Unsteady Natural Convection in Near-Shore Waters

Thesis submitted by

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Abstract

In the near-shore waters of natural water bodies, an increasing depth in the offshore direction is a geometric factor which results in differential heating or cooling across the shore. Exposed to daylight radiation, the volumetric heating rate in the shallow region is greater than in the deep region, generating a warm surface layer flowing offshore. At night, a circulation in the opposite direction is induced by differential cooling owing to heat loss from the water to the atmosphere. Field experiments demonstrate that this natural convection in calm near-shore waters plays a significant role in cross-shore exchanges with significant biological and environmental implications. This thesis aims to provide detailed quantification of this thermal flow for various thermal forcing conditions.

Based on a wedge model, an improved scaling analysis is proposed in the present study to reveal more detailed features of the flow than the previous scaling analysis, especially the dependency of flow properties on offshore distance. Four different types of thermal forcing (radiative heating, isoflux cooling, constant and ramped isothermal heating) are considered in the scaling analysis, and the scaling results are verified by the corresponding numerical simulations.

For heating induced by absorption of radiation, two critical functions of the Rayleigh number with respect to offshore distance are derived from scaling analysis to identify the distinctness and stability of the thermal boundary layer at any local position. These two functions reveal four possible flow scenarios, depending on the bottom slope and the maximum water depth. For each flow scenario, the flow domain may be composed of multiple subregions with distinct thermal and flow features, depending on the Rayleigh number. The dividing positions between neighboring subregions and the flow properties in each subregion are quantified by scaling. The scenario of relatively large bottom slope and shallow water is examined in detail and classified into three flow regimes based on the Rayleigh number. For the unstable flow regime, the entire flow domain is composed of three subregions, with the dominant mode of heat transfer changing from conduction to stable convection and finally to unstable convection as offshore distance increases. Characteristics of instability are investigated through a comprehensive spectral analysis which reveals the dependency of spectral properties on water depth, offshore distance and the Rayleigh number.

For isoflux cooling on the water surface, flow scenarios revealed by scaling analysis share similarity with that of the radiation heating, although the mechanisms of these two cases are significantly different. The distinctness and stability of the thermal boundary layer are identified by two critical Rayleigh number functions, a comparison between which reveals two possible flow scenarios depending on the bottom slope. The scenario with relatively large bottom slopes is examined in detail and further classified into three possible flow regimes depending on the Rayleigh number.

For constant and ramped (increasing linearly with time) isothermal heating at the water surface, a hybrid of approximate analytical solutions and scaling analysis is used to quantify the flow in the conductive region and scaling analysis is developed for the convective region. For the conductive region, the problem is simplified into a one dimensional conduction problem with a variable local water depth. The analytical solutions of the simplified problem agree well with the numerical results obtained by solving the full Navier-Stokes equations. It is revealed that for the conductive region, when the thermal boundary layer reaches the local bottom at time t_{sp}, the local velocity reaches a maximum value for constant heating, and for ramp heating, the flow becomes steady at time t_{sp} if the ramp duration is larger than t_{sp}. For both constant and ramped heating, flow in the conductive region eventually becomes isothermal and stationary. The dependency of the maximum velocity and steady state velocity on various flow parameters is quantified by scaling. For the convective region, a comparison between the ramp duration P and the time it takes for convection to balance conduction reveals two scenarios: if the ramp finishes before the balance, no steady state is reached within the ramp duration, and after the ramp finishes, the flow velocity continues to increase and gradually becomes steady, whereas if the ramp finishes after the balance, a quasi-steady state is reached within the ramp duration, and the flow becomes steady soon after the ramp finishes. For both scenarios, the flow reaches the same steady state velocity as the corresponding constant heating case.

