Discretisation of Constant Rate Loading

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ABSTRACT

An analytical expression incorporating constant rate loading in the determination of consolidation settlements has been summarised. An alternative method to account for time-dependent loading has also been considered, where the total applied load was divided into equal increments which were each applied at known intervals of time using the program Matlab. To establish the decay of excess pore pressure and thus, the percentage consolidation, Terzaghi's original one-dimensional solution was used, whereby the incremental loads were each instantaneously applied, giving rise to small 'initial' uniform distributions in excess pore water pressure. It was found that for increments of time factor less than 0.0143, this discretised approach effectively becomes a constant rate loading problem. The post-construction settlements that were generated using this discretised approach were compared with those generated using the original constant rate loading approach, which was found to be inaccurate for small loading periods. Finally, simple exponential approximations were developed to describe the percentage consolidation that occurs during construction for any period of loading.

Keywords: clays, consolidation, pore pressures, settlement, theoretical analysis.

1 INTRODUCTION

In certain geotechnical applications, it is necessary to estimate the consolidation settlement of a soil layer that is subjected to an increase in vertical total stress that occurs over a prolonged period of time. Foundation construction loading, or surcharge loading on a clay layer, are cases where time-dependent loading must be considered – the assumption of an instantaneously applied load no longer applies. Over the years, several methods for calculating primary consolidation settlements that occur under time-dependent loading have been developed (Gibson 1958, Olson 1977, Lee and Sills 1981 Zhu and Yin 1998, Conte and Troncone 2006, Zhu and Yin 2005, Hsu and Lu 2006, Hanna et al. 2011). Olson (1977) derived a mathematical solution to one-dimensional consolidation for constant rate loading (or ramp loading) in which the vertical total stress is assumed to be uniform with depth. That is, a uniform initial excess pore pressure distribution was considered. Studies conducted by Gibson (1958) and Lee and Sills (1981) account for the compression of the soil layer during deposition. Zhu and Yin (1998) extended the investigation into ramp loading by considering an excess pore water pressure distribution that varies linearly with depth and time.

Hanna et al. (2011) recently proposed a simple and easily applicable method for calculating construction and post-construction settlements, which is further explored within this paper. Here, the initial distribution of excess pore water pressure is assumed to remain constant over the depth of the soil layer.

2 ANALYTICAL SOLUTION

2.1 Constant rate loading

The Terzaghi equation that describes the decay of excess pore pressure that occurs when a load is applied instantaneously to a saturated clay stratum is shown as follows:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \tag{1}$$

where u is the excess pore water pressure at time t, z is the depth measured downward from the top of the clay layer, and c_v represents the coefficient of consolidation. In reality, the load is rarely applied instantaneously, but instead is applied in steps as construction or preloading takes place. By reducing the loading to a constant rate, and discretising the ramp loading into infinitesimal pressure increments, Hanna et al. (2011) showed that the degree of consolidation at the end of loading (U_L) can be given by:

$$U_L = 1 - \frac{1}{T}\sum_{n=0}^{\infty}\frac{2}{M^4}\left[1 - \exp(-M^2 T)\right] \tag{2}$$

where:

$$M = \frac{(2n+1)\pi}{2} \tag{3}$$

and T is the time factor[1] corresponding to the loading period (t_L), and is based on layer thickness H irrespective of drainage path length:

$$T = \frac{c_v t}{H^2} \tag{4}$$

The constant rate loading relationship in equation (2) is based on the assumption that the infinitesimally applied loads all generate a uniform distribution of excess pore water pressure with depth. If this assumption of uniform 'initial' excess pore pressure is valid, the proposed expression for constant rate loading can be used as the baseline for determining the construction and post-construction consolidation settlements for any loading period (t_L). Here, the following adjustments are required;

