WATER RESOURCES RESEARCH, VOL. 17, NO. 4, PAGES 827–832, AUGUST 1981

Performance Evaluation of a Stochastic Optimization Model for Reservoir Design and Management With Explicit Reliability Criteria

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A seasonal, chance-constrained linear programing model, which facilitates the development of reservoir system designs and operation policies and which incorporates multiple linear decision rules (LDR) conditioned upon the streamflows in other seasons, is evaluated with variations in the numbers of seasons per year and decision rules per season. The performance is also compared with the original, single-LDR model. The motivations for the expected results and the numerical solutions obtained for the examples considered are presented. The multiple-LDR model is shown to be superior to the single-LDR model: for equivalent restrictions on the reservoir operation and performance the multiple-LDR model gives a smaller capacity reservoir than the single-LDR model. Furthermore, as the number of LDR's per season is increased, the required capacity decreases. And when the operating rules defined by the models are tested in a simulation of actual reservoir operation, the multiple LDR's perform more closely to the specifications prescribed in the linear program than the single LDR.

INTRODUCTION

During the last 10 years the problems of sizing and operation of water reservoir systems have received great attention with the introduction of systems analysis and operations research methodologies. These attempts are aimed at optimal decision making in reservoir regulation, together with simultaneous explicit statements on the degree of risk or reliability with which the various objectives can be met in the long run.

One approach to finding optimal operating rules, as proposed by *ReVelle et al.* [1969], requires the release rule to be a linear function of reservoir storage. As applied to reservoir operation, the form of this linear decision rule (LDR) is $X = S - b$, where X denotes the release during a period of reservoir operation, S denotes the storage at the end of the previous period, and b is the decision parameter chosen to optimize a criterion function. This rule may be interpreted as an aid to the actual operator's judgment in selecting a release commitment.

Since the introduction of linear decision rules in water reservoir management problems by *ReVelle et al.* [1969], many modifications, extensions, and discussions of the rule have been reported in the literature [e.g., *Sobel,* 1975; *Loucks,* 1976; *Sniedovich,* 1980]. *Loucks and Dorfman* [1975] proposed a LDR that is a function of present storage volume and future inflow; the future inflow is assumed to be a known quantity. *Gundelach and ReVelle* [1975] and *ReVelle and Gundelach* [1975] proposed a LDR that is a function of present and past storage volumes. Both of these extended LDR's, when employed in an optimization model with an objective of minimizing reservoir storage capacity subject to satisfying several performance criteria, require smaller reservoirs than the original LDR. Other investigators have demonstrated the ease of finding optimal linear decision rules under certain conditions [*Eastman and ReVelle,* 1973], the means of including economic efficiency as the criterion for determining optimal LDR's [*Houck et al.,* 1980], and the use of LDR's in multiple-reservoir operations [*Joeres et al.,* 1971; *Nayak and Arora,* 1971]. *Loucks and Dorfman* [1975] presented a comparative study of some of the existing forms of the linear decision rules. Still other investigators have discussed the physical meaning and implications of reservoir operation reliabilities in an attempt to develop a probability theory of reservoirs based entirely on statistical techniques [*Phatarford,* 1976; *Klemeš,* 1969, 1970]. Methods of finding storage size–probability of failure–draft relationships for reservoirs with Markovian seasonal inputs and methods of accounting for the seasonal and annual correlation of streamflows are discussed in these works.

Models to define optimal operating rules that do not use linear decision rules have also been reported in the literature. *Colorni and Fronza* [1976], *Simonovic and Marino* [1980], and others have employed reliability programing, a form of chance-constrained dynamic programing, to find optimal operating rules. *Young* [1967], *Bhaskar and Whitlatch* [1980], and others have proposed using deterministic optimization models and regression analysis to develop operating rules. *Butcher* [1971], *Buras* [1966], and others have assumed a discrete Markov structure for streamflows and used stochastic dynamic programing to define operating rules. *Loucks* [1969], *Houck and Cohon* [1978], and others also assumed a discrete Markov structure for the streamflows but found operating rules using linear programing. And numerous investigators have reported other methods for finding optimal reservoir-operating rules.

