised using the appropriate scale value.

For exact similarity it is necessary to have equality of the nondimensional coefficients (quantities in brackets) shown in Equations 3.1, 3.2 and 3.3 for the model and the atmosphere. These requirements are: undistorted scaling of geometry and equality of Rossby number $[R_0 = U_0/L_0\Omega]$, Richardson number $[R_i = (\Delta T)_{o}/T_{o}]$, Reynolds number $[R_e = U_{o}L_{o}/v_{o}]$, Prandtl number $[P_r = v_o \rho_c C_o / k_o]$ and Eckart number $[E_c = U_o^2 / C_{po} (\Delta T)_o]$. In the current project only isothermal conditions are being considered and hence Richardson, Prandtl and Eckert number scaling can be neglected. In addition, equality of Rossby numbers cannot be obtained as this would require modelling the turning of the mean wind direction with height. Thus, the only remaining dynamic scale criterion for the air flow is equality of Reynolds numbers.

As well as obtaining correct dynamic scaling of the air flow, similar consideration must be given to the surface water waves. Once waves grow to a sufficient extent that they are no longer in the capillary range, the wave phase speed ceases to be governed by the effects of surface tension and becomes dependent only upon gravity. Thus waves with a wave length greater than a few centimetres are properly termed gravity waves. This fact, coupled with the assumption of inviscid flow, common to almost all

207*

207*

and

wave theories, clearly illustrates the dominance of the gravity forces over the viscous forces. Indeed, wind-generated ocean surface waves have frequently been modelled by laboratory gravity waves using undistorted Froude scaling (99, 135, 54) $[F_r = U_o/\sqrt{g L_o}]$.

Thus the requirement for exact modelling in a wind-wave facility is for equality of both Froude and Reynolds numbers in the field and the laboratory. The futility of attempting to model using multiple scales has been well documented and invariably results in a scale ratio of one. Cermak (19), however, indicates that inequality of the Reynolds numbers does not seriously limit capabilities for modelling the atmospheric boundary layer, as the significant flow features are only weakly dependent upon the Reynolds number, provided the flow is turbulent. It is therefore possible to neglect the effects of Reynolds number changes under flow conditions where the Reynolds number is larger than the critical value at which transition from laminar to turbulent flow takes place. To insure independence of the laboratory flow from Reynolds number effects, the flow must be aerodynamically rough. A Reynolds number lower limit has been determined for aerodynamically rough flows over sand grain roughness, but a comparable limit has not been found for flow over random water waves. It can be speculated that aerodynamically smooth flow (characterised by a viscous sublayer) almost never exists over an ocean and, moreover, that in a laboratory wind-wave flume the mere presence of locally wind-generated waves on the water surface insures aerodynamically rough flow. Thus, since the laboratory air flow is almost certainly turbulent, the Reynolds number scaling requirements can be relaxed and an airflow structure similar to the prototype conditions can still be maintained.

On the basis of the remaining requirement of Froude number similitude, length, velocity and time scales can be defined as

$$L_{r} = \frac{L_{m}}{L_{p}} = n^{-1}$$
(3.4a)

$$V_r = \frac{m}{V_p} = n^{-\frac{\pi}{2}}$$
(3.4b)

and $T_r = \frac{T_m}{T_p} = n^{-\frac{1}{2}}$ (3.4c)

where the subscripts m and p refer to the model and prototype respectively. Therefore, if the scale ratio n equals 50 a typically ocean wave with H = 4m and T = 10s in a 20 ms⁻¹ wind would be modelled by a laboratory wave with H = 80 mm and T = 1.41s in an air flow of 2.83 ms⁻¹.

In addition to the requirements of dynamic similarity, the conditions of geometric similarity must also be maintained between the model and real conditions. Such requirements are particularly important in regard to modelling the marine atmospheric boundary layer. The three areas which require particular attention are the boundary layer shape, the turbulence intensity and the structure of the longitudinal velocity spectrum.

Plate (97) has shown that in analogy to the turbulent boundary layer along a flat plate the atmospheric surface layer can be considered as two regions: (a) an outer sublayer, whose mechanics are governed by the interaction of pressure gradient and Coriolis force and whose characteristics are determined mostly by the conditions near the edge of the surface layer; and (b) an inner sublayer, whose structure is determined by the flux of momentum to ground (or ocean) which depends on the nature of the surface. It is this inner region which is of particular interest in a windwave facility. The velocity distribution at zero pressure gradient is fully specified for the inner layer by a velocity scale, u_{\star} , and length scale, z_{o} , such that

$$\overline{u}/u_{\star} = f(z/z_{o}) \tag{3.5}$$

This functional relationship can be shown (97) to follow the classical logarithmic form

$$\frac{\overline{u}}{u_{\star}} = \frac{1}{\kappa} \ln \left(\frac{z - d_{o}}{z_{o}} \right)$$
(3.6)

where d_0 is a zero-plane displacement and κ is von Karman's constant = 0.4. In the laboratory situation, the boundary layer is similar to that over a flat plate, for which a logarithmic layer exists in the lower 15%, obtained by setting $d_0 = 0$ in Equation 3.6

$$\frac{\overline{u}}{u_{\star}} = \frac{1}{\kappa} \ln \left(\frac{z}{z}\right)$$
(3.7)

plate (97) states that this is the only portion of the naturally generated laboratory boundary layer which is an exact counterpart of the corresponding sublayer of the planetary boundary layer. From Equations 3.6 and 3.7, scaling requires that \bar{u}/u_{\star} be the same in the field and laboratory such that z_{s}/z_{o} be constant in model and prototype, where z_{s} is a scale length.

Rather than allow the laboratory boundary layer to develop naturally, it is common practice to use artificial devices such as fences and screens to accelerate its development. By trial and error a velocity profile that is similar in the laboratory and in the field over a significant fraction of its height can be achieved. Of particular advantage in modelling by this method is the use of the power law approximation to Equation 3.7

$$\frac{u}{u_s} = \left(\frac{z}{z_s}\right)^{\alpha}$$
(3.8)

where α is an exponent that depends on the surface roughness and u_s is the velocity as height z_s . Davenport (23) reports that for open sea conditions α is approximately 0.1. Equation 3.8 represents a good engineering approximation to the planetary boundary layer and for this reason both Plate (97) and Cermak (19) recommend its use.

In addition, to realistically simulate field conditions, the thickness of the logarithmic region should be at least a few multiples of a characteristic physical roughness height such as the standard deviation, σ . Bole and Harris (12) indicate that the log region should extend to at least 5σ above the mean water level. For the example mentioned earlier with H = 80 mm this would mean a boundary layer of thickness greater than 100 mm.

As well as correctly modelling the mean boundary layer shape, the turbulent structure of the atmospheric boundary layer must also be reproduced in the laboratory. It is commonly assumed (113) that in the high frequency portion of the velocity spectrum the influence of viscosity is small. In this subrange, known as the initial subrange, the eddy motion may be assumed to be independent of viscosity and thus determined soley by the rate of energy transfer from larger eddies. From this assumption known as Kolmogorov's second hypothesis, it follows from dimensional considerations that

$$E_{uu}(f) \alpha f^{-5/3}$$
 (3.9)

where E₁₁₁(f) is the spectrum of velocity fluctuations in the x direction.

The mean square value of the velocity fluctuations may be expressed (113) as

$$\overline{u'^2} = \beta u_{+}^2$$
(3.10)

where β is an empirical constant, independent of height and approximately equal to 6.0. It follows from Equations 3.10 and 3.7 that

$$I_{u} = \frac{\sqrt{u'^{2}}}{\bar{u}} = \frac{\sigma_{u}}{\bar{u}} \approx \frac{1}{\ln(z/z_{o})}$$
(3.11)

where σ_u is the standard deviation of the x component of velocity and I_u is the turbulence intensity. As for the power law for the mean velocity profile, Equation 3.11 can be approximated by (121)

$$I_{u} = 0.097 (z_{g}/z)^{\alpha}$$
(3.12)

where Z is the gradient height corresponding to the edge of the boundary layer and again α is approximately 0.1 over the ocean.

3.2 DESIGN OF WIND-WAVE FLUME

3.2.1 Existing Facility

The wind-wave flume used for this study was a development of an existing wave flume in the main hydraulics laboratory of the Department of Civil and Systems Engineering of the James Cook University of North Queensland, and has been described in detail by Mitchell et al (83). The wave flume, which is shown in Figures 3.1 and 3.2, consisted of an open channel 0.41 m square in section and of length 14 m. Along its length the flume was supported and the sides laterally braced by a mild steel framework constructed of channel and angle sections. The supports were located at 2.5 m intervals and, with the exception of the working section, were fixed to the laboratory with adjustable connections to facilitate flume levelling. The complete working section was supported by adjustable rubber mountings to minimise transmission of laboratory vibrations to any instrumentation. At each end of the flume were steel tanks which rested directly on the laboratory floor. These were originally part of an estuarine dispersion experiment for which the flume was originally used and are only relevant as end supports.

The sides and bottom of the flume were fabricated from 3 mm mild steel plate, with the exception of the working section, where the walls were of 6 mm plate glass for a length of 4.85 m to allow close observation of fluid flow. In a previous experiment, coarse grained sand and paint had been applied to the floor of the flume over its entire length. The resulting flume bed equivalent sand grain roughness, k_s , has been evaluated (83) at 2.0 mm. The dimensions of the flume are shown in Figures 3.1 and 3.2.

The beach at the downstream end of the flume was designed to minimise wave reflection and consisted of a 30 mm thick layer of 12 mm aggregate constrained above and below by 6 mm square wire mesh. This was supported in a steel frame sloping at an angle of 14° to the flume floor, as illustrated in Figure 3.3. Estimates (83) of the reflection coefficient of 2.0% show that the beach is effective.

Waves were generated in the flume by the motion of a wedge-shaped piston along an axis inclined at 11° to the longitudinal axis of the flume, as illustrated in Figure 3.4. A wedge was chosen in preference to a flapper as it allows better approximation to the vertical profile of horizontal water velocity, and effectively eliminates all backwash problems and wave form contamination from flow around the wave maker. Movement of the wedge was controlled by an analog command signal generated by a PDP-11 mini-computer from a digitised synthetic wave record. Displacement of the wave generator was maintained in relation to the command signal by means of an an electro-hydraulic servo-control system. The control system is shown in Figure 3.5.

The servo-control system was of a negative feedback type, with the feedback being an electrical analog of the piston displacement from a mean position. This was provided by a Schaevitz DC-D 10000 LVDT, having a maximum output of ±10.00 volts over its operating displacement of ±254 mm. The command signal and feedback were inputs to a differential amplifier, the output providing an error signal which was converted to a current control signal suitable for the operation of a Mooq A 076-102 servo-valve. Incorporated in the control signal was a 240 Hz square wave dither signal which ensured smooth servo-valve operation. The servo-valve directed pressurised low viscosity Mobil DTE-24 hydraulic fluid to the appropriate end of a modified Pongrass H 7-13-B DACEDE 5F-21 double-acting hydraulic cylinder. The cylinder in its original form exhibited unacceptable shudder at low frequencies. The use of Sperry Vickers T seals, which necessiated reconstruction of the cast iron piston and cylinder end blocks, eliminated virtually all the shudder. Pressurised hydraulic fluid was supplied at 6.89 MPa by a Sperry Vickers V10, fixed vane 18 1 min⁻¹, 5.6 kW hydraulic power supply. In order to maintain constant operating characteristics the hydraulic fluid temperature was kept at $35 \pm 2.5^{\circ}$ C by monitoring the fluid temperature with a YSI520 thermistor probe. and using this to switch a flow of water through or to bypass an oil cooler as appropriate.

The piston wave generator was designed in the shape of a wedge of overall dimensions as shown in Figure 3.6. In an effort to prevent leakage, rubber flaps were attached around the perimeter of the front surface, these brushed against the flume walls and provided an effective seal. The wedge was constructed from 5 mm marine plywood, assembled to form a closed hollow prism which fitted snugly into the flume cross-section. Directly underneath the wedge, the floor sloped upwards at 11° as shown in Figure 3.6.

The wedge was suspended at four points from two parallel stainless steel 25 mm diameter rods sloping at 11° to the horizontal, as shown in Figure 3.6, and aligned along the flume axis. These rods passed through lubricated longitudinal bearings, attached to the wedge by a simple framework, thus allowing single degree of freedom movement of the wedge in the direction of the supporting rods. The hydraulic cylinder piston rod was attached to the wedge at one point on the centre line of the top of the wedge, with the line of action of the hydraulic piston parallel to, and

hidway between, the two mounting rods. Attached to an adjustable bracket on one side of the wedge was the sliding rod or core of the LVDT. The rod passed through the LVDT, itself attached to the supporting framework and aligned parallel to the hydraulic piston and supporting rods.

3.2.2 Modified Facility

The major alteration to the existing facility for the present project was the enclosure of the water channel with a hood and the installation of The hood was rectangular in shape and extended 880 mm above the a fan. wave channel. The vertical walls of the hood were attached to the flume through a series of small angle brackets. Except for the observation section, which consisted of four 5 mm thick and 1 m long glass panels, the hood was constructed from 12 mm thick plywood. For a water depth of 320 mm, used throughout this project, a clear air flow region of 0.97 m was available above the mean water level. With a maximum wave height of 100 mm, this arrangement provided for a ratio of wave height to mean air flow depth of 0.10, which was considered sufficient to prevent blockage effects by the waves (12). The hood was constructed in five separate sections to allow ease of construction and entry to the flume. From the beach end of the flume these sections were 2.9 m, 1.7 m, 4 m, 2.4 m and 1.2 m long; the 4 m section being the observation section. In an effort to prevent the growth of the side and top boundary layers as well as to reduce the horizontal pressure gradient, these sections were not sealed at their joints, air being allowed to leak from the hood at these joins to help in restricting the growth of these unwanted boundary layers. This system had the additional advantage of minimizing the transmission of vibrations along the flume.

In designing the fan, the major decision to be made was whether a sucking or blowing configuration was required. In aerodynamic wind tunnels a sucking arrangement is generally recognised as superior as there is less swirl in the air caused by the fan. Such an arrangement would have required the fan to be placed at the wave maker end of the flume where laboratory space was limited and a complicated ducting system would have been required to avoid the moving wave maker. A blowing configuration with the fan mounted at the beach end of the flume was consequently adopted. As the existing flume was constructed with the beach against the western wall of the

laboratory, it was necessary to mount the fan on a high frame and lead the air flow into the hood via a 90° vertical bend. The ideal arrangement would have been to have a long horizontal section before the entry to the flume, in which to develop the boundary layer.

The fan, motor and transmission assembly were mounted atop a heavy steel frame, 3.7 m high. The fan was a Phoenix 270 SW Centrifugal fan in standard fan arrangement Number 1, driven by a 7.5 kW, 1440 r.p.m. electric motor via a Dunstan Series 700 hydraulic power transmission. The fan was connected to the transmission through a triple V-belt drive with a 170 mm pulley at the transmission and a 280 mm pulley at the fan. This arrangement provided a speed reduction of 0.61, giving a maximum fan speed of 874 r.p.m., consistent with the manufacturer's recommendations. The Dunstan power transmission provided continuous speed control from 0 to 1440 r.p.m. The complete motor, transmission and fan assembly was mounted on a single steel frame and was connected to the larger supporting frame by a series of spring mounts, which provided vibration isolation of the fan and drive system from the laboratory floor. The full assembly is shown in Figure 3.7.

A vertical transition duct 540 mm long expanded the 680 mm x 580 mm fan outlet to 880 mm x 410 mm consistent with the dimensions of the flume hood. A coarse wire mesh was placed at the fan outlet as an initial measure towards achieving a more uniform fan flow. Following the transition section was a vertical rectangular section 400 mm long, filled with 50 mm I.D. cardboard tubes to assist in removing the swirl in the air produced by the fan and in strengthening the air flow. A 100 mm rubber gusset connected the transition and flow straightening sections of the ducting, isolating the flume from any fan vibrations. After the flow straightening section, the ducting passed through a 90° vertical bend to enter the flume hood section. The bend was circular with an outer radius of 1200 mm and an inner radius of 320 mm. In an effort to achieve even flow around the bend, five evenly spaced galvanised iron turning vanes were used. The vanes extended around the full 90° of the bend. Although originally intended only to provide an even air flow transition from the vertical section of ducting to the horizontal flume, the vanes also assisted in developing the boundary layer. Since the outer vanes were considerably longer than the inner vanes, they provided greater frictional retardation to the flow, producing a lower wind velocity around the outside of the bend.

As mentioned in Section 3.1 a boundary layer of thickness greater than 100 mm was desirable. Yu and Lin (137) have indicated that provided a fence is used to trip the flow, the surface skin friction coefficient, $c_{_{f}}$, and the boundary layer thickness, δ , are given by

$$C_{f} = \frac{2u_{\star}^{2}}{U_{\infty}^{2}}$$

$$= 0.003 \left(\frac{g \times 1}{U_{\infty}^{2}}\right)^{-1/5}$$
(3.13a)
(3.13b)

and
$$\frac{g\delta}{U_{\omega}^2} = 0.025 \left(\frac{g x}{U_{\omega}^2}\right)^{4/5}$$
 (3.14)

where x is the fetch along the flume. Values of δ and u_* at the flume working section (x = 7 m), as determined from Equations 3.13 and 3.14, are shown in Table 3.1.