<u>During Construction</u> ($t < t_L$) – The 'final' consolidation settlement at the end of loading (U_L) is proportionally reduced to reflect the fraction of load being applied at t. If the load accumulated at time t is represented by $q(t)$ and the total load at the end of loading is given by q_L, the consolidation settlement during construction can be calculated as follows:

$$U_{t<t_L} = U_L\left(\frac{q(t)}{q_L}\right) \tag{5}$$

<u>Post-construction</u> ($t > t_L$) – The problem can now be treated as an instantaneous case, where the 'initial' excess pore pressure distribution (at $t = t_L$) is assumed to be sinusoidal or half-sinusoidal for doubly and singly drained layers, respectively. This assumption is based on the knowledge that the excess pore pressure isochrones take a sinusoidal or half-sinusoidal shape during consolidation (when considering a uniform initial excess pore water pressure distribution). The post-construction consolidation settlements can thus be calculated using the following equation:

$$U_{t>t_L} = U_L + (1 - U_L)U_{t-t_L} \tag{6}$$

where U_L is the average degree of consolidation at the end of constant rate loading (t_L). The U_{t-t_L} values are those generated by a sinusoidal/half-sinusoidal initial distribution (due to an instantaneously applied load), which are commonly available in literature. These values should be selected and used in equation (5) according to the following time factor:

$$T_{t-t_L} = \frac{c_v(t - t_L)}{H^2} \tag{7}$$

The complete post-construction settlement can be plotted in relation to construction settlement by then plotting all $U_{t>tL}$ values at T values that have been shifted to account for the loading period (i.e. $T = T_{t-t_L} + T_L$). The settlement-time curves for various loading periods are shown in Figure 1 for one- and two-way drainage cases. The constant or ramped loading curve (RL base curve) is also provided as a base-line.

[1] The dimensionless time factor referred to in this paper is not dependent upon drainage path length, and is only expressed in terms of layer thickness, the advantages of which are discussed in Lovisa et al. (2011). As a result, separate time factors are required for singly and doubly drained cases.

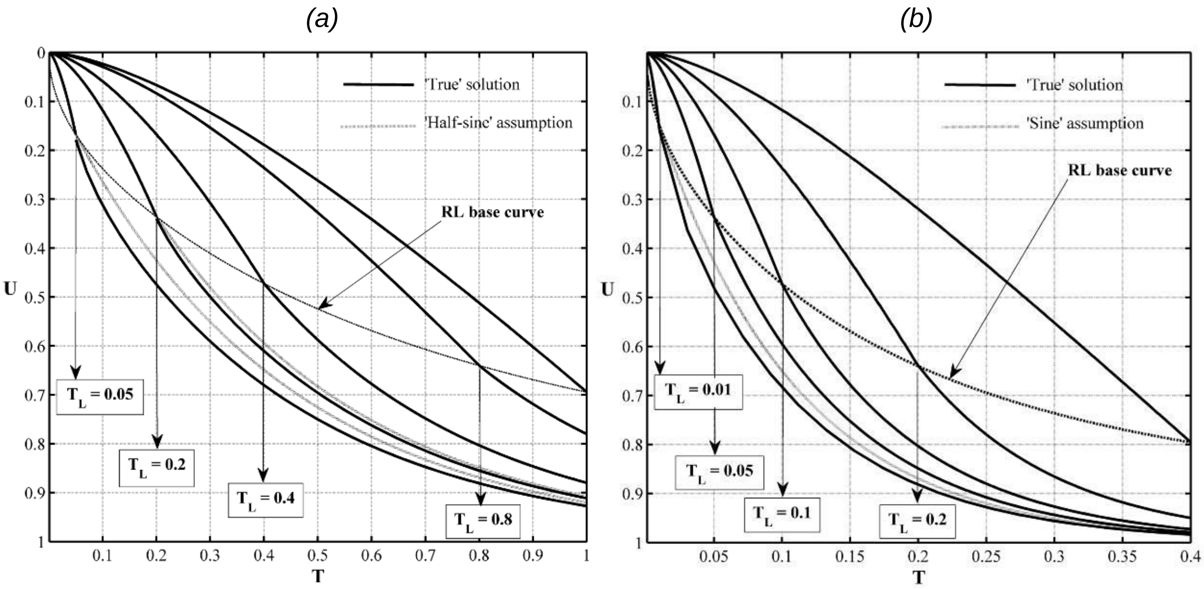


Figure 1. Pre- and post-construction consolidation settlement for various construction times (T_L) with (a) one-way or (b) two-way drainage permitted