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Nomenclature

A	bottom slope
$\vec{A}$	surface area vector enclosing the control volume in (A.13)
a, b	coefficient of the discredited equation (A.19)
$Conv(x)$	convection integrated over the local depth at offshore distance x
$\overline{Conv(x)}$	convection averaged over the local depth at offshore distance x
C_p	specific heat
f	the lowest frequency of harmonic mode
f_d	the most dominant frequency
$f_1(x)$	critical function of Rayleigh number for distinct thermal boundary layer
$f_2(x)$	critical function of Rayleigh number for unstable thermal boundary layer
F_y	buoyancy force in the y momentum equation
g	acceleration due to gravity
h	the maximum water depth
$H(x)$	horizontal heat transfer rate averaged over the local depth at offshore distance x
H_0	volumetric heating intensity at the water surface (= $I_0/\rho C_p$)
I_0	radiation intensity at the water surface
k	thermal conductivity
m	coefficient of the QUICK scheme (A.14)
$\hat{n}$	coordinate normal to the sloping bottom
p	pressure
P	the duration of the ramp heating
P_d	power of the most dominant frequency
Pr	Prandtl number
$Q(x)$	Flow rate integrated over the local depth at offshore distance x
$\overline{Q(x)}$	Flow rate averaged over the local depth at offshore distance x
Ra	global Rayleigh number
Ra_c	critical Rayleigh number of instability
$Ra_c(0°)$	critical Rayleigh number of instability for horizontal fluid layers
Ra_L	local Rayleigh number of the thermal boundary layer

S	Internal heating source
S_ϕ	source of ϕ per unit volume in the general transport equation (A.13)
std	standard deviation of the time series at quasi-steady state
t	time
t_B	time for the onset of instability
t_c	time for the thermal boundary layer to reach steady state
t_d	time for the thermal boundary layer to diffuse over the local water depth
t_m	time for the local flow to reach the maximum velocity and the maximum horizontal temperature gradient
t_{sp}	steady state time within the ramp duration
T	temperature
T_a	temperature increase through absorption of radiation
T_b	temperature increase through bottom heat flux in the radiation heating case
T_S	temperature decrease through surface heat flux in the cooling case
T_0	initial temperature
T_1	Difference between the local temperature and the average temperature
$\bar{T}$	the average temperature over the local water depth
u	component of velocity in the x direction
u_{max}	the maximum horizontal component of velocity
u_p	velocity at the time that ramp finishes for the short ramp scenario
u_{sp}	steady state velocity within the ramp duration
u_s	velocity at quasi-steady state for the long ramp scenario
v	component of velocity in the y direction
$\vec{v}$	velocity vector
V	control volume
x	coordinate in the horizontal direction
x_0	dividing position between distinct and indistinct thermal boundary layer
x_1	dividing position between stable and unstable subregions
x_2	dividing position between unstable middle region and stable offshore region for the deep water case in radiation heating
y	coordinate in the vertical direction

Greek symbols

α	under-relaxation factor
β	thermal expansion coefficient
ΔT	temperature difference between the water surface and the initial temperature for constant isothermal heating
Δt	time step
$\Delta \phi$	variation of ϕ at each iteration
δ_T	thickness of the thermal boundary layer
δ_d	thickness of the distinct thermal boundary layer at steady state for constant isothermal heating
ϕ	quantity to be solved in the general transport equation
ϕ_{old}	value of ϕ in the previous iteration
Γ_ϕ	diffusion coefficient for ϕ in the general transport equation
η	bulk attenuation coefficient for water
κ	thermal diffusivity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	density
ρ_0	initial density
θ	inclination angle of the thermal layer
τ	Normalized temperature difference between the local position and the water surface
$\bar{\tau}$	Normalized average temperature over the local depth
$\bar{\tau_s}$	Normalized steady state average temperature over the local depth
$\left.\frac{\partial \bar{\tau}}{\partial x}\right\|_s$	Temperature gradient at steady state within the ramp duration

Subscript

f	face of the control volume
i, j	tensor indices
p	cell P
nb	neighbouring cells of cell P

Superscript

n	Time level of iteration