The purpose of this paper is to evaluate a proposed multiple-LDR model [*Houck,* 1979]. Instead of a single LDR per season, several LDR's per season, each conditioned on the previous season's streamflow, are defined. Thus the feasible region of the operation rules is relaxed, allowing improvement in the reservoir operation. The multiple-LDR model is solved for various numbers of LDR's per season and seasons per year for a hypothetical reservoir on the Gunpowder River in Maryland. The solutions are compared with each other and with the solution of the single-LDR model proposed by *ReVelle et al.* [1969]. The results of simulating the optimum operation policies of the single- and multiple-LDR models are also compared.

LDR MODELS

Because the single-LDR model is a special case of the more general multiple-LDR model, the latter will be presented first. The objective function of the model is chosen to be minimization of the capacity of the reservoir, which may be considered as a surrogate for initial economic costs. Physical op-

Paper number 1W0453.
0043-1397/81/001W-0453$01.00

erating restrictions and performance requirements are incorporated as constraints in the linear programing model.

The following notation is used in the model:

X_t^{ij} release in season t, conditioned upon streamflows in interval i in season $t-1$ and interval j in season $t-2$;

S_t^j storage volume at the beginning of season t, conditioned upon streamflow in interval j in season $t-2$;

b_t^i decision parameter in season t, conditioned on streamflow in interval i in season $t-1$;

R_t^i streamflow in season t, conditioned on streamflow in interval i in season $t-1$, a random variable;

$F_{jt-1}(\ )$ cumulative distribution function of streamflows in season $t-1$, conditioned on streamflow in interval j in season $t-2$;

P_t^j probability of streamflow in interval j in season $t-2$;

P_t^{ij} probability of streamflows in intervals i and j during seasons $t-1$ and $t-2$, respectively;

α_t^{ij} probability with which X_t^{ij} equals or exceeds $X_{\min}$;

β_t^j probability with which S_t^j equals or exceeds $S_{\min}$;

γ_t^j probability with which S_t^j does not exceed CAP;

CAP reservoir capacity;

i, j indices of streamflow interval in seasons $t-1$ and $t-2$;

I, J number of streamflow intervals in seasons $t-1$ and $t-2$;

$S_{\min}$ minimum permissible storage volume;

t index of seasons;

T number of seasons per year;

$X_{\min}$ minimum allowable release volume per season;

α probability with which releases in a season exceed or equal $X_{\min}$;

β probability with which storage volume in a season equals or exceeds $S_{\min}$;

γ probability with which storage volume in a season does not exceed CAP;

$F_{jt-1}^{-1}(\ )$ inverse cumulative distribution function of streamflows in season $t-1$ conditioned on streamflow in season $t-2$ occurring in interval j.

The continuity equation, or mass balance equation, for storage in the reservoir is

$$S_{t+1}^i = S_t^j + R_t^i - X_t^{ij} \quad (1)$$

The form of the linear decision rule to be used is

$$X_t^{ij} = S_t^j - b_t^i$$

For each season t and for each range or interval i of streamflow volume in season $t-1$, there will be a decision parameter b_t^i. Thus the release in season t will be dependent on the streamflows of previous seasons. Substituting the linear decision rule yields

$$X_t^{ij} = R_{t-1}^j + b_{t-1}^j - b_t^i \quad (2)$$

Therefore storage and release are reduced to relations between the inflow and the decision parameter only. The performance criteria are defined in terms of minimum and maximum required storages and the minimum permissible release.

The first set of chance constraints, to define the probability that each possible release in a season exceeds the minimum allowable release, is

$$P[X_t^{ij} \geq X_{\min}] = \alpha_t^{ij} \quad (3)$$

or

$$P[R_{t-1}^j + b_{t-1}^j - b_t^i \geq X_{\min}] = \alpha_t^{ij} \quad (4)$$

or

$$P[R_{t-1}^j \leq X_{\min} - b_{t-1}^j + b_t^i] = 1 - \alpha_t^{ij} \quad (5)$$

Because R_{t-1}^j is a random variable, the left-hand side of the equation is the cumulative distribution function (CDF) of the streamflows in season $t-1$, conditioned on the streamflow in season $t-2$ occurring in interval j. This CDF, denoted by $F_{jt-1}(\ )$, can be written as

