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**THINKING, REASONING AND WORKING
MATHEMATICALLY:
A TEACHER'S RESPONSE TO
CURRICULUM CHANGE**

Thesis submitted by

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in January, 2010

Doctor of Philosophy Degree

in the School of Education

James Cook University

For
Primary School Mathematics Teachers

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I, Kerry Lee Smith, declare that this is my own work and has not been submitted in any form for any other degree or other award at any other university or other institution of tertiary education. Information derived from the published or unpublished work of others has been acknowledged in the text and a list of references is given.

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Acknowledgements

This case study research has been a learning experience that has been truly enlightening both academically and personally. Like my personal life, I have learned that the practice of research, too, is not static; it is forever evolving and is influenced by many interrelated factors, people, beliefs and attitudes. I have been fortunate to work with many inspirational people throughout this project all of whom were, and are, motivated by a need to advance mathematics education through better understanding the teacher's and the students' experiences.

First, and foremost, I would like to acknowledge Associate Professor Mary Klein who supervised this project. I have said this many times to Mary, but words cannot express the gratitude I feel towards the guidance and encouragement she has given me over the three years of my candidature, as well as her belief in my chosen project, and me as a researcher.

Second, I cannot thank Reagan enough for allowing me into her classroom, and into her life as a teacher. Throughout this project we both grew as professionals together, which would not have occurred if Reagan had not been open and willing to investigate her teaching practice and new curriculum ideals. Third, thanks must go to the students who were so courageous in changing their views of mathematics and mathematics learning, and in engaging with the discourse of mathematics in new and productive ways. Fourth, my gratitude goes to the principal of the school for granting permission for this research to be conducted in Reagan's classroom. I appreciate his support towards Reagan as she embarked on this project to help us both better inform the practice of teaching.

Next, I would like to thank Dr Hilary Whitehouse and Dr Michelle Lasen for their guidance, encouragement and belief in me as an academic researcher. My gratitude also goes to Dr Rosemary Dunn, along with others who attended and contributed to the thesis writing group. Rosemary's guidance provided a structural frame that assisted my writing process from beginning to end. Also, Jill Barber undertook the editing process, for which I am extremely grateful.

Finally, I wish to thank two other special influences in my life who continue to support my growth in every way. The first is my spiritual guide without which I could not have maintained a steady focus throughout this project. The second is my husband Tony, for his love, patience, kindness and understanding.

ABSTRACT

Research consistently points out that the quality of mathematics teaching and learning needs improvement if students' numeracy outcomes are to be improved. Curriculum documents, such as the *Years 1-10 Mathematics Syllabus* (QSA, 2004) and *Queensland Curriculum and Assessment Reporting Framework* (DETA, 2005), recommend teachers adopt an inquiry based 'process approach' where there is an increased focus on student learning. The students should be encouraged to think and reason, communicate and reflect with and about mathematical ideas to construct and validate them in ways that make sense to themselves and others. This approach is intended to improve numeracy outcomes by focussing on the productive development of mathematical understandings, practices and dispositions. However, while past research has examined teachers' understanding of the key messages of reform, research about the practical implementation is limited. Thus a gap exists between the intended and practised curriculum in mathematics classrooms and this gap needs to be more fully explained and understood for numeracy outcomes to improve.

This 'descriptive case study' (Shank, 2006) focussed on that gap to better understand the practical implementation of curriculum ideals. The study was conducted in a year six classroom of a small private school in North Queensland. It investigated, in detail, one teacher's attempt to implement curriculum change to reveal how and why certain experiences challenge, inspire or motivate the teacher's facilitation and students' uptake of learning processes comprising thinking, reasoning and working mathematically. The researched change involved the teacher's adoption of certain mathematics practices that would arguably result in more effective instructional strategies and investigative learning processes. Student pre- and post-questionnaires were used to determine changes to disposition and willingness to engage in mathematics learning. Pre- and post-questionnaires were also used to explicate changes to the teacher's pedagogical beliefs or understandings as a result of implementing the curriculum change. Further data were obtained from semi-structured interviews and detailed

classroom observations and all data were analysed through a qualitative content data analysis (Lankshear & Knobel, 2005). Whilst this study is limited to a sample size of one teacher, the rich, thick data revealed that change is complex; it is worthwhile, yet slow and abounds with challenges. The teacher's practice changed and the classroom atmosphere altered to enhance more collaborative mathematical inquiry. Student engagement and disposition started to improve in relation to reform ideals. Further research is needed to document, collaborate and perform cross case analyses to highlight exemplary practices and to examine the effect of reform oriented teaching on student learning outcomes and achievement. The results of this study will inform policymakers and researchers regarding future research directives and acquaint other teachers with some of the successes and challenges of implementing new policy directives.

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