U _∞ (ms ⁻¹)	ک (mm)	u _* (ms ⁻¹)
1	75	0.036
2	99	0.082
3	117	0.134
4	131	0.189
5	143	0.248
6	154	0.308

Table 3.1 Estimated values of boundary layer thickness at the working section with the aid of a fence to trip the flow. After Yu and Lin (137).

Based on these figures, a 40 mm high galvanised iron fence was placed at the beginning of the horizontal flume section; the fence height was a recommendation by Yu and Lin (137). Initial measurements indicated the turbulence intensity was quite low. Hence, to increase the turbulence in the air flow, a fine mesh screen was inserted 200 mm downstream from the fence. A plan of the complete wind-wave flume is shown in Figure 3.8.

3.2.3 Fan Design

One of the major design considerations was the selection of the fan. The design constraints were that it must deliver a flow of not less than $2.5 \text{ m}^3 \text{s}^{-1}$, which is equivalent to a velocity of 6 ms⁻¹ in the working section, and that it must not require a motor larger than 7.5 kW, since such a motor was already available.

Normal practice is to base design upon an estimation of the energy ratio, the ratio of the kinetic energy in the air passing through the test section per unit time to the power required at the fan blades (100)

E.R. =
$$\frac{\frac{1_2}{P} \rho u_0^3 A_0}{A_0 u_0 \Delta P_t}$$

= $\frac{\frac{1_2}{P} \rho u_0^2}{\Delta P_t}$ (3.15)
= $\frac{Dynamic pressure in working section}{Total pressure loss along the flume}$

where u and A are the average velocity and cross-sectional area in the working section respectively. The energy ratio is also commonly inverted and quoted as a total loss coefficient Σk_0 where

$$\Sigma k_{o} = \frac{1}{E \cdot R \cdot} = \frac{\Delta p_{t}}{\frac{1}{2} \rho u_{o}^{2}}$$
 (3.16)

To obtain $\sum_{i=0}^{\infty} k_{i}$, the total pressure loss coefficient k_{i} in each section of the flume is estimated and summed:

$$k_{oi} = \frac{\Delta p_{i}}{\frac{1}{2} \rho u_{o}^{2}} = \frac{\Delta p_{i}}{q_{o}} = \frac{\Delta p_{i}}{q_{i}} \left(\frac{A_{o}}{A_{i}}\right)^{2} = k_{i} \left(\frac{A_{o}}{A_{i}}\right)^{2}$$
(3.17)

where q_i and A_i are the dynamic pressure and cross-sectional area at section i respectively and k_i is the loss coefficient based on the local dynamic pressure q_i . Table 3.2 shows these calculations for the present flume with the origin of the estimated values of k_i .

Flume Section	k _i	A (m ²)	$\left(\frac{A}{A_{i}}\right)^{2}$	k oi	Source of k i
Inlet	0.5	0.45	0.87	0.44	Phoenix Fan Catalogue (95)
Coarse Grid	0.3	0.40	1.10	0.33	Pankhurst & Holder (88)
Fine Grid	1.0	0.42	1.00	1.00	Pankhurst & Holder (88)
Flow Straighteners	0.3	0.36	1.36	0.41	Pope & Harper (100)
. Turning Vanes	5x0.2	0.36	1.36	1.36	Pope & Harper (100)
Corner	0.3	0.36	1.36	0.41	Pope & Harper (100)
Exit	1.0	0.42	1.00	1.00	-
Friction	~1.0	0.42	1.00	1.00	Moody Diagram

Table 3.2 Estimated section loss coefficients.

The estimated values of Table 3.2 give $\Sigma k_0 = 5.95$. Based on this value for Σk_0 , the flume load characteristic as a relationship between total pressure loss and volume flow can be obtained. This is plotted in Figure 3.9 together with the fan characteristics of the chosen fan as provided by the manufacturer. The intersection of the fan characteristics with the load curve gives the volume flow rate corresponding to this particular fan speed of 600 r.p.m. This point represents a delivery volume of 2.5 m³s⁻¹ or 6 ms⁻¹ in the working section, which satisfies the design requirements.

3.2.4 Performance Evaluation of Air Flow

Before any air-sea interaction experiments were performed, an extensive set of evaluation measurements were undertaken to ensure that the air flow satisfied the similarity conditions described in Section 3.1. Once the wind had reached the water section of the flume little could be done to alter its flow characteristics. Indeed one of the objects of this project was to determine the precise nature of this flow in an opposing wind-wave situation. Therefore, the goal was to obtain reasonable flow conditions at the entrance to the water section, any subsequent changes being assumed to be a result of the physics of air-sea interaction. For this reason the evaluation measurements were taken at a point 2.7 m downstream from the inlet bend and flow trip fence. The mean velocity values were obtained using a standard Pitot-static tube and a Thies precision inclined manometer, whilst the turbulence data was obtained with a hot film anemometer. The hot film anemometer system is described in detail in Section 4.4.

The measurements were made with a fan speed of 400 r.p.m. and a corresponding free stream velocity of 4.6 ms⁻¹ and concentrated on determining the structure of the turbulent boundary layer. Mean velocities were measured at a number of heights above the mean water level, the subsequent velocity profile being shown in Figure 3.10. A least squares curve approximation to this data gives

$$\overline{\underline{u}}_{s} = (0.091 \pm 0.022) \ln z + (1.1 \pm 0.1)$$
(3.18)

where the errors represent 95% confidence limits. Equation 3.18 together with Equation 3.7 yield $u_* = 0.17 \text{ ms}^{-1}$ and $z_0 = 3.5 \times 10^{-6} \text{m}$. These values scale to full scale terms of $u_* = 1.2 \text{ ms}^{-1}$ and $z_0 = 1.8 \times 10^{-4} \text{m}$, within the z_0 range of $3.0 \times 10^{-6} \text{m}$ to $4.0 \times 10^{-3} \text{m}$ reported (23) for calm seas. It should, however, be recognised that determining z_0 from such a curve fit is quite inaccurate as a small change in the slope of the curve can significantly change the intercept value z_0 . A variety of empirical formulas have been proposed for determining u_* above ocean waves. Based on data from a number of sources Amorocho and De Vries (1) have proposed

$$u_{\star} = d \left\{ (C_{10}_{\text{max}} - C_{10}_{\text{min}}) \left[1 + \exp \left(- \frac{U_{10} - m}{S} \right) \right]^{-1} + C_{10}_{\text{min}} \right\}^{\frac{1}{2}} U_{10}$$
(3.19)

where d = 0.97 \pm 0.10, C₁₀ max = 0.00225, C₁₀ min = 0.00104, m = 12.5 ms⁻¹ and S = 1.56 ms⁻¹. Equation 3.19 predicts u_{*} values in the range 0.88 ms⁻¹ to 1.01 ms⁻¹ for the full scale conditions, comparing favourably with the value of 1.2 ms⁻¹ obtained from the evaluation measurements. The full velocity profile predicted by the log law (Equation 3.7) and the equivalent power law (Equation 3.8) are also shown in Figure 3.10. These plots clearly indicate the mean boundary layer shape has been reproduced to the correct scale in the flume. The above results indicate u $z_0/\nu \approx 0.04$, indicating a "smooth wall" flow. This is an unavoidable consequence of the reduced scale of the modelling.

In addition to the mean velocity profile, turbulence intensities were measured at various heights above the mean water level. The vertical profile of turbulence intensities, along with the theoretical profile, Equation 3.11 are shown in Figure 3.11, the agreement again being acceptable. The data used to determine the turbulence intensities is also presented in spectral form in Figure 3.12 and shows that at higher frequencies the spectrum decays at approximately $f^{-5/3}$ as predicted for the inertial subrange. These measurements of the structure of the boundary layer clearly indicate that both the mean flow structure and the turbulence levels are consistent with correctly scaled field measurements.

In addition to these boundary layer measurements, a number of other experiments were conducted to ensure the flow was symmetric about the vertical centre line of the flume. Velocity measurements were made across the cross section of the flume both at the entrance and at the working section. Contour plots of the time-mean wind velocities are presented in Figure 3.13. The flow pattern near the flume inlet is quite uniform with only thin boundary layers on the side walls. The higher flow in the upper core region is due to the fine screen which was located immediately upstream of this location and extended only part of the way up the flume. This core region is much more diffuse at the working section and the side wall boundary layers are thicker. There is some asymmetric flow, with a slightly higher flow on left side of the flume, due to blockage from a wave height probe located upstream of the working section and described in Chapter 4. The degree of asymmetry is not serious, however, and the flow is considered adequate for the purposes of this project.

As an aid for later experiments, the centre line free stream velocity was also measured as a function of the fan speed. This data, which appears in Figure 3.14, indicates a linear relationship between the fan speed and U_{∞} , the free stream wind velocity. A least squares approximation to the data yields

$$U_{m} = (0.0118 \pm 0.0006) * (Fan r.p.m.)$$
(3.20)

where the errors represent 95% confidence limits. There is no apparent reason why this relationshp should be linear, since it merely represents the locus of the intersection points of the fan characteristics for various fan speeds with the flume load curve. The relationship is, however, very useful since the fan speed is much easier to measure than U_{∞} .

3.3 WAVE MAKER PERFORMANCE

3.3.1 Theoretical Transfer Functions

The concept of wave generation in the laboratory is not a new one, and even before the last world war numerous designs were in operation. Many of them were remarkable for their use of complex mechanical linkages in an attempt to produce satisfactory waves in flumes and basins. Whichever design is used, knowledge of the transfer function relating the wave generator motion to the wave motion is of fundamental importance.

Biesel and Suquet (8) have calculated a theoretical transfer function, based on linear wave theory, for a piston wave generator as

$$a = |H|e \tag{3.21}$$

where a is the wave amplitude, e is the piston stroke and H is the complex transfer function, the amplitude of which is

$$|H| = \frac{2 \sinh^2(kd)}{\sinh(kd) \cosh(kd) + kd}$$
(3.22)

where k and d are the wave number and depth respectively. The assumption of linear wave theory used in developing Equations 3.21 and 3.22 has been tested experimentally by Ursell, Dean and Yu (130). Their results showed that for small wave slope, ak, agreement between theoretical and realised wave amplitudes was good, whereas for larger wave slope agreement was poorer. It was suggested that this was due to finite amplitude effects, and a proposed upper limit for validity of first order theory was suggested in terms of wave slope as ak \geq 0.09. Keating and Webber (55) have confirmed this value but suggested that it could be extended to ak = 0.25 at the expense of only a slightly greater error.

The preceeding studies have focussed attention on the ratio of the stroke to wave height and have regarded the actual wave profile to be of secondary importance. However, it has been observed (7) that, when finite amplitude waves are produced by a sinusoidally moving wave generator, the resulting wave, rather than being of permanent form, breaks down into a primary and secondary wave. These two waves travel at different phase speeds and the resulting wave profiles will, of course, exhibit the presence of these secondary waves, depending upon the distance from the generating surface. The elimination of these secondary waves would be advantageous and has been considered by Madsen (77). Madsen's results show that the water surface profile for a wave produced by a sinusoidally moving piston wave generator is given by

$$\eta(t) = -a \sin(kx - \omega t) - a_{p}^{(2)} \cos 2(kx - \omega t) + a_{L}^{(2)} \cos(k'x - 2\omega t)$$
(3.23)

where

$$a = \frac{\xi \tanh kd}{n'}$$
(3.24)

$$a_{p}^{(2)} = -\frac{ka^{2}}{4} \frac{(2 + \cosh 2kd) \cosh kd}{\sinh^{3} kd}$$
 (3.25)

$$a_{\rm L}^{(2)} = \frac{1}{2} a^2 \frac{\coth kd}{d} \left(\frac{3}{4 \sinh^2 kd} - \frac{n}{2}\right) \frac{\tanh k'd}{n'}$$
(3.26)

$$n = \frac{1}{2} \left(1 + \frac{2kd}{\sinh 2kd} \right)$$
 (3.27)

$$n' = \frac{1}{2} \left(1 + \frac{2k'd}{\sinh 2k'd} \right)$$
(3.28)

where $\omega^2 = gk \tanh kd$ and $4\omega^2 = gk' \tanh k'd$. ξ is the amplitude of the wave maker motion and the superscript ⁽²⁾ indicates the terms are of the second order. Equation 3.23 shows that the water surface consists of a primary component, a Stokes second order progressive wave and a free second harmonic. In addition, Madsen showed that, in order to eliminate second order waves, the wave-maker motion must be prescribed by

$$\xi'(t) = -\xi \left[\cos \omega t + \frac{1}{2} \frac{a}{nd} \left(\frac{3}{4 \sinh^2 kd} - \frac{n}{2} \right) \sin 2\omega t \right]$$
(3.29)

Equation 3.29 is identical to the periodic part of the depth averaged horizontal water particle motion beneath a Stokes II progressive wave. As pointed out by Madsin, one would intuitively expect that, to produce a wave of permanent form, the generating surface should be given a motion which corresponds to the water particle motion under the desired wave. Since the two terms of Equation 3.23 with a frequency of 2ω have different wave numbers, the two waves will propagate at different speeds. Hence, the resulting amplitude a⁽²⁾ of the second harmonic terms will vary with distance from the generator.

In order to assess the relative magnitudes of the primary wave and it higher harmonics at the test section of the flume, a series of water surface level experiments were made with a sinusoidal generator motion. The water surface elevation was measured for wave maker stroke frequencies of 0.5 Hz, 1.0 Hz, 1.5 Hz and 2.0 Hz. The resultant time series appear in Figure 3.15 and their corresponding variance spectra in Figure 3.16. An examination of the time series indicates that there is no evidence of the secondary waves separating out from the primary waves. Indeed the wave profile appears to be approximately sinusoidal. The spectra of these records, however, indicate that not only is there a second harmonic present but also higher harmonics; the second harmonic is in all cases at least an order of magnitude smaller than the primary wave and the higher harmonics are smaller still.

3.3.2 Measured Transfer Functions

The transfer function, H, represented in Equations 3.21 and 3.22 was developed from the assumption that the wave maker motion was sinusoidal. The transfer function can, however, be defined in a more convenient and general form for a broader band input.

Consider a linear system with a single, well defined, input x(t) and single output y(t). The input and output of the system can be related by (87)

$$Y(f) = H(f) X(f)$$
 (3.30)

$$E_{yy}(f) = |H(f)|^2 E_{xx}(f)$$
 (3.31)

and
$$E_{xy}(f) = H(f) E_{xx}(f)$$
 (3.32)

where Y(f) and X(f) are the Fourier transforms of y(t) and x(t) respectively, E_{yY} (f) and E_{xx} (f) are the variance spectra of y(t) and x(t) respectively, E_{xY} (f) is the cross spectrum between x(t) and y(t) and H(f) is the complex transfer function of the system. H(f) is complex, containing information about both the amplitude and phase response of the system.

In the present context the wave maker is not a single system but it has been represented as an n-stage linear system, for which the realised water surface elevation spectrum $E_{\eta\eta}(f)$ can be related to the input command signal spectrum $E_{cc}(f)$, by a series of complex transfer functions $H_i(f)$ as

$$E_{nn}(f) = |H_1(f)|^2 \cdot |H_2(f)|^2 \dots |H_n(f)|^2 \cdot E_{cc}(f)$$
(3.33)

Four distinct stages are immediately recognizable and are shown in Figure 3.17. The first of these transfer functions, $H_1(f)$ is a direct result of the discrete representation of a given wave record. Digitisation of a finite time series of length $t_R = N\Delta t$ at a time interval Δt , discerns only those frequencies in the range

$$\frac{1}{N\Delta t} < f < \frac{1}{2\Delta t}$$
(3.34)

where the frequency resolution $\Delta f = 1/N\Delta t$ and the Nyquist frequency $f_N = 1/2\Delta t$. Any energy at frequencies greater than the Nyquist frequency will be folded back below $f_N(5)$. Provided that the input or command signal and the digitising frequency are chosen so as to avoid folding about the Nyquist frequency, the transfer function $H_1(f)$ has the form of an ideal low pass filter with complete cutoff at f_N .

The first major distortion of the command signal occurs as a result of the frequency response of the hydraulically-driven wave maker. This relationship between the command signal and the wave maker motion is described by $H_2(f)$. A second modification of the command signal occurs as a result of the piston stroke to wave height transfer function $H_3(f)$. The final transfer function $H_4(f)$ is the most difficult to evaluate. It must include wave dispersion, reflection at the beach, wave breaking or "white capping", viscous dissipation, non-linear wave interaction and free second harmonic components as discussed in Section 3.3.1. In fact it should not be referred to as a transfer function at all as it includes a number of non-linear effects.

In an effort to determine these various transfer functions, an extensive set of experiments were devised in which the wave maker motion and the water surface evaluation at the test section and near the beach were measured. Such measurements do not allow all of the transfer functions to be determined but the products $H_1(f) \cdot H_2(f)$ and $H(f) = H_1(f) \cdot H_2(f) \cdot H_3(f) \cdot H_4(f)$ can be deduced. The determination of H(f) is of particular importance.

For direct evaluation of the complex transfer functions, the convenient choice for an input signal is 'white noise', having a uniform variance spectrum and a random phase spectrum. To prevent excessive inertial loads being placed on the supporting framework and to prevent aliasing, an upper limit of 4.0 Hz was placed on the uniform input spectrum $E_{\rm CC}(f)$. The generation of the command signals from the specified variance spectrum was based upon the inverse Fourier Transform method and is described by Mitchell et al (83). Seven input spectra were used, each with a different variance and for each spectrum ten different input records were synthesized, each corresponding to a different random phase spectrum. A total of seventy individual experiments were performed to reduce the confidence limits on the resultant transfer functions (see Appendix B). The command signal spectral parameters are shown in Table 3.3 and the input spectral variances in Table 3.4.

