Vertical Stresses within Granular Materials in Silos

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Keywords: Silo, vertical stress, earth pressure, Janssen's effect, numerical modelling

ABSTRACT

Cylindrical silos are used to store granular materials, such as grain, sugar, flour and pharmaceutical items. They undergo different types of loadings during their operational lifetime. There are several techniques used by the design engineers to evaluate the vertical stresses within these silos. This paper summarises the numerical analysis of vertical normal stresses within the granular material stored in the silos, and validation of simulations through model tests in the laboratory. Analytical methods were developed in the past to quantify the vertical stresses in silos. Due to arching, the vertical stresses within the granular material contained within the silos are significantly less than that expected from the overburden weight. The dependence of arching on material properties and the geometry of the silo is studied through several laboratory trials. Sand, placed at 30% relative density, was used in the laboratory model tests. Numerical simulations were carried out using FLAC, a commercially available finite difference software, and the vertical stress variation with depth was compared with test results, then the implications were discussed. Although an aspect ratio of six (height/diameter) was used in the laboratory study, extrapolation for higher aspect ratios is discussed using the FLAC simulations.

INTRODUCTION

Silos are used for mass storage of many products like grain, sugar, cement, flour and pharmaceutical items in chemical, agriculture, and food processing industries. In underground mining, granular backfills such as hydraulic fills are stored in square or rectangular stopes, where the stress variation with depth is similar to that in a silo. Operational failures of silos have been reported over the time (Dogangun et al. 2009). Understanding the stress distribution in these cylindrical silos, is necessary for carrying out their structural designs. The correct understanding of vertical stresses will improve design criteria for silos, hoppers as well as foundations and other supporting structures. This study aims to identify the vertical stress variation when the vessel is filled with granular material and how the variation is influenced by the diameter and wall roughness conditions. Model laboratory tests were undertaken and test results are compared with those from numerical simulations. Additionally, vertical stresses derived from analytical formulations are compared with numerical model and laboratory test results.

Arching occurs when granular material is placed in rigid containers or behind retaining walls. Shear stress acting at the interface between the wall and the granular material leads to arching and results in lower vertical stresses at any depths with considerable amount of loads transferred to the walls. Arching in silos was identified by Janssen (1895) and he developed an expression for stress distribution in a silo filled with corn. Janssen observed a significant stress reduction at the bottom of the silo, and identified that phenomenon as arching. Additionally Janssen recognized the influence of silo width on the stress distribution. This exponential stress variation is referred to as the Janssen effect and later this expression was extended by others into backfilled trenches (Marston 1930), retaining walls (Handy 1985; Paik and Salgado 2003), backfilled mine stopes (Pirapakaran and Sivakugan 2007a).

Analytical expressions serve as basis for stress calculations in designs. In all the derivations developed so far, the quasi-static equilibrium for the silo was assumed and equations were derived accordingly (Pipatpongsa and Heng 2010). Janssen made two assumptions that (1) vertical stress is distributed uniformly over the horizontal cross section and (2) the vertical and horizontal stresses are principal stresses. The second assumption is incorrect, since the shear stresses must be zero at principal planes. In order to treat the non-linear stress distribution, Walker (1966) introduced a distribution factor, which is defined as the ratio between the axial stress at the wall to mean axial stress. Nedderman (1992) discussed that this distribution factor is complex and it doesn't affect the results significantly. Because of the reasons stated above, in the present studies, a uniform vertical stress is generally assumed over the diameter (Pipatpongsa and Heng 2010).

Though it assumed that the vertical stress is uniform over the entire width, intuitively from the understanding of arching, it can be seen that the vertical stresses increase towards the centre. This is due to shear stresses at the walls, resulting in greater load transfer to walls and therefore vertical stress is minimized near the wall and maximised at the centre. Hence, the predicted average vertical stress with these analytical expressions may exceed the actual vertical stress at centre, and this was noted by Janssen (1895) and many others. Therefore, to address this complex stress distribution under static conditions, numerical modelling was used and verified with laboratory tests.