2.2 Discretised loading

To assess the assumption of a sinusoidal/half-sinusoidal 'initial' excess pore pressure distribution at $t = t_L$, the consolidation settlement resulting from time-dependent loading was determined using an alternative approach. Here, the constant rate loading was actually simulated by applying finite but very small 'instantaneous' loads. The process was still analytically examined, but using the solution to Terzaghi's consolidation theory in equation (1) for an instantaneously applied load. Here, the total load q_L, was divided into a large number of increments (each with magnitude Δq) which were applied in a discrete fashion over the course of t_L. In each case, the load increment was allowed to decay for some fraction of time (ΔT), upon which the next load increment would be added, and subsequent pore pressure decay allowed. By increasing the number of increments (i.e. reducing the magnitude of Δq and ΔT), it is possible to determine the point at which this discretised loading effectively becomes constant rate loading.

An example of this process is shown in Figure 2 for a loading period of T_L = 0.3, where the number of loading increments was increased until the consolidation settlement approached the settlement generated using the 'true' constant rate loading expression.

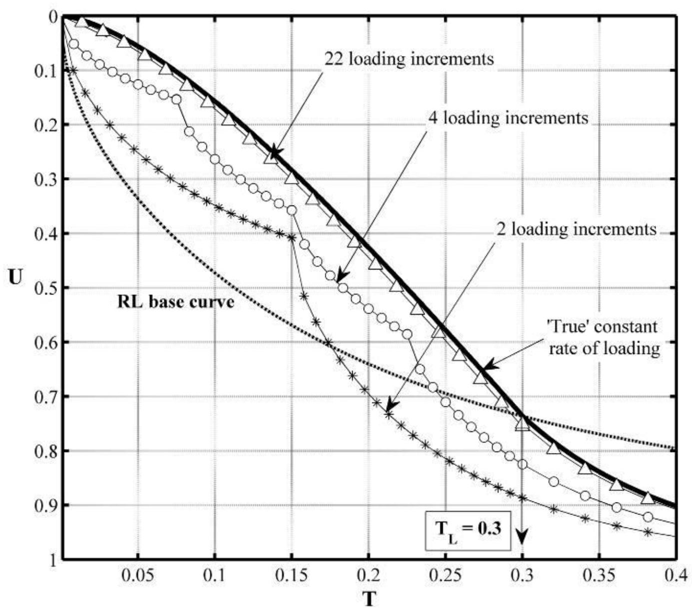


Figure 2. Effect of varying number of loading steps within construction period of T_L

This procedure was repeated for a number of other values of T_L to confirm the limiting value of ΔT. It was found that if the time factor between loading increments was less than 0.0143, the discretised loading could be considered constant rate loading.

Using this discretised approach, the validity of the sinusoidal/half-sinusoidal assumption used in equation (5) was examined. It was found that for values of $T > 0.2$ and $T > 0.05$ for one- and two-way drainage, respectively, this assumption is completely valid. However, for values of T less than 0.2 and 0.05 (for one- and two-way drainage), the post-construction settlements are underestimated, and consolidation actually proceeds slightly faster than anticipated as shown in Figure 1. This can be attributed to the shape of the excess pore pressure isochrones – in the early stages of consolidation, the shapes of pore pressure isochrones resulting from a uniform initial excess pore pressure distribution are actually parabolic rather than sinusoidal. This is highlighted in Figure 3, where the actual pore pressure isochrones resulting from a uniform initial distribution operating under one-way drainage are shown along with half-sinusoidal approximations. As suggested by the constant rate loading comparison, the isochrones do not become sinusoidal in shape until T = 0.2 (or T = 0.05 in the case of two-way drainage).