Time series of the wave maker displacement and of water surface elevation at the test section and near the beach were recorded. Spectra and cross spectra were then determined and the resulting transfer function

Number of points	N	2048
Time length of record	t _R .	102.4s
Sampling interval	Δt	0.05s
Upper frequency limit	f	4.0 Hz
Frequency increment	Δf	0.00977 Hz
Nyquist frequency	f _N	10.0 Hz

Table 3.3 Command signal spectral parameters

Run Nos.	Variance (V ²)	Variance (m ²)
1-10	0.70	4.52×10^{-4}
11-20	1.00	6.45×10^{-4}
21-30	1.50	9.68 x 10 ⁻⁴
31-40	2.00	1.29×10^{-3}
41-50	2.50	1.61×10^{-3}
51-60	3.00	1.94×10^{-3}
61-70	3.50	2.26×10^{-3}

Table 3.4 Command signal variances

obtained from Equation 3.32. Average transfer functions were then obtained for each of the seven inputs with different spectral variances by frequency averaging the transfer functions from each group of ten command signals. Thus twenty-one transfer functions were obtained; seven relating the command signal to wave maker motion, seven between the command signal and water surface elevation at the test section and the remainder between the command signal and the water surface elevation at the beach.

The seven transfer functions between the command signal and the wave maker motion, $H_1(f) \cdot H_2(f)$, were almost identical and were averaged to give the final result shown in Figure 3.18. The gain of the transfer function is one at zero Hz but gradually decreases with increasing frequency. The phase relationship also indicates considerable distortion of the command

signal, the phase lag of the wave maker increasing with increasing frequency. The full transfer function indicates that the wave maker acts as a typical low pass filter. A least squares curve fitting scheme was used to approximate the transfer function. It was found that the amplitude of the transfer function could be accurately represented by the single curve

$$\left| H_{1}(f) \cdot H_{2}(f) \right| = \frac{1}{(1 + \alpha f^{p})} \quad (\pm 10\%) \quad (3.35)$$

where $\alpha = 0.89$ and p = 1.5. A piece-wise approximation was necessary for the phase relationship, with the final result

$$\phi$$
 (f) = 120f + 4.0 (±8%) , 0 \leq f $<$ 0.25 Hz (3.36a)

$$\phi_{12}(f) = \frac{180 f^n}{1 + f^n} (\pm 2\%) , \quad 0.25 \leqslant f < 1.5 Hz$$
 (3.36b)

$$\phi_{12}(f) = \frac{180 f^{n}}{1 + f^{n}} + 21f - 33(\pm 2\%) , \quad 1.5 \leqslant f < 4.0 \text{ Hz} \qquad (3.36c)$$

where n = 1.2 and the errors in Equations 3.35 and 3.36 represent 95% confidence limits. These results confirm the result obtained by Mitchell et al in an earlier attempt to determine the transfer functions for this facility.

The complete transfer function, |H(f)|, relating the input command signal to the water surface elevation at the test section is shown for each of the input spectral variances in Figure 3.19. A typical result for the phase relationship is shown in Figure 3.20, indicating that the phase is completely random. Figure 3.19 indicates that, although the transfer functions are similar, they are not identical, confirming that the system is slightly non-linear. The differences are not significant, however, and the seven results have been averaged to yield a final average transfer function, illustrated in Figure 3.21. The transfer function has a steeply rising forward face with a peak at about 1.4 Hz. At frequencies above 1.4 Hz, |H(f)| gradually decreases in value, except for a small secondary peak near 2.0 Hz. An explanation of the |H(f)| shape can be obtained by examining the theoretical transfer function $|H_3(f)|$ defined by Equation 3.22. This result is shown in Figure 3.22 along with the product $|H_1(f) \cdot H_2(f)| \cdot |H_3(f)|$ in Figure 3.23, where $|H_1(f) \cdot H_2(f)|$ was obtained from Equation 3.35. A comparison of Figure 3.21 and 3.23 reveals that they are remarkably similar, the slight differences being presumably the effect of the $H_4(f)$ term which is not included in the theoretical prediction. Based on the theoretical predictions, however, it is clear the steep forward face of the transfer function is a result of the machine to wave term $H_3(f)$, whereas the higher frequency face is governed by the command signal to machine term $H_2(f)$. The small secondary peak is less easily explained and its cause is not immediately obvious. Figure 3.19 indicates that the magnitude of this peak increases with increasing input spectral variance, indicating that it may be a finite amplitude effect. It is interesting to note, however, that the natural frequency of the flume in its first transverse mode is

$$f_0 = \frac{\sqrt{gd}}{2l} = 2.2 \text{ Hz}$$
 (3.37)

where l = 0.41 m is the flume width. As this value corresponds almost exactly with the secondary peak it is very likely that it is a result of excitation of a natural frequency of the flume. Indeed, in subsequent tests with sinusoidal waves it was obvious that the natural frequency oscillations could easily be excited by waves with a frequency near 2 Hz.

The average transfer function H(f) at the beach is shown in Figure 3.24. Again this is similar to the transfer function at the test section, although it has a slightly reduced magnitude. This is presumably due to the effects of the $H_4(f)$ term, which will be different for different positions along the flume. The secondary peak near 2 Hz is smaller than at the test section and there are a number of additional secondary peaks on the low frequency face of the transfer function. The origin of these peaks is not clear but it is likely that they are finite amplitude effects.

In order to test the accuracy of the measured transfer function H(f), an attempt was made to generate a particular spectrum at the test section. The target spectrum was a Pierson-Moskowitz spectrum with a variance $\sigma^2 = 5.625 \times 10^{-5} m^2$ and a peak frequency f_p = 1.5 Hz. The required input

spectrum was determined from Equation 3.31 and the corresponding command signal generated in the manner outlined by Mitchell et al (83). The measured wave spectrum, together with 95% confidence limits (Appendix H), and the target spectrum are shown in Figure 3.25. The variance of the measured spectrum is $4.815 \times 10^{-5} m^2$, approximately 14% lower than the target spectrum, although the target spectrum generally lies within the confidence limits of the measured spectrum. The agreement is not perfect but it is adequate for the present purposes.

3.4 WAVE REFLECTIONS

3.4.1 Reflection of Mechanically-Generated Waves from the Beach

In the theoretical analysis of Section 3.3, it was assumed that the wave flume was infinitely long or that the wave energy was completely absorbed at the end of the flume. In practice this is not the case and the mechanically-generated wave train will be partially reflected from the beach. This reflected wave, known as the primary reflected wave, generally has an amplitude of only a small fraction of the incident amplitude. The primary reflected wave is reflected (almost completely) from the vertical face of the wave maker, as a secondary incident wave; this is reflected from the beach as a secondary reflected wave, and so on. The higher reflections from the beach have progressively smaller amplitudes and can be neglected. Using these assumptions, Ursell, Dean and Yu (130) have shown that the variation in wave amplitude along the flume is given by

$$a(x) = a_0 \left[1 + \zeta_R \cos(kx + \phi_R) + \zeta_R \cos\phi_R \right]$$
(3.38)

where ζ_R is a reflection coefficient and ϕ_R a phase angle. Ursell, Dean and Yu (130) have also shown that this equation is applicable when written in terms of wave height, provided the reflection coefficient is small. Equation 3.38 predicts that the wave height will vary sinusoidally along the flume with a wave length equal to half that of the incident wave. The mean of this variation is

$$H = 2a_{O} (1 + \zeta_{R} \cos \phi_{R})$$
(3.39)

and the reflection coefficient may be found from

$$\zeta_{\rm R} = \frac{\frac{{\rm H}_{\rm max} - {\rm H}_{\rm min}}{{\rm H}_{\rm max} + {\rm H}_{\rm min}} \tag{3.40}$$

Mitchell et al (83) have measured the variation in wave height along the length of the present facility and determined the reflection coefficient $\zeta_p = 2.12$ %. This value is sufficiently small that reflection can be neglected.

3.4.2 Reflection of Wind-Generated Waves from the Wave Maker

Because of the unique design of the present facility, wind-generated waves will propagate towards the wave maker. Thus, in addition to the mechanically-generated waves being reflected from the beach, the windgenerated waves will also be reflected from the wave maker. This is a potentially serious problem although the wind generated waves are small, as the wave maker will have a reflection coefficient near one. It is possible for these waves to be reflected back along the flume and contaminate the wave field at the test section. A theoretical analysis similar to that of Section 3.4.1 is not possible. As the waves propagate towards the wave maker they will be receiving energy from the wind and will grow. Once reflected, however, they will experience an opposing wind and will lose energy. The magnitude of these reflected waves at the test section was determined by measuring the water surface elevation with and without a temporary beach included at the wave maker end of the flume. In both cases the wave maker was kept stationary and the fan speed was constant at approximately 600 r.p.m. The temporary beach was assumed to have a low reflection coefficient, being constructed of sheet metal and making an angle of 30° with the horizontal. A total of twenty experiments were performed, ten with the temporary beach and ten without, each time series consisting of 16,384 points sampled at 20 Hz. The ten resulting variance spectra for each of the two cases were averaged and the resultant spectra smoothed using an 80 point block average (Appendix H). The large number of points in the time series and the repetition of experiments was used to increase the number of degrees of freedom in the spectral estimate and reduce the confidence limits.

The final variance spectra, with and without the beach, are shown in Figure 3.26 together with their 95% confidence limits. The two spectra are almost identical except for the small secondary peak at 2 Hz for the case without the beach. As this frequency corresponds to the fundamental natural frequency of the flume in the transverse direction, it appears that the reflected waves have excited this natural mode. Even including this difference in the spectra at the natural frequency, the variances for the two cases differ by only 2.3%. Hence, it can be assumed that the influence of reflections from the wave maker end of the flume are insignificant.

3.5 WATER LEVEL SETUP

A steady wind blowing over the water surface exerts a mean horizontal shear stress at the water surface, in addition to the fluctuating stresses that are responsible for wind wave growth and decay. In a confined body of water like the wave flume, this steady surface shear stress τ_s will force and maintain a vertical circulation and water level setup along the flume.

The governing equations are the long wave equations (139) in one spatial dimension

$$\frac{\partial n}{\partial t} + \frac{\partial U}{\partial x} = 0 \tag{3.41}$$

$$\frac{\partial U}{\partial t} + \frac{\partial}{\partial x} \left(\frac{U^2}{d + \eta} \right) = -g(d + \eta) \frac{\partial \eta}{\partial x} + \frac{1}{\rho_w} \left(\tau_s - \tau_b \right)$$
(3.42)

where η is the water surface elevation, U is the depth-integrated flow per unit width, d is the mean water depth, τ_s is the shear stress at the water surface and τ_b is the shear stress at the bottom. Shear stresses on the flume sides are neglected. For the steady state situation, Equation 3.42 reduces to

$$\frac{d\eta}{dx} = \frac{\tau_s - \tau_b}{\rho_w g(d + \eta)}$$
(3.43)

Equations 3.42 and 3.43 assume uni-directional flow throughout the water depth with τ_s and τ_b in opposite directions. In the wave flume, mass conservation requires that there be a vertical circulation with a return flow in the lower section of the flume. Therefore τ_s and τ_b will be in the same direction and Equation 3.43 becomes

$$\frac{d\eta}{dx} = \frac{\tau_s + \tau_b}{\rho_w g(d + \eta)}$$
(3.44)

where $\tau_s = \rho_a u_*^2$ and $\tau_b = f \rho_w \bar{u}^2 / 8$ with f the Darcy-Weisbach friction factor and \bar{u} the average velocity in the bottom boundary layer. Saville (138) has found that for an enclosed lake system $\tau_b \approx 0.1 \tau_s$, which upon substitution into Equation 3.44 yields

$$\frac{dn}{dx} = \frac{1.1 \tau_s}{\rho_w g(d+\eta)}$$
(3.45)

The simulataneous mass conservation equation may be integrated along the flume to give

$$\int_{0}^{L} \eta \, d\mathbf{x} = 0 \tag{3.46}$$

Equations 3.45 and 3.46 were solved simultaneously using a Runge-Kutta algorithm for Equation 3.45 with assumed initial $\eta(x = 0)$. The correct initial condition was determined by a trial and error solution of Equation 3.46.

Solutions are presented in Figure 3.27 for typical u_* values of 0.10 ... 0.25 ms⁻¹ (see Section 8.1). The maximum water surface setup is approximately 2 x 10⁻⁴ m. In addition, Figure 3.27 indicates that the water surface profile is approximately linear with distance along the flume, consistent with equation 3.45 if the assumption $\eta << d$ is made.

These results indicate that the water level setup in the flume is extremely small and the effect of the sloping water surface would be insignificant. In all subsequent analysis, it has been assumed that the mean water level is horizontal.

4. INSTRUMENTATION

The aim of the present project is to determine the energy flux between the wind and the surface waves in an opposing wind situation. Chapter 2 has shown that this flux can be obtained from simultaneous measurements of water surface elevation, surface pressure and turbulent wind velocity. It is important to determine not only the magnitude of these quantities accurately but also their phase relationships, for which a sophisticated laboratory instrumentation system is required. The following sections will describe the present system, including an evaluation of its accuracy and performance.

4.1 WATER LEVEL MEASUREMENT SYSTEM

The water surface elevation was measured by a twin wire resistance The probe consisted of two fine parallel stainless steel wires probe. (diameter 0.25 mm, spacing 10 mm) tensioned in a perspex frame as shown in Figure 4.1. The conduction of electricity between the wires varies with the depth of submergence and hence the electrical resistance of the gauge will decrease with increasing depth of submergence. The lower section of the perspex frame has an elliptical cross-section in order to minimise flow disturbance. A schematic illustration of the apparatus is given in Figure 4.2, and shows the wave probe incorporated as the active element in an A.C.-excited Wheatstone bridge. In operation the bridge was balanced to give a zero voltage output at the mean water level position. The bridge excitation of 5 kHz at 2.5 V r.m.s. was applied by a Sanei M52 carrier strain amplifier. The resulting demodulated amplified output was suitable for direct input to the minicomputer analog to digital (A/D) converters.

Initial tests with the wave gauge system indicated that a lone gauge worked satisfactorily but, when more than one gauge was placed in the flume, there was interference between the gauges, even when the gauges were separated by several metres. Based on advice from Dr. J.L. Hammack of the University of Florida, small audio transformers were included as isolaters on the input side of the Wheatstone bridge. Isolation transformers were already built into the demodulator electronics on the output side. The isolation transformers proved to be very successful and

completely eliminated interference between probes. An electrical circuit diagram showing the Wheatstone bridge and input isolation transformer appears in Figure 4.3.

The calibration system consisted of a long slender vertical rod upon which the wave gauge was mounted and an adjustable pointer gauge mounted atop the flume. The mounting rod was connected to the pointer gauge at one end and protruded through the bottom of the flume at the other. A rubber seal, through which the rod ran, prevented water leaking from the flume. By adjusting the pointer gauge the wave gauge could be raised or lowered, thus altering its depth of submergence. A vernier scale on the pointer gauge provided accurate positioning of the wave gauge to \pm 0.1 mm. The calibration and mounting system for the wave

The calibration procedure consisted of firstly balancing the Wheatstone bridge with the probe submerged in still water to a physical zero reference. This reference was provided by a line marked approximately half way up the vertical leg of the perspex frame. The probe was then moved in 10 mm increments over a range of 50 mm above and below the mean level by manual operation of the pointer gauge. At each level the output voltage was sampled by the minicomputer, the final value being the average of 200 readings taken over a 10 s period. In addition to this mean value, the standard deviation of the 200 points was also calculated to confirm correct instrument behaviour. As the minicomputer was located some distance from the laboratory, the sampling sequence was initiated remotely from the laboratory. Completion of sampling was signalled by a status indication on a small control module.

A typical example of a calibration curve is shown in Figure 4.5. It is very nearly linear and water surface records were reduced to physical units by using the calibration data as a linear look-up table. It was found that the calibration curve for the wave gauges varied slightly from day to day, presumably depending upon impurities in the water, and a fresh calibration curve was obtained at the beginning and end of each day's experiments. Individual wave records were reduced assuming a linear variation with time between these two calibration curves. Such variations were, however, invariably very small, provided the gauges were regularly cleaned with alcohol. If not, dirt tended to accumulate on the wires and

the gauges would exhibit a calibration drift with time. The stability of the gauges can be seen from Figure 4.6 which shows the output voltage from the gauge in still water over a period of four hours. The maximum variation in output voltage over this period is only 1%. Based on these results, it is believed that the wave gauges can measure the water surface elevation to better than ± 0.5 mm.

4.2 WATER LEVEL FOLLOWER

Water level follower or wave follower describes a range of mechanisms which can be used to maintain instruments in close proximity to an undulating water surface. The simplest type of wave follower is a simple float. Mechanically-driven systems with a water level sensor and a negative feedback circuit generally give better frequency response. Although the details of individual systems vary, their basic design is similar and has been reviewed by Shemdin and Tober (112). The following section describes the wave follower used in this project to determine normal and shear stresses a small distance above the water surface.