DEVELOPMENT OF ANALYTICAL SOLUTIONS

Shear plane method (also known as the method of differential slices) is often used in designs, where the static stability of an infinitesimal horizontal layer element is considered (Figure 1). This expression is integrated over the entire depth, to estimate the average vertical stress at bottom. When calculating the shear stress on walls imposed by the layer element, it is assumed that the maximum shear stress is mobilized along the walls throughout the entire height. The normal stress on wall is taken as the horizontal stress at that height and is calculated with the lateral stress ratio *K*, which is defined as the ratio between the horizontal stress to vertical stress. In defining *K*, Janssen (1895) used the horizontal stress at the wall with vertical stress at the centre, but Jáky (1948) used the horizontal and vertical stresses at the centre. These assumptions were studied by Pipatpongsa and Heng (2010) who confirmed that Jáky's assumption can be applied assuming 'at rest' earth pressure conditions. In the study reported herein, *K* was taken as the coefficient of earth pressure at rest, suggested by Jáky (1948), which is defined by $K_o = 1 - sin\varphi$, where φ is the effective friction angle of the material.

For the vertical equilibrium of a layer element within the cylindrical silo (Figure 1),

$$\sigma_z.\frac{\pi D^2}{4} + \gamma.\frac{\pi D^2}{4}.\mathrm{d}h = (\sigma_z + \mathrm{d}\sigma_z).\frac{\pi D^2}{4} + K\sigma_z\, tan\delta.\pi D.\mathrm{d}h \qquad (1)$$

where σ_z is the average vertical stress, *D* is diameter of the silo, γ is the unit weight of stored material, d*h* is the height of layer element, *K* is the lateral pressure coefficient and δ is the interface friction angle.

Equation (1) can be rearranged as follows,

$$\int_{\mathrm{q}}^{\sigma_z} \frac{\mathrm{d}\sigma_z}{\gamma - (4tan\delta.K\sigma_z)/D} = \int_0^z \mathrm{d}h \qquad (2)$$

Assuming no surcharge pressure (q) is applied, σ_z can be derived as,

$$\sigma_z = \frac{\gamma D}{4K.tan\delta}\left[1 - exp\left(-\frac{4K.tan\delta.z}{D}\right)\right] \qquad (3)$$

The main contributors to the vertical stresses in cylindrical vessels can be identified as γ, δ, D, and z in Equation 3. The vertical stress varies proportionally with the unit weight of the stored material (Equation 3). The vertical stress can be expressed in dimensionless form ($\sigma_z/\gamma D$), after normalizing with the multiple of unit weight and diameter of the vessel. The height was normalized with the diameter *D*, whereas the maximum height is 6*D* in this study. Moreover Singh et al.(2010) showed that diameter vertical stress variation is almost same, when the friction angle of stored material varies from 25° to 45°. Therefore, results of this study can be applied to other materials with the effective friction angle in the above range. When the results are expressed in normalized units as given above, it is possible to estimate vertical stress for other dimensions and in vessels containing different materials.

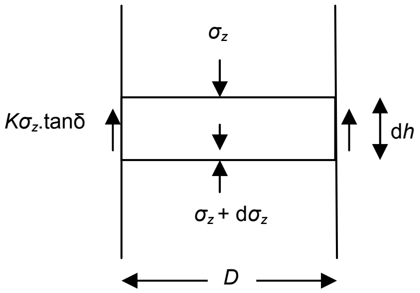

Figure 1. Schematic diagram of a horizontal layer element used in shear plane method

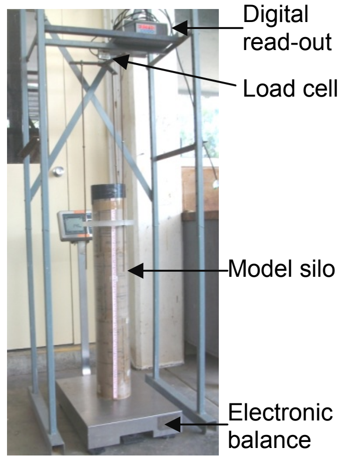

Figure 2. Laboratory test setup

LABORATORY TESTS

This study investigates the variations of vertical stress in cylindrical silos, with diameter and the wall roughness, through a series of laboratory tests. The tests were carried out at James Cook University Geomechanics Laboratory. Two cylindrical model silos, with outer diameters of 100 mm and 150 mm, were made out of Perspex material. The total height of the models was limited to six times the diameter. Pirapakaran & Sivakugan (2007a) identified that the full development of arching takes place within this height where the vertical stress reaches an asymptotic value. Sand was used as the granular filling material. The grain size distribution of the sand was determined, from Malvern Mastersizer X, as median grain diameter of 0.34 mm, effective grain size (D_{10}) of 0.115 mm and uniformity coefficient (C_u) of 2.47. In the unified soil classification system, the sand would be classified as uniformly graded sand, with symbol of SP.