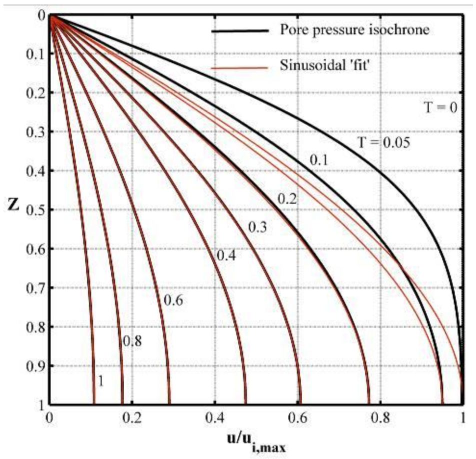


Figure 3. Pore pressure isochrones as they become sinusoidal

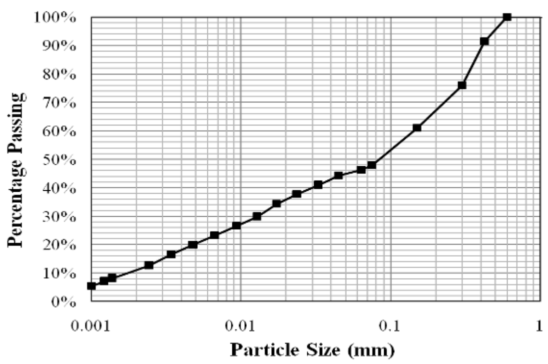
This discretised approach has many applications beyond constant rate loading. For example, surcharge preloading usually takes place in large 'steps', where a quantity of fill is deposited and allowed to sit for weeks before another deposition occurs. As a result, an assumption of constant rate loading in these cases might be unreasonable. Instead, the discretised approach outlined in this study can be used to assess the percentage consolidation that will occur given any variety of fill history.

3 EXPERIMENTAL STUDY

Using the knowledge that applying load increments at time factor intervals of less than T = 0.0143, an experimental study was conducted to complement the analytical results. Time-dependent loading tests were carried out using a sandy, highly plastic clay, the properties of which are provided in Table 1.

Table 1: Properties of soil

Property	Value
Specific Gravity	2.36
Liquid limit	71%
Plastic limit	33%
Plastic index	38%
Linear shrinkage	5%
Effective grain size, D_{10} (mm)	0.0018
D_{60} (mm)	0.145
D_{30} (mm)	0.013
Coefficient of uniformity, C_u	81
Coefficient of curvature, C_c	0.64
Coefficient of consolidation, c_v (m^2/yr)	0.5



Two independent oedometer tests were conducted simultaneously (labelled Sample A and B). The relevant consolidation parameters including applied stress, total primary consolidation settlement and initial thickness are provided in Table 2.

Table 2: Consolidation parameters

Sample	q (kPa)	ΔH (mm)	H_0 (mm)
A	107.6	0.206	18.576
	215.1	0.272	18.241
B	107.6	0.257	18.887
	215.1	0.38	18.429

The total load (or more accurately, pressure) was applied over a period of two hours, which corresponds to a time factor of approximately 0.3, based on c_v values established previously using standard oedometer tests. The load was divided into 240 increments which ensured a small enough ΔT of 0.00125 (i.e. constant rate loading could be reasonably assumed). Physically, this required spooning sand into a hanging bucket every 30 seconds. The results for each sample are shown in Figure 4, and demonstrate a close agreement between the theoretical constant rate loading curve and experimental discretised loading curve.

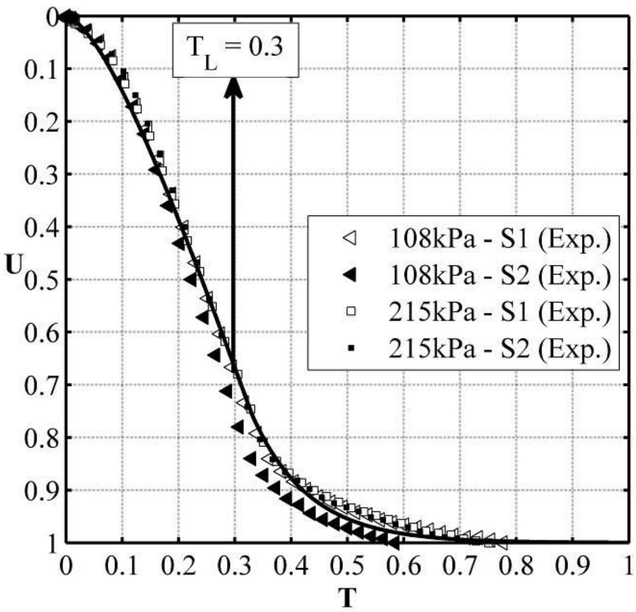


Figure 4. Experimental vs theoretical time-dependent loading

4 APPROXIMATIONS FOR CONSTANT RATE LOADING

It is widely known (Taylor 1948, Fox 1948) that the early stages of consolidation (for $U < 0.52$) can be mathematically approximated using an exponential function of the form:

$$U = AT^B \tag{8}$$

where $A = 1.128$ and $B = 0.5$ for a case of uniform initial excess pore pressure where the load is applied instantaneously.