4.2.1 Wave Follower Description

The wave follower system is illustrated in Figures 4.7 and 4.8 and was powered by a 24 V Electro-Craft Corporation D.C. servo-motor. The rotational motion of the motor was converted to a vertical reciprocating motion through a chain drive. The chain ran about two sprokets; the bottom sproket was mounted in bearings and free to rotate whilst the top sproket was connected directly to the motor shaft. A 5 mm diameter vertical stainless steel shaft was connected to the chain. The shaft was supported by two linear bearings, one at the top of the flume and the other approximately mid-way between the water and the flume top. Connected to the end of the shaft was a 200 mm length of 1.5 mm outside diameter hypodermic tubing. This tubing proved rigid enough to prevent buffeting by the wind yet slender enough not to cause significant flow disturbance. Limit switches, connected in series with the power supply to the motor, were located adjacent to the top and bottom chain sprokets. These switches prevented any possibility of an excessive vertical displacement damaging the drive assembly.

The feedback system for the motor control was provided by the water acting as a switch to complete an electrical circuit. One terminal of the circuit was permanently immersed in the water. The second terminal was the end of the thin tubing attached to the vertical shaft. When the tubing touched the water surface, the circuit was made and when it broke contact with the water surface the circuit was broken. When the circuit was complete, the controlling electronics provided a voltage to the drive motor, causing it to lift the shaft and when contact with the water surface was broken, the motor was directed to drive in the opposite direction. Thus the wave follower hunted for the water surface. The full wave follower assembly was mounted such that the contact point with the water surface was beside the wave gauge at the test section. The wave gauge and the wave follower were separated across the flume by approximately 80 mm and, provided the waves were two dimensional, the wave gauge and the wave follower would be responding to the same water level change.

4.2.2 Wave Follower Performance

The efficiency of the wave follower is measured by the transfer function between the water surface and the wave follower motion. Ideally, such a transfer function should have a gain of one and a phase of zero, indicating that the wave follower followed the water surface perfectly.

To determine the transfer function it was necessary to record simultaneously the water surface elevation and the wave follower position. The water surface elevation was easily measured with the wave gauge. The wave follower position was determined by attaching a ten turn linear potentiometer to the shaft of the drive motor. Thus, when a constant input voltage was applied to the potentiometer, the output voltage was proportional to the wave follower position. This voltage was then sampled directly by the minicomputer. The calibration relationship between the output voltage of the potentiometer and the wave follower position was obtained by manually raising and lowering the wave follower shaft and recording the output voltage.

As mentioned before, the ideal input signal for determining a transfer function is a signal with a white noise spectrum. The input signal in this case was the water surface elevation. Due to the frequency response

of the wave maker such a wave spectrum cannot be generated. Instead, a white noise command signal was used for the wave maker. The resulting wave spectrum and wave follower position spectrum are shown in Figure 4.9. These spectra represent the average of ten experiments, each with a different random phase spectrum for the wave maker command signal. As before, the experiments were repeated to obtain acceptable confidence limits on the spectral estimates. The final transfer function between the water surface elevation and the wave follower position is shown in Figure 4.10 and the corresponding coherence function in Figure 4.11.

The gain of the transfer function is one for frequencies below approximately 1.9 Hz. Above this frequency the gain behaves quite erratically, reaching a maximum of 1.1 at 2.2 Hz, falling to a minimum of 0.75 at 3.0 Hz before rising again to 1.1 and 3.9 Hz. The phase of the transfer function is much better behaved being zero for frequencies below 1.6 Hz. Above this value the phase is positive, indicating the wave follower leads the water surface. The maximum phase difference, however, is only 12° which occurs at 2.8 Hz. This transfer function indicates that the wave follower performs well for frequencies below about 2.0 Hz.

The irregular behaviour of the transfer function is not easily explained but some insight can, however, be gleaned from the corresponding coherence function of Figure 4.11. The coherence function has a value of one for frequencies below 2.0 Hz. Above this value the coherence function decreases rapidly in value. For a linear system, the coherence function $\gamma_{xy}^{2}(f)$ is the fractional portion of the mean square value at the output y(t) which can be directly attributed to the input x(t) at frequency f. If the value of the coherence function falls below unity, one or more of three possible situations exist (5):

- (a) Extraneous noise is present in the measurements
- (b) The system relating x(t) and y(t) is not linear
- (c) y(t) is an output due to other inputs as well as x(t).

It is unlikely that the system would suddenly become non-linear above 2.0 Hz when it behaved quite well below this value but the other two explanations for a reduction in the value of $\gamma^2(f)$ are, however, quite possible. Indeed, it has been observed that above 2.0 Hz the mechanically generated waves

lose their two dimensional nature and become short crested. Since the wave gauge and the wave follower are separated by 80 mm in the across flume direction, the wave measured at the wave gauge may not be the input driving the wave follower. This will be especially true if the waves are short crested. In addition, the wave spectrum (Figure 4.9) which provides the input to the system has a peak at 1.5 Hz and decreases rapidly in magnitude for higher frequencies. Therefore, as the frequency increases, the signal to noise ratio will decrease and noise may well contaminate the system. These two factors tend to indicate that the behaviour of both the transfer function and the coherence function are results of the experimental technique and not instrument response.

In conclusion, it can be said that the wave follower accurately tracks waves up to a frequency of at least 2.0 Hz. The instrument response above this frequency cannot be accurately determined. In view of the observed short crested behaviour of waves above 2.0 Hz, the experimental study was limited to frequencies below 2.0 Hz.

An additional experiment was conducted to determine the frequency at which the wave follower hunted for the water surface. With a stationary water surface, the wave follower position was sampled at 100 Hz for a period of 2.73 min, yielding a time series of 16,384 points. The resulting spectrum appears in Figure 4.12. This figure clearly indicates that the hunting frequency is 16 Hz. Since this value is far above the wave frequencies used in this project, it is reasonable to assume that the high frequency oscillations of the wave follower will have no significant effects on experimental results.

4.3 PRESSURE MEASUREMENT SYSTEM

4.3.1 Description of System

The pressure measurement system was designed to measure the very small wave-induced static pressure above the waves. The system has two parts, a probe section located within the flume and a sensing section located externally. The probe system consisted of three individual probes. A disk probe and a total head probe were mounted on the wave follower close to the water surface whilst a static probe was located in the free
stream flow, 0.7 m above the mean water level. The sensing system consisted of two Setra Systems Model 239 E capacitance type low range differential pressure transducers with a full scale range of ± 69 Pa. The probe and sensor systems were connected through a system of valves and thin tubing, allowing the transducers to be switched to one of three positions. The transducers could be switched either to the probe system or a calibration system and when not in use they could be vented to atmosphere. A schematic diagram showing the full system appears in Figure 4.13.

The disk probe was constructed from perspex and was 9.4 mm in diameter with a thickness of 2.6 mm. A small hole of diameter 0.5 mm passed through the disk in a transverse direction. This small passage was intersected mid-way through the disk by a radial passage. The radial passage was in turn connected to the pressure tubing. The disk probe is illustrated in Figure 4.14. Because of the finite thickness of the probe, it compresses the adjacent streamlines and will record a pressure slightly below the true static pressure. Bryer and Pankhurst (16) indicate that the pressure recorded by the disk, p_p, follows the relationship

$$p_{s} - p_{D} = K \frac{1}{2} \rho v^{2}$$
 (4.1)

where p_s is the static pressure, v is the wind velocity and K is a constant which must be determined by calibration.

Equation 4.1 indicates that it is necessary to also know the instantaneous wind velocity at the probe, which required the installation of a total head probe on the wave follower adjacent to the disk. This probe consisted simply of a stagnation tube constructed from 1.62 mm outside diameter hypodermic tubing aligned into the flow. The pressure sensed by the total head probe is

$$p_t = p_s + \frac{1}{2}\rho v^2$$
 (4.2)

where p_t is the total pressure. The disk and total head probes were connected to either side of one of the differential pressure transducers. The differential pressure sensed by the transducer is then

$$p_{\rm D} - p_{\rm t} = -\frac{1}{2}\rho v^2 (1 + K)$$
(4.3)

Therefore, since K is known from calibration, the velocity can be determined.

This single differential pressure record is sufficient to determine v but an additional measurement is required to determine p_s . This additional measurement was achieved by placing a Pitot-static probe in the free stream of the flow. The pressure sensed by the static port of this probe was connected across a second transducer with the disk output. This second transducer then sensed the differential pressure

$$p_{\rm D} - p_{\rm so} = (p_{\rm s} - p_{\rm so}) - K^{1}_{2}\rho v^{2}$$
 (4.4)

where p_{SO} is the free stream static pressure. Since $\frac{1}{2}\rho v^2$ was obtained from the first transducer output in conjunction with Equation 4.3, the output of the second transducer, along with Equation 4.4, yields the quantity $p_{S} - p_{SO}$. As p_{SO} is merely the mean static pressure within the flume, this term represents the wave-induced pressure. The three pressure probes are shown in Figure 4.15.

The voltage output from the two pressure transducers was ± 2.5 V for the full scale range of ± 69 Pa. As the pressures to be measured could be as small as 1 Pa, it was necessary to amplify the output signals. It was also important to ensure that extraneous noise did not contaminate the pressure signals. Although the manufacturer's specifications indicated that electrical noise levels were less than 0.02% of the full range output, the signals were passed through 5 Hz low pass filters. The filtered signals were then fed into D.C. amplifiers with selectable gains of 1, 5 or 10. The output from the amplifiers was then recorded directly by the minicomputer. A schematic diagram of this system appears in Figure 4.16.

4.3.2 Pressure Transducer Calibration

Calibration of pressure transducers with such extreme sensitivity poses considerable problems. Merely applying a differential pressure of order 10 Pa, only 0.01% of atmospheric pressure, is no trivial task. Initially, an extremely accurate manometer system was designed and constructed for this purpose but it proved unsuitable. Any system which creates a static differential pressure requires the transducer to be connected to a closed volume, where a change in temperature ΔT will cause a change in pressure Δp as predicted by the ideal gas law

$$\Delta T = \frac{\Delta p}{p} T$$
(4.5)

where p and T are the initial pressure and temperature in the closed volume. For $p = 10^5$ Pa (1 atmosphere) and T = 298°K (25°C), a temperature change of only 0.2°C is required to produce a pressure change of 69 Pa, equal to the transducer full scale value. Such large changes in the pressure were observed with the manometer system as a result of small air temperature fluctuations; they persisted even when the manometer system was housed in thick thermal insulation.

To avoid these problems, a system which generated a dynamic pressure, and hence avoided the problems associated with a closed volume, was designed. The final system relied upon measuring the pressure distribution about a cylinder in a uniform flow. A cylinder of diameter 16 mm was placed in a small aerodynamic wind tunnel capable of producing a wind velocity of 7 ms⁻¹, and shown in Figure 4.17. A pressure tap was placed on the surface of the cylinder and this tapping was connected to one port of the transducer to be calibrated. The second transducer port was connected to a static pressure tap located on the wall of the wind tunnel, the transducer measuring the dynamic pressure at the cylinder's surface. By rotating the cylinder, the differential pressure ranges from approximately +25Pa at the forward stagnation point ($\theta = 0^\circ$) through 0 Pa at about $\theta = 50^\circ$ to -18Pa for θ beyond about 80° for a centreline wind velocity of 6.5 ms⁻¹.

The calibration procedure consisted of firstly rotating the cylinder until the surface pressure port faced directly into the air flow. The wind velocity was then gradually increased until a precision inclined manometer, connected in parallel with the pressure transducers, indicated that the differential pressure was approximately equal to the transducer full scale positive value. The cylinder was then rotated in small increments, the manometer reading noted and the minicomputer prompted to sample the

transducers. The prompt to the minicomputer was initiated from the remote module described in Section 4.1. Each sample consisted of an average over ten seconds. This procedure was continued until the manometer indicated that a pressure equal to the transducer full scale negative value had been reached. A schematic diagram of this calibration system appears in Figure 4.18.

A typical calibration curve, obtained through the above procedure is shown in Figure 4.19. The relationship is linear to good approximation. As with the wave gauges, the pressure transducers were calibrated at the beginning and end of each session of experiments (typically 2 hrs), but variations in the calibration curves were quite small. Hence, no calibration changes were necessary during a session of experiments.

4.3.3 Disk Pressure Probe Calibration

Use of the disk pressure probe system to measure static pressures required the determination of the calibration constant K in Equation 4.1. The disk probe was mounted in the small aerodynamic wind tunnel mentioned previously, with a standard Pitot-static tube directly alongside. two pressure transducers were then connected such that one measured $p_s - p_p$ and the other $p_{+} - p_{s} = \frac{1}{2}\rho v^{2}$, where p_{s} and p_{+} are the pressures recorded by the static and total pressure ports of the Pitot-static tube and p_{D} is the pressure measured by the disk probe. The outputs from the transducers were then recorded for wind velocities ranging from 0 ms⁻¹ to 7 ms⁻¹. This range of wind velocities corresponds to the range used in subsequent experiments and eliminates the need to investigate any Reynolds number dependence in the calibration result. The final calibration curve appears in Figure 4.20. It is clear from this figure that the linear relationship predicted by Equation 4.1 is a reasonable approximation to the data. A least squares approximation to the data yields $K = 0.22 \pm$ 0.02. This value is higher than typical values quoted by Bryer and Pankhurst (16) but this was expected as the present probe has a relatively high thickness to diameter ratio.

In the above calibration procedure, the disk was aligned so that it lay in a vertical plane with its edge pointing into the air flow. To assess the probes complete usefulness, however, it was also necessary

to measure its response to pitch (rotation about the transverse horizontal axis) and yaw (rotation about the vertical axis). The results are presented in Figure 4.21. Because of the symmetric nature of the disk it has a flat response to variations in pitch. This is a considerable advantage in the present context since the pitch angle will vary due to the undulating nature of flow over the surface waves. As expected, the probes response to yaw is more dramatic. The results show a region of approximately $\pm 5^{\circ}$ in which the response is flat. For larger yaw angles the pressure increases rapidly, presumably as a result of flow separation from the edge of the disk. Fortunately, the probe should experience. little variation in the angle of yaw as the flow in the wind-wave flume is two dimensional. The flat section of the curve is, however, sufficiently wide to prevent significant errors due to probe misalignment with the flow.

4.3.4 Calibration of Total Probe for Pitch and Yaw

The effects of pitch and yaw are also important for the total head probe, although there is no distinction for a symmetric probe. The same procedure as previously described for the disk probe was again used for the total probe, the results appearing in Figure 4.22. The calibration curve is relatively flat for angles up to 21°, but beyond this the pressure sensed by the probe decreases rapidly. The maximum slope possible for gravity waves is $\pi/7$, which corresponds to an angle of 24° to the horizontal. It is reasonable to assume that the maximum angle of pitch that the probe would experience would also be of a similar value. Since the probe response is relatively flat for angles less than 21° any errors due to the angle of pitch of the probe relative to the air flow would be quite small.

4.3.5 The Effects of Flow Turbulence

When placed in a turbulent flow, pressure probes will not only respond to the static and dynamic pressures but also to the normal Reynolds stress terms $-\rho u''u''$, $-\rho v''v''$ and $-\rho w''w''$. Goldstein (37) has considered the effects of turbulence on a total pressure or Pitot tube. He concludes that the reading of a Pitot tube will exceed the total pressure by an amount corresponding to the mean kinetic pressure of the turbulent velocity fluctuations, u", v", w". The magnitude of the increase in Pitot reading

is given simply by $\frac{1}{2}\rho_{a}(\overline{u^{"}u^{"}} + \overline{v^{"}v^{"}} + \overline{w^{"}w^{"}})$. For a high turbulence intensity (113)

$$I_{u} = 0.15 = \frac{\sqrt{u''u''}}{\bar{u}}$$
 (4.6)

 $\overline{u^{"}u^{"}}$ equals 0.0225 u. In addition, measurements have shown that, in the surface layer (113) $\overline{u^{"}u^{"}}$: $\overline{v^{"}v^{"}}$: $\overline{w^{"}w^{"}} \rightarrow 1$: 0.6 : 0.3, from which the total effect of the turbulence would be only 4% of the dynamic pressure $\frac{1}{2}\rho_{a}\overline{u^{2}}$. This error has been neglected.

There is almost no mention in the literature of the effects of turbulence on a disk probe. It is reasonable to assume, however, that the turbulent fluctuations will be quite small at the centre of the disk. A similar assumption is commonly made for a surface pressure tapping in parallel streamline flows where Shaw (107) has shown that the error caused is less than 1% of $\frac{1}{2}\rho_{a}\bar{u}^{2}$. Since the flow past the disk is not dissimilar to that near a wall, turbulence effects on the disk probe can be neglected with reasonable confidence.

4.3.6 The Dynamic Response of Pressure Tubing

Long lengths of tubing can have a considerable influence on the measured fluctuating pressures, the classical example being organ pipe resonance (105). An extensive theoretical analysis of the problem has been presented by Bergh and Tijdeman (6) and is outlined in Appendix A. Each of the pressure probes was connected to the pressure transducers by a 2 m length of 2 mm inside diameter flexible plastic tubing followed by a 1.5 length of 3 mm inside diameter P.V.C. tubing. Along the length of this second section of tubing there were a number of valves and manifolds which are not included in Bergh and Tijdeman's analysis. Their theoretical prediction of the transfer function appears in Figure 4.23. Figure 4.23a shows the transfer function for the frequency range 0 Hz to 4 Hz. The gain of the transfer function is relatively flat in this region, steadily increasing from 1.0 at 0 Hz to 1.03 at 4 Hz. The phase lag of the transfer function steadily increases and reaches a value of 19.4° at 4 Hz. Although the high frequency response of the tubing system is of no immediate interest in this project, Figure 4.23 b shows the transfer function for frequencies

up to 300 Hz. This figure clearly illustrates the presence of resonance peaks at a number of frequencies as well as the general fall off in response due to frictional losses to the tube walls.