The interface characteristics depend on wall roughness and internal friction angle of the material being stored. The effective friction angle of sand was determined with direct shear test (AS 1289.6.2.2-1998) in the laboratory at a relative density of 30% (Table 1). Two different wall roughness conditions were used to study the wall roughness as (1) low wall roughness-Perspex wall, (2) high wall roughness-glued sandpaper [KMCA Garnet G62 P40 Garnet electro coated dry sanding abrasive paper]. Interfacial friction angles were determined in the laboratory conditions by, replacing the lower half of the direct shear box with a Perspex block, where a sand paper was attached to the Perspex block on the side which is shearing with sand in the test (Ting et al. 2012). The interfacial friction angles, for selected wall roughness conditions, are tabulated in Table 1.

Table 1*. Material properties for sand and Perspex, with wall roughness details for numerical simulations*

Properties	Sand at relative density (30%)	Perspex wall
Young's Modulus, E (MPa)	0.42	3200
Poisson's ratio, ν	0.2	0.3
Density, ρ (kg/m^3)	1496	1190
Effective friction angle of sand, (°)	38	-
Interface friction angle – low wall roughness, (°)	27	-
Interface friction angle – high wall roughness, (°)	38	-
Angle of dilation, (°)	2.5	-

The Perspex silo was held vertically by three equally spaced supports. Then supports were attached to a load cell (Revere Transducers, type 9363-D3-100kg_20T1; precision 0.001 kg), where the load transferred to walls was measured. The model silo was lowered, leaving a grain size gap between the bottom of the model silo and the electronic balance (maximum reading 60 kg, precision 0.02 kg). The load transferred to bottom after each filling, was measured with the balance (Figure 2). The apparatus has been described in detail by Pirapakaran and Sivakugan (2007b). The weight of sand (relative density= 30%) required to fill the silo was calculated and model silo was filled in 6 layers. After each stage of filling, the readings from the balance and load cell were taken and checked whether the summation of the load on the load cell and bottom scale, matched the weight of poured sand. This check ensured the performance of the apparatus after each stage of filling. After filling each layer, the average stress at bottom was calculated, dividing the electronic balance reading by the cross sectional area of the model silo. Tests were performed on two model silos with outer diameters of 150 mm and 100 mm and for two wall roughness conditions, mentioned above. The variation of the vertical stress with depth for every case is presented in Figure 3.

NUMERICAL SIMULATIONS

Numerical simulation is an effective tool, when the calculations are not straightforward or where closed form solutions are not available. Fast Lagrangian Analysis of Continua (FLAC), a commercially available finite difference software package, can model the filling of the silo and the system behaviour after each filling stage. As discussed earlier the arching behaviour can be effectively modelled through numerical (FLAC) simulations, and the non-linear vertical stress variation along the width could be identified. In addition, vertical stress at any point can be calculated with numerical simulation results.

Axisymmetric configuration was used to model silos. Dimensions of the finite difference models were the same as those of the model silos tested in the laboratory, where the outer diameters of 100 mm and 150 mm with a wall thickness of 3 mm. Height of each model was taken as 6D. The model was discretized into 1 mm x 1 mm grid. A homogenous linear elastic material was used to model the Perspex wall but the granular fill was modelled as a Mohr-Coulomb material. All the material parameters used for modelling are tabulated in Table1. Along the side of the model, wall was fixed for both y and x-displacement. Y-displacement was fixed at the bottom of the silo. To model the interface between the wall and the granular fill, interface elements were used. Low rough Perspex interface and rough sand paper interface were modelled with appropriate interface parameters (Table 1).

The silo was filled with equal thickness layers of filling material in the simulation. Pirapakaran and Sivakugan (2007a) discussed the effect of layer height on the vertical stress distribution in FLAC with various filling layer heights. The simulation software treats the filling material as a continuum; hence the silo was not filled instantly. The model silos were filled in 36 layers and solved for equilibrium after each filling. The average vertical stresses at particular heights were computed so that the values can be compared with the laboratory results.

