

APPENDICES

Appendix 2.1

2.1.1 Bending and shear stresses for a rigid cylinder

The maximum shear stress produced at the base of the model coral colony is the hydrodynamic force per unit cross-sectional area, given by

$$\tau_{\max} = \frac{4F_h}{\pi d^2} \quad (2.A1)$$

Substituting hydrodynamic force (Eq. 3) gives Eq. 5. The maximal compressive and tensile stresses produced by bending rigid cylinder, σ_{\max} , is given by:

$$\sigma_{\max} = \frac{M_h d}{2I} \quad (2.A2)$$

where M_h is the bending moment, d is the diameter of the cylinder and I is the second moment of area for a cylinder about the substrate (Gere and Timoshenko 1987). In the absence of a velocity gradient, the bending moment is equal to the product of hydrodynamic force applied to the cylinder (Eq. 3) and the average moment arm (half the cylinder's height) giving:

$$M_h = \frac{\rho_w h^2 du^2}{4} \quad (2.A3)$$

Assuming that the cross-sectional area at the base of the cylinder can be approximated by a circle, the second moment of area is expressed as (Gere and Timoshenko 1987):

$$I = \frac{\pi d^4}{64} \quad (2.A4)$$

Substituting Eq. A1, A2 and A3 gives Eq. 2.6.

2.1.2 Unattached colony force threshold

This threshold exists for an unattached model colony where the hydrodynamic bending moment (2.A3) exceeds the gravitational bending moment and the colony topples. The average gravitational bending moment acts at the middle of the base and is given by

$$M_g = \frac{\rho_a g \pi h d^3}{8} \quad (2.A5)$$

The threshold exists when M_h equals M_g , giving

$$c^2 = \frac{A}{S^3 u^4} \quad (2.A6)$$

where

$$c = \frac{2\rho_w}{\rho_a \pi g}$$

Appendix 2.2

Specific energy required to displace a unit volume of material is given by (Poulos and Davies 1980):

$$SE = \left(\frac{m + n^2 m'}{m + m'} \right) \left(\frac{mghN}{0.1A_r} \right) \quad (2.B1)$$

where m is the dropped mass, m' is the mass of the rod, n is the coefficient of restitution (~ 0.5), g is the gravitational constant, h is the drop height, A_r is the cross-sectional area of the rod and N is the number of drops per 10 cm interval. Substituting the known masses and dimensions for the DCP and the modified DCP gave:

$$N_{DCP} = 1.97N_{mDCP} \quad (2.B2)$$

Substrate density:

$$\rho_s \propto \log N_{mDCP} \quad (2.B3)$$

Substrate compressive strength:

$$\sigma_s \propto N_{mDCP} \quad (2.B4)$$

Coefficients of proportionality were calculated by assuming the average density and strength of laboratory samples was approximately equal to the average of the dynamic probing results.

Appendix 3.1

Bending stress for a coral colony

The maximal compressive and tensile stresses produced by bending a coral colony, σ_{bend} , is given by:

$$\sigma_{bend} = \frac{Md_{para}}{2I_{xx}}, \quad (3.A1)$$

where M is the bending moment, d_{para} is the width of the cylinder parallel to incident water flow and I_{xx} is the second moment of area for the colony about the substrate (xx, see Fig. 1) (Meriam and Kraige 1987). Assuming that the cross-sectional area at the base of the colony can be approximated by an ellipse, the second moment of area is expressed as:

$$I_{xx} = \frac{\pi(d_{para}^3 d_{perp})}{64}, \quad (3.A2)$$

where d_{perp} is the colony width perpendicular to d_{para} . Substituting Eqs. 3.A1 and 3.A2 gives:

$$\sigma_{bend} = \frac{32M}{\pi(d_{para}^2 d_{perp})} \quad (3.A3)$$

Appendix 3.2

Consider a normally-distributed random variable y with a probability density function $f(y)$ specified as:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-\mu}{\sigma}\right)^2}, \quad -\infty \leq y \leq \infty \quad (3.A9)$$

Standard linear regressions model the mean as a linear function of some independent variable x (i.e. $\mu = a + bx$), but a constant standard deviation:

$$f(y) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-(a+bx)}{\sigma}\right)^2}, \quad -\infty \leq y \leq \infty \quad (3.A10)$$

This is more commonly written $y = a + bx + \varepsilon(0, \sigma^2)$. However, one may also model the standard deviation parameter σ as a function of the dependent variable (i.e., $\sigma = c + gx$).

The probability density for such a model is:

$$f(y) = \frac{1}{(c + gx)\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{y-(a+bx)}{c+gx}\right)^2}, \quad -\infty \leq y \leq \infty \quad (3.A11)$$

or $y = a + bx + \varepsilon(0, (c + gx)^2)$. Parameters a , b , c and g can be found using a maximum likelihood optimisation function, where $y = \log(MPS)$.

Truncated normal distribution

The probability density function for a truncated normal distribution is given by:

$$f_{trunc}(y) = \begin{cases} \frac{f(y)}{\int_{-\infty}^{y_{trunc}} f(y) dy}, & -\infty \leq y \leq y_{trunc} \\ 0 & y_{trunc} < y < \infty \end{cases} \quad (3.A12)$$

If the mean, standard deviation, and truncation point are all functions of an independent variable, x (such as distance from the reef crest), then the appropriate model is:

$$f_{trunc}(y) = \begin{cases} \frac{f(y)}{h+kx} & -\infty \leq y \leq h+kx \\ \int_{-\infty}^{h+kx} f(y)dy, & h+kx < y < \infty \\ 0 & h+kx < y < \infty \end{cases} \quad (3.A13)$$

where $f(y)$ is given by Eq. 3.A11.