Using the procedure outlined in Lovisa et al. (2012), this exponential approximation was also found to apply to the entire region of construction settlement, regardless of T_L. Adjusting the constants A and B was sufficient to adequately capture the entire settlement-time curve during construction. The variation in these approximation constants with time is shown in Figure 5. For cases where one-way drainage is permitted, assuming a constant value of B = 1.5 as the power constant and adjusting A accordingly will result in root mean square (RMS) errors less than 0.002.

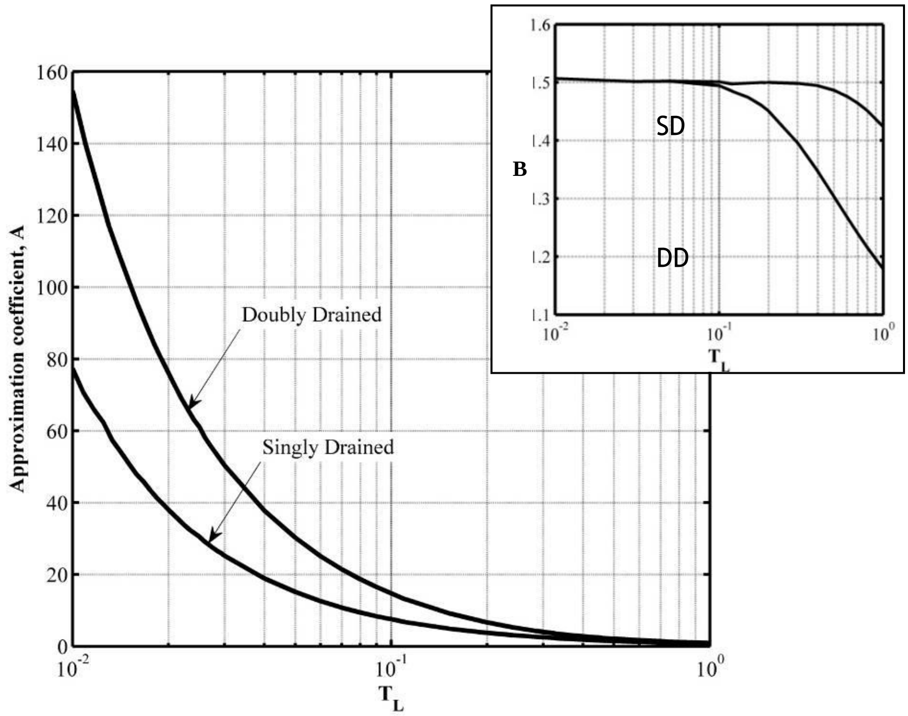


Figure 5. Approximation constants

5 CONCLUSION

The analytical expression for consolidation settlement due to constant rate loading proposed by Hanna et al. (2011) has been summarised. An alternative approach using a discretisation technique has been proposed, where the total load was divided into a large number of increments, each of which was 'instantaneously' applied at select time intervals. When these time factor intervals became less than 0.0143, this discretised approach effectively becomes a constant rate loading problem. Once this alternative method was validated, the construction settlements for small construction periods (T < 0.2 for one-way drainage and T < 0.05 for two-way drainage) were evaluated and compared with those determined using the true constant rate loading approach. It was found that the assumption of a sinusoidal/half-sinusoidal distribution of excess pore pressure at the end of construction for these cases was unrealistic. It is more likely that the excess pore pressure distribution takes a parabolic shape, which accounts for the discrepancy between results.

The rate of consolidation during construction can be easily approximated using a simple exponential expression. The form of the approximating equation remains the same, and equation constants are simply adjusted to account for varying loading periods.

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