RESULTS AND DISCUSSION

The vertical stresses obtained from numerical simulations and laboratory tests are compared in Figure 3. The vertical stress predictions from the analytical equations (Equation 3) also included in the same figure. In analytical equations, the average stress occurred in cylindrical silos increases exponentially with depth and reach an asymptote, which is observed in numerical simulations and laboratory tests. Wall roughness conditions significantly affects the vertical stress variation as predicted in analytical expressions, whereas more load is transferred to walls when the wall becomes rough (Figures 3a vs. 3b).

Vertical stress variation with analytical expressions for low wall roughness conditions shows a good agreement with the laboratory and numerical model results. Adherence among the three approaches could be seen in Figures. 3a and 3c, when the wall surface is low rough. But there is some deviation in the case of rough wall conditions as in Figures. 3b and 3d. When the wall surface becomes rough with sand papers, the average vertical stress distribution becomes quite different from analytical expressions. Since the analytical equation tends to underestimate the vertical stresses with the rough wall conditions and incorporating additional parameters such as angle of dilation in the analytical formulations may improve the predictions.

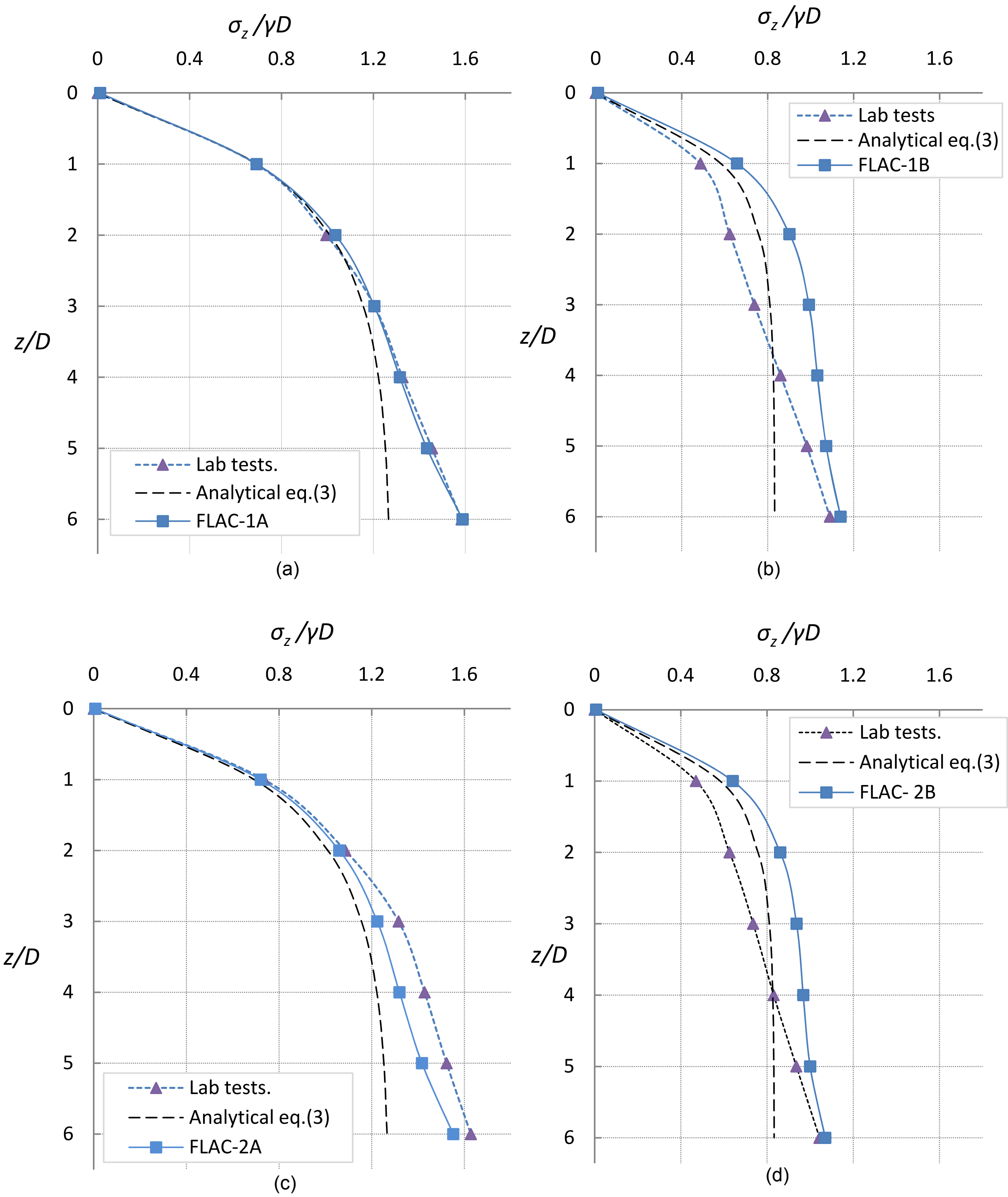

Figure 3. Variation of vertical stress compared to analytical, lab tests and FLAC simulations.(a) 100 mm diameter and low wall roughness, (b)100 mm diameter and high wall roughness, (c)150 mm diameter and low wall roughness, (d)150 mm diameter and high wall roughness

When comparing above three approaches, significant scatter is observed near the base, in all four situations. Towards the bottom of the silo the vertical stress variation, from numerical and laboratory model test approaches, becomes linear deviating from analytical solutions (Figure 3). Shear plane method assumes continuous layer elements in calculations (Figure 1). Furthermore, elements within the fill can yield due to the stress from top layers. However, close to the stiff base, the movements of the elements are restricted which limit the degree of mobilisation and hence the shear stress is less mobilised along the wall. Therefore it can be deduced that the arching is not fully developed very close to the bottom, which is possibly the reason that the laboratory and numerical model results deviate from the analytical expressions near the bottom.