As the phase lag is particularly critical to the present experiments and as the valves and manifolds of the tubing system are not included in the above analysis, it was desirable to confirm, or otherwise, this theoretical prediction of the transfer function. To measure the actual transfer function for the various tubes, an apparatus capable of generating small dynamic pressures in the frequency range from 0 Hz to 4 Hz was designed and constructed. It consisted of a metal drum of volume 60 l connected to a small piston and cylinder of diameter 25 mm. The cylinder was a modified bicycle pump, the pump bucket having been removed and replaced with a piston sealed with "O" rings. Thus pushing the piston in would increase the pressure in the drum whilst pulling the piston out would reduce the pressure in the drum. The piston was driven by a crank shaft and connecting rod system, the crank shaft being driven through a double reduction 16 to 1 chain drive powered by a variable speed electric motor. By varying the length of the crank shaft, the piston stroke length could be varied and hence oscillating pressure signals of various amplitudes were generated within the drum. The frequency of the pressure signal was varied by altering the speed of the electric drive motor. To prevent pressure changes within the drum due to atmospheric temperature variations, a small air leak to atmosphere was provided. Since this leak has a relatively long time constant, it had no effect on the generated pressure signals. The full dynamic pressure generating system is illustrated in Figure 4.24.

To determine the response of the pressure tubing system, one pressure transducer was placed flush against the drum and a second was connected to the end of the pressure tubing with the pressure probe sealed within the drum. The first transducer recorded the actual dynamic pressure signal within the drum whereas the second transducer recorded the pressure signal after modification by the tubing. The transfer function can be determined from these two signals. If the pressure in the drum is described by A coswt, the signal at the end of the tubing is $A|H|\cos(\omega t + \phi)$, where |H| and ϕ are the gain and phase of the transfer function respectively. Each record yields the transfer function for only one frequency and it was necessary to repeat the process a number of times with different frequency signals to determine the full transfer function.

A basic assumption in this analysis is that the pressure sensed by the probe and that by the flush mounted transducer are equal. This will only be true if the pressure response within the drum is instantaneous. Obviously such a response would not occur and thus there would be a further transfer function relating the two pressures. The drum was 0.37 m in diameter and 0.56 m long, whilst the flush transducer tapping and the probe were separated by only 50 mm. Hnece, in relation to the overall drum dimensions the two pressures were sensed at approximately the same point. Therefore the effects of the flow distribution within the drum would be minimised and the flush transducer and probe can be assumed to be sensing the same pressure to reasonable accuracy.

To apply the above analysis, it is necessary that the pressure generated in the drum be sinusoidal; this became the major design constraint for the dynamic calibration system. Applying the universal ideal gas law, the pressure in the drum is given by

$$\Delta p = \frac{p_1 \Delta V}{V_1 - \Delta V} \tag{4.7}$$

and since $\Delta V << V$ for this design

$$\Delta p \approx \frac{p_1 \Delta V}{V_1} \tag{4.8}$$

where Δp is the change in pressure caused by the volume change ΔV , p_1 is the initial pressure and V_1 is the initial volume of the drum-cylinder system. Therefore, if ΔV varies sinusoidally and the pressure response is immediate and uniform, Δp will also be sinusoidal. A sinusoidal variation of ΔV implies a sinusoidal variation of the piston. For the crankshaft-connecting rod system, the motion of the piston is described by (84)

$$x = r \left[\cos\theta + q - \frac{1}{2q} \sin^2\theta - \frac{1}{8q^3} \sin^4\theta - \dots \right]$$
(4.9)

where r is the crankshaft length, qr is the connecting rod length and θ is the angle the crankshaft makes with the horizontal. The motion will

only be sinusoidal if the connecting rod is of infinite length but sufficient accuracy can be obtained if q > 4.0. For the present case q = 4.4 which produces a second order term in Equation 4.9 of only 5% of the sinusoidal term.

Transfer functions were evaluated for each of the four probe-tube systems used in subsequent experiments. These systems were the disk probe line to transducer number one, H_{Di} , the total probe line to transducer number one, H_{T_1} , the disk probe line to transducer number two, H_{D_2} , and the static probe line to transducer number two, H_{S_2} . The four resulting transfer functions appear in Figure 4.25 and differ quite markedly from the theoretical predictions of Figure 4.23. These differences reflect the result of neglecting the presence of the probe and the various valves and manifolds in the pressure lines in the theoretical analysis. All of the transfer functions are of similar shape showing a continual decrease in |H| and increasing phase lag with increasing frequency. Least squares polynomial approximations were applied to the data with the results

$$|H_{D_1}| = 1 + 1.69 \times 10^{-2} f - 6.20 \times 10^{-2} f^2 + 1.17 \times 10^{-2} f^3$$

- 4.45 x 10⁻⁴ f⁴ (4.10a)

$$\phi_{D_1} = -15.6f$$
 (4.10b)

$$|H_{T_1}| = 1 + 2.53 \times 10^{-2} f - 4.85 \times 10^{-2} f^2 + 1.39 \times 10^{-2} f^3$$

- 1.38 x 10⁻³ f⁴ (4.11a)

$$\Phi_{T_1} = -10.4f$$
 (4.11b)

$$|H_{D_2}| = 1 - 6.02 \times 10^{-2} f + 2.46 \times 10^{-2} f^2 - 3.05 \times 10^{-2} f^3 + 5.71 \times 10^{-3} f^4$$
(4.12a)

$$\phi_{\rm D_2} = -20.5f$$
 (4.12b)

$$|H_{S_2}| = 1 - 8.35 \times 10^{-2} f + 1.08 \times 10^{-2} f^2 - 5.76 \times 10^{-3} f^3$$

+ 4.68 × 10⁻⁴ f⁴ (4.13a)

$$\phi_{s_2} = -19.4f$$
 (4.13b)

with 95% confidence limits of $\pm 5\%$ for the gain equations and $\pm 7\%$ for the phase. The pressure tubing system has a very significant impact on the recorded pressure signal. If such effects has been ignored, the results of any measurements would be in serious error.

4.3.7 Wave-Follower-Induced Pressure

The vertical oscillatory motion of the wave follower will cause the column of air within the tubing to be accelerated, resulting in an oscillatory pressure that will be sensed by the pressure transducers. If the motion of the wave follower is $z = a \cos \omega t$, then the acceleration is $\ddot{z} = -a\omega^2 \cos \omega t$. For a rigid tubing system between the probe and pressure transducer, it is then reasonable to assume that the induced pressure will be of the form $p_{WF} \alpha \omega^2 \cos \omega t$ as proposed by Shemdin (108). In the present experimental setup, the tubing is not rigid and the wave follower induced pressure may well deviate from the above relationship. A series of experiments were consequently conducted to determine the transfer function relating the motion of the wave follower to the induced pressure.

The wave follower was driven in a sinusoidal fashion by a signal generator in the absence of waves and the motion of the wave follower was recorded by the potentiometer on the drive motor. Initial experiments with commercial signal generators proved unsuccessful as any slight D.C. bias in the signal would cause the wave follower to gradually move beyond its vertical traversing limits. This problem was overcome by using an extremely sensitive signal generator designed by Dr. C. Kikkert of the Department of Electrical and Electronic Engineering at James Cook University. Rather than measure the induced pressure for each line, the differential pressures between the disk and total probe lines and the disk and static probe lines were recorded. These were the only pressures of relevance, being the differential pressures subsequently measured in experiments. The resulting transfer functions for these two system appear in Figure 4.26.

The gain of both transfer functions are rather similar and both approximate to a power law relationship with frequency. Least squares curve approximations give for the disk-static system

$$|H_{\rm DS}| = 11.6 \ f^{1.82}$$
 (4.14a)

whereas for the disk-total system

$$|H_{DT}| = 1.70 f^{2.66}$$
 (4.15a)

with 95% confidence intervals of ±5%. Thus, neither transfer functions follows the f^2 relationship expected for a rigid tubing system, but neither are they greatly removed from this relationship. As well as the differences in slope of the two relationships, it is significant that the magnitude of the disk-static system is considerably larger than the disk-total This is apparently caused by the disk and total probes being system. mounted beside each other and their pressure tubes being taped together for the major part of their length. Hence the flexing of the tubing as the wave follower moved was almost identical for each system. If the induced pressures in each length of tube were almost identical the resulting induced differential pressure would be small. Examination of the individual pressure signals revealed that this was the case. The induced pressures in the disk and total lines were very similar whereas the induced pressure for the static line was considerably smaller, thus accounting for the observed magnitudes of the transfer functions.

The phase relationships of the two transfer functions are also similar, exhibiting phase lags which increase with increasing frequency. The actual magnitudes of these phase angles, however, are quite different. Least squares curve approximations gave

$$\phi_{\rm DS} = -2.55 - 17.2f + 3.56f^2 - 0.580f^3 \tag{4.14b}$$

and

$$\phi_{\rm DT} = -116 - 1.48f - 4.69f^2 + 0.615f^3 \tag{4.15b}$$

with 95% confidence intervals of ±6%. The phase relationship for the disk-

static system is as expected, with the phase lag approaching zero at f = 0. In contrast, the behaviour of the disk-total system is rather unusual with an apparent phase lag of approximately 116° at f = 0. Such a value is clearly not realistic. This behaviour can, however, be explained by considering the induced pressures in each line and their differences. If the pressures in two lines are given by A sin ω t and B sin $(\omega t + \Delta \phi)$, the differential pressure between the two is given by

$$C \sin(\omega t + \alpha) = A \sin \omega t - B \sin(\omega t + \Delta \phi)$$
(4.16)

whence

$$C = (A - B \cos \Delta \phi) / \cos \alpha$$
(4.17a)

and
$$\tan \alpha = \frac{-B \sin \Delta \phi}{A - B \sin \Delta \phi}$$
 (4.17b)

When A and B differ considerably the resultant differential phase angle varies almost linearly with $\Delta\phi$. If A and B are of similar magnitude, however, a small phase difference, $\Delta\phi$ produces a large differential phase angle α . As $\Delta\phi$ increases, however, α reduces considerably in magnitude as can be seen in Figure 4.27. The unusual behaviour of the phase relationship for the disk-total system is most likely a consequence of the similar magnitudes of the pressure signals in each line. The disk-static system behaves completely differently since the pressure signals in the individual lines are of significantly different magnitude.

4.3.8 Response of Electronic Systems

The output voltages from the pressure transducers were firstly passed through 5 Hz low pass filters to remove any high frequency noise and then amplified to obtain reasonable resolution on the minicomputer's 12 bit A/D converters. Recursive analog filters can introduce considerable phase lags as well as having non-ideal cut-off behaviour, and a set of experiments was performed to determine the transfer functions for the low pass filters. The transfer functions for the D.C. amplifiers were also found experimentally.

The experimental procedure consisted of firstly generating a digital white noise signal on the minicomputer. This signal was then fed through a D/A converter to produce an analog signal. This signal was passed through the filter or amplifier and the output sampled by the minicomputer through an A/D converter. The transfer function was then found from Equation 3.32. The resulting transfer functions for the two 5 Hz low pass filters appear in Figure 4.28. They are quite similar, although not identical, with a gradual fall off in the gain of the transfer function with increasing frequency, becoming considerably steeper above 5 Hz. There are very large phase lags; for example, at approximately 3.75 Hz the output signal lags the input by 180° .

The transfer functions for the D.C. amplifers showed completely flat gain characteristics and zero phase errors for the frequency range of interest in this project. Indeed, the response was flat up to frequencies of 10 k Hz.

4.4 VELOCITY MEASUREMENT SYSTEMS

4.4.1 Hot Film Anemometer System

Mean velocity measurements were made using a standard Pitot-static tube and an inclined manometer but such a system is unsuitable for the measurement of turbulence. Turbulent velocity traces were obtained with a dual channel I.S.V.R. constant temperature hot film anemometer (24). For the measurement of single velocity components, the probe was a TSI Model 1210-20 standard hot film probe and for two component measurements the probe was a TSI Model 1243-20 boundary layer cross film probe. Both these probes have a diameter of 51 microns and a sensing length of 1 mm, giving a length to diameter ratio, $\ell/d = 20$. A special mounting bracket on the wave follower enabled both the yaw and pitch angles of the probe to be individually adjusted to ensure accurate alignment of the probe. This mounting system is shown in Figure 4.29. The analog outputs from the hot film anemometer was fed through low pass anti-aliasing filters and recorded either by the PDP-11 minicomputer or the HP spectrum analyser (see Section 4.5).

4.4.2 Probe Calibration

The calibration of a hot film probe simply requires recording the anemometer output voltage for various known air velocities. Additional insight can, however, be gained by considering some practical anemometer cooling laws. The heat loss from the film includes both the convective loss to the flow and conductive loss to the supports. In normal constant temperature operation, the probe resistance is maintained substantially constant. The probe forms one arm of a Wheatstone bridge which is automatically maintained in balance by a specially designed amplifier. The heat supplied to the wire is proportional to E^2 , the square of the anemometer bridge output voltage. The heat lost due to forced convection can be expressed by the difference $E^2 - E_0^2$, where E_0 corresponds to the bridge output at zero flow. A practical correlation between flow velocity and convective heat loss can be expressed by the empirical law

$$E^{2} = E_{0}^{2} + KU^{n}$$
 (4.18)

where the constant K and the exponent n are functions of the flow speed and the probe geometry. A number of so-called universal values have been proposed for n, the best known being "King's law" (22) in which n = 0.5. More recent work has shown that an exponent of 0.45 gives a better correlation than an exponent of 0.5 in the Reynolds number range usual in hot-wire anemometry (13) although there is some evidence that the value of n is dependent upon the probe Reynolds number (24). Therefore, it is good practice to determine n from calibration results. In addition, the intercept value of Equation 4.18 is generally dependent upon the temperature of the fluid in which the probe is immersed. Hence the calibration curve can drift as a result of room temperature changes.

The calibration curve parameters E_0 , K and n were determined, in situ, in the flume free stream flow against a standard Pitot-static tube. In addition, the probes were recalibrated at intervals not exceeding one hour to prevent excessive drift of E_0 due to temperature changes. Such temperature effects were further reduced by performing experiments either in the evening or the middle of the day, when the room temperature was relatively stable. The parameters of the calibration curve were determined from a least squares approximation to the data, as outlined in Appendix I. A typical calibration curve appears in Figure 4.30.

4.4.3 Probe Alignment

According to the equations of motion, the flow over an infinitely long wire in the plane normal to its axis should be independent of the velocity along the axis of the wire. The heat transfer, on the other hand, depends upon both the axial and the normal components of flow but to first approximation is independent of the axial component of velocity. This cosine law for the effective cooling velocity U_{eff} is (13)

$$U_{\text{eff}} = U \cos \psi \tag{4.19}$$

where ψ is the angle between the flow direction and the plane normal to the wire axis. In addition to this simple relationship, a number of empirical laws, which include axial cooling, have been proposed. Champagne et al (20) have suggested that for hot wire probes

$$U_{eff} = U(\cos^2\psi + k_1^2 \sin^2\psi)^{\frac{1}{2}}$$
(4.20)

where k_1 falls from 0.2 at l/d = 200 to zero at l/d = 600, the range of validity being $25^{\circ} < \psi < 60^{\circ}$. Friehe and Schwartz (33) have proposed

$$U_{eff} = U \left[1 - k_2 (1 - \cos^{\frac{1}{2}} \psi) \right]^2$$
(4.21)

where $k_2 = 1-2600 (d/l)^2$ for hot wires and 1-2.2 d/l for cylindrical hot films, the range of validity being $0^{\circ} < \psi < 60^{\circ}$. For the hot film probes used in this project $k_2 = 0.89$. To assess the sensitivity of these probes to the longitudinal cooling component, Equations 4.19 and 4.21 are compared in Figure 4.31. The deviation from the cosine relationship is very small for this probe and the cosine law has been adopted in all subsequent analyses.

Based on the cosine relationship, it is clear that alignment is not critical for a probe placed approximately perpendicular to the air flow, as the cooling velocity is relatively invariant for $-15^{\circ} < \psi < 15^{\circ}$. For cross film probes, which are at approximately $\pm 45^{\circ}$ to the flow, however, alignment is critical. For such probes an alignment error of 5° can result in an error of 9% in U_{eff}. Because of the sensitivity of the cross film probes to alignment, the housing bracket which connects the probe to the wave follower was fitted with a yaw adjustment (see Figure 4.29). Using this adjustment the probe was accurately aligned $(\pm 0.5^{\circ})$ against a theodolite.

4.5 DATA ACQUISITION AND EXPERIMENTAL CONTROL

4.5.1 Minicomputer System

All experimental control and most data acquisition was handled by the PDP-11 minicomputer. The computer served the dual role of simultaneously controlling the wave generator and sampling from up to eight channels. Although the experimental control was largely automatic, remote control of each stage was available from a control module located in the laboratory.

In almost all experiments, there were three individual stages. The first stage involved moving the wave maker forward to its maximum position and halting. The piston was moved forward with a steady linear motion to its maximum foward position, corresponding to the starting point of the wave record at its lowest trough. At this point the gradient of the wave record was zero enabling smooth transition from rest into the wave record signal. On a signal from the control module, the wave maker began to cycle through its programmed wave record. This phase allowed the wave pattern to stabilise before sampling commenced; a period of five minutes was typical. The third and final stage of the experiment was the data acquisition. While continuing to cycle, sampling took place at a selected rate and, upon completion, the wave maker was moved back to its initial position with a steady linear motion.

