



**AN ANALYTICAL AND EXPERIMENTAL INVESTIGATION OF
ACTIVE VIBRATION CONTROL TECHNIQUES ON COUPLED
PLATE STRUCTURES**

**Thesis submitted by
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In the School of Engineering
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ABSTRACT

This PhD thesis presents a theoretical and experimental investigation using active control to attenuate the vibration responses associated with coupled plate structures. Three plate structures were examined, which corresponded to an L, T and X shaped plate. The plate theory used to determine the dynamic and controlled responses of the coupled plate structures is presented for a generic structure consisting of four finite plates joined together at right angles in a X-shape. The theory for active vibration control of the coupled plate using single and multiple control actuators and error sensors is also presented for both dependent and independent control.

The use of multiple actuators and error sensors in various arrangements to attenuate the response of various coupled plate structures is demonstrated. The number and location of the control forces and error sensors are varied, and their effects on the control performance are compared. In addition, the effect of the control forces driven dependently and independently was investigated. For active control at discrete resonance frequencies, the global response of the structure was observed. Experiments were conducted in order to validate theoretical results on the active control of the global response at a low resonance frequency. The results showed excellent correlation, validating the effectiveness of the active control application.

An energy method to predict the vibrational response and its transmission between coupled structures in the medium to high frequency ranges is Statistical Energy Analysis (SEA). In

this thesis, SEA is used to model several built-up structures and estimate their vibrational response using energy flow relationships. Energy levels of the L, T, X-shaped plates, and a 7-plate structure, predicted from the exact analytical waveguide model are compared with those of conventional SEA models. A hybrid approach between the two techniques is also presented. The hybrid method uses the analytical waveguide method to estimate the input power and coupling loss factors used in the conventional SEA equations. The energy levels in individual plate subsystems using the exact analytical method, SEA, and the hybrid technique are compared.

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LIST OF SYMBOLS

ω	Angular frequency	[rad/s]
ρ	Material density	[kg/m ³]
ν	Poisson's ratio	
η	Structural loss factor	
η_{ij}	Coupling loss factor	
$n(\omega)$	Modal density	[modes.s/rad]
σ	Plate thickness ratio	
$\langle \tau_{ij} \rangle$	Power transmission coefficient	
$[\alpha_p], [\alpha_s]$	Matrices of the primary and secondary flexural displacements	
$[\mathbf{X}]$	Coefficient matrix	
$[\mathbf{F}_p], [\mathbf{F}_s]$	Force matrices	
A_i, B_i, C_i, D_i	Coefficients of the plate flexural propagating and near field waves for forced vibration	
c_B	Phase velocity	[m/s]
c_L	Longitudinal wave speed	[m/s]
c_g	Group velocity	[m/s]
D	Flexural rigidity of the plate	[Nm]
E	Young's modulus	[N/m ²]
E	Energy stored in the subsystem.	[J]
E_k	Kinetic energy	[Nm]
f	Frequency	[Hz]
F_p	Magnitude of the primary force	[N]
F_s	Magnitude of the secondary force	[N]
$F_s _{opt}$	Optimal control force amplitude	[N]
G_p, G_s	Primary and secondary transfer functions	[m/N]
h	Plate thickness	[m]
k_x, k_y, k_n, k_p	Wave numbers	[1/m]
L	Length of the junction line	[m]

L_a	Acceleration level, reference level $a_{ref} = 10^{-6} \text{ m/s}^2$	[dB]
L_{xi}	Length of the plate along the x-axis	[m]
L_y	Length of the plate along the y-axis	[m]
m, m'	Mode number	
M	Mass of each individual plate	[kg]
M_x	Bending moment	[Nm]
M_{xy}	Twisting moment	[Nm]
N_x	Longitudinal force	[N]
P_{in}	Input power	[J]
P_{dis}	Dissipated energy	[J]
P_{ij}	Energy flow from subsystem i to subsystem j	[J]
Q_x	Net vertical shear force	[N]
F_x	Bending shear force	[N]
S_i	Surface area of the subsystem i	[m ²]
u	Longitudinal displacement of the plate	[m]
w_p	Displacement due to primary force	[m]
w_s	Displacement due to secondary force	[m]
w_{tot}	Total displacement due to primary and secondary forces	[m]
x_e	x co-ordinate for the error sensor	[m]
x_p	x co-ordinate for the primary force	[m]
x_s	x co-ordinate for the secondary force	[m]
y_e	y co-ordinate for the error sensor	[m]
y_p	y co-ordinate for the primary force	[m]
y_s	y co-ordinate for the secondary force	[m]
Z	Impedance of an infinite plate	[kg/s]