$$F_{jt-1}(X_{\min} - b_{t-1}^j + b_t^i) = 1 - \alpha_t^{ij} \quad (6)$$

Denoting the inverse CDF by $F_{jt-1}^{-1}(\ )$ allows (6) to be written as

$$X_{\min} - b_{t-1}^j + b_t^i - F_{jt-1}^{-1}(1 - \alpha_t^{ij}) = 0 \quad (7)$$

$$i = 1, 2, \cdots, I \qquad j = 1, 2, \cdots, J \qquad t = 1, 2, \cdots, T$$

The convex portion of the inverse CDF can be piecewise linearized so that the regular simplex algorithm or its common variants may be used. *Houck* [1979] has shown that in the range of interest of the model the inverse CDF can be taken as convex without incurring formidable errors. However, with an increase in the number of decision rules per season, or in the number of seasons considered per year, some accuracy may have to be sacrificed in order to keep the feasible region convex. The actual reliability with which release in season t exceeds $X_{\min}$ is the expected value of the α_t^{ij}; this reliability is specified to exceed α, which may assume values like 0.8, 0.9, or 0.99, depending on the actual planning or management problem under consideration. The reliability constraint on release is

$$\sum_i \sum_j P_t^{ij} \alpha_t^{ij} \geq \alpha \qquad t = 1, 2, \cdots, T \quad (8)$$

The same procedure is followed in formulating the reliability constraint on the minimum storage requirement:

$$P[S_t^j \geq S_{\min}] = \beta_t^j \quad (9)$$

or

$$S_{\min} - b_{t-1}^j - F_{jt-1}^{-1}(1 - \beta_t^j) = 0 \quad (10)$$

$$j = 1, 2, \cdots, J \qquad t = 1, 2, \cdots, T$$

The reliability with which minimum storage is exceeded is specified as β, and the reliability constraint is given as

$$\sum_j P_t^j \beta_t^j \geq \beta \qquad t = 1, 2, \cdots, T \quad (11)$$

The other restriction on the maximum storage is given as

$$P[S_t^j \leq CAP] = \gamma_t^j \quad (12)$$

or

$$CAP - b_{t-1}^j - F_{jt-1}^{-1}(\gamma_t^j) = 0 \quad (13)$$

$$j = 1, 2, \cdots, J \qquad t = 1, 2, \cdots, T$$

The reliability with which the storage must not exceed the dam capacity is specified as γ, and the reliability constraint is given by

$$\sum_j P_t^j \gamma_t^j \geq \gamma \qquad t = 1, 2, \cdots, T \tag{14}$$

The entire multiple-LDR model comprises an objective function of minimize reservoir capacity (minimize CAP) and constraints on minimum storages ((10) and (11)), maximum storages ((13) and (14)), and minimum releases ((7) and (8)). As was stated previously, the model can be solved as a linear program.

The single-LDR model is obtained by restricting the indices i and j to a single value $i = j = 1$. Then there is only one LDR per season, $p_t^1 = 1.0$, $P_t^{11} = 1.0$, and the release in one season is now independent of the streamflows in previous seasons. The loss of the dependence of release on previous streamflows and the reduction in the number of release rules per season as incorporated in the multiple-LDR model are expected to result in the single-LDR model requiring a larger reservoir capacity than the multiple-LDR model to achieve the same performance criteria.

TESTING THE LDR MODELS

The multiple-LDR model was extensively tested and compared with the single-LDR model. The operating rules specified by the solutions of the LDR models were tested in a reservoir simulation model. The LDR models were constructed for, and all testing was performed on, a hypothetical reservoir located on the Gunpowder River in Maryland.

In the first phase of testing, the number of seasons (i, j) considered per year was varied, starting with two seasons of 6 months each, in both the multiple-LDR and the single-LDR models. For the multiple-LDR model the number of intervals per season, that is, the number of decision rules considered per season, was also two; the median streamflow value of the historical record was used to partition the intervals, so that the frequency of occurrence of streamflows in each season and each interval was equal. In a similar manner the number of seasons per year was increased up to six. The solutions were obtained for both types of LDR models with equivalent numbers of seasons and for different specified values of minimum release and/or minimum storage. In the second phase the number of intervals or decision rules in the multiple-LDR model was increased to three and tested in a four-season model.