Each of these successive phases was initiated by an enable signal sent to the minicomputer from the control module. The enable signals consisted of simple, manually selected discrete D.C. voltage levels. At any stage during execution, the minicomputer program status was indicated on the control module by a series of lights, successively illuminated by a signal from the minicomputer upon successful entry into the next phase of the program. The module also incorporated eight manually selectable low pass filters, with cut-off ranging from 1.0 to 24 Hz, to reduce input of high frequency noise to the servo-amplifier, thus smoothing wave maker operation.

The wave maker transfer functions described in Section 3.3.2 were obtained with the module low pass filters set at 6 Hz. If another filter setting had been used it would have been necessary to obtain a new wave maker transfer function corresponding to this setting. An automatic cut out ensured that, should the minicomputer crash, the resulting voltage surge would not damage the wave maker.

The sampled data was transferred from the computer's memory to floppy disks for storage and subsequently to the University's main frame computer system, a DEC-SYSTEM 1091, for further analysis. The full control and data acquisition system was shown schematically in Figure 3.5.

4.5.2 Spectrum Analyser

The maximum sampling rate which could be obtained from the minicomputer using FORTRAN was approximately 300 Hz. Although this was more than adequate for most requirements, it was not adequate for determining the turbulent structure of the air flow close to the water surface. Consequently the turbulence data from the hot film anemometer was recorded using a Hewlett Packard HP 3582 A Spectrum Analyser, capable of sampling at selected rates up to 50 k Hz and determining amplitude spectra, coherence functions and transfer functions automatically. The Spectrum Analyser is shown in Figure 4.32 along with much of the other instrumentation described in this chapter.

5. SURFACE DRIFT CURRENT

When the wind blows over a water surface, fluctuating stresses can lead to the growth or decay of waves. In addition the mean shear stress will cause a momentum transfer to the water which induces a surface current. provided the wind duration is relatively short, such currents are confined to the region near the water surface and are called wind-induced surface drift currents. The wave flume is an enclosed basin and the conservation of mass requires a return flow in the lower section of the flume. Since such a recirculating flow will influence the phase speed of the surface waves, the magnitude of the surface drift current was determined experimentally.

Small styrofoam floats were timed over a 4.6 m length of the flume. The floats were flat disks to keep them low to the water surface and to minimize additional drift due to direct wind-induced drag on the float. In addition, the depth of sumbergence of the floats was very small and their velocity could be assumed to quite closely approximate the surface drift velocity, rather than a depth-integrated value. The experiments were conducted for free stream wind velocities ranging from 1 to 6 ms⁻¹ and in the absence of mechanically-generated waves. Each experiment was repeated ten times and the results averaged. The data is presented in Table 5.1 and plotted in Figure 5.1.

U _∞	u _c
(ms ⁻¹)	(ms ⁻¹)
0.85	$(1.26 \pm 0.13) \times 10^{-2}$
1.60	$(2.50 \pm 0.24) \times 10^{-2}$
2.08	$(3.15 \pm 0.20) \times 10^{-2}$
2.62	$(3.85 \pm 0.35) \times 10^{-2}$
3.25	$(4.75 \pm 0.38) \times 10^{-2}$
3.75	$(5.65 \pm 0.30) \times 10^{-2}$
4.12	$(6.53 \pm 0.47) \times 10^{-2}$
4.89	$(7.32 \pm 0.41) \times 10^{-2}$
5.89	$(8.63 \pm 0.33) \times 10^{-2}$

Table 5.1 Surface drift current versus free stream wind velocity.

From Figure 5.1 there appears to be a linear relationship between u_c and U_{ω} , the free stream wind velocity. A least squares approximation yields the result

$$u_c = (0.015 \pm 0.002) U_{\infty}$$
 (5.1)

where the error represents 95% confidence limits. Both the magnitude of this drift current and the linear dependence upon the wind velocity are consistent with the results of Keulegan (58), Van Dorn (131) and Shemdin (108).

One potential consequence of the wind-induced drift current is to alter the phase speed of the surface waves. Lilly (68) has considered this problem, assuming the current profile to have a parabolic shape with zero net mass transport and the flow to be laminar. He shows that the phase speed becomes

$$\bar{c} = c \left\{ 1 + \frac{u_c}{c} \left[1 + \frac{3}{2(kd)^2} - \frac{1 + 2 \cosh(2kd)}{kd \sinh(2kd)} \right] \right\}$$
(5.2)

where $C^2 = (g/k) \tanh(kd)$ and d is the water depth. In the present context C and u_c are in opposite directions and Equation 5.2 indicates that the wave phase speed will be reduced by the presence of the current.

In Table 5.2 Equation 5.2 has been applied to the wind wave flume for a free stream wind velocity of $U_{\infty} = 6 \text{ ms}^{-1}$ to assess the relative importance of the drift velocity in altering the wave phase speed.

It is clear that the effects of the surface drift current become more important as the wave frequency increases, the wave orbital velocity field becoming concentrated closer to the surface where the drift current is a maximum. The 6 ms⁻¹ wind velocity used in Table 5.2 is the maximum wind velocity used in this research. The magnitude of the surface drift current and its effects on the phase speed will be reduced for lower wind speeds. In addition, Table 5.2 gives only a rough estimate of the effects of the surface current, since Lilly (68) assumed laminar flow in deriving Equation 5.2. Dye tracer experiments, however, indicated the flow in the experimental facility was in fact turbulent. Despite this, Table 5.2

f (Hz)	kd (d = 0.32 m)	C (ms ⁻¹)	$C - \overline{C}$ (ms ⁻¹)	% difference between C and C
0.75	0.97	1.56	1.26×10^{-2}	0.8
1.00	1.44	1.40	2.25×10^{-2}	1.6
1.25	2.08	1.21	3.33×10^{-2}	2.8
1.50	2.91	1.04	4.41×10^{-2}	4.2
1.75	3.95	0.89	5.31×10^{-2}	6.0
2.00	5.15	0.78	6.06×10^{-2}	7.8

Table 5.2 Effect of surface drift current on wave phase speed.

should provide a reasonable order-of-magnitude estimate, the alteration in phase speed being, at most, 10% with an average value of approximately 3%. Based on these figures, the reduction in phase speed caused by the surface drift current should be a secondary effect and has been neglected in all subsequent analyses.

It was shown in Section 2.4 that an opposing current can lead to very substantial wave decay. Using Equation 2.63, the ratio of the wave energy after encountering a current to that before can be calculated. These values have been evaluated for the experimental facility and appear in Table 5.3.

f (Hz)	ω _ο (Hz)	u † (ms ⁻¹)	ພ ູ /ພ	E ₂ /E
0.75	4.71	0.090	0.993	0.952
1.00	6.28	0.090	0.991	0.937
1.25	7.85	0.090	0.988	0.922
1.50	9.42	0.090	0.986	0.906
1.75	10.99	0.090	0.984	0.891
2.00	12.56	0.090	0.981	0.876

 \dagger Value based on U_m = 6 ms⁻¹

Table 5.3. Wave attenuation due to opposing current.

Again the figures of Table 5.3 provide only an order-of-magnitude estimate of the effects of the opposing current, since, Equation 2.63 was derived for a uniform current and its applicability to a recirculating flow, such as that of the experimental facility, is difficult to assess. assumption of a uniform current velocity profile is equivalent to placing the waves in a moving frame of reference. In such a reference frame, the wave frequency will be Doppler shifted but the flow structure within the wave will remain unchanged. For a current with a depth-dependent velocity profile, the orbital motion beneath the wave will be distorted and presumably also the wave profile. In addition, the attenuation described by Equation 2.63 was derived for an active wind sea with an f^{-5} saturation spectrum, whereas monochromatic sinusoidal waves have been used in the present study. Although the applicability of the specific values presented in Table 5.3 is questionable, they indicate that measured attenuation rates cannot be attributed completely to air-sea interaction, since wave-current interaction and possibly other effects may be substantial.

Plate, Chang and Hidy (98) have indicated that, for a following wind, the presence of a surface drift current can also alter the air-sea energy flux. The surface current will alter the air boundary layer profile and hence change the height of the critical layer, whose position is vital in the Miles instability theory. For an opposing wind, no critical layer can exist and energy flux measurements inferred from surface stress measurements should not be effected by the presence of the surface drift current.

6. SPATIAL WAVE DECAY

In order to determine the wave decay with fetch along the flume, wave heights were measured at the mid-flume working section (x = 7.49 m), and near the beach (x = 3.30 m). These experiments were repeated for a number of free stream velocities and for wave frequencies of 0.75, 1.00, 1.25, 1.50 and 1.75 Hz. From the time series at both locations, the wave amplitudes were determined using the procedure described in Appendix C. It was then assumed the decay followed an exponential relationship

$$\frac{E_2}{E_1} = \exp(b\Delta x/C_q) \tag{6.1}$$

where E_1 and E_2 are the wave energies at the mid-flume test section and the beach respectively, $\Delta x = 4.19$ m is the distance between the two measurement points, C_g is the wave group velocity and b is an exponential decay coefficient. Therefore, b can be determined as

$$b = \ln (a_2^2/a_1^2) C_{q}/\Delta x$$
 (6.2)

where a1 and a2 are the wave amplitudes at the two points. Since measurements were made at only two points, the assumption of an exponential decay cannot be confirmed from the measurements. The assumption is not completely without basis, however, since Bole (11) and numerous others have shown that most growth and decay processes for waves in wind-wave flumes follow such a relationship The variation in the exponential decay factor, b, as a function of $U_{\rm m}/C$ is presented in Figure 6.1 for each of the wave frequencies. The trend in each plot is quite similar, with b approximately equal to $-0.02 \pm 10\%$ s⁻¹ at U_{m} /C = 0 and increasing in magnitude as an apparent power law function of U_/C. In interpreting these results, it would be wrong to attribute this rate of wave decay completely to the effects of air-sea interaction, since processes such as viscous dissipation, wave-current interaction, wavewave interaction, wave reflection and dispersion and turbulent shear, will also be active in this situation. Determining the individual contributions from these various processes is no trivial task and, as pointed out in Chapter 2, the effects of wave-current and short wave-long wave interactions are still not fully understood. When U_{m}/C is zero, neither of these processes will be active and air-sea interaction will probably be very small

(117). Therefore, the value of $b = -0.02 \pm 10\%$ at $U_{\infty}/C = 0$ can reasonably be assumed to be a result of the other processes. It was shown in Section 3.4 that wave reflections within the flume are quite small and, since sinusoidal waves are being used, wave dispersion should not be significant. The effects of viscous dissipation and turbulent shear, however, will be active. Hunt (49) has considered viscous damping of wave energy within the wall and floor boundaries of rectangular flumes and derived the relationship

$$\frac{E_2}{E_1} = \exp\left[-\frac{4k}{B}\left(\frac{\nu}{2\omega}\right)^{\frac{1}{2}} \frac{kB + \sinh\left(2kd\right)}{2kd + \sinh\left(2kd\right)} \Delta x\right]$$
(6.3)

where B is the flume width and v the kinematic viscosity of water. Equation 6.3 is, however, only an order-of-magnitude estimate of the effects of viscous dissipation as a laminar boundary layer assumption was used in its derivation. Using Equation 6.3 together with Equation 6.1, the theoretical exponential decay factor for viscous dissipation can be determined for the present experimental facility. These results are presented in Table 6.1.

f (Hz)	ω (Hz)	k (m ⁻¹)	C g (ms ⁻¹)	b (viscous decay) (s ⁻¹)
0.75	4.73	3.03	1.22	-0.039
1.00	6.28	4.50	0.92	-0.040
1.25	7.85	6.49	0.68	-0.040
1.50	9.42	9.11	0.54	-0.041
1.75	10.99	12.33	0.45	-0.043

Table 6.1 Theoretical viscous decay rates of waves

When compared with the measured values of b at $U_{\infty}/C = 0$, it can be seen that the values of Table 6.1 are almost twice the magnitude of the experimental data. Despite this discrepancy, the relative invariance of the b values from Table 6.1 is consistent with the trend observed in the present experiments.

7. SURFACE PRESSURE FIELD

The wave-induced pressure field was determined (see Appendix E) from a total of 336 pressure experiments. Three parameters could be freely varied: the wave frequency f, the free stream wind velocity U_{∞} and the height of the wave following probe above the water surface, z. The values chosen for these three parameters were f = 0.75, 1.00, 1.25, 1.50, 1.75 and 2.00 Hz, z = 20, 35, 63 and 120 mm and U_{∞} was taken at various but not fixed values between 0 and 6 ms⁻¹. Using the recorded time series in conjunction with the reduction techniques of Appendix E and analysis techniques of Appendix C, it was possible to determine the amplitudes of the wave and pressure signals and the phase relationship between them for each experiment.

An examination of this data immediately reveals that the water surface and pressure records are almost exactly 180° out of phase. This is clearly apparent in Figure 7.1, which shows time series of water surface elevation and pressure at a variety of wind speeds, for 1 Hz waves and a probe height of 20 mm. In addition, this trend continues across the entire data set and appears to be uninfluenced by changes in the wave frequency, probe height or wind velocity. Figure 7.2 shows the phase angle between the two records as a function of U_/C. The mean of these values is 183° with 95% confidence intervals of $\pm 6^{\circ}$. This result is quite different from that consistently obtained by others (see Figure 2.7) for a following wind where a marked phase shift from 180° has been reported. It would have been desirable to confirm these following wind results in this experimental flume. The fan design however, would not allow the air flow direction to be reversed, thus precluding such experiments. In view of the differences from the following wind case, it is unlikely that a Miles-type instability mechanism is active in the opposing wind situation. Certainly a critical layer cannot exist.

A phase angle of 180° is consistent with the predictions of potential theory as outlined in Section 2.2.1. Figure 7.3 shows $amp(\tilde{p})/\rho_{a}g$ plotted against a e^{-kz} (1 - U_{∞}/C)²; potential theory (Equation 2.25) predicts

$$\frac{\operatorname{amp}(\tilde{p})}{\rho_{a}g} = \operatorname{a} e^{-kz} \left(1 - U_{\infty}/C\right)^{2}$$
(7.1)

Figure 7.3 shows a definite linear trend, but falling slightly below the potential flow solution. A least squares approximation yields the result

$$\frac{\operatorname{amp}(\tilde{p})}{\rho_{a}g} = (0.8 \pm 0.2) \operatorname{ae}^{-kz} (1 - U_{\infty}/C)^{2}$$
(7.2)

where the error represents the 95% confidence limits. Although the potential flow prediction does lie within these bounds, a slight divergence from the theoretical result is indicated, presumably due to the violation of one or more of the assumptions inherent in the potential flow solution. As indicated in Section 2.2.1, these assumptions are those of incompressible, irrotational and inviscid flow. The assumptions of irrotational and inviscid flow are very questionable when considering flow within a boundary layer and it would be surprising if they were not violated. It is perhaps surprising that the experimental results do agree so well with the potential flow situation.

The implications of such a potential flow result for the pressure field are very significant, since, as indicated by Equation 2.105, it is the pressure component 90° out of phase with the water surface which causes an energy flux. If the pressure signal is in anti-phase with the waves as indicated by the potential flow solution, there will be no energy flux and hence neither wave growth nor decay due to normal stresses. In addition, there is no obvious deviation in the data trend at high values of U_{∞}/C and it appears unlikely that there was any air flow separation, even at high wind velocities. This seems to preclude a decay mechanism similar to that proposed by Jeffreys (52) for wave growth.

8. SURFACE VELOCITY FIELD

8.1 BOUNDARY LAYER PROFILES ABOVE WAVES

The physics of air-sea interaction and the wind-wave energy flux are largely dependent upon the nature of the air flow near the water surface and measurements of the mean air velocity profile were a routine part of the experimental program. A stationary hot film anemometer was used to measure the mean air velocity at various heights above mechanically generated waves of frequencies 0.75, 1.00, 1.25, 1.50, 1.75 and 2.00 Hz and for six different fan speeds, a total of thirty-six velocity profiles being determined. These velocity profiles are plotted in Figure 8.1. In the boundary layer region, the profile is clearly logarithmic and the logarithmic boundary layer profile, Equation 3.7, was least-squares curve fitted to this part of the profiles. The alternate power law profile, Equation 3.8, could also have been used to approximate these measurements. Although the power law is useful in modelling the shape of the atmospheric boundary layer profile, it provides no insight into the physics of the boundary layer. In contrast, the logarithmic profile has a sound physical basis and was hence adopted in this analysis.