When the diameter of silo is changed, the vertical stresses show agreement in tests and simulations (Figures 3a vs. 3c and 3b vs. 3d) as they are presented in terms of dimensionless variables. This agreement shows the possibility of extending these results to silos with large diameters, representing dimensions of the prototype ones. Extrapolation is possible with the use of numerical simulations with appropriate parameters representing the field situations. For higher *z/D* ratios, laboratory studies will not be feasible. However, these situations can be conveniently modelled through numerical simulations. Considering Figure 3, it is clear that the vertical stress have reached the asymptote at *z/D* equals 6. It is useful to use this condition as a guide, but it is recommended to simulate the specific condition numerically, in order to estimate the stress conditions within the silo.

Since the confining stresses were very low in the lab model (maximum 3.5 kPa), it was suggested to use a low Young's modulus with FLAC simulations. Therefore, to represent the laboratory conditions in the model silo, a low Young's modulus was used in the numerical simulations. In real-life applications, a higher Young's modulus appropriate to the stress levels needs to be considered. Since the in-situ Young's modulus of sands is 10 MPa to 40 MPa, FLAC can be used to simulate the actual site conditions with suitable parameters but the modelling procedure will be the same. Furthermore, this apparatus can be modified with the use of earth pressure cells to measure the vertical stress at various locations on bottom. The non-linear stress variation at bottom in the horizontal direction can be studied further by these additional earth pressure cells.

CONCLUSIONS

To estimate the vertical stresses in cylindrical silos, the Janssen's (1895) equation gives some basis but is not sufficient. The effects of the silo diameter and the wall roughness on the vertical stresses were studied through laboratory model tests which enabled the measurements of average vertical stresses. It is shown that numerical simulations conservatively estimate the vertical stress at any depth. Therefore, numerical simulations can be adopted to predict the stress conditions in cylindrical vessels, when filled with a granular material. The laboratory setup proposed herein can be used effectively to determine the average vertical stress at any depth within the fill. By placing an earth pressure cell at the centre of the base, it is possible to determine the vertical stress variation with depth along the centreline of the model.

The use of normalized dimensionless variables offer greater flexibility to adopt these results for any other situation where the height is less or equal to six times of diameter or for another material. Since the vertical stresses reached an asymptote after this height, for taller silos extrapolation is suggested. Furthermore, numerical simulations would be an effective tool to estimate vertical stresses for these situations.

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