The most time-consuming portion of this study was the actual formulation of the model, which also included the computation of joint and conditioned probability distributions from the historical streamflow data. The commonly adopted method of frequency interpretation was used, and Weibull's plotting formula was utilized to find the cumulative distribution functions. The number of intervals into which a particular season could be partitioned was limited by the length of the historical record available. Even a relatively long record of 81 years can provide only 27 plotting positions per season if streamflows are divided into three intervals. This is a vital limitation because the multiple-LDR model is a linear program that necessitates the linear approximation of the cumulative conditioned distribution functions. All the models tested were found to be highly sensitive to even small variations in plotting positions for very low and high probabilities. However, this does not undermine the comparative judgments presented here.

TABLE 1. Solutions for Two-Season LDR Models

	Minimum Required Capacity (CAP^*), 10^6 m^3	
X_{min}, 10^6 m^3/season	Single LDR	Multiple LDR (Two Intervals)
0–62.5	239.1	189.3
73.1	241.6	189.3
83.6	252.0	189.3
93.9	262.5	189.9
96.2	264.6	192.0
99.2	infeasible	…*
104.5	infeasible	200.4
115.1–∞	infeasible	infeasible

*The value of CAP^* was not determined for certain X_{min} values.

The number of linear segments used to piecewise linearize the nonlinear conditioned CDF's was determined by actual eye estimation, based on the shape of the individual CDF's; no hard and fast rules were followed. Only the convex regions in decision space given by the CDF's could be considered because a convex decision space is necessary to ensure a global optimal solution.

Some sample solutions of the single-LDR and multiple-LDR models for the different variations as described earlier are summarized in Tables 1–3. The LDR models are solved without any restrictions on maximum release and minimum freeboard. The value of S_{min} equals $52.2 * 10^6$ m^3 and the reliability levels (α, β, γ) are set equal to 0.9 in all models unless stated otherwise. Also, the seasons within any particular model are of equal length.

In an effort to improve the four-season, three-interval, multiple-LDR model's performance the constraints corresponding to events of very low joint probability were removed from the model. No significant improvement resulted, especially for very low specified releases. This is due mostly to the fact that for very low values of X_{min} the constraints on α_t^{ij} are rarely binding; hence the constraints on CAP and S_{min} determine the solution.

The models were also solved with different values of S_{min}. As was found by *ReVelle et al.* [1969], the optimal value of CAP changed by an amount equal to the change in S_{min}. It was also found, as expected, that the feasible range of X_{min} in-

TABLE 2. Solutions for Four-Season LDR Models

	Minimum Required Capacity (CAP^*)		
X_{min}, 10^6 m^3/season	Single LDR, 10^6 m^3	Multiple LDR (Two Intervals), 10^6 m^3	Multiple LDR (Three Intervals), 10^6 m^3
0–31.4	148.5	146.1	142.4
41.6	156.2	146.1	142.4
44.3	161.5	146.1	142.4
45.4	…	146.1	142.4
46.9	166.7	148.5	142.4
47.3	…	…	142.4
49.2	…	…	147.8
51.1	…	…	153.5
52.2	infeasible	159.7	…
53.0	infeasible	162.8	159.8
54.1	infeasible	infeasible	165.6
54.5–∞	infeasible	infeasible	infeasible

TABLE 3. Solutions for Six-Season LDR Models

X_{min}, 10^6 m^3/season	Minimum Required Capacity (CAP^*): Single LDR, 10^6 m^3	Multiple LDR (Two Intervals), 10^6 m^3
0–22.7	125.1	123.5
26.5	133.2	123.6
30.3	148.4	138.0
32.2	infeasible	145.5
33.3	infeasible	150.1
34.1–∞	infeasible	infeasible

creases with a decrease in reliability levels (α_t, β_t, γ_t). And the minimum capacity required for a specified minimum release increases as the reliability levels increase: for the six-season, two-interval, multiple-LDR model, with X_{min} equal to $26.5*10^6$ m^3, the minimum capacity is $123.6*10^6$ m^3 with reliability levels of 90% but is $164.9*10^6$ m^3 with reliability levels of 95%.

For the typical multiple-LDR model just mentioned the solution of the linear program, by the multipurpose optimization system resident on a CDC6500, required approximately 60 CPU seconds and cost about $2; the number of constraints equals 199; the number of lower or upper bounds specified equals 96; and the number of variables was 126. However, it should be noted that the size of the linear program may be substantially increased by increasing the number of straight-line approximations of the CDF's. The performance evaluation of the model through simulation is discussed next.

PERFORMANCE EVALUATION

A simulation study was performed with the release policies specified by the solution of the multiple-LDR and single-LDR models. The program was made as simple as possible to simulate the way a hypothetical, single reservoir may operate if the decision rules are blindly applied. Thus whenever possible, the reservoir release equaled the relevant LDR. A failure was defined as any time the LDR could not be followed because too little water was available (actual release is less than LDR) or spilling occurred (actual release is greater than LDR) and any time the release failed to satisfy the minimum required release. Most of the simulations (all of those included in Table 4) were performed for an 80-year period, using the historical streamflows as inputs to the reservoir.

Examination of Tables 1–3 and column 6 of Table 4 shows a significant decrease in the capacity required for a given minimum release when the multiple-LDR model and single-LDR model are compared. When two seasons are considered (models 1 and 2 of Table 4), the single-LDR model specifies a capacity 38% greater than that required by the multiple-LDR model. With four seasons (models 4, 5, and 6 of Table 4) the single-LDR model specifies a capacity 12% and 17% greater than the capacities of the two-interval and three-interval multiple-LDR models, respectively, and for six seasons (models 8 and 9 of Table 4) a capacity 8% larger is required by the single-LDR model.

Although it may appear that the percentage advantage of the multiple-LDR model over the single-LDR model decreases with an increase in the number of seasons, these data may not fully support this conclusion. At the same time that the number of seasons increased, the value of X_{min} with respect to the maximum feasible value of X_{min} diminished. As the minimum release becomes very small, some of the advantage of the multiple-LDR model is lost. Thus two competing effects are included in these data.

This phenomenon can be seen more clearly by examining the results for models 2 and 3 in Table 4. Here the only difference in performance requirements is the value of X_{min}: in model 2 the value of X_{min} is very near the maximum feasible value; in model 3 the value of X_{min} is sufficiently small that it does not even affect the optimal reservoir capacity. If X_{min} were increased from $52.3*10^6$ m^3/season, the value of CAP^*, as shown in Table 1, does not change. One conclusion that could be drawn from these facts is that in a simulation, model 2 will have more failures in meeting the minimum release than model 3, and model 2 will have more storage plus release failures than model 3. This is exactly what occurs, as can be seen by the last three columns of Table 4. The same effect is seen in models 5 and 7 for the four-season case.

Because all reliability levels (α, β, γ) were set equal to 0.9, it is expected, in the simulation, that the percentage of releases less than X_{min} will not be greater than 10%. Also, the percentages of storages equal to the reservoir capacity or less than S_{min} are expected to be no greater than 10%. Because of a slight difference in the way that the LDR model and the simulation model represent actual operation (the LDR model assumes that storage volumes greater than the reservoir capacity could actually exist, whereas the simulation model does not), it is possible that the percentages of various failures could be traded between the types of failure. For example, it would be possible that the percentage of storages equaling capacity would be 12%, while the percentage of releases satisfying the

TABLE 4. Simulation Results

Model	Model Type	Number of Intervals	Number of Seasons	X_{min}, 10^6 m^3/season	CAP^*, 10^6 m^3	Percentage of Releases Less than X_{min}	Percentage of Storages Less Than S_{min} or Equal to CAP^*	Sum of Percentages of Release and Storage Failures
1	single LDR	1	2	96.2	264.6	4.0	12.0	16.0
2	multiple LDR	2	2	96.2	192.0	8.0	26.0	34.0
3	multiple LDR	2	2	52.3	189.3	2.0	24.0	26.0
4	single LDR	1	4	46.9	166.7	2.0	8.0	10.0
5	multiple LDR	2	4	46.9	148.5	11.2	12.2	23.4
6	multiple LDR	3	4	47.3	142.4	10.3	8.8	19.1
7	multiple LDR	2	4	23.0	146.1	6.3	13.1	19.4
8	single LDR	1	6	26.5	133.2	6.8	7.0	13.8
9	multiple LDR	2	6	26.5	123.6	7.5	6.8	14.3

minimum release requirement would be 8%. Also, the percentages may be slightly different because of the finite length of the simulation. Perhaps, with a simulation over thousands of years, the percentage values could be refined.

Another difference between the multiple-LDR models and the single-LDR models is that the percentages of failures in a simulation correspond more closely to the reliability levels specified in the LDR models. This is evident for all the models of Table 4 but is especially evident when the fullest use is made of the reservoir.

For example, models 1 and 2 differ only in the number of LDR's per season and the capacities of the reservoir; the value of X_{min} is close to the maximum feasible value of minimum release. It would be expected therefore that the sum of percentages of failures would approximately equal 30%. The single-LDR model specifies a larger reservoir capacity ($264.6*10^6$ m^3) and has a 34% failure rate. Although the 34% failure rate exceeds the expected failure rate, in longer simulations, using synthetically generated data, both the 16% and 34% failure rates are slightly reduced. Hence the multiple-LDR model specifies an operations policy and reservoir capacity that correspond to the desired performance criteria more closely than the single LDR model.

The same effect is seen in the other models of Table 4 but to a lesser degree. This is because the reservoir considered in these models is not being used to its fullest potential. For example, in the four-season case, minimum releases up to 15% greater (i.e., 54.1 versus $46.9*10^6$ m^3/season) are feasible within the reliability criteria specified in the LDR models. Hence it is expected that the sum of percentages of failures might be less than 30%.

SUMMARY, CONCLUSIONS, AND DISCUSSION

A comparison of a single-LDR model and a multiple-LDR model which is explicitly stochastic has been presented. By varying the number of seasons considered per year, the number of LDR's per season, and the values of minimum storage and minimum release in the LDR models, a wide range of reservoir operating conditions were studied. The operating policies and the reservoir capacities specified by the LDR models for different conditions were tested in a simulation model. All of this work was done for a hypothetical reservoir on the Gunpowder River in Maryland.

The multiple-LDR models were shown to be superior to the single-LDR model. Under identical operating requirements the multiple-LDR models specify significantly smaller reservoir capacities than the single-LDR model. As the number of LDR's per season increases, the optimal reservoir capacity decreases. When the operating policies of the LDR models are tested in a simulation, the multiple-LDR models perform more closely to expected performance criteria than the single-LDR model.

The drawbacks of the multiple-LDR models compared to the single-LDR model are mostly due to data limitations and model construction. The cumulative distribution functions of streamflow for one season, conditioned on the streamflow for the previous season, are needed for the multiple-LDR models. Only several points on unconditioned CDF's are needed for the single-LDR model. Therefore limited data sets may be more easily accommodated by the single-LDR model. Once the conditioned CDF's are estimated, they must be piecewise linearized for inclusion in a linear program. Although this procedure is straightforward, it involves the manipulation of many numbers, increasing the chance of errors in the linear programing inputs.

The single- and multiple-LDR models are similar in several important ways. *Loucks and Dorfman* [1975] and *Gundelach and ReVelle* [1975] propose different forms for the linear decision rules. All of these can be accommodated in the single- and multiple-LDR models. An important property of the single-LDR model is its size. Large multiple-reservoir systems responding to multiple uses with an objective of economic efficiency can be included in a single-LDR model [*Houck et al.*, 1980]. The multiple-LDR models are also small and can be used to consider large reservoir systems. They are larger than the single-LDR model, and their size increases as the number of LDR's per season increases, but they remain within the present limits of computability for multiple-reservoir, multiple-purpose systems.

Acknowledgments. This material is based upon work supported by the National Science Foundation under grant CME 7916819. The Purdue Research Foundation also offered generous support under grant XR0340. Hasan Yazicigil of Southern Illinois University, Stephen Burges of the University of Washington, and Jery Stedinger of Cornell University motivated us to complete this work, and we appreciate their interest, criticisms, and assistance. Of course, we reserve all credit for errors and confusion for ourselves.

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(Received October 20, 1980;
revised March 4, 1981;
accepted March 10, 1981.)