Phillips (94) and Brooke-Benjamin (15) have considered the air flow above waves and predicted theoretically that, near the water surface, the atmospheric boundary layer will be "bent" to follow the large-scale surface undulations. Although the velocity profiles of Figure 8.1 appear to be logarithmic, Brooke-Benjamin (15) has shown that an oscillatory vertical coordinate $\zeta = z - a e^{-kz}$ must be used to obtain a truly logarithmic relationship. Adopting this coordinate transformation, the boundary layer profile becomes

$$\frac{\overline{u}(\zeta)}{u_{\star}} = \frac{1}{\kappa} \ln \left(\frac{\zeta}{\zeta_{0}}\right)$$
(8.1)

where ζ_0 is the roughness length. When a stationary probe is used, as is the case in the present context, the expected velocity profile can be obtained by expanding Equation 8.1 in a Taylor series about $\zeta = z$. The final result becomes (94)

$$\frac{\overline{u(z)}}{u_{\star}} \approx \frac{1}{\kappa} \ln \left(\frac{\overline{z}}{\zeta}\right) - \frac{1}{2\kappa z^2} \int_{0}^{\infty} F(\underline{k}) e^{-2kz} d\underline{k}$$
(8.2)

where $F(\underline{k})$ is the directional wave number spectrum. The second term in this expression arises from the curvature of the velocity profile, together with the fact that a constant height does not correspond to a constant distance above the water surface. This departure from a simple logarithmic profile is significant only near the surface, where kz is small. Measurements well clear of the surface, particularly by Roll (104) and Brocks (14), have confirmed the logarithmic nature of the velocity profile over waves. Takeda (126) has however measured departures from a logarithmic form very close to the surface that may be associated with the second term of Equation 8.2.

For the present experiments, the minimum height above the mean water level was 50 mm and the maximum wave amplitude was approximately 40 mm. The results do not indicate any obvious deviation from the logarithmic relationship and it must be assumed that the second term of Equation 8.2 is negligible at these heights.

An additional effect which could lead to a deviation from the logarithmic profile is the presence of the surface drift current. To include the effects of the drift current, the boundary layer equation should be written as

$$\bar{u}(z) = \frac{u_{\star}}{\kappa} \ln \left(\frac{z}{z_{o}}\right) + 1.5 \times 10^{-2} U_{\infty}$$
(8.3)

where the second term is the surface drift current as given by Equation 5.1. This second term in Equation 8.3 is quite small and, except very close to the water surface, would be negligible in comparison to the first term.

The values of u_* obtained from the least squares curve approximations to the boundary layer profiles are presented as a function of U_{∞} in Figure 8.2 for each of the six wave frequencies used. For all frequencies, the values of u_* increase with U_{∞} until approximately $U_{\infty} = 4 \text{ ms}^{-1}$. For values of U_{∞} between 4 ms⁻¹ and 5 ms⁻¹, the u_* curves reach a plateau region with a mean values of approximately $u_* = 0.16 \text{ ms}^{-1}$. For free stream wind velocities above 5 ms⁻¹, there is some evidence that u_* again increases. This unusual behaviour of the shear velocity is presumably related to changes in the surface roughness. Simiu and Scanlan (113) have shown that the surface roughness length z_0 and the shear velocity u_* are not independent parameters and an increase in the surface roughness will cause a corresponding increase in u_* . The plateau in the u_* , U_∞ curve may well be caused by a decrease in the surface roughness. It is perhaps possible that the mechanically-generated waves control the surface roughness at low wind speeds. As the wind velocity increases, the mechanically-generated waves decay in magnitude and their influence on the surface roughness decreases. At the same time the wind generated waves are increasing in magnitude with the wind velocity. The plateau region possibly marks the transition between these two roughness regimes. Below 4 ms⁻¹, the roughness may be determined by the mechanically-generated waves whereas above 5 ms⁻¹ the roughness could be governed by the wind generated waves.

The values of u_{\star} as a function of $U_{0.1}$, the velocity 100 mm above the mean water surface, are presented in Figure 8.3 along with data for growing laboratory wind waves obtained from a number of different sources and collated by Amorocho and De Vries (1). The comparison between this data and the current results is surprisingly good, despite possible differences in surface roughness, with the present data lying slightly below the mean of the following wind data.

8.2 WAVE-INDUCED VELOCITIES

Turbulent velocities above the water surface were measured with both stationary and wave following probes, involving 132 experiments with the stationary probe and 264 experiments with the wave following probe. The same wind speed and frequency ranges used for the pressure experiments were again adopted. For the stationary probe, measurements were taken at $z_s = 60$ and 80 mm above the mean water level, and for the wave following experiments heights of z = 30, 55, 77 and 100 mm above the oscillating water surface were used. This data was analysed as described in Appendix G to determine the u and w velocity fields and the various Reynolds stress terms.

As indicated in Section 2.7, the u and w velocity signals can be expressed as the sum of a mean component, a wave-induced component and a turbulent component

$$u = \bar{u} + \tilde{u} + u' \tag{8.4a}$$

and
$$w = \overline{w} + \widetilde{w} + w^{*}$$
 (8.4b)

Removing the mean from the records yields the oscillating components of velocity

$$\mathbf{u}^{\mathbf{u}} = \widetilde{\mathbf{u}} + \mathbf{u}^{\mathbf{u}} \tag{8.5a}$$

and
$$w'' = \tilde{w} + w'$$
 (8.5b)

Typical time histories of these velocity components are shown in Figure 8.4 together with the water surface records for a range of wind speeds blowing in opposition to 1 Hz mechanically-generated waves. A stationary velocity probe at $z_{2} = 60 \text{ mm}$ was used for all these experiments. The wave-induced components \tilde{u} and \tilde{w} are clearly visible in these records at the 1 Hz wave frequency; they have considerably greater magnitude than the random turbulent components u' and w'. Rather than being purely sinusoidal, these wave-induced components exhibit a saw-tooth structure in many of these plots. This is particularly evident in the w" records. The most likely explanation for this structure is the presence of higher harmonics in the velocity records. This is discussed in more detail in Section 8.3. These velocity records show that \tilde{u} and η are approximately 180° out of phase, whereas \tilde{w} leads η by approximately 90°. These trends extend across the entire stationary probe data set and can also be seen in Figure 8.5, which shows the phase angle between \widetilde{u} and η , $\varphi_{\widetilde{u}\eta}$, as a function of U_{∞}/C and Figure 8.6, which shows the phase angle between \widetilde{w} and n, $\varphi_{\widetilde{w}n}$, as a function of U_/C. There is little scatter in this data, the mean phase angles being constant despite variations in wind velocity, wave frequency and probe elevation. The mean values of $\phi_{\widetilde{u}n}$ and $\phi_{\widetilde{w}n}$ are 182° and 89° respectively, compared with potential flow predictions (Equations 2.26 and 2.27) of 180° and 90°

With knowledge of the potential flow predictions for the wave-induced velocities (Equations 2.26 and 2.27), $\operatorname{amp}(\tilde{u})/U_{\omega}$ and $\operatorname{amp}(\tilde{w})/U_{\omega}$ have been plotted against $a/U_{\omega}(\omega - kU_{\omega})e^{-kz}$ in Figures 8.7 and 8.8. There are clear linear trends in both cases, with least square curve approximations yielding

$$amp(\tilde{u}) = (0.65 \pm 0.16) a (\omega - kU_{\omega})e^{-kz}$$
 (8.6)

and $amp(\tilde{w}) = (0.59 \pm 0.14) a (\omega - kU_{\infty})e^{-kz}$ (8.7)

where the errors represent 95% confidence limits.

The phase relationships for the data obtained with the wave follower are similar to those for the stationary probe and are presented in Figures 8.9 and 8.10. Although the data is more scattered (perhaps caused by vortex-induced lateral vibrations of the stem of the wave follower) it is clear that both phase angles are constant. The data yields mean values of $\phi_{\tilde{u}\eta} = 186^{\circ}$ and $\phi_{\tilde{w}\eta} = 86^{\circ}$ which again agree well with the potential flow solutions. The nondimensional, wave-induced velocities $amp(\tilde{u})/U_{\infty}$ and $amp(\tilde{w})/U_{\infty}$ obtained in the wave following frame of reference are shown in Figures 8.11 and 8.12 as functions of the potential flow solution. Although there is some scatter in the data, linear trends are still apparent with least squares approximations of

$$amp(\tilde{u}) = (0.80 \pm 0.20) a (\omega - kU_{\infty})e^{-KZ}$$
 (8.8)

and
$$amp(\tilde{w}) = (0.30 \pm 0.07) a (\omega - kU_{\omega})e^{-kZ}$$
 (8.9)

where the errors again represent 95% confidence intervals. As expected, these results differ quite considerably from the stationary probe results, the largest difference being for the \tilde{w} component. For the wave following results, the probe is always a fixed distance above the water surface but the effective height of the stationary probe varies with the phase of the waves. The actual differences in the velocities will depend on the streamline pattern above the water surface. If the streamlines at various heights exactly reflected the shape of the water surface, the wave following probe would follow a streamline and record a constant velocity. In contrast, the stationary probe would be continuously "cut" by various streamlines and would sense a fluctuating velocity with the passage of waves. The wave follower results are more relevant, more closely reflecting the true influence of the oscillating water surface.

The wave-induced velocity measurements discussed above consistently fall below the predictions of potential theory. To ensure that this was not the result of an instrumentation or calibration error, the measurement system and analysis techniques were carefully checked and found to be satisfactory. It can only be concluded that this difference is the result of a different flow structure to that predicted by potential theory. The general qualitative predictions of potential theory are, however, confirmed and it is interesting to speculate why this should be the case. Indeed it is possible that this agreement occurs, for no other reason, than that the potential flow functions are dimensionally correct. This is insured by the potential flow result at z = 0. This argument of the equations being dimensionally correct is confirmed in other areas of fluid mechanics. A typical example is why eddy viscosity works in some turbulent closure models.

Although no similar measurements in opposing wind situations could be found in the published literature, both Stewart (123) and Chao and Hsu (21) have measured wave-induced velocities above waves moving slower than the wind (i.e. $0 < U_m/C < 1$), where U_m is the wind velocity outside the boundary layer. This situation has some similarity to the present case, since no critical layer can exist, and provides an interesting comparison with the present results. The magnitude and phase of their wave-induced velocities are shown in Figure 8.13, for 0<U_/C<1 and for U_/C>1. For U_/C<1, their data indicates constant phase relationship of $\phi_{\widetilde{u}n} \approx 180^{\circ}$ and $\phi_{\widetilde{w}n} \approx 90^{\circ}$, which is consistent with both the present research and the predictions of potential theory. The corresponding amplitude data in Figure 8.13 is less easily compared with the present result, since neither the wave height nor U have been reported by either Stewart or Chao and Hsu. For constant wave amplitude a and constant U_/C however, their wave-induced velocities increase at a possibly exponential rate with decreasing nondimensional height kz, which is consistent with the e^{-kz} behaviour predicted by potential theory. The data for U_{∞}/C > 1 is significantly different. Above the critical layer, $\phi_{\widetilde{w}n}$ is constant at

approximately -40° whereas below the critical layer the phase angle increases very rapidly towards 100° . In contrast, $\phi_{\widetilde{u}\eta}$ increases in a roughly linear fashion from -160° to 30° with increasing nondimensional height and exhibits no marked change in behaviour at the critical height. The amplitudes of the wave-induced velocities are also quite different to those for $U_{\infty}/C < 1$ with amp(\widetilde{w}) decreasing in magnitude with decreasing kz. These comparisons indicate that a marked change occurs at $U_{\infty}/C = 1$ in the air flow pattern above waves. Above this value, a critical layer exists and strongly influences the flow in its vicinity. For $U_{\infty}/C < 1$ no critical layer can exist and the flow appears to follow, at least qualitatively, the predictions of potential flow theory.

8.3 REYNOLDS STRESSES

In Section 2.7 it was shown that, when the momentum flux terms u"u", w"w" and u"w" in the momentum equation are time-averaged, they take on the properties of stresses, hence their common name of Reynolds stresses. For each of the experiments performed, both in the stationary and wave following coordinate systems, the u"u", w"w" and u"w" products and their time means (Reynolds stresses) were evaluated (see Appendix G).

By considering the structure of the Reynolds stress terms, some insight can be gained into their possible form. It was shown in Section 2.7 that these Reynolds stress terms can be expressed as the sum of a waveinduced Reynolds stress and a turbulent Reynolds stress

u"u"	=	$\tilde{u}\tilde{u}$ + $u'u'$	(8.10a)

$$\overline{w^{\prime\prime}w^{\prime\prime\prime}} = \overline{\widetilde{w}\widetilde{w}} + \overline{w^{\prime}w^{\prime\prime}}$$
(8.10b)

$$\overline{u^{\prime\prime}w^{\prime\prime}} = \overline{\widetilde{u}\widetilde{w}} + \overline{u^{\prime}w^{\prime}}$$
(8.10c)

The wave-induced velocities in Equations 8.10 are sinusoidal with frequency ω and can be expressed as

$$u = amp(u) \cos(\omega t + \phi_{u})$$
(8.11a)
and $\tilde{w} = amp(\tilde{w}) \cos(\omega t + \phi_{u})$ (8.11b)

If the trigonometric identity

$$2 \cos A \cos B = \cos (A - B) + \cos (A + B)$$
 (8.12)

where A and B are general angles, is considered the wave-induced product terms in Equations 8.10 become

$$\widetilde{u}\widetilde{u} = \frac{1}{2} \left[\operatorname{amp}(\widetilde{u}) \right]^{2} \left[1 + \cos\left(2\omega t + 2\phi_{u}\right) \right]$$
(8.13a)

$$\widetilde{w}\widetilde{w} = \frac{1}{2} \left[\operatorname{amp}(\widetilde{w}) \right]^2 \left[1 + \cos\left(2\omega t + 2\phi_{w}\right) \right]$$
(8.13b)

$$\widetilde{u}\widetilde{w} = \frac{1}{2} \operatorname{amp}(\widetilde{u}) \operatorname{amp}(\widetilde{w}) \left[\cos(\phi_{u} - \phi_{w}) + \cos(2\omega t + \phi_{u} + \phi_{w}) \right]$$
(8.13c)

The wave-induced Reynolds stress terms consequently are

 $\widetilde{\widetilde{u}}\widetilde{\widetilde{u}} = \frac{1}{2} [\operatorname{amp}(\widetilde{u})]^2$ (8.14a)

$$\widetilde{\widetilde{ww}} = \frac{1}{2} [\operatorname{amp}(\widetilde{w})]^2$$
(8.14b)

and
$$\widetilde{\widetilde{u}}\widetilde{\widetilde{w}} = \frac{1}{2} \operatorname{amp}(\widetilde{u}) \operatorname{amp}(\widetilde{w}) \cos(\phi_{u} - \phi_{w})$$
 (8.14c)

The structure of the turbulent Reynolds stresses is more difficult to estimate since it will depend on the nature of the turbulent flow. The turbulent Reynolds stress is most conveniently represented as the variance of the turbulent velocity spectrum (Section 3.1)

$$\overline{u^{\prime}u^{\prime}} = \int_{0}^{\infty} E_{u^{\prime}u^{\prime}} df \qquad (8.15a)$$

$$w'w' = \int_{0}^{\infty} E_{w'w'} df \qquad (8.15b)$$

$$\overline{u'w'} = \int_{0}^{\infty} E_{u'w'} df \qquad (8.15c)$$

To determine the relative magnitudes of these terms as compared to the waveinduced Reynolds stress, the spectrum analyser was used to measure the velocity spectra for a number of cases. Figure 8.14 presents a sample of

these results for 1 Hz mechanically-generated waves, a stationary probe height of 50 mm and for six wind speeds between 0 and 6 ms⁻¹. The figure shows the u" and w" autospectral functions as well as the phase between u" and w", $\phi_{\mu\nu}$ and the coherence between u" and w", $\gamma_{\mu\nu}$. At high frequencies the velocity spectra decay at a rate proportional to $f^{-5/3}$, characteristic of the inertial subrange as described by Equation 3.9. Because of this rapid decay with frequency, the major contribution to the Reynolds stress will be from the low frequency components of the spectrum. The low frequency regions of both the u" and w" spectra are characterized by a large spike at 1 Hz, corresponding to the wave-induced velocity component \tilde{u} and \tilde{w} , and by smaller spikes at the harmonics of this value. In general, this wave-induced spike in the spectrum is approximately two orders of magnitude greater than the background levels. Although the spike is quite narrow banded, the area under this section of the spectrum is significantly larger than the area of the remainder of the spectrum. Hence it can be concluded that the wave-induced Reynolds stress will be larger than the turbulent Reynolds stress, which can be neglected as an initial approximation.

Mention was made earlier of the saw-tooth structure of many of the velocity time series shown in Figure 8.4. Examination of the Fourier series expansion for a saw-tooth wave indicates that it consists of the sum of the primary component and each of its harmonics with a reduction in their amplitude with frequency. The presence of such higher harmonics is confirmed by the spectra of Figure 8.14. The magnitude of the harmonics is more pronounced in the w" spectra than the u" spectra. This is consistent with the stronger saw-tooth trend in the w" time series. The higher harmonics are present in the velocity records since the mechanically generated water waves were not purely sinusoidal. Although the harmonics are relatively insignificant in the water surface record the velocities they induce are relatively more significant. This occurs since the waveinduced velocity is proportional to ω^2 .

Figure 8.14 also shows the phase and coherence between u" and w". The wave-induced component is again clearly evident in both these functions. The phase relationship appears random at all frequencies except near the frequency of the mechanically-generated waves were the phase difference is

consistently 90° as indicated in Section 8.2. The coherence function is characterised by a spike of magnitude one at this same frequency, indicating that the wave-induced components \tilde{u} and \tilde{w} are highly correlated.

Neglecting the effects of the turbulent Reynolds stresses, Equations 8.10 yield

$$\overline{u^{\prime\prime}u^{\prime\prime}} \approx \overline{\widetilde{u}\widetilde{u}} = \frac{1}{2} [\operatorname{amp}(u)]^2$$
(8.16a)

and $\overline{w^{"}w^{"}} \approx \overline{\widetilde{w}\widetilde{w}} = \frac{1}{2} [amp(w)]^{2}$ (8.16b)

Equation 8.14c indicates that the contribution to the $\overline{u^{"}w^{"}}$ Reynolds stress by the wave-induced Reynolds stress is a function of $(\phi_u - \phi_w)$. In Section 8.2 this phase difference was shown to be approximately 90°. Hence from Equation 8.14c

and therefore

$$\overline{u''w''} \approx \overline{u'w''}$$
(8.18)

In view of the relationships given by Equations 8.16, and since it has shown that \tilde{u} and \tilde{w} follow the general trends of potential flow theory, $\overline{u^{"}u^{"}}$ and $\overline{w^{"}w^{"}}$ obtained with the fixed probe have been plotted in Figures 8.15 and 8.16 against $[a(\omega - kU_{\infty})e^{-kZ}]^2$. This function is the square of the wave-induced velocity predicted by potential theory. Both plots show an increasing and approximately linear trend with increasing values of the potential flow function. There is considerable scatter at low values of the independent variable, presumably because, at these values, the waveinduced Reynolds stresses are quite small, a significant portion of the total Reynolds stress coming from the turbulent components $\overline{u'u'}$ and $\overline{w'w'}$. As the wave-induced contribution increase in magnitude, however, they begin to dominate and the scatter reduces. Least squares curve approximations to these plots yield

$$\overline{u^{"}u^{"}} = (0.4 \pm 0.2) [a(\omega - kU_{\omega})e^{-kz}]^2$$
(8.19)

and
$$\overline{w''w''} = (0.3 \pm 0.1) [a(\omega - kU_{\omega})e^{-kz}]^2$$
 (8.20)

Figure 8.17 also shows the Reynolds shear stress term, $\overline{u^{"}w^{"}}$, measured with the stationary probe, as a function of this potential flow function. As predicted by Equation 8.18 and confirmed by this figure, $\overline{u^{"}w^{"}}$ is independent of the wave-induced Reynolds stress.

Similar plots can be presented for the data obtained with the wave follower and these are shown for $\overline{u"u"}$, $\overline{w"w"}$ and $\overline{u"w"}$ in Figures 8.18, 8.19 and 8.20 respectively. Again the $\overline{u"u"}$ and $\overline{w"w"}$ terms have approximately linear relationships with the potential flow function, given by

$$\mathbf{u}^{*}\mathbf{u}^{*} = (0.7 \pm 0.2) \left[\mathbf{a}(\omega - k\mathbf{U}_{\omega}) \mathbf{e}^{-\mathbf{K}\mathbf{Z}} \right]^{2}$$
(8.21)

and
$$\overline{w^{"}w^{"}} = (0.2 \pm 0.1) [a(\omega - kU_{\infty})e^{-kz}]^2$$
 (8.22)

As for the fixed probe data, the term $\overline{u''w''}$ appear to be independent of the wave-induced velocity. When plotted as a function of U_{∞}^{2} (Figure 8.21), however, there is a tendency for $\overline{u''w''}$ to increase with U_{∞}^{2} , although scatter suggests that $\overline{u''w''}$ is not solely a function of U_{∞}^{2} . Since this term is determined by the product of the turbulent components u' and w', it is reasonable to assume that $\overline{u''w''}$ is also a function of surface roughness and elevation and probably other parameters. This term, however, is considerably smaller than either $\overline{u''u''}$ or $\overline{w''w''}$ and the accuracy of the experiments precludes the determination of a more involved functional relationship. Nevertheless, Figure 8.21 can be approximated by

$$\overline{u^{"}w^{"}} = (6 \pm 2) \times 10^{-4} U_{m}^{2}$$
 (8.23)

As shown earlier, it is only the mean components of the momentum flux terms u"u", w"w" and u"w" which have the properties of stresses and hence potentially cause an air-sea energy flux. Chao and Hsu (21) and Wu et al (134) have defined these terms in a similar manner to Equations 8.4, as
$$u^{"}u^{"} = \overline{u^{"}u^{"}} + u^{"}u^{"} + turbulence \qquad (8.24a)$$

$$w''w'' = \overline{w''w''} + \overline{w''w''} + \text{turbulence}$$
(8.24b)

and
$$u''w'' = \overline{u''w''} + u''w'' + turbulence$$
 (8.24c)

where the second terms represent the components of the records with frequency ω . These terms vanish from the mean-flow momentum equations when the equations are time-averaged and consequently are not apparent stresses. Yet they are termed wave-induced oscillating stresses (at frequency ω) by the above authors. Nevertheless, it is interesting to examine the time series of u"u", w"w" and u"w" to determine whether the $\widehat{u"u"}$, $\widehat{w"w"}$ and $\widehat{u"w"}$ components are present.

Figure 8.22 shows typical spectra of the u"u", w"w" and u"w" products. A peak at a frequency of 2ω is clearly apparent and is caused by the product of the wave-induced velocities as predicted by Equations 8.13. In addition, a smaller peak is also present at frequency ω . The following wind data of Chao and Hsu (21) (Figure 8.22) shows the reverse trend with a major peak at ω and a smaller peak at 2ω . A spectral peak at frequency ω must be caused by the product of random turbulent components, which according to Kendall (57) must be a result either of variations in u' or w' or by the variation in the correlation between them. Whatever the reason, the cyclic behaviour indicates that there must be some controlling processes organising these otherwise random turbulent quantities at the wave frequency. Since the peak at ω in the present data is considerably less marked than in the following wind case, it appears that these controlling processes are weaker in the opposing wind situation.

9. THE WIND-WAVE ENERGY FLUX

As discussed in Section 2.2.4, surface wave generation by air-sea interaction involves an initial linear phase followed by an exponential phase, described respectively by Equations 2.43 and 2.44. If expressed as source terms of the Radiative Transfer Equation (Equation 2.10), these energy fluxes become

$$S_{A}(f,\theta) = a_{A} + b_{A} E(f,\theta)$$
(9.1)

where

$$a_{A}(f,\theta) = \frac{2\pi^{2} \omega \Pi(k,\omega)}{\rho_{w}^{2}C^{3}C_{g}}$$
(9.2)

and
$$b_{A}(f,\theta) = \mu \omega$$
 (9.3)

In the present context it is the exponential term defined by Equation 9.3 which is of interest. The coupling coefficient, μ , has already been shown (Section 2.2.4) to be the sum of two terms μ_1 and ν_2 given by Equations 2.105 and 2.106, respectively. Assuming the viscous stresses are negligible, Equation 2.105 can be simplified to

$$\mu = \left[-\operatorname{amp}(\tilde{p}) \sin \phi_{1} - \operatorname{amp}(\partial \eta / \partial x) \rho_{a} \overline{u'' u''} - \operatorname{amp}(\partial \eta / \partial x) \rho_{a} \overline{u'' w''}\right] / \rho_{w} C^{2} ka$$
(9.4)

since $amp(\partial \eta / \partial x) = ak$, Equation 9.4 further reduces to

$$\mu = \left[-\operatorname{amp}(\tilde{p}) \sin\phi_1 - \operatorname{ak}\rho_a \overline{u''u''} - \operatorname{ak}\rho_a \overline{u''w''}\right] / \rho_w C^2 \operatorname{ak}$$
(9.5)

Using the experimental result, Equation 7.2 with kz = 0, and φ_1 = 183 \pm 6° the first term of Equation 9.5 becomes

$$\mu^{(1)} = (5.5 \pm 10.5) \times 10^{-2} \rho_{a} / \rho_{w} (1 - U_{w} / C)^{2}$$
(9.6)

Similarly, using Equation 8.21, the second term becomes

$$\mu^{(2)} = (-0.7 \pm 0.2) \rho_{a} / \rho_{w} (ak)^{2} (1 - U_{w} / C)^{2}$$
(9.7)

and from Equation 8.23 the final term is

$$\mu^{(3)} = (-6 \pm 2) \times 10^{-4} \rho_a / \rho_w (U_w / C)^2$$
(9.8)

A comparison of these three terms is complicated by the square of the wave slope in Equation 9.7, as this introduces a free parameter not present in either of the other terms. In addition, when the experimental errors are considered $\mu^{(1)}$ could in fact be zero; this is highly probable since it is unlikely that an opposing wind would cause a positive energy flux. The $\mu^{(3)}$ term is extremely small and the wave slope would need to be very small, of order 10^{-2} , for it to be of comparable magnitude to $\mu^{(2)}$. A reasonable approximation to the energy flux coefficient is then

$$\mu = (-0.7 \pm 0.2) \rho_{a} / \rho_{m} (ak)^{2} (1 - U_{m} / C)^{2}$$
(9.9)

Equation 9.9 is plotted in Figure 9.1 for values of the wave slope ak of 0.10, 0.20 ... 0.45 together with μ for a following wind as reported by Snyder et al (117) and summarised in Equation 2.74 with $\gamma = 0.25$. The strong dependence on the wave slope is clear, indicating that steep waves will be quickly attenuated whereas less steep waves will almost be uneffected by an opposing wind. From Equation 9.9 the decay of a particular wave component can be determined. A typical deep water swell component with f = 0.05 Hz and H = 3 m and propagating into an adverse wind of 15 ms⁻¹ will be reduced in height to 2.99 m over a period of 20 hours. This is a reduction in height of less than 1% over this long duration. In contrast, a short 0.20 Hz wave with the same initial 3 m height will be reduced in height to 1.5 m over the much shorter duration of 1 hour. This represents a reduction of 50% in wave height. These examples clearly illustrate the importance of the wave slope ak and ratio of wind velocity to the wave phase speed. This point is illustrated in Figure 9.2, which shows the decay of waves with various wave slopes against fetch or duration, as predicted by Equation 9.9.

10. VISUAL OBSERVATIONS

Figures 10.1 show a series of photographs of the water surface profile for mechanically generated wave frequencies of f = 0.75, 1.00, 1.25, 1.50, 1.75 and 2.00 Hz and various opposing free stream wind velocities between 0 and 7 ms⁻¹. In these photographs, the mechanically generated waves are propagating from left to right, whereas the wind is in the opposite direction.

For a fan speed below approximately 300 rpm ($U_{\infty} = 3.5 \text{ ms}^{-1}$), there is little apparent alteration of the wave profile due to the opposing wind. Above this velocity, however, wind generated waves can be seen propagating from right to left. In Section 3.2.4 it was noted that there was a plateau region in the U_{∞} versus u_{\star} relationship at $U_{\infty} \approx 4 \text{ ms}^{-1}$ and postulated that this plateau may mark a transition in the surface roughness. Below the transition the surface roughness may be determined by the mechanically generated waves, whereas above the transition the wind generated waves may govern the surface roughness. The appearance of wind waves at $U_{\infty} \approx 3.5 \text{ ms}^{-1}$ is consistent with this hypothesis; the plateau occurring at a slightly higher wind velocity where the wind generated waves had grown in height.

When long and short waves interact, the short waves should have enhanced amplitudes at the crests of the long waves (see Section 2.3.2). Such an effect can be seen in Figures 10.1c to 10.1f. In some of these photographs there also appears to be some "white capping" of the wind generated waves at the crests of the mechanically generated waves. The presence of such wave interaction processes indicates that not all the wave decay observed can be attributed to the effects of air-sea interaction. This illustrates the importance of using surface stress measurements to determine the air-water energy flux rather than inferring it from rates of spatial wave decay.

In Chapter 9 it was shown that the surface stress measurements indicated that the rate of wave decay increases very rapidly with increasing wave frequency. This result is confirmed by Figures 10.1. Waves of frequency 1 Hz (Figure 10.1b) appear to experience no obvious decay even at high wind velocity. In contrast, waves of frequency 2 Hz (Figure 10.1f) are attenuated very rapidly and for $U_{\infty} > 4.7 \text{ ms}^{-1}$ (400 rpm) it is difficult to discern any remaining mechanically generated waves. This flattening of the water surface has also been reported by fishermen in Northern Australian waters who observed the passage of a tropical cyclone through the relatively shallow and protected Great Barrier Reef waters of the Whitsunday Passage. Because the region is protected by numerous coral reefs only locally generated high frequency wind waves would be present. After the passage of the eye of the storm and the accompanying wind direction reversal, they report that the waves were completely flattened by the strong winds.

112.

11. COMPARISON WITH OTHER RESULTS

As discussed in Chapter 2, there are only four reliable sets of measurements which can be used as a comparison with the present results. These experiments have been conducted by Snodgrass et al (114), King and Shemdin (59), Stewart and Teague (124) and Snyder et al (117).

Snodgrass et al (114) observed the propagation of deep water waves across the Pacific Ocean from south of Australia past Hawaii to Alaska. They found that, for frequencies below 0.075 Hz, the waves decayed so slowly that the rate could hardly be measured. Above this frequency the decay rates were still very small and increased in magnitude with frequency. Interpreting these results is not straight forward as the waves would have encountered various local wind fields during their passage. As an approximation, however, it can be assumed that $U_{\infty}/C = 0$, in which case these results are consistent with the present findings. Considering the same swell component as previously, with f = 0.05 Hz, H = 3 m and $U_{\infty}/C = 0$, over a fetch of 15,000 km (approximate propagation distance for Snodgrass et al experiment), the wave height is reduced by only 3%.

In interpreting these results it has been assumed that $U_{\infty}/C = 0$ is, in fact, an opposing wind situation. Although this assumption is not immediately obvious it can be reconciled by considering a frame of reference moving with the wave form. By considering such a reference frame it can be seen that $U_{\infty}/C = 1$ marks the transition between following and opposing winds.

King and Shemdin (59) have observed swell with frequencies near 0.08 Hz propagating ahead of hurricanes and seemingly uneffected by the very strong cross and adverse winds encountered. For such situations the wind velocity, U_{∞} may be as high as 40 ms⁻¹. Because the wave frequency is so low, however, U_{∞}/C is still only of order -2 and the $(1 - U_{\infty}/C)^2$ term in Equation 9.9 will be of order 10. The (ak)² term, however, will be very small and at the fetches investigated by King and Shemdin (~100 km) almost no swell attenuation would be expected. Thus, it can be seen that, despite the very strong adverse winds which can be encountered in parts of hurricanes or tropical cyclones, low frequency waves remain almost uneffected. In contrast, high frequency waves will be quickly attenuated by such winds and this is apparently confirmed by the absence of high frequency wave images ahead of the storm in the King and Shemdin radar data.

The most comprehensive field measurements of wave decay in opposing winds have been obtained by Stewart and Teague (124) who observed the growth and decay of approximately 0.14 Hz waves before and after the passage of a frontal system. The wind velocity was approximately 13 ms⁻¹ and the wind shift was almost exactly 180°. Their measured growth rate was 6.7 times the decay rate. In comparing these results with the present findings it is necessary to make some assumption about wave height. They report that the spectral variance of the waves was $\sigma^2 = 0.093 \text{ m}^2$; assuming that the waves are sinusoidal, this yields a wave amplitude a = 0.43 m. The wave slope is then ak = 0.034 and $\left|U_{_{\rm CD}}/C\right|$ = 1.17. Equation 2.74 with γ = 0.2 gives $\mu = 0.034 \rho_a / \rho_w$ for growing waves whereas Equation 9.9 yields a corresponding decay rate of μ = -0.0038 ρ_a/ρ_w ; the growth rate is predicted to be 8.9 times larger than the decay rate, compared with the value of 6.7 obtained by Stewart and Teague. Such a difference is well within the experimental uncertainty of both the data of Stewart and Teague and of the present project.

The data of Snyder et al (117) provides some careful field measurements of the surface normal stress distribution for opposing winds. Their measurements indicate that the surface pressure and the water surface elevation are essentially 180° out of phase, in agreement with both the predictions of potential flow theory and the results of this project. This supports the Chapter 7 experimental result that there is no air-sea energy flux due to normal stresses. 12. CONCLUSIONS

An extensive set of laboratory experiments have determined the windwave energy flux in an opposing wind-wave situation. The experiments were conducted for a variety of wind speeds and wave frequencies, covering quite a wide parameter range.

The wave-induced pressure above the waves was approximately in antiphase with the water surface, consistent with the predictions of potential theory. The magnitude of the wave-induced pressure fell below the potential theory prediction but still followed the same qualitative trend. These relationships between the water surface elevation and the surface pressure indicate that there is no wind-wave energy flux due to normal stresses, in sharp contrast to that obtained in following wind situations, where pressure forces : are the dominant source of the wind-wave energy flux. It appears likely that this difference is related to the non-existence of a critical layer in the opposing wind situation.

Measurements of the near surface velocity field indicated that the Reynolds stress $\overline{u^{"}u^{"}}$ was the dominant source of the wind-wave energy flux in an opposing wind. The resulting wave decay is proportional to the wave slope, ak, squared and the function $(1 - U_{\infty}/C)$ squared. As a result, high frequency waves will be attentuated much more rapidly than low frequency waves. Typical examples indicate that low frequency swell can propagate vast distances in an opposing wind and experience negligible decay. In contrast, high frequency waves under the same conditions will be attenuated quite rapidly.

This research was designed to obtain a broad understanding of the processes involved in the decay of waves due to opposing wind. Now that it has been determined that it is the Reynolds stress $-\rho_a$ u"u" which is responsible for wave decay in such situations, a more extensive set of experiments aimed specifically at determining the structure of this term would be most useful.

Despite the preliminary nature of this research, it has resulted in a source term for the Radiative Transfer Equation which should assist in achieving reliable wave predictions in